DYNAMICS OF ENERGY LOSS OF A BUNCH INTERSECTING A BOUNDARY BETWEEN VACUUM AND DIELECTRIC IN A WAVEGUIDE*

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Abstract

We analyze radiation of a small bunch crossing a boundary between two dielectrics in a cylindrical waveguide. The main attention is paid to investigation of dynamics of a charge energy loss and the effect of the boundary on the electromagnetic field (EMF). Algorithms of computations for the field and the energy loss are founded upon certain transformations of integration path. We consider two instances in detail: the bunch is flying from dielectric into vacuum and from vacuum into dielectric. In both cases we compare the energy losses by transition radiation (TR) and by Cherenkov one (CR). Our investigation shows, for example, that energy loss is negative at certain segments of the bunch trajectory.

INTRODUCTION

One of the problems being important for the wakefield acceleration technique and for new methods of generation of microwave radiation consists in analysis of effect of the boundary on the wave field when a bunch flies into a dielectric structure or from one. It should be noticed that energetic characteristics of TR at a single boundary in a waveguide and in the case of a dielectric plate were investigated in papers [1,2]. However, the most attention was paid to study the energetic spectrums of generated modes. Dynamics of an energy loss as well as an EMF structure has not been analyzed.

Our research is based on original approach used previously for the case of the vacuum-plasma boundary [3]. But Cherenkov radiation is not generated in such situation; therefore it varies radically from the case under consideration. The electromagnetic field structure of the point charge was partially investigated in our works [4,5]. Now we analyze dynamics of the energy losses which allows of better understanding physical phenomena in this situation.

We consider a point charge q moving in a metal circular waveguide of radius a along its axis (z-axis) and intersecting the boundary ($z=0$) between two homogeneous isotropic non-dispersive dielectrics with permittivity ε_1 ($z<0$) and ε_2 ($z>0$) at the moment $t=0$. The charge passes uniformly with a velocity $\vec{V}=c\beta\vec{e}_z$ (c is a light speed in vacuum).

The analytical solution of the problem is traditionally found for the spectral harmonics of the vector potentials as an expansion into a series of eigenfunctions of the transversal operator [3,6]. Expressions for components of the EMF can be easily derived from the formulae for the general case of the boundary between two arbitrary homogeneous isotropic media [3].

We investigate the dynamics of the charge energy loss per unit length of the charge motion:

$$W=-qE_z\Big|_{\substack{z\to\beta ct\\ r\to 0}} . \tag{1}$$

As it follows from general expressions for EMF the charge energy losses in both media have two summands: $W_{1,2}=W_{1,2}^q+W_{1,2}^b$. The first one $\left(W_{1,2}^q\right)$ is connected with so-called by V. L. Ginzburg [7] "forced" field that is EMF of the charge in a regular waveguide. It contains CR if $\beta>\beta_{C1,2}$, $\beta_{C1,2}=\varepsilon_{1,2}^{-1/2}$. The second summand $\left(W_{1,2}^b\right)$ is connected with the "free" field that is determined by the influence of the boundary and includes TR. Each summand is decomposition in an infinite series of normal modes:

$$W_{1,2}^{q,b}=\sum_{n=1}^{\infty}W_{n1,2}^{q,b} . \tag{2}$$

The charge energy loss in a regular waveguide with homogeneous filling analysed in many papers is equal to [8]

$$W_{n1,2}^q=\begin{cases}2q^2\left(a^2\varepsilon_{1,2}J_1^2(\chi_{0n})\right)^{-1}, & \beta>\beta_{C1,2}\\ 0, & \beta<\beta_{C1,2}\end{cases} , \tag{3}$$

where χ_{0n} is the n^{th} zero of the Bessel function $J_0(x)$. For analysis of the charge energy loss by TR $\left(W_{n1,2}^b\right)$ we use the exact integral representation. We investigate it with analytical and computational methods. Analytical research is an asymptotic investigation with the steepest descent technique. Computations are based on original algorithm using some transformation of the integration path.

We study two cases in detail: the bunch is flying from dielectric ($\varepsilon_1>1$) into vacuum ($\varepsilon_2=1$) and from vacuum ($\varepsilon_1=1$) into dielectric ($\varepsilon_2>1$).

THE CASE OF FLYING FROM DIELECTRIC INTO VACUUM

Numerical Approach

Efficient algorithm used for numerical calculation is based on a certain transformation of the initial integration path in the complex plane of ω. Earlier such an algorithm was used for calculating of the forced field in different dispersive media [9] and in a waveguide with the boundary between vacuum and cold plasma [3]. We demonstrate this method for the vacuum area.

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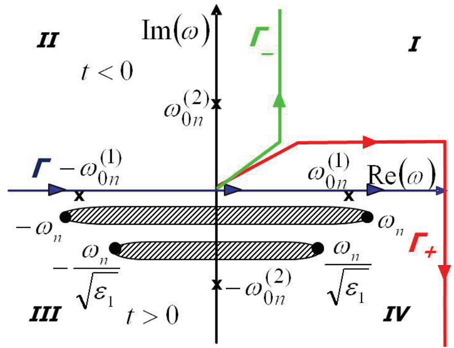


Figure 1: Disposition of the singularities of the integrands, corresponding branch cuts and integration path in a complex plane of ω for W_n^b in vacuum (at $t>0$) if $\beta > \beta_{C1}$.

The first step of the investigation consists of a study of the singularities of integrands. Disposition of these singularities is presented in Fig. 1. Four branch points $\pm\tilde{\omega}_n^{(1)} = \pm\omega_n \varepsilon_1^{-1/2} - i\delta_1$ and $\pm\tilde{\omega}_n^{(2)} = \pm\omega_n - i\delta_2$ are pointed with black circles; two pairs of poles $\pm\omega_{0n}^{(1)} = \pm\beta\omega_n\left(\varepsilon_1\beta^2 - 1\right)^{-1/2} - i\delta_3$ (situated on the real axis if $\beta > \beta_{C1}$) and $\pm\omega_{0n}^{(2)} = \pm i\beta\omega_n\left(1-\beta^2\right)^{-1/2}$ are shown with crosses. Here $\delta_1, \delta_2, \delta_3$ are positive infinitesimal quantities which tend to zero if we do not take into account absorption in a medium. It is convenient to have the branch cuts as it is shown in Fig. 1. The integrand of W_n^b decreases in the upper half-plane of ω at $t<0$ and in the lower half-plane of ω at $t>0$ for any value of β.

As the integration path Γ goes through the poles $\omega_{0n}^{(1)}$ this leads to rather abrupt behavior of integrands. The numerical algorithm is adapted for overcoming this difficulty. First, solution can be easily written as an integral on a half-infinite contour Γ. Further, we can transform this contour in an upper half plane (ω) into the green contour Γ_- (Fig. 1) for $t<0$ and in a lower half-plane into the red contour Γ_+ for $t>0$. The new contours should bypass all the singularities and then go parallel to the steepest descending path. The integrands have regular behavior along these new contours. Note that, in this way, the charge energy loss can be computed in the domain both near and far from the boundary. We can also optimize the parameters of contours for each computation.

Results and Discussions

Dynamics of the total energy loss per unit length for the 1st mode is presented in Fig. 2. We consider the total energy loss per the length unit for the n^{th} mode in dimensionless unities:

$$\tilde{W}_n = W_n / W_n^q, \tag{4}$$

where W_n^q is the energy loss for Cherenkov mode (3).

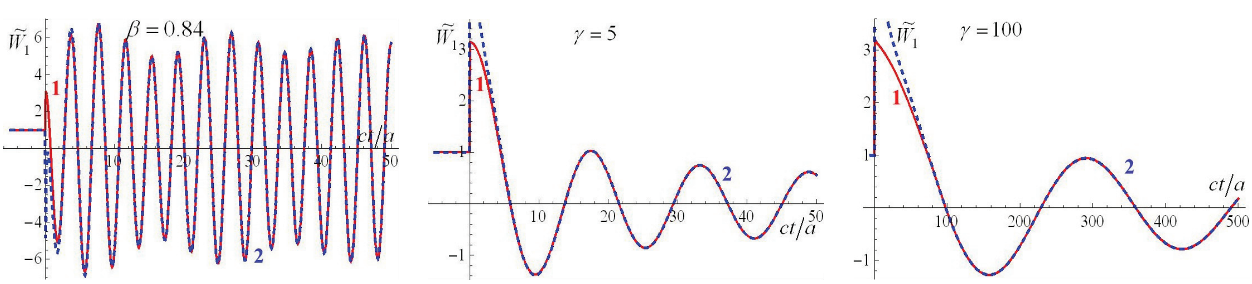
If $\beta > \beta_{C1}$, Cherenkov radiation emerges in dielectric. CR is reflected and refracted at the boundary and, as a result, so-called Cherenkov transition radiation (CTR) is generated [4,5]. In vacuum, CTR exists under conditions $\beta_{C1} < \beta < \beta_{CT1} \equiv (\varepsilon_1 - 1)^{-1/2}$ in the area $z < z_1 = ct\sqrt{\left|1-\beta^2(\varepsilon_1 - 1)\right|}\Big/\beta$. The dimension of this zone z_1 increases with group velocity of waveguide waves

$$V_{g1} = \sqrt{1-\beta^2(\varepsilon_1 - 1)}\Big/\beta. \tag{5}$$

The limit speed β_{CT1} is connected with total internal reflection of CR from the boundary. So, if $\varepsilon_1 < 2$ CTR emerges for ultra-relativistic particles with $\beta \approx 1$. If the group velocity (5) is more than the charge velocity, that is

$$\beta < \beta_0 = \sqrt{\left(\sqrt{(\varepsilon_1 - 1)^2 + 4} - \varepsilon_1 + 1\right)\Big/2},$$

the charge interacts with CTR (Fig. 2 *a*). One can see that the energy loss oscillates with approximately constant amplitude in the vacuum area. The situation changes if $\beta > \beta_0$ (Fig. 2 *b,c*), when the charge leaves the CTR behind (when the charge interacts with TR only).



(*a*) (*b*) (*c*)

Figure 2: The case of flying from dielectric into vacuum. Dependence of the normalized energy loss for the first mode of the whole field on dimensionless time *ct/a* for different charge velocities β (or $\gamma = 1\big/\sqrt{1-\beta^2}$). Red solid line 1 corresponds to the exact calculation. The asymptotic approximations are given as well (blue dashed line 2); $\varepsilon_1 = 1.5$, $\varepsilon_2 = 1$, $\omega_0 = 2\pi \cdot 10\text{GHz}$, $a = 5\text{mm}$, $\beta_{C1} = 0.816$, $\beta_0 = 0.884$.

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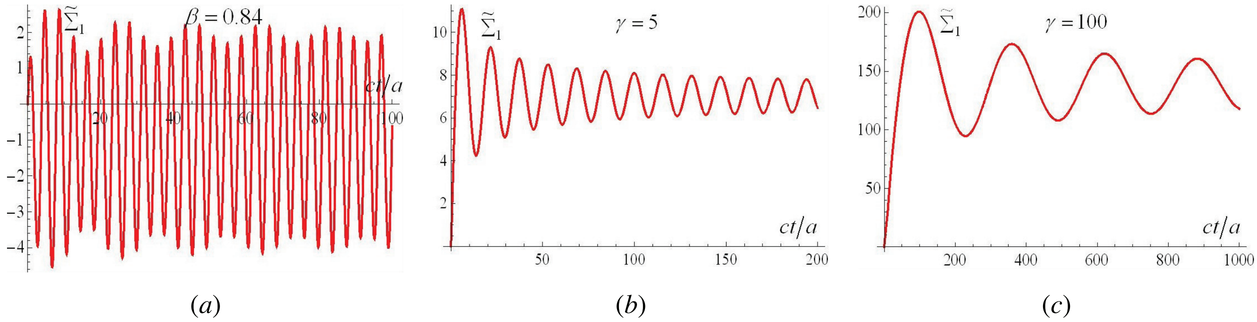

Figure 3: The case of flying from dielectric into vacuum. Dependence of the total normalized charge energy loss for the first mode on dimensionless time *ct/a* for different charge velocities; the same in Fig. 2.

In this case, the oscillation amplitude lessens as $1/\sqrt{t}$, and the oscillation period $T_0 = a\gamma(c\chi_{on})^{-1}$ is getting larger with increase in γ.

In Fig.2 one can also see a good coincidence between the exact solution (red solid line 1) and the analytical asymptotic expressions (dashed blue line 2) in some domain outside the boundary.

We investigate the total energy loss by TR in vacuum defined by the integral $\Sigma = \int_0^{Vt} W(z)dz$ as well. Figure 3 presents the total normalized energy loss for the first mode $\widetilde{\Sigma}_1 = \Sigma_1\, a\varepsilon J_1^2(\chi_{01})(2q^2)^{-1}$. If $\beta < \beta_0$, the total energy loss has a vibrating character (Fig. 3 *a*). If $\beta > \beta_0$, this interaction decreases with t (Fig. 3 *b,c*), and Σ_1 tends to some constant which is proportional to γ.

THE CASE OF FLYING FROM VACUUM INTO DIELECTRIC

Analogous investigation of the charge energy loss can be made for the case of flying from vacuum ($\varepsilon_1 = 1$) into dielectric with ε_2. Here we discuss only physical results obtained. The dynamics of the energy loss per unit length for the first mode of the total field is presented in Fig.4, top. One can see that the energy loss is negative in some area near the boundary (where the bunch is attracted to it). The dimension of this zone decreases with increase in β and in ε_2. The total energy loss for the time interval $(-\infty, t)$ determined by the integral $\Sigma = \int_{-\infty}^{Vt} W(z)dz$ is shown in Fig.4, bottom. One can see that the total energy loss is negative up to the moment $t_0 \approx a\left[\chi_{0n}c\gamma(\varepsilon_2 - 1)\right]^{-1}$. So, this moment t_0 decreases with increase in γ and ε_2. The total energy loss is getting positive only for $t > t_0$ when the Cherenkov loss is getting dominant.

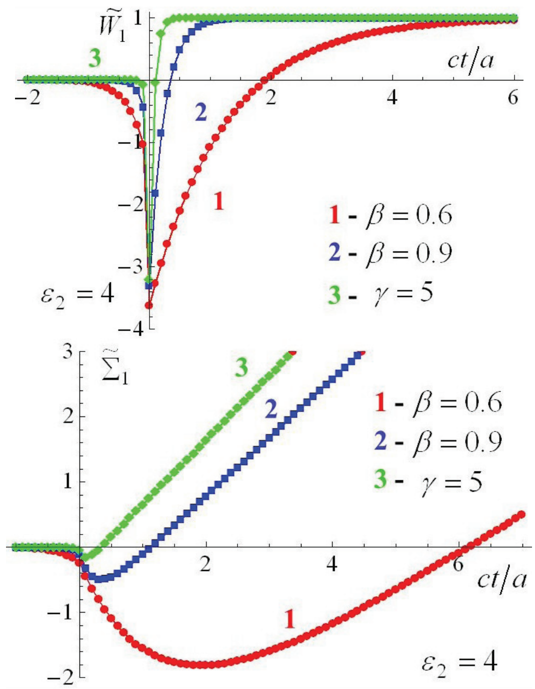

Figure 4: The case of flying from vacuum into dielectric. Dependence of the normalized energy loss per unit length (top) and the normalized total energy loss (bottom) of the first mode of the total field on dimensionless time *ct/a* for different charge velocities; $\omega_0 = 2\pi \cdot 10\text{GHz}$, $a = 5$ mm.

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