DESIGN OF SRF CAVITIES WITH CELL PROFILES BASED ON BEZIER SPLINES*

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Abstract

Elliptical cavities have been a standard in SRF linac technology for 30 years. In this work, we present a novel approach [1] using Bezier spline profile curves. By using different degrees of spline curves, the number of free parameters can be varied to suit a given problem (endcell tuning, basecell figures of merit), thus leading to a high flexibility of the spline approach. As a realistic example, a cubic spline SRF multicell cavity geometry is calculated and the figures of merit are optimized for the operational mode. We also present an outline for HOM endcell optimization that can be realized using available 2D solvers.

INTRODUCTION

In modern ring-based and linac facilities, high acceleration gradients and duty cycles are achieved using superconducting radio frequency (SRF) cavities. To protect these cavities from quenching and RF breakdown, a sufficiently small ratio of peak surface electric field and accelerator gradient $E_{\text{surf}}/E_{\text{acc}}$ is required. Therefore the shapes must not contain edges, and thus a smooth profile curve for the figure of rotation is needed.

As had been done in normal-conducting cavities, edges of cylindrical/iris-loaded cavities were rounded off in a first step. To further enhance performance, the rounding circles were substituted by ellipses [2] (Fig. 1), leading to modern SRF “re-entrant”, “low-loss” and “TESLA-shape” [3] cavities, in which resonant multipacting of secondary electrons is highly suppressed in contrast to cylindrical shapes. While elliptical shapes have shown superior performance, this parameterization is used by historical contingency, as to the authors' knowledge, no other parameterizations of similar smoothness (modulated sinusoid curves, splines) have been researched for ultra-relativistic particles.

Spline Geometry Parameters

The spline cavity geometry is such an alternative shape with additional desirable properties like continuous curvature, and a number of free parameters that can be chosen lower than for elliptical shapes (but may be arbitrarily increased).

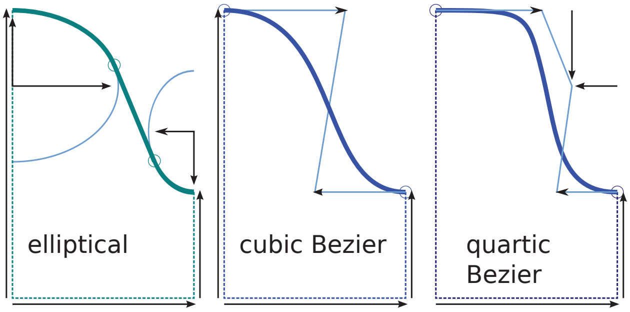

Figure 1: Sketch and cavity parameters (arrows) for different cavity shape parameterizations.

A Bezier spline is a parameterized curve [4]

$$\vec{s}(t) = \vec{a}_0 + \vec{a}_1 t + \vec{a}_2 t^2 + \ldots = \sum_{n=0}^{N} \vec{s}_n b_{n,N}(t) \qquad (1)$$

with Bernstein polynomials $b_{n,N}$ and $t \in [0,1]$. The $N+1$ points $\vec{s}_n$ define the so-called control polygon and contain the free parameters of the spline. Let us consider the simplest possible case of a cubic ($N = 3$) Bezier spline as shown in Fig. 1. The 8 geometry parameters are reduced by a) zero slope of the curve at iris and equator radius of the cavity, and b) fixed relation of the coordinate system to the equator plane. 5 cavity parameters remain, compared to 7 for an elliptical cavity.

Applying two further elementary RF problem constraints c) operation frequency and d) transit-time factor (cell length is $\beta\lambda/2$), only three optimization parameters remain for the specific application. Instead of specifying these constraints, the low dimensionality of the remaining problem allows to scan the remaining parameter space, thus creating a “map” for the cubic Bezier cavity that can be used to speed up complex design optimization tasks.

IMPLEMENTATION

For the single cell computations with periodic boundary conditions, the 2d code SUPERFISH [5] was employed. Since splines are not implemented in SUPERFISH, a small wrapper for MATLAB was developed. The spline geometry was discretized by calculating a 200 point (halfcell) polygon from its parameters (Fig. 2), which was then used as input in SUPERFISH, also specifying a special localized mesh that matches the polygon discretization accuracy.

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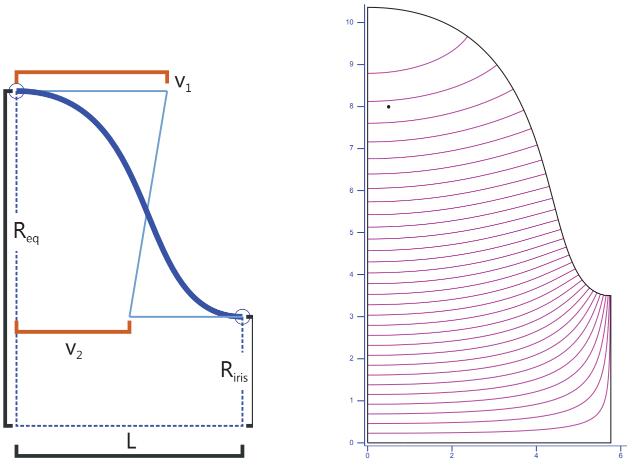

Figure 2: Parameterization for cubic Bezier shape (left) and exemplary result for electric field lines ($v_1 = 0.9, v_2 = 0.65, R_{\mathrm{iris}} = 35$ mm, $R_{\mathrm{eq}} = 103.60$ mm, right).

Frequency Tuning and Parameter Scan

Since the spline parameterization is not known to SUPERFISH, the R_{eq} tuning procedure had to be reimplemented in MATLAB. The procedure almost exactly imitates the SUPERFISH procedure [5] by reading out $D(k^2)$ (from OUTFIS.TXT) after each SUPERFISH run. After fitting a polynomial $D(R_{\mathrm{eq}})$ to the data of previous iteration values, the outer equator radius is tuned to the frequency (π-mode) $\nu_\pi = 1\ 3$ GHz.

For a fixed iris radius, the $(v_1, v_2)/L$ parameter space (Fig. 2) was scanned in an arbitrary range $v_i/L \in [0.05, 0.95]$ that was partly chosen due to meshing considerations (no "re-entrant"-type shapes in this region). The sequence proceeds in a spiral pattern, starting from the central point $(0.5, 0.5)$ in 0.05 steps, so that the changes of the tuned equator radii and their slopes between the steps were small and fast convergence (1-3 iterations) could be achieved. Knowing the matching equator radii for the π-mode boundary condition, a second scan for the 0-mode was performed to obtain the corresponding eigenmode frequency ν_0 needed for the intercell coupling constant (see Results). The procedure for one fixed iris radius took approx. 3.5 hours on a standard PC.

RESULTS

Single Cavity Map

The results of the parameter scan for four different figures of merit [6] with two different iris radii are shown in Figs. 3 and 5. These figures are the shunt impedance per quality factor R/Q (wall-loss independent shunt impedance parameter), the surface electric field to effective acceleration gradient factor $E_{\mathrm{surf}}/E_{\mathrm{acc}}$ up to 5.0, the Geometry factor G (inversely related to cryogenic losses) and the intercell coupling constant $\kappa = 2(\nu_\pi - \nu_0)(\nu_\pi + \nu_0)$.

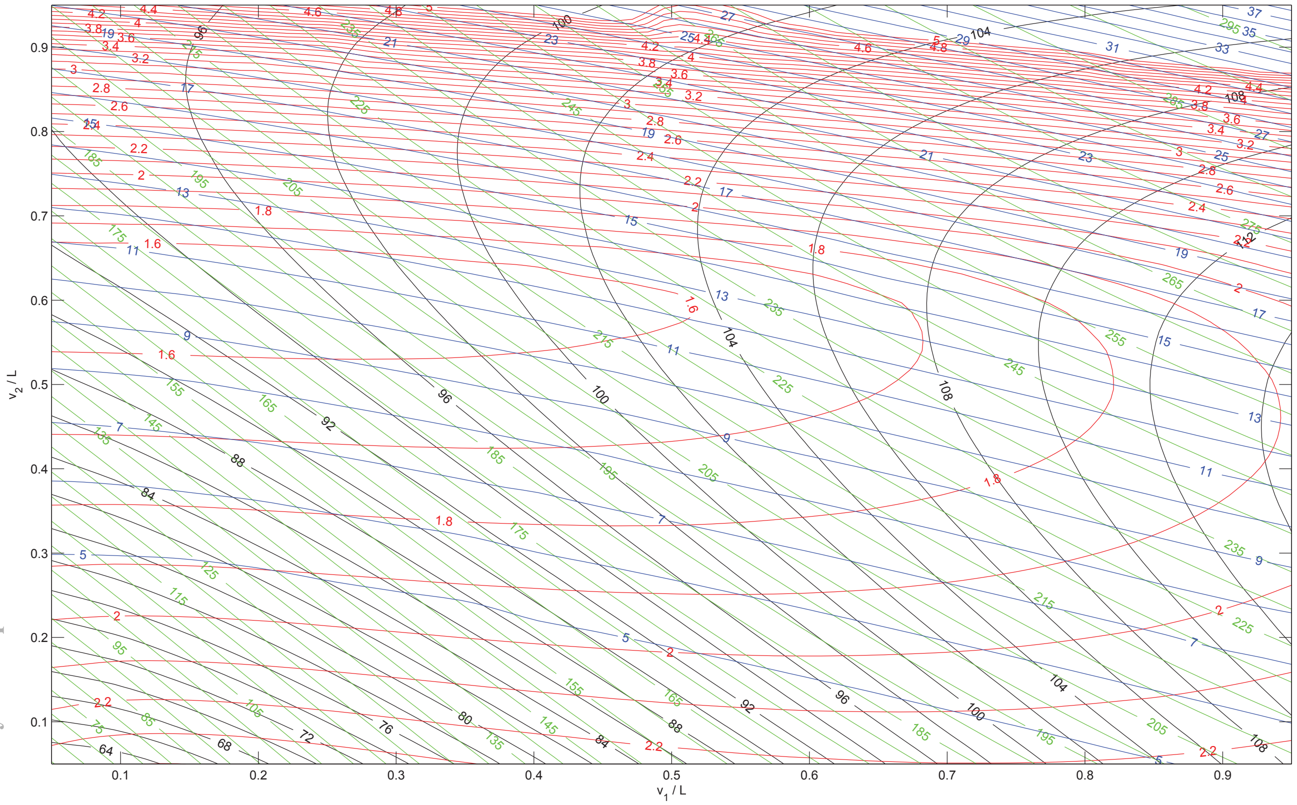

Figure 3: Cubic Bezier map for a 1.3 GHz cavity with iris radius 35 mm. The map shows the cubic interpolated isolines of R/Q [Ω] in black, $E_{\mathrm{surf}}/E_{\mathrm{acc}}$ in red, the Geometry factor [Ω] in green, and the intercell coupling constant [10^{-3}] in blue.

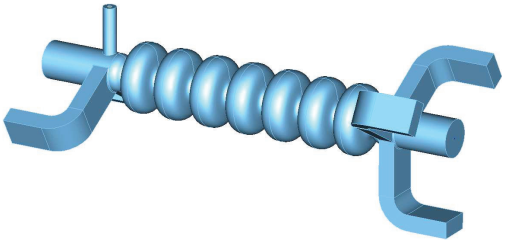

Figure 4: Rendering of the HOM-damped seven-cell Spline cavity.

It can be seen by comparison e.g. with [3] that the cubic spline shape geometry performance is in roughly the same range as the standard elliptical cavity geometries, although as stated, the cubic shape has two less free parameters.

By inspecting the maps, the trade-off between different figures of merit in the design process becomes visible (see e.g. red and black lines). The differences in $E_{\text{surf}}/E_{\text{acc}}$ (e.g. the 1.6 plateau) and intercell coupling strength for different radii are also of interest.

Multicell Structure

Within the scope of BERLinPro main linac research [7, 8, 9], a heavily HOM-damped 7 cell 1.3 GHz cubic Bezier spline design with an iris radius of 36mm was computed and optimized for the operational passband. Due to the fact that the BERLinPro linac uses waveguide absorbers for HOM damping (no rotational symmetry), CST MWS [10] was used to calculate the structure's eigenmodes.

For the cavity with radius-tuned endcells to reach field homogenity, $R/Q = 768\,\Omega$ could be achieved. The frequency difference of the operational mode to the nearest mode of the passband is 1.3 MHz. With these values, the Spline multicell structure has almost equal performance for the operational mode as the elliptical BERLinPro cavity [8].

OUTLOOK

It was shown that cubic Bezier spline cavity geometries yield roughly comparable figures of merit to elliptical cavities and can be used for rapid cavity prototyping using the map approach.

The initial parameters can then be used as starting points for optimization of higher order Bezier splines, due to the possibility of increasing the order of a given spline without changing its actual shape (degree elevation). This property may allow for further optimization of central cavity cells. By iterative degree elevation and a subspace optimization scheme, it may be possible to approximate an idealized free-form cavity for a given goal function.

Furthermore, degree elevation is of special interest for HOM tuning of endcells, where a large number of eigenmode fields need to be optimized to allow sufficient HOM damping, e.g. for ERLs. While asymmetric endcells are often used in elliptical cavities (each HOM mode couples out only on one side), the flexible number of spline parameters could circumvent this problem, allowing stronger damping.

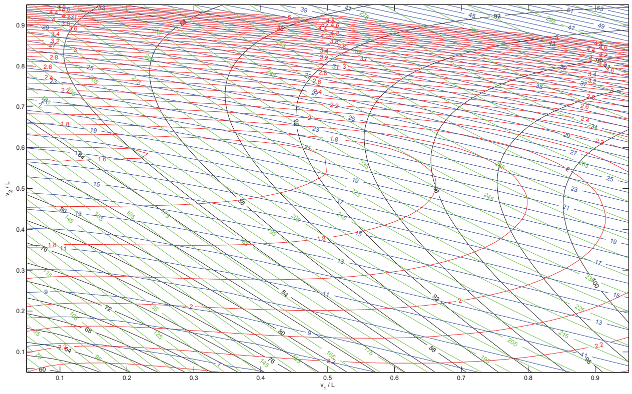

Figure 5: Cubic Bezier map for a 1.3 GHz cavity with iris radius 40 mm. The figures may be scaled for design of 1.5 GHz cavities with ≈ 35 mm radius. Isoline legend is the same as for Fig. 3.

REFERENCES

[1] B. Riemann et al., "SRF multicell cavity design using cubic and higher order spline cavity profiles", T 80.9, Verhandlungen DPG Göttingen 2012

[2] P. Kneisel, R. Vincon and J. Halbritter. "First results on elliptically shaped cavities", *Nuclear Instrum. Methods*, 1981, 188, 669 - 670

[3] Aune et al., "Superconducting TESLA cavities", *Phys. Rev. ST Accel. Beams* Vol. 3 pp. 092001 (2000)

[4] Paul De Casteljau. "Courbes à pôles", National Industrial Property Institute (France), 1959

[5] K. Halbach and R.F. Holsinger. "SUPERFISH - a computer program for evaluation of RF cavities with cylindrical symmetry", *Part. Acc.*, 7:213–222, 1976.

[6] T. Wangler, *RF Linear Accelerators*, 2nd ed., Wiley-VCH, 2007

[7] A. Jankowiak et al., "BERLinPro - a compact demonstrator ERL for high current and low emittance beams", Proc. LINAC 10, Tsukuba, Japan, 2010.

[8] A. Neumann et al., "Status of the HOM calculations for the BERLinPro main linac cavity", FRAAC3, Proc. ICAP'12, Rostock, Germany, 2012.

[9] T. Galek, "Traveling Poles Elimination Scheme and Calculations of External Quality Factors of HOMs in SC Cavities", WEP07, Proc. ICAP'12, Rostock, Germany, 2012.

[10] CST Microwave Studio (CST Studio Suite), CST AG, 2012.