STOCHASTIC RESPONSE SURFACE METHOD FOR STUDYING MICROPHONING AND LORENTZ DETUNING OF ACCELERATOR CAVITIES*

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Abstract

The dependence of the resonant frequency of an RF cavity on its geometry is represented by a stochastic response surface model, which is constructed on the basis of a few eigenmode solutions extended with sensitivity information. The response surface model is used for statistic analysis and for calculating the effect of Lorentz detuning.

INTRODUCTION

High energy cavities are used within a very small frequency range. Any mechanical deformation, albeit small, may lead to an unacceptable shift of the frequency of the applied eigenmode [1]. To achieve a design that is robust against microphoning and Lorentz detuning [2–5], the simulation tool should deal with small changes in geometry in a consistent way and should be able to calculate the resonance frequencies with a relative accuracy of 10^{-5}. The simple and straightforward procedure which changes and remeshes the geometry and repeats the eigenmode solving, turns out to be inefficient because the introduced discretisation errors will mask the small changes in eigenfrequency, unless an prohibitively fine mesh is used [6]. This paper introduces two techniques to overcome this problem: (a) the eigenmode solver also delivers the sensitivities of the eigenfrequency and thereby increases the amount of information obtained for a single set of geometric parameters; (b) the eigenfrequency is modelled by a stochastic response surface method which allows reliable interpolation and uses the concept of uncertainty to deal with errors introduced by remeshing.

CAVITY EIGENMODE SOLVERS

The eigenmodes of the cavity are calculated by solving one of

$$\nabla\times\left(\frac{1}{\mu}\nabla\times\vec{E}\right) = \omega^2\varepsilon\vec{E} \quad (1)$$

$$\nabla\times\left(\frac{1}{\varepsilon}\nabla\times\vec{H}\right) = \omega^2\mu\vec{H} \quad (2)$$

with $\vec{E}$ and $\vec{H}$ the electric and magnetic field strengths, ω the angular frequency, ε the permittivity and μ the permeability [7, 8]. Only the eigenmodes with the lowest eigenfrequencies are relevant. When discretised by the finite-element (FE) method or the finite-integration technique (FIT), (1) and (2) become

$$\mathbf{K}_{\frac{1}{\mu}}\widehat{\mathbf{e}} = \omega^2\mathbf{M}_\varepsilon\widehat{\mathbf{e}} \quad (3)$$

$$\mathbf{K}_{\frac{1}{\varepsilon}}\widehat{\mathbf{h}} = \omega^2\mathbf{M}_\mu\widehat{\mathbf{h}} \quad (4)$$

and will further be expressed generically by $\mathbf{K}_\alpha\mathbf{u} = \omega^2\mathbf{M}_\beta\mathbf{u}$ where $(\alpha,\beta,\mathbf{u})$ either stands for $(\frac{1}{\mu},\varepsilon,\widehat{\mathbf{e}})$ or $(\frac{1}{\varepsilon},\mu,\widehat{\mathbf{h}})$ and $\widehat{\mathbf{e}}_j$ and $\widehat{\mathbf{h}}_j$ are the degrees of freedom for the electric and magnetic field strengths respectively. The matrix coefficients are

$$\mathbf{K}_{\alpha,i,j} = \int_V \alpha\,(\nabla\times\vec{w}_i)\cdot(\nabla\times\vec{w}_j)\,\mathrm{d}V \quad (5)$$

$$\mathbf{M}_{\beta,i,j} = \int_V \beta\vec{w}_i\cdot\vec{w}_j\,\mathrm{d}V \quad (6)$$

with $\vec{w}_i(x,y,z)$ FE or FIT shape functions and V the computational domain.

In many situations, the cavity has a cylindrical symmetry and only modes with $\vec{E}=(E_r,0,E_z)$ and $\vec{H}=(0,H_\theta,0)$ are relevant. Then, a substantial saving of computation cost is achieved by only triangulating the rz-cross-section of the cavity and discretising (2) by the shape functions

$$\vec{w}_j = \frac{N_j(r,z)}{2\pi r}\vec{e}_\theta \quad (7)$$

$$N_j^{(k)}(r,z) = \frac{a_j^{(k)}+b_j^{(k)}r^2+c_j^{(k)}z}{2S^{(k)}} \quad (8)$$

where $\vec{e}_\theta$ is the peripheral unit vector, $N_j(r,z)$ are nodal shape functions associated with mesh node j and (8) expresses $N_j(r,z)$ in element k as a function of the coefficients $a_j^{(k)}$, $b_j^{(k)}$ and $c_j^{(k)}$. $N_j(r,z)^{(k)}$ is features a quadratic dependence on r such that a homogeneous electric field can be represented exactly on the mesh [9].

SENSITIVITIES OF THE RESONANCE FREQUENCY

As will become clear below, highly accurate sensitivities of the cavity eigenmode with respect to geometric parameters are of paramount importance for studying microphoning and Lorentz detuning and for a stochastic analysis or optimisation of the design. The sensitivities of the eigenfrequency ω_p of eigenmode $(\omega_p,\mathbf{u}_p)$ with respect to the geometric parameters ζ_q are obtained directly from the eigenvalue solver by [10–12]

$$\frac{\mathrm{d}\omega_p}{\mathrm{d}\zeta_q} = \frac{1}{2\omega_p}\frac{\mathbf{u}_p^H\left(\frac{\mathrm{d}\mathbf{K}_\alpha}{\mathrm{d}\zeta_q}-\omega_p^2\frac{\mathrm{d}\mathbf{M}_\beta}{\mathrm{d}\zeta_q}\right)\mathbf{u}_p}{\mathbf{u}_p^H\mathbf{M}_\beta\mathbf{u}_p} \quad (9)$$

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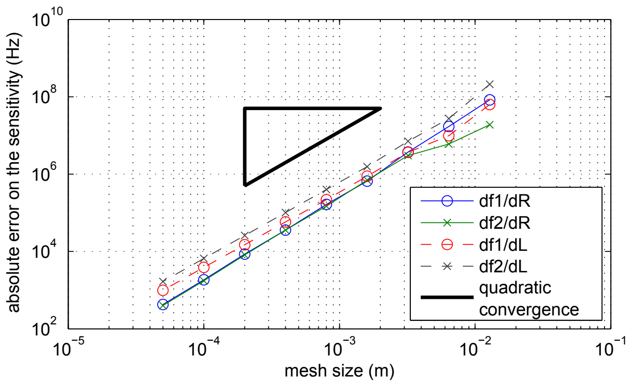

Figure 1: Convergence of the absolute discretisation error for the sensitivities of the first two eigenfrequencies (f_1, f_2) of a pill-box resonator with respect to changes in the radius R and length L of a pill-box resonator.

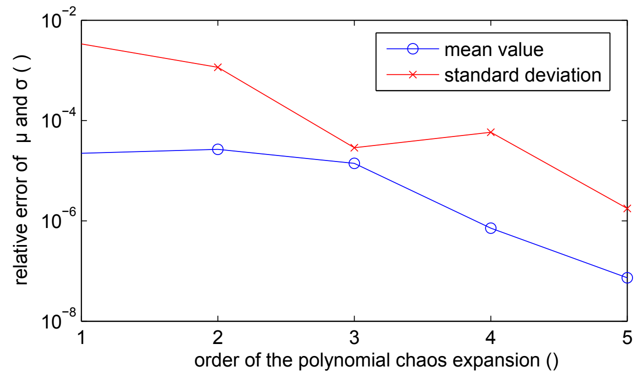

Figure 2: Convergence of the approximation error for the mean value μ and the standard deviation σ with respect to the order of the polynomial chaos expansion.

The calculation of $\frac{\mathrm{d}\mathbf{K}_\alpha}{\mathrm{d}\zeta_q}$ and $\frac{\mathrm{d}\mathbf{K}_\beta}{\mathrm{d}\zeta_q}$ involves the derivatives $\frac{\mathrm{d}\mathbf{w}_j}{\zeta_q}$ of the shape functions with respect to the geometric parameters [13]. The convergence of the discretisation error for the sensitivity calculated by (9) is of the same order as the one for the eigenvalues themselves. An exemplary validation for a 2D eigenmode solver based on (7) and applied to a pill-box resonator is shown in Fig. 1.

STOCHASTIC RESPONSE SURFACE METHOD

The study of microphoning or Lorentz detuning of a resonating cavity and the optimisation of the design require a large number of eigenmode solutions. A direct calculation may become too time consuming. Instead, we solve the eigenmodes for a restricted number of geometries, use these to construct a surrogate model, which is then exploited for calculating the properties for slightly modified geometries. Moreover, we prefer a *stochastic* response surface method (RSM) instead of a standard RSM, in order to account for both model uncertainties and simulation inaccuracies [14, 15]. The uncertain geometric parameters are based on Gaussian variables and the eigenfrequencies are approximated by a series expansion of multi-dimensional Hermite polynomials. Their coefficients are determined by regression.

As a rule of thumb, the data of $2N$ eigenmode solutions are sufficient to obtain a robust estimate of N coefficients. The sensitivities of the resonant frequency with respect to M input parameters can be exploited to diminish the number of required collocation points to $2N/(M+1)$. This is particularly beneficial because the additional calculation of M sensitivities is substantially less time consuming than the eigenmode calculation for a single geometry.

The insertion of the stochastic RSM between the field solver and an outer calculation procedure is illustrated for a stochastic analysis of a cylindrical cavity with radius $R = 30$ mm and length $L = 100$ mm. The standard deviations are $\sigma_R = 1.5$ mm and $\sigma_L = 5$ mm respectively. A reference solution for the eigenfrequency $f = 3.849$ GHz and its standard deviation $\sigma_f = 194.1$ MHz is obtained by 1000 2D FE eigenmode solves. The stochastic RSM preserves the high accuracy of the FE solver. The relative numerical error falls below 10^{-5} for a polynomial chaos expansion of fifth order (Fig. 2). A stochastic RSM of fifth order accuracy set up with 42 FE eigenmode solves achieves the same accuracy as the reference solution. When, additionally, the FE solver provides the sensitivities of the eigenfrequency to R and L, a stochastic RSM for only using 14 sets of geometric parameters is sufficient.

The stochastic RSM allows to organise an analysis with changing geometric parameters as long as their range is covered by the RSM. This is commonly the case for statistical analyses and for studying microphoning and Lorentz detuning.

EXAMPLE: LORENTZ DETUNING

The accelerating eigenmode of the TESLA cavity [16] is computed for a single cell by a 2D eigenmode solver based on (2) discretised by the shape functions (7) and (8). The Lorentz force density or radiation pressure acting on the inner cavity wall is

$$f_{\mathrm{Lor}} = \frac{1}{2}\mu \left(\sum_j \widehat{\mathbf{h}}_j \frac{N_j(r,z)}{2\pi r} \right)^2 \tag{10}$$

The mechanical deformation is calculated by a 1D axisymmetric shell solver on the boundary of the 2D mesh, yielding the radial displacements $w(z)$, axial displacements $u(z)$ and bending angles of the mesh nodes (Fig. 3) [17,18]. Assuming elastic behaviour, the deformation changes linear with the radiation pressure. However, the radiation pressure is quadratically dependent on the magnitude of the electric field. Also the dependence of the eigenfrequency on the magnitude of the deformation is nonlinear. It is assumed that the pattern of the deformation is

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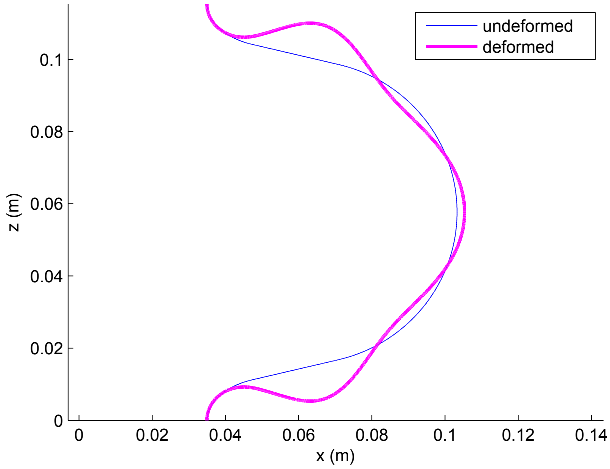

Figure 3: Displacement wall TESLA cavity due to radiation pressure with exaggeration factor 100.

only marginally influenced by the loading such that the deformed cavity wall can be parametrised by

$$\begin{cases} R(z) & \leftarrow & R(z) + \alpha w(z) \\ z & \leftarrow & z + \alpha u(z) \end{cases} \tag{11}$$

where $(R(z), z)$ is the design geometry, $(w(z), u(z))$ is the deformation pattern and α represents the size of the deformation. α is sized such that $\alpha = 1$ corresponds to the deformation when an energy of 1 J is stored in a single cell.

The sensitivity of the eigenfrequency to the deformation pattern is determined without changing the topology of the FE mesh. For numerical reasons, the deformation is spread out over all mesh nodes.

$$\begin{cases} r_j & \leftarrow & r_j + \alpha u(z_j) \frac{r_j}{R(z_j)} \\ z_j & \leftarrow & z_j + \alpha w(z_j) \frac{r_j}{R(z_j)} \end{cases} \tag{12}$$

The dependence of the FE shape functions on α causes $\frac{\mathrm{d}\vec{w}_j}{\mathrm{d}\alpha} \neq 0$ and hence $\frac{\mathrm{d}\mathbf{K}_{\frac{1}{\varepsilon}}}{\mathrm{d}\alpha} \neq 0$ and $\frac{\mathrm{d}\mathbf{K}_{\mu}}{\mathrm{d}\alpha} \neq 0$. The sensitivity of the eigenfrequency on the magnitude of the deformation is calculated from (9) and found to be $\frac{\mathrm{d}f}{\mathrm{d}\alpha} = 75.615$ Hz.

CONCLUSIONS

Significant shifts of the eigenfrequency of a cavity because of small geometric deformations are adequately calculated by an eigenmode solve delivering sensitivity information combined with a stochastic response surface model.

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