# UNDULATOR RADIATION INSIDE A DIELECTRIC WAVEGUIDE

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*Abstract*

We investigate the radiation from a charge moving along a helix around a dielectric cylinder immersed in a homogeneous medium. The radiation intensity in the exterior medium at large distances from the cylinder has been considered previously and here we are mainly concerned with the radiation propagating inside the cylinder. Numerical examples are given for a dielectric cylinder in the vacuum. It is shown that the presence of the cylinder can lead to the considerable increase of the radiation intensity. The insertion of a dielectric waveguide provides an additional mechanism for tuning the characteristics of the undulator radiation by choosing the parameters of the waveguide. The radiated energy inside the cylinder is redistributed among the cylinder modes and, as a result, the corresponding spectrum differs significantly from the homogeneous medium or free-space results. This change is of special interest in the low-frequency range where the distribution of the radiation energy among small number of modes leads to the enhancement of the spectral density for the radiation intensity. The radiation emitted on the waveguide modes propagates inside the cylinder and the waveguide serves as a natural collector for the radiation.

## INTRODUCTION

The motion of charged particle along a helical orbit is used in helical undulators for generating electromagnetic radiation in a narrow spectral interval at frequencies ranging from radio or millimeter waves to X-rays. The unique characteristics, such as high intensity and high collimation, have resulted in extensive applications of this radiation in a wide variety of experiments and in many disciplines (see, for instance, [1] and references given therein). These applications motivate the importance of investigations for various mechanisms of controlling the radiation parameters. From this point of view, it is of interest to consider the influence of a medium on the spectral and angular distributions of the radiation.

In [2, 3] we have investigated the radiation generated by a charge moving along a helical orbit around/inside a dielectric cylinder enclosed by a homogeneous medium. It has been shown that under certain conditions strong narrow peaks appear in the angular distribution of the radiation intensity in the exterior medium. At these peaks the radiated energy exceeds the corresponding quantity in the case of a homogeneous medium by several orders of magnitude. In these investigations we have considered the radiation at large distances from the cylinder. In the present paper we consider the radiation intensity inside a dielectric cylinder emitted by a charge moving along a helical trajectory around the cylinder (for the radiation from a charged particle rotating inside a dielectric cylinder see [4]).

## RADIATION INSIDE A DIELECTRIC WAVEGUIDE

Consider a point charge $q$ moving along the helical trajectory of radius $\rho_0$ outside a dielectric cylinder with radius $\rho_1$ and with permittivity $\varepsilon_0$. We will assume that this system is immersed in a homogeneous medium with dielectric permittivity $\varepsilon_1$ (magnetic permeability will be taken to be unit). We denote with $\omega_0 = 2\pi / T = v_\perp / \rho_0$ the angular velocity of the charge, $v_\parallel$ and $v_\perp$ are the particle velocities along the
axis of the cylinder and in the perpendicular plane, respectively.

Electromagnetic fields inside the cylinder can be presented in the form of the Fourier expansion

$$F_l(r,t) = 2\,\mathrm{Re}\sum_{m=0}^{\infty}{}' e^{im(\phi-\omega_0 t)} \int_{-\infty}^{\infty} dk_z e^{ik_z(z-v_\parallel t)} F_{ml}(k_z,\rho), \qquad (1)$$

where $F = E$ and $F = H$ for electric and magnetic fields respectively. The radiation parts in the field are determined by the singular points of the integrand. The only poles of the Fourier components $F_{ml}(k_z,\rho)$ are the zeros of the function

$$U_m(k_z) = V_m\left(\varepsilon_0|\lambda_1|\rho_1\frac{J_m'}{J_m} + \varepsilon_1\lambda_0\rho_1\frac{K_m'}{K_m}\right) - m^2\frac{\lambda_0^2+|\lambda_1|^2}{\lambda_0^2|\lambda_1|^2}\left(\varepsilon_0|\lambda_1|^2+\varepsilon_1\lambda_0^2\right), \qquad (2)$$

where $J_m = J_m(\lambda_0\rho_1)$ and $K_m = K_m(|\lambda_1|\rho_1)$ are the Bessel and Macdonald functions and the prime means the differentiation with respect to the argument of the function,

$$\lambda_j^2 = \omega_m^2(k_z)\varepsilon_j/c^2 - k_z^2,\ \omega_m(k_z) = m\omega_0 + k_z v_\parallel,$$

and

$$V_m = |\lambda_1|\rho_1\frac{J_m'}{J_m} + \lambda_0\rho_1\frac{K_m'}{K_m}.$$

The corresponding modes are exponentially damped in the region outside the cylinder. These modes are the eigenmodes of the dielectric cylinder and propagate inside the cylinder.

We denote by $\lambda_0\rho_1 = \lambda_{m,s}$, $s = 1,2,\ldots$, the solutions to equation $U_m = 0$ with respect to $\lambda_0\rho_1$ for the modes with $m \neq 0$. The corresponding modes $k_z = k_{m,s}^{(\pm)}$ are related to these solutions by the formula



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$$k_{m,s}^{(\pm)} = \frac{m\omega_0\sqrt{\varepsilon_0}}{c(1-\beta_{0\|}^2)}\left[\beta_{0\|} \pm \sqrt{1+b_{m,s}^2(\beta_{0\|}^2-1)}\right],$$
$$b_{m,s} = \frac{c\lambda_{m,s}}{m\omega_0\rho_1\sqrt{\varepsilon_0}}, \quad (3)$$

where $\beta_{0\|} = v_\|\sqrt{\varepsilon_0}/c$. For $\beta_{0\|} < 1$, the condition for $k_{m,s}^{(\pm)}$ to be real defines the maximum value for $s$, wich we will denote by $s_m$:

$$\lambda_{m,s_m} < m\omega_0\rho_1\sqrt{\varepsilon_0}(1-\beta_{0\|}^2)^{-1/2}/c < \lambda_{m,s_m+1}.$$

If the Cherenkov condition $\beta_{0\|} > 1$ is satisfied, the upper limit $s_m$ for $s$ is determined by the dispersion law for the dielectric permittivity via the condition $\varepsilon_0(\omega_m) > c^2/v_\|^2$.

Under the condition $\beta_{0\|} < 1$, for the radiation parts in the fields one finds

$$F_l^{(rad)}(\boldsymbol{r},t) = \sigma F_l^{(\sigma)}(\boldsymbol{r},t)$$
$$= \sigma 4\pi \,\mathrm{Re}\left[i\sum_{m=1}^{\infty} e^{im(\phi-\omega_0 t)} \sum_{s=1}^{s_m} \underset{k_z=k_{m,s}^{(\sigma)}}{\mathrm{Res}}\, e^{ik_z(z-v_\| t)} F_{ml}(k_z,\rho)\right], \quad (4)$$

where $\sigma = +(-)$ for $z - v_\| t > 0$ $(z - v_\| t < 0)$. This expression describes waves propagating along the positive direction of the axis $z$ for $\sigma = +$ and for $\sigma = -$, $b_{m,s} < 1$, and waves propagating along the negative direction to the axis $z$ for $\sigma = -$, $1 < b_{m,s} < 1/\sqrt{1-\beta_{0\|}^2}$. Under the condition $\beta_{0\|} > 1$ for the radiation fields one has $F_l^{(rad)}(\boldsymbol{r},t) = -\sum_{\sigma=\pm} F_l^{(\sigma)}(\boldsymbol{r},t)\theta(v_\| t - z)$, where $\theta(x)$ is the Heaviside unit step function. In this case the radiation field is behind the charge. The latter formula describes the waves propagating along the positive direction of the axis $z$ for $\sigma = +$ and for $\sigma = -$, $b_{m,s} > 1$, and waves propagating along the negative direction of the axis $z$ for $\sigma = -$, $b_{m,s} < 1$.

The radiation intensity inside the dielectric cylinder can be obtained by evaluating the energy flux through the cross-section of the dielectric cylinder:

$$I_\sigma^{(in)} = \sum_{m=1}^{\infty} I_{\sigma,m}^{(in)} = \frac{q^2c^2}{2\varepsilon_0}(\varepsilon_1-\varepsilon_0)^2$$
$$\times \sum_{m=1}^{\infty}\sum_{s=1}^{s_m} \frac{J_m^{-2}}{K_m^2} \frac{\lambda_{m,s}^2}{U_m'^2(k_{m,s}^{(\sigma)})} \frac{k_{m,s}^{(\sigma)}\rho_1^2}{\omega_m(k_{m,s}^{(\sigma)})}$$
$$\times \sum_{p=\pm 1} G_{m,s}^{(p\sigma)}\left[\left(\frac{\omega_m^2(k_{m,s}^{(\sigma)})\varepsilon_0}{c^2}\right)G_{m,s}^{(p\sigma)} - \frac{\lambda_{m,s}^2}{\rho_1^2}G_{m,s}^{(-p\sigma)}\right], \quad (5)$$
$$\times \left[J_{m+p}'^2 + J_{m+p}^2\left(1-\frac{(m+p)^2}{\lambda_{m,s}^2}\right)\right]$$

where

$$G_{m,s}^{(p\sigma)} = \frac{v_\perp}{2c}(V_m + pmu)\sum_{l=\pm 1}\frac{lK_{m+l}(\lambda_{m,s}^{(\sigma)}\rho_0/\rho_1)}{V_m - lmu}$$
$$+ \frac{v_\|}{c}\frac{\lambda_{m,s}^{(\sigma)}V_m}{k_{m,s}^{(\sigma)2}\rho_1^2} \quad (6)$$
$$\times \left(\frac{k_{m,s}^{(\sigma)}\rho_1}{V_m - pmu} - \frac{\lambda_{m,s}}{k_{m,s}^{(\sigma)}\rho_1^2}\right)K_m(\lambda_{m,s}^{(\sigma)}\rho_0/\rho_1),$$

with $u = \lambda_{m,s}/\lambda_{m,s}^{(\sigma)} + \lambda_{m,s}^{(\sigma)}/\lambda_{m,s}$ and

$$\lambda_{m,s}^{(\sigma)} = \sqrt{(1-\varepsilon_1/\varepsilon_0)k_{m,s}^{(\sigma)2}\rho_1^2 - \varepsilon_1\lambda_{m,s}^2/\varepsilon_0}.$$

In the numerical examples below we consider the case $v_\| = 0$. In this case $I_+^{(in)} = I_-^{(in)}$. In Figures 1 and 2, by black points we display the number of the radiated quanta at a given harmonic $m$ per period of the charge rotation, $N_m = 2TI_{+,m}^{(in)}/(\hbar m\omega_0)$. The graphs are plotted for $\varepsilon_0 = 3$, $\varepsilon_1 = 1$ and for the electron with the energy 2 MeV. For Figure 1 we have taken $\rho_1/\rho_0 = 0.95$ and for Figure 2 one has $\rho_1/\rho_0 = 0.99$. The red points correspond to the radiation in the vacuum in the absence of the cylinder. As it is seen from the graphs, for certain harmonics the radiation intensity inside the dielectric cylinder is essentially larger than the corresponding intensity for the synchrotron radiation in the vacuum. At these harmonics new modes of the dielectric cylinder appear. For example, in the case $\rho_1/\rho_0 = 0.95$ there are no modes for $m = 7$ and a single mode appears for $m = 8$. Similarly, for $\rho_1/\rho_0 = 0.99$, the number of eigenmodes increases by one for the harmonics $m = 7$ and $m = 26$.

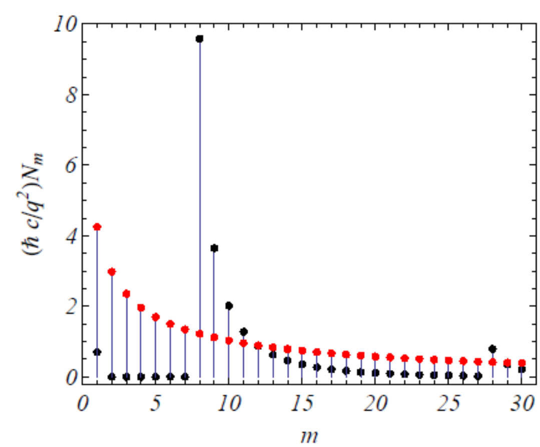


Figure 1: Number of the radiated quanta on a given harmonic $m$ propagating inside the dielectric cylinder for $\rho_1/\rho_0 = 0.95$ (black points). The red points correspond to the radiation intensity in the absence of the cylinder.



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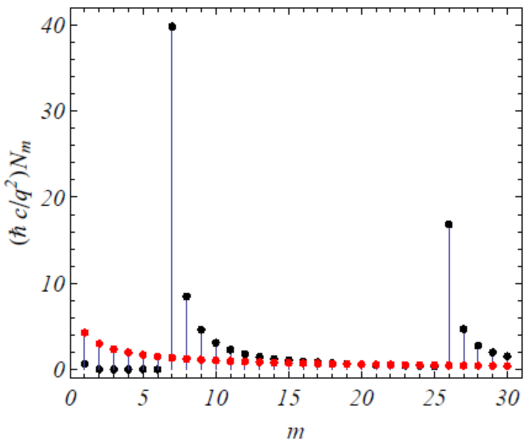


Figure 2: The same as in Figure 1 for $\rho_1 / \rho_0 = 0.99$.

## CONCLUSION

In the present talk we have investigated the radiation from a charged particle moving along a helical trajectory around a dielectric cylinder immersed into homogeneous medium. The radiation intensity at large distances from the cylinder has been studied before in [3] and here we have considered the part of the radiation propagating inside the cylinder. We have specified the conditions for the existence of such radiation. Radiated energy inside the cylinder is redistributed among the cylinder modes which are solutions of Eq. (2). The spectral distribution of the radiation intensity differs significantly from that for the radiation in homogeneous medium or in free-space. Radiation emitted on the waveguide modes propagates inside the cylinder and the waveguide serves as a natural collector for the radiation. This eliminates the necessity for focusing to achieve a high-power spectral intensity. Note that, in addition to the part of the radiation propagating inside the cylinder, there are radiation fields in the exterior region localized near the surface of the dielectric cylinder. These fields are the tails of the eigenmodes for the dielectric cylinder and exponentially decrease with the distance from the cylinder. The corresponding energy flux through the plane perpendicular to the cylinder axis can be investigated in the way described above for the interior region. The details will be presented in the forthcoming paper.

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