SIMULATION OF MULTIBUNCH INSTABILITIES WITH THE HEADTAIL CODE

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Abstract

Multibunch instabilities due to beam-coupling impedance can be a critical limitation for synchrotrons operating with many bunches. To study these instabilities, the HEADTAIL code has been extended to simulate the motion of many bunches under the action of wake fields. All the features already present in the single-bunch version of the code have remained available, in particular synchrotron motion, chromaticity, amplitude detuning due to octupoles and the ability to load any kind of wake fields through tables. The code has been then parallelized in order to track thousands of bunches in a reasonable amount of time, showing a linear scaling with the number of processors used. We show benchmarks against Laclare's theory in simple cases, obtaining a good agreement. Results for bunch trains in the LHC and comparison with beam-based measurements are also exhibited.

INTRODUCTION

Transverse coupled-bunch instabilities occur in general when several bunches interact with their surroundings, creating wake fields that act back on the bunch train in such a way as to give rise to an exponentially growing oscillation. To evaluate the rise times of such instabilities, several theories exist, such as Sacherer's [1], Laclare's [2] and Scott Berg's [3], as well as a macroparticle simulation code, MTRISM [4]. All have some limitations: MTRISM and Scott Berg's theory do not take into account quadrupolar impedance which is quite significant in e.g. the LHC due the high-impedance flat collimators [5]; Sacherer's and Laclare's formalisms assume a machine entirely filled with equidistant bunches, which is not the case in e.g. the LHC and even less in the SPS where only about 30% of the machine is filled to produce the nominal LHC beam. Therefore, to be able to study multibunch instabilities in the LHC and the SPS, we chose to extend the single-bunch macroparticle code HEADTAIL [6]. A first simplified version, accounting only for rigid bunch oscillations with no longitudinal motion, was already developed in Ref. [7]. We present here a new extension that can handle both multibunch and intrabunch motion, benchmarking it with respect to theory, and showing various results in the case of the LHC. Finally, we compare HEADTAIL simulations using the LHC impedance model with beam-based experiments.

DESCRIPTION OF HEADTAIL MULTIBUNCH

HEADTAIL is a macroparticle simulation code where each individual macroparticle i is tracked through a ring subdivided into several kick sections. After initialization with Gaussian (or uniform) distributions, with the possibility to enforce longitudinal matching, macroparticles are tracked in mainly three steps per kick section: 1) the bunches are sliced longitudinally, 2) wake fields kicks are applied to each macroparticles, and 3) their transverse phase space coordinates are linearly transported to the next kick section. Once per turn, the synchrotron motion update is applied, separately for each bunch. For the second step, the kicks $\Delta x'_i$, $\Delta y'_i$ and $\Delta\delta_i$ are computed as

$$\Delta x'_i = \mathcal{C} \sum_{z_S > z_{S_i}} n_S W_x\left(z_{S_i} - z_S, x_S, y_S, x_{S_i}, y_{S_i}\right),$$

$$\Delta y'_i = \mathcal{C} \sum_{z_S > z_{S_i}} n_S W_y\left(z_{S_i} - z_S, x_S, y_S, x_{S_i}, y_{S_i}\right),$$

$$\Delta\delta_i = \mathcal{C} \sum_{z_S \geq z_{S_i}} n_S W_{||}\left(z_{S_i} - z_S\right),$$

where $\mathcal{C} = -\frac{e^2}{E_0\beta^2\gamma}$, γ being the Lorentz factor, $\beta = \sqrt{1-\gamma^{-2}}$, E_0 the rest mass of the elementary particles (protons or electrons) and e the elementary charge. S_i is the slice containing the macroparticle i, and n_S, x_S, y_S, z_S are the number of particles, and the transverse and longitudinal positions of each slice S (z decreases when going toward the tail of the bunches). In the above expressions the sums run over all slices and bunches before the slice of the macroparticle considered, neglecting thus any wake emitted in the forward direction. The sums continue up to a certain number of turns, i.e. the wakes of preceding turns are taken into account. $W_{||}(z)$ is the longitudinal wake function, while $W_x(z)$ and $W_y(z)$ are given by

$$W_x(z, x_S, y_S, x_{S_i}, y_{S_i}) = W_x^{dip}(z)x_S + W_{xy}^{dip}(z)y_S + W_x^{quad}(z)x_{S_i} + W_{xy}^{quad}(z)y_{S_i}, \quad (1)$$

$$W_y(z, x_S, y_S, x_{S_i}, y_{S_i}) = W_y^{dip}(z)y_S + W_{xy}^{dip}(z)x_S + W_y^{quad}(z)y_{S_i} + W_{xy}^{quad}(z)x_{S_i}, \quad (2)$$

where dip stands for "dipolar" and $quad$ for "quadrupolar". Note that coupled terms – i.e. linear wakes in the x direction but proportional to the y position and vice versa, are taken into account. The wake functions above ($W_x^{dip}(z)$, $W_x^{quad}(z)$, etc.) are provided in a table given in input.

The code has been parallelized over the bunches, which is quite efficient since all bunches can be treated almost independently, the only requirement being that after each slicing the processors exchange for all the bunches the positions and number of particles of each slice, such that the wakes can be computed in all bunches. This represents a limited amount of data since the number of slices usually does not exceed a few hundreds. Indeed, Fig. 1 shows

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that the computational time is inversely proportional to the number of processors used, i.e. the parallelization is linear with the number of processors.

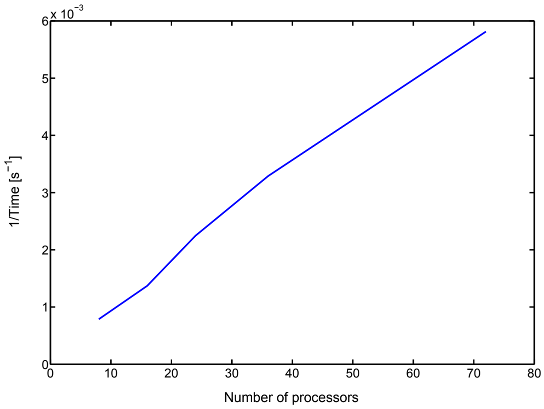

Figure 1: Inverse of the computational time vs. number of processors used, for 72 bunches with 10 slices per bunch.

COMPARISON WITH THEORY

The code has been compared to Laclare's formalism [2], in which the complex angular frequency shifts $\delta\omega$ of all possible modes are found as the eigenvalues of an infinite matrix. The most critical instability is then the one whose $\delta\omega$ has the lowest (negative) imaginary part. The formalism has been implemented in a code that automatically checks that the necessary matrix truncation still gives accurate eigenvalues (within 0.1%) by testing convergence with respect to the matrix size. The formalism assumes no linear coupling, a completely filled machine with equidistant bunches, as well as dipolar transverse impedances and a linear longitudinal bucket without distortion. We used therefore the same conditions in HEADTAIL. Also, longitudinal Gaussian distributions cut at $2\sigma_z^{rms}$ were used in both the theory and HEADTAIL. Note that in the simulations the longitudinal parameters were initially matched.

To obtain the rise times from the simulations, we compute thanks to SUSSIX [8] the highest spectral lines of the beam average transverse position and momenta, on a sliding window along the simulation, and fit the amplitude of the highest spectrum line as a function of time by an exponential.

We study here the case of the CERN SPS filled with a 25 ns beam. The impedance of its vacuum pipe is computed from Zotter's theory for an infinitely long axisymmetric cylindrical structure [9], assuming the pipe has 2 cm radius, 2 mm thickness and is made of stainless steel surrounded by vacuum [10]. The wake function is then obtained thanks to a Fourier transform with an uneven sampling [11]. Since the beam pipe is actually of elliptical cross section with the horizontal semi-axis significantly larger than the vertical one, to obtain the final dipolar impedances and wake functions we multiply by the Yokoya factors [12] for a flat chamber, i.e. $\frac{\pi^2}{24}$ in x and $\frac{\pi^2}{12}$ in y.

Table 1 shows the beam parameters used in both the simulation and the theory, and in Fig. 2 we can see the excellent agreement on the rise times, with a slight discrepancy only for the highest positive chromaticity considered.

Table 1: Beam Parameters for the SPS

Nb. of bunches	N_b	924
Bunch population	N	$5 \cdot 10^{10}$
RMS bunch length	σ_z^{rms}	0.19 m
Momentum spread	σ_δ	0.002
RF voltage	V_{rf}	3 MV
Harmonic number	h	4620
Bunch spacing	ΔT_b	25 ns
Circumference	C	6911 m
Tunes	$Q_{x,y,s}$	26.13, 26.16, 0.0073
Beta functions	$\beta_{x,y}$	42, 42 m
Norm. emittances	$\varepsilon_{x,y}$	4, 4 μm.rad
Mom. compaction	α_p	$1.92 \cdot 10^{-3}$
Lorentz factor	γ	27.7

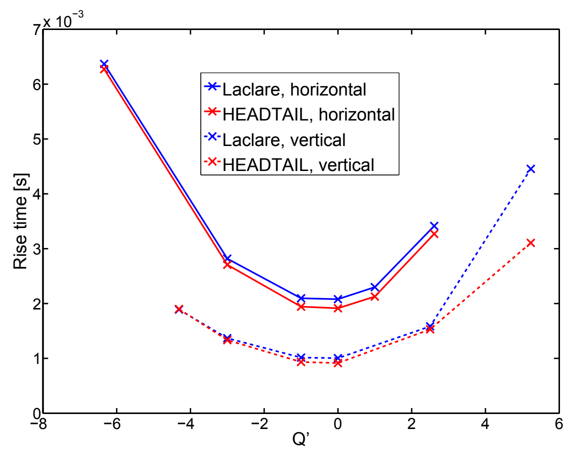

Figure 2: Rise times vs. Q' for 924 bunches in the SPS: Laclare's theory and HEADTAIL multibunch.

SIMULATIONS RESULTS FOR THE LHC

For these simulations, we use HEADTAIL with a non linear longitudinal bucket and the impedance model presented in Ref. [11], which presently includes the resistive-wall impedance of the 44 collimators (some being in graphite), of the copper-coated beam screens covering 86% of the ring, and of the copper vacuum pipe for the remaining 14%, together with a broad band impedance model to account for most of the smooth transitions around the ring [13]. Wake functions are obtained thanks to a Fourier transform on a uneven sampling [11]. Dipolar, quadrupolar and coupled terms are all taken into account, and weighted by the corresponding beta functions in order to be lumped into a single kick.

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Comparison Between Short and Long Trains

During current physics operation in the LHC, the bunches are not all equidistant. Indeed, due to the limited capacity of the injectors, the rise time of the injection kickers and that of the LHC dump system kicker, the bunches are grouped in batches of 36 bunches all separated by 50 ns, these batches being separated between them by empty gaps of various sizes (from 225 ns to 3 μs) [14]. Note that due to an additional constraint on the injection system, a low intensity pilot bunch plus a batch of 12 bunches are currently always injected before the batches of 36 bunches.

The multibunch HEADTAIL code presented above now allows to study any filling pattern. This is particularly interesting for small trains which are in principle far from the situation of a completely filled machine accessible to theories such as the ones from Sacherer [1] or Laclare [2]. We study here the case of a train of 36 bunches spaced between them by 50 ns and compare it to the case of 1782 equidistant bunches with the same 50 ns spacing. We show in Table 2 the parameters used in the simulations, which are close to those of normal operation at 3.5 TeV/c in 2011. In Fig. 3 the average x positions of the full beam is shown. The beam with 1782 bunches gets unstable more quickly than the other one, but the rise time is only about a factor 2.5 higher (in the vertical plane - not shown here, the factor is 4). To analyse the coupled-bunch nature of the instabilities, a singular value decomposition (SVD) [15, 16] was performed on the bunch centroid data (vs. bunch number and turn). In Fig. 4 it appears that the spatial pattern of the most critical "mode" from the SVD has a larger wavelength with 1782 bunches than with 36 bunches.

Table 2: LHC Parameters for the Comparison Between 36 and 1782 Bunches

N	$1.2 \cdot 10^{11}$	Q_x	64.28	Q_y	59.31
σ_δ	$1.6 \cdot 10^{-4}$	σ_z^{rms}	0.09 m	V_{rf}	12 MV
α_p	$3.2 \cdot 10^{-4}$	h	35640	ΔT_b	50 ns
C	26659 m	γ	3730.3	Q_s	0.0025
ε_x	3.75 μm.rad	Q'_x	0	β_x	66 m
ε_y	3.75 μm.rad	Q'_y	0	β_y	71.5 m

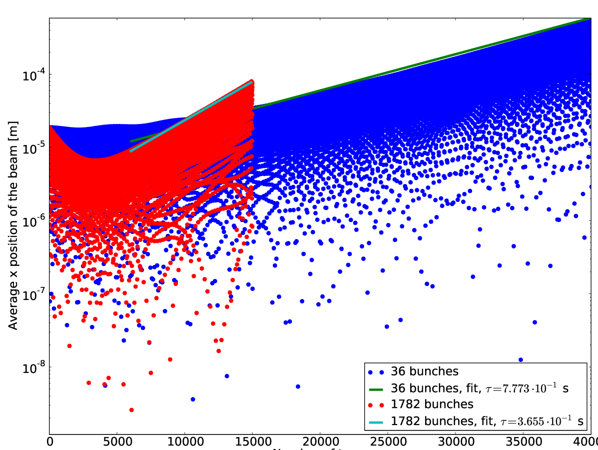
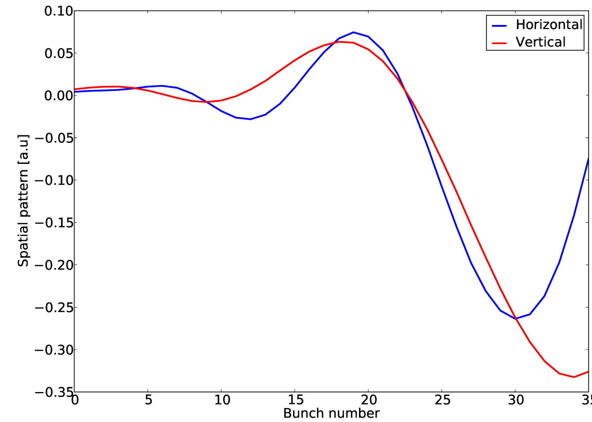
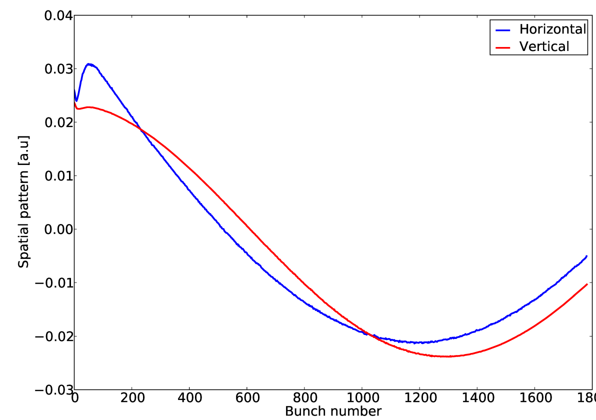


Figure 3: Average horizontal beam position vs. turns for 36 and 1782 bunches.



(a) Case with 36 bunches.



(b) Case with 1782 bunches.

Figure 4: SVD spatial pattern along the bunches for short and long bunch trains.

Coupled-Bunch Headtail Instabilities

Single-bunch headtail modes are well known instabilities occurring in synchrotrons, and they are currently damped in the LHC thanks to Landau damping provided by octupoles [17]. When many bunches are in the machine, coupled-bunch headtail modes (i.e. coupled-bunch modes with intrabunch motion) could be stronger than the single-bunch headtail modes. Using the parameters of Table 2 except for $Q'_x = Q'_y = 6$ and $N = 3 \cdot 10^{11}$ protons per bunch, we compare in Table 3 the tune shifts (with respect to the synchrotron sideband) and the rise times of the $m = -1$ headtail mode, for the single-bunch and 36-bunches cases. The values were obtained thanks to a spectral analysis on complex frequencies described in Ref. [11], performed on the time pattern from the SVD decomposition (see above) for the 36-bunches case. Clearly, the rise times are lower and the tune shifts significantly higher in the 36-bunches case with respect to the single-bunch ones. For the 36 bunches case, an headtail mode with one node is

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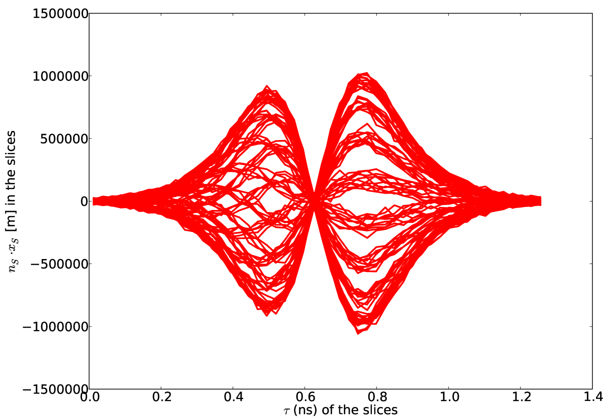
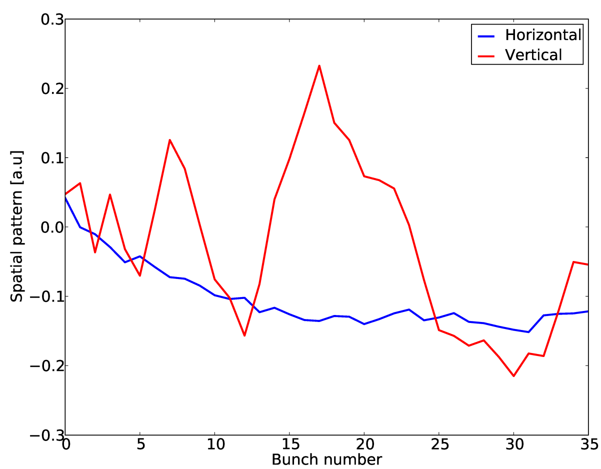
clearly visible in Fig. 5a, and the coupled-bunch nature of the spatial pattern from the SVD in Fig. 5b.

Table 3: Rise Times and Tune Shifts of the $m = -1$ Mode

	τ_x [s]	τ_y [s]	ΔQ_x	ΔQ_y
1 bunch	2.25	2.45	$4.1 \cdot 10^{-6}$	$4.1 \cdot 10^{-5}$
36 bunches	1.74	1.95	$2.9 \cdot 10^{-5}$	$5.7 \cdot 10^{-5}$



(a) Horizontal bunch profile of the last bunch.



(b) SVD spatial pattern along the bunches.

Figure 5: Intrabunch and coupled-bunch motion for 36 bunches.

COMPARISON WITH LHC BEAM-BASED MEASUREMENTS

To check the accuracy of both the LHC impedance model presented above and the HEADTAIL code, a dedicated experiment [18] was carried on to measure transverse instability rise times of the rigid-bunch modes, as well as the loss of Landau damping threshold in terms of octupole current at top energy. This was done on a beam of 48 nominal bunches (a batch of 36 50 ns spaced bunches preceded by 12 bunches also 50 ns spaced) and a low intensity pilot bunch.

The idea of the experiment was to switch the feedback off during a long enough time window, in order to allow transverse coupled-bunch instabilities to develop. At top energy, it was also necessary to reduce Landau damping by decreasing the current in the octupoles, which was done in steps. Note that the defocusing octupoles were set to a positive current and the focusing ones to its opposite. Bunch centroid data were acquired during the time window when the feedback was off, and all beam and machine parameters (intensity, bunch length, collimator half gaps and emittances) continuously monitored and used in the simulations.

Results at Injection

In Fig. 6 the average instability rise times (obtained with two different methods) of the 8 last bunches of beam 2, are compared to HEADTAIL simulations for various chromaticities. The agreement between the model and the measurements is remarkable for Q' close to zero or negative. The only significant discrepancy appears in vertical when $Q'_y = 2$. Note that in this latter case only one set of data and one fitting method could be used, hence the absence of error bar. In the same figure single-bunch simulations results are also shown and seem to rule out the possibility that the instabilities observed were actually single-bunch. This is confirmed by the spatial pattern from the SVD (see above) along the batch of 36 bunches in Fig. 7.

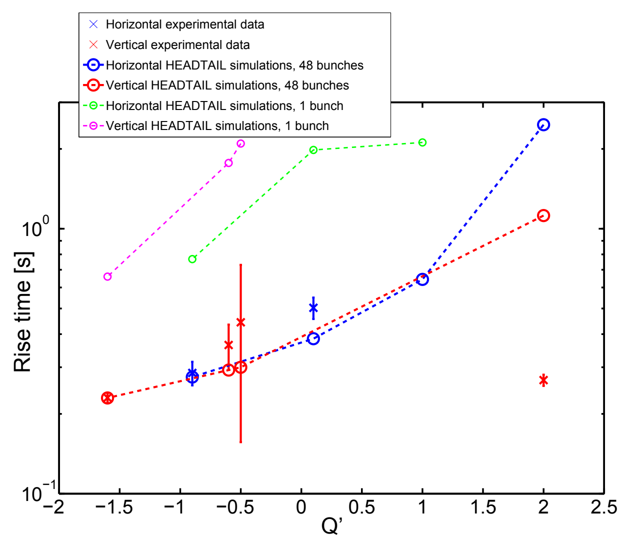


Figure 6: Beam 2 rise times vs. Q' at injection, from measurements and HEADTAIL (single-bunch and multibunch).

Results at Flat Top

At 3.5 TeV$/c$ some instabilities were observed in the vertical plane only. On the contrary, HEADTAIL simulations exhibit instabilities only in the horizontal plane for non zero octupole currents, which can be explained by the fact that $Q'_x = 0$ while $Q'_y \geq 1$ (the impedances in both planes being quite similar [11]). Also, a discrepancy of factor $2 - 3$ between simulations and measurements is visible on the rise times of the 8 last bunches of the train, as shown in Fig. 8. Note the quite large error bars on the simulation data, due to the different fitting methods used, which probably indicates that a higher number of simulated turns would be better for a higher accuracy of the fit.

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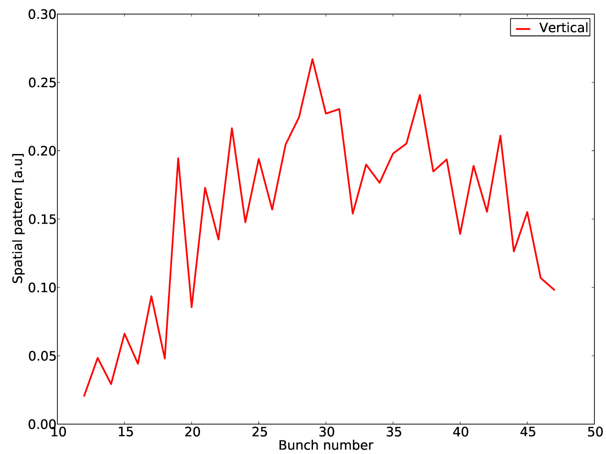

Figure 7: Vertical spatial pattern of the highest "mode" from the SVD of 36-bunches batch, for beam 1 with $Q'_y = 0.3$, at injection.

In Fig. 9 we compare beam 1 vertical rise times from the measurements with HEADTAIL simulations results when the octupoles are switched off. The discrepancy between HEADTAIL and the measurements already mentioned appears clearly. On the other hand, the accuracy of the measurement of Q' at 3.5 TeV$/c$ is rather large (of the order of one unit), which could explain the discrepancies observed: if $Q'_y = 0$, these are much reduced.

Finally, measurements show that when the damper is off 60 A (resp. 70 A) in the octupoles are enough to stabilize beam 1 (resp. beam 2), which is less than foreseen in the model (resp. 120 A and 100 A). A possible explanation of the discrepancy is that some sources of non-linearity have been neglected in the simulations, in particular Q'' (second derivative of the tune with respect to the momentum deviation), which is quite high when octupoles are on. The effect of Q'' on beam stability is currently under study.

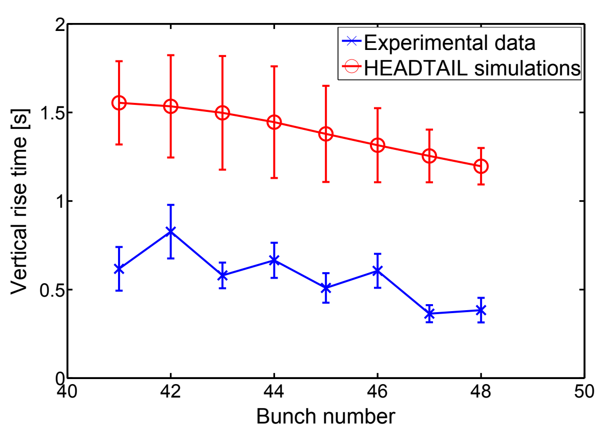

Figure 8: Vertical rise times of the last 8 bunches of beam 2 when octupoles are off at 3.5 TeV$/c$, compared to HEADTAIL simulations.

CONCLUSION

The wake fields code HEADTAIL is now able to simulate multibunch trains, and was successfully benchmarked with theory in simple cases, for instability rise times. Using the LHC impedance model, the code was also compared to an LHC measurement, giving reasonable agreement.

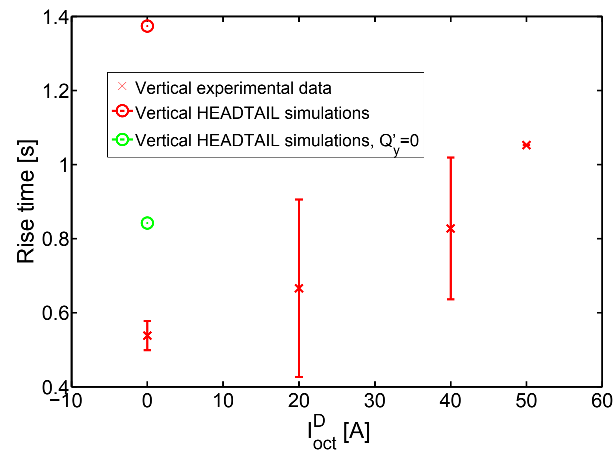

Figure 9: Measured vertical rise times vs. octupole current for beam 1 at 3.5 TeV$/c$, compared to HEADTAIL simulations, both with the measured Q'_y (≈ 2) and with $Q'_y = 0$.

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