Simulation of Microwave Instability in LER of KEKB and SuperKEKB

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KEK & SOKENDAI

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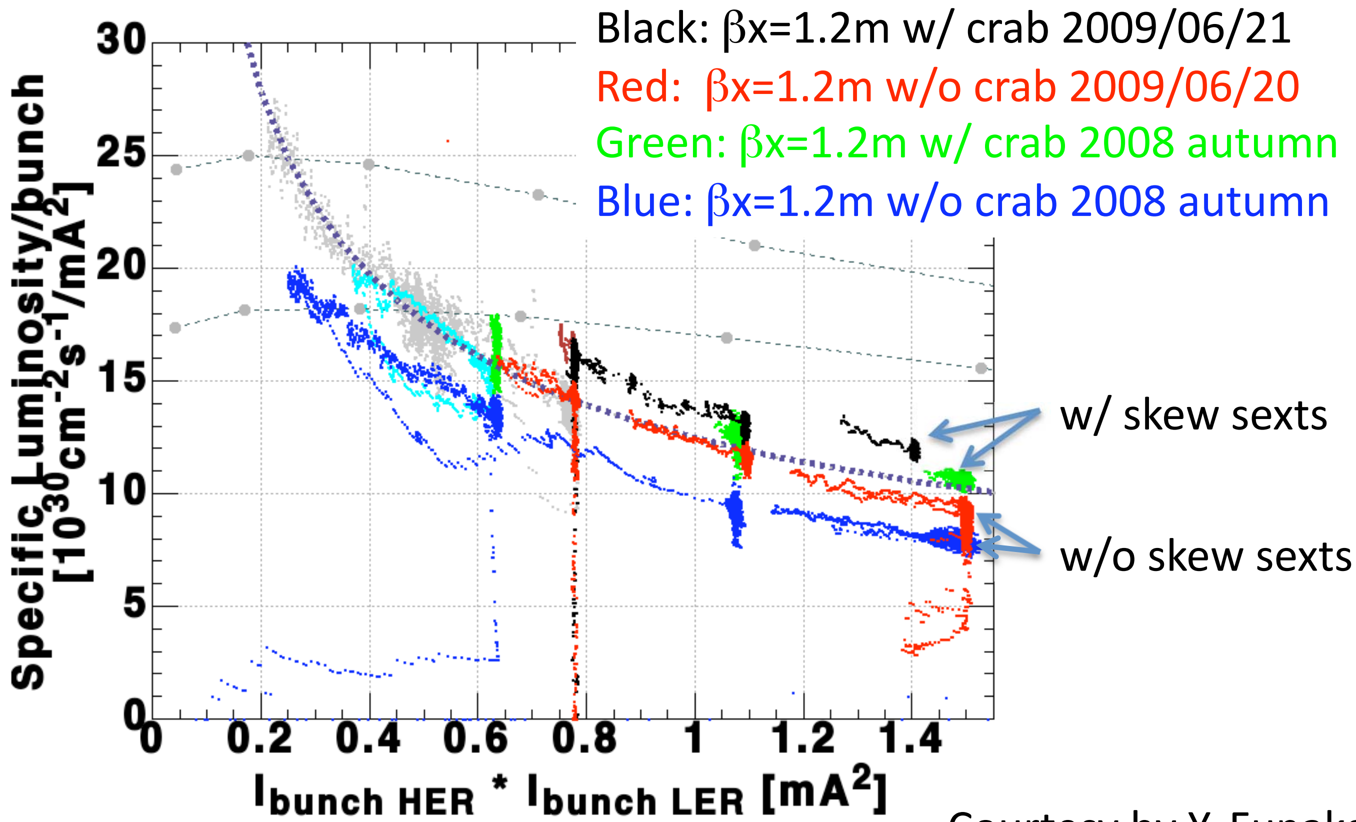
Outline

- Introduction
- Impedance calculation for KEKB LER
- Codes development
  - CSR calculation
  - VFP solver
- Simulation results
- Summary

Introduction (1)

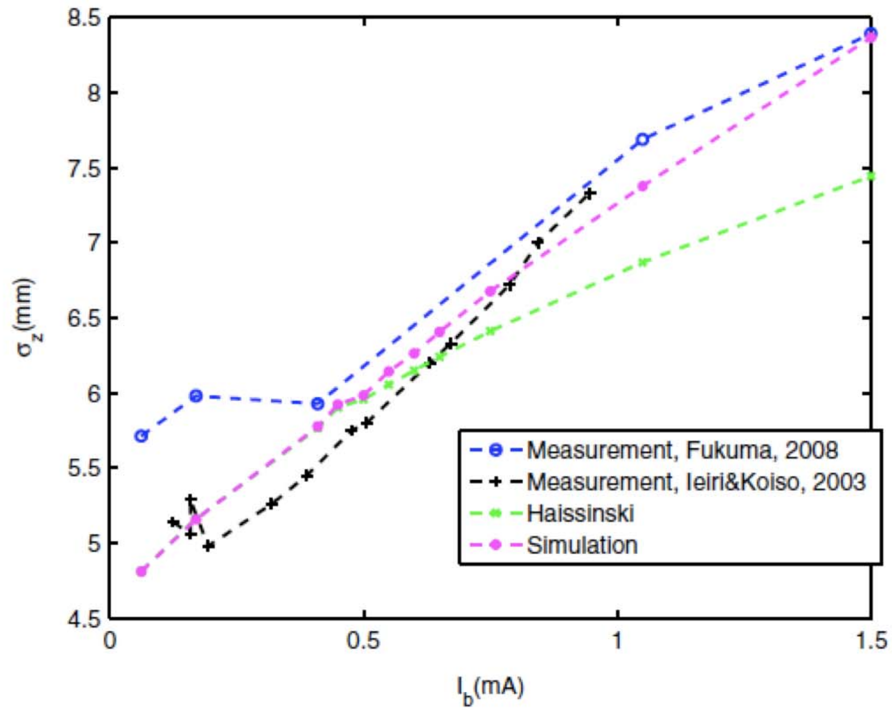
- Commissioning with crab cavities at KEKB was successful. But at high bunch currents, specific luminosity is still much lower than prediction.

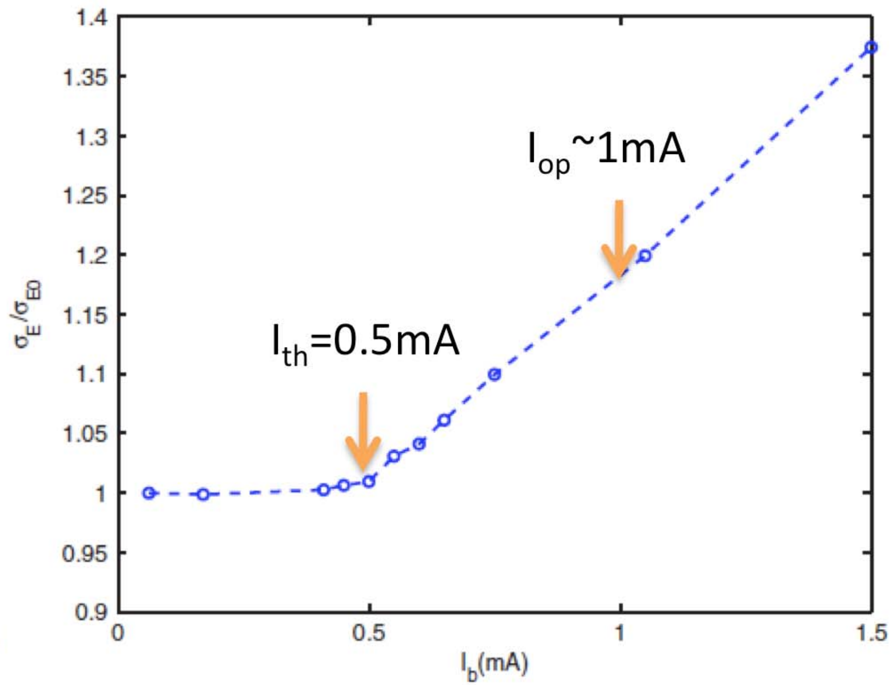


Courtesy by Y. Funakoshi

Introduction (2)

- The study based on a broadband resonator impedance model (Y. Cai) showed I_{th}=0.5mA at KEKB-LER.





Y. Cai, et al., "Potential-Well Distortation, Microwave Instability, and Their Effects with Colliding Beams at KEKB", Phys. Rev. ST Accel. Beams 12, 061002 (2009).

Parameter	Description	LER
L (nH)	Inductance	116.7
R (KΩ)	Resistance	22.9
C (fF)	Capacitor	0.22

Introduction (3)

- Enlarged energy spread due to MWI enhances Synchro-betatron resonances, even drive Sawtooth instability at high bunch currents
- To achieve higher luminosity at KEKB, MWI should be studied in detail.
  - Well-defined impedance model is needed
  - MWI simulations
- The design of SuperKEKB is ongoing. For nano-beam option, $N_p=10^{11}$ and $\sigma_z=5mm$ may be chosen with luminosity ~$8x10^{35}$ $cm^{-2}s^{-1}$.

Introduction (4)

- GdfidL on a cluster with large memory (256 GB) was used to calculate ultra-short wake potentials.
- Several codes were developed at KEK to study MWI for KEKB and SuperKEKB.
  - CSR codes were developed by T. Agoh and K. Oide independently, their results agree well in some senses.
  - PIC tracking and VFP solver were developed.

Impedance calculation for KEKB LER (1)

- Vacuum components: Taking into account as many as possible: Crab cavity, TFB, tapers, gate valves…
- Careful modeling: SR masks, flange gaps, pumping ports…

Component	Number	Software
ARES cavity	20	GdfidL
Movable mask	16	GdfidL
SR mask (arc/wiggler)	1000 (905/95)	GdfidL
Bellows	1000	GdfidL
Flange gap	2000	GdfidL
BPM	440	MAFIA
Pumping port	3000	GdfidL
Crab cavity	1	ABCI
FB kicker/BPM	1/40	GdfidL
Tapers ARES/Crab/Abort/Injection IR(IP/QCSL/QCSR)	 4/2/2/2 6(2/2/2)	GdfidL
Gate valves f94/f150/94x150	26/13/2	GdfidL

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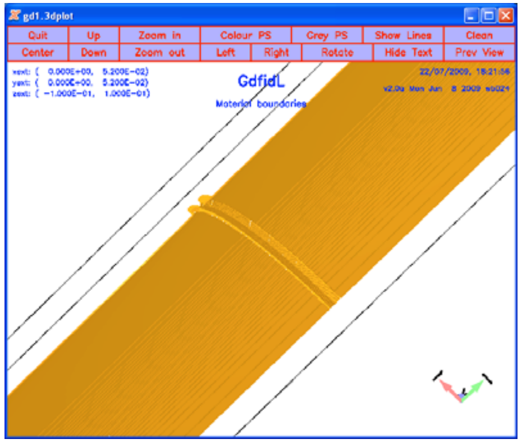
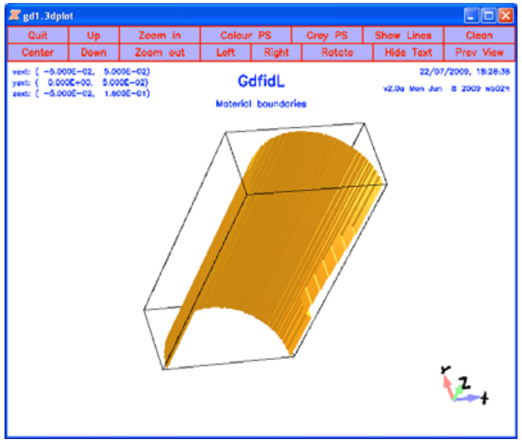
Impedance calculation for KEKB LER (2)

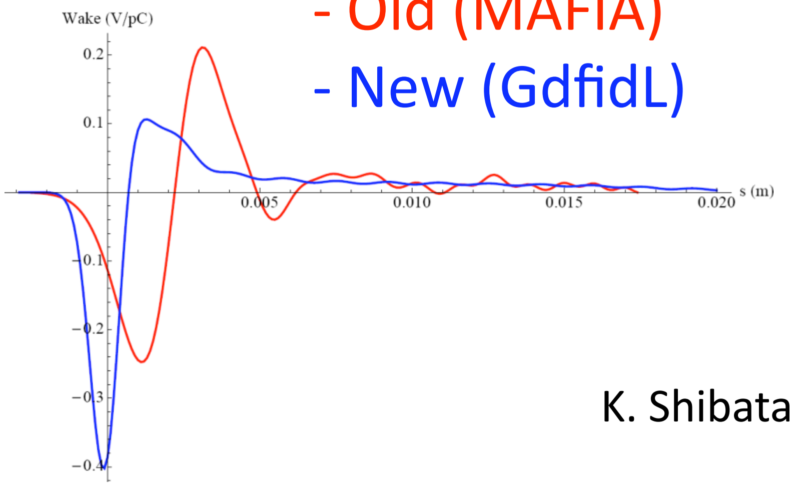
Flange gap



SR mask



- Old (MAFIA)
- New (GdfidL)

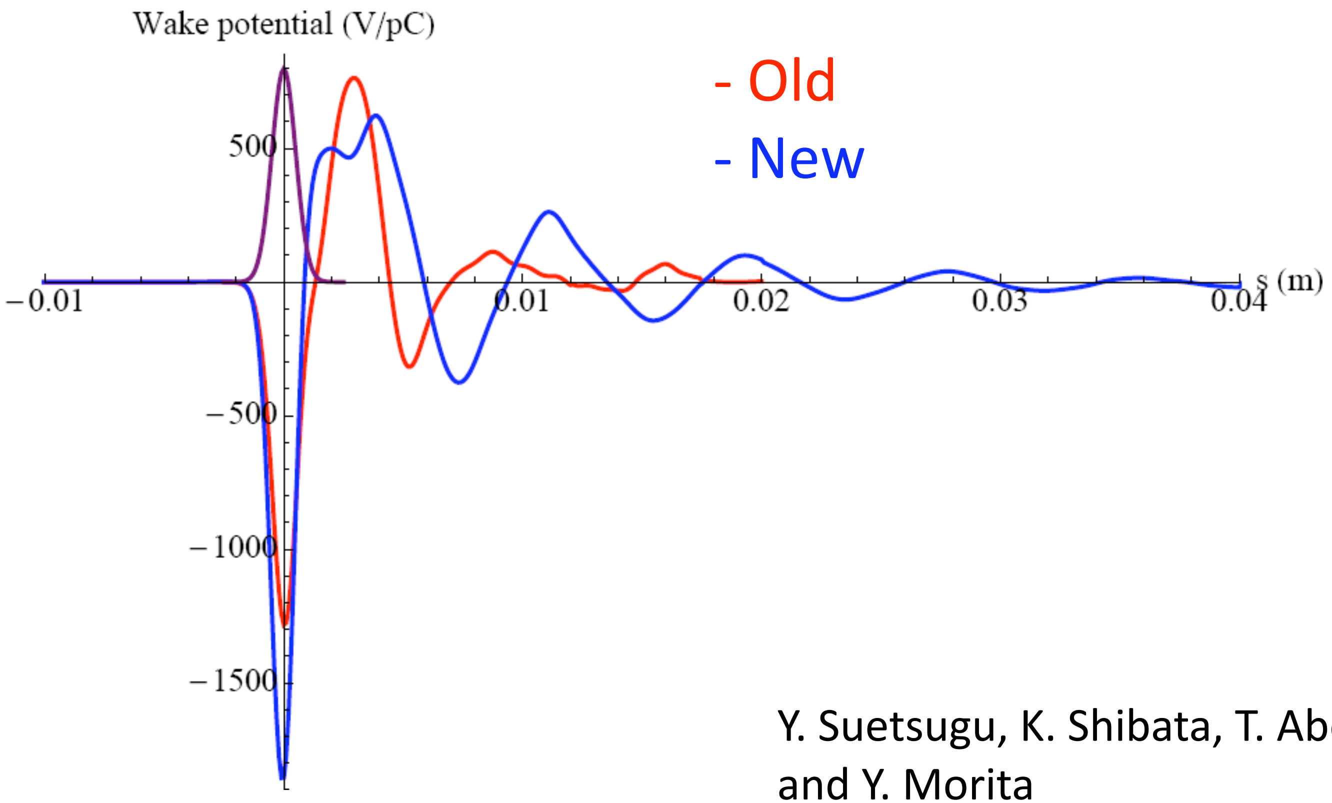


- Old (MAFIA)
- New (GdfidL)

K. Shibata

Impedance calculation for KEKB LER (3)

- Geometrical wake (GW) potential of 0.5mm bunch
  - Main improvements are wakes of SR masks and flange gaps
  - Contributions from crab cavity, FB, tapers and gate valves are relatively small



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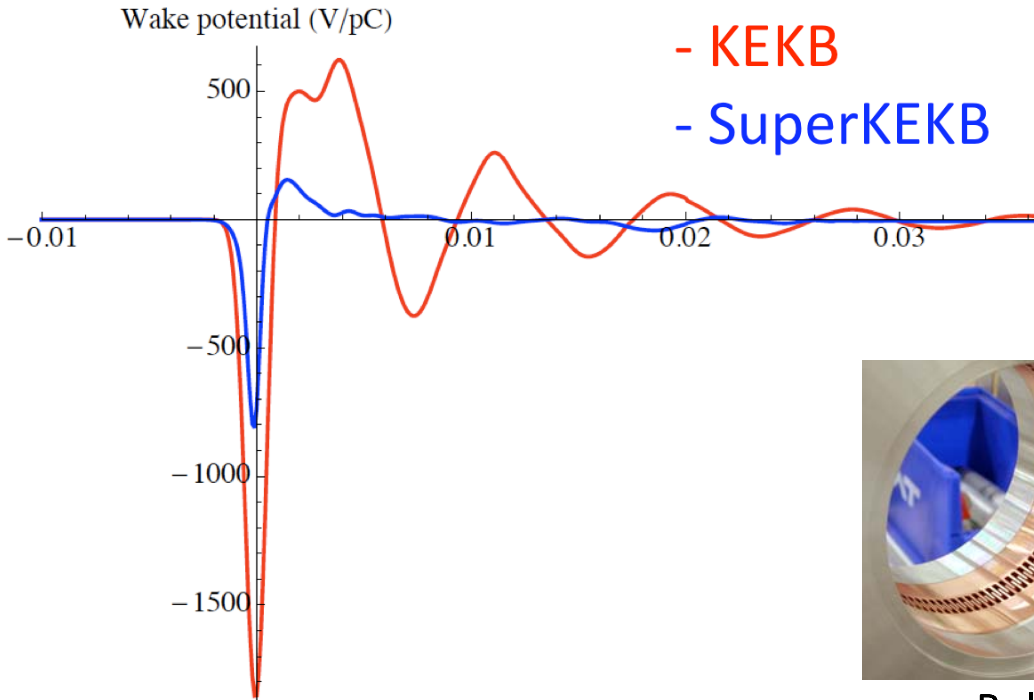
Impedance calculation for SuperKEKB

- There will be significant improvements on vacuum components to reduce beam impedance



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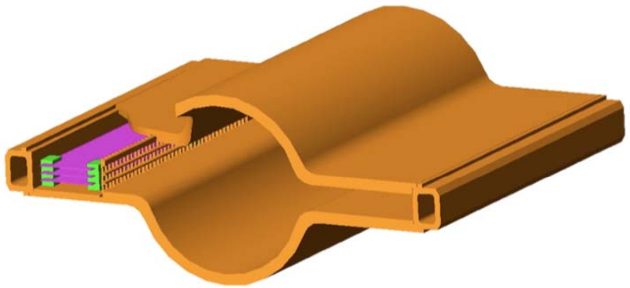
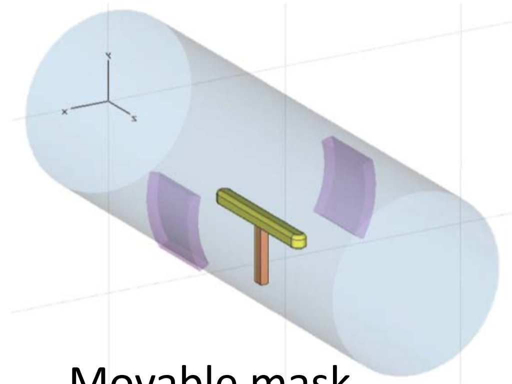


Antechamber

Bellows



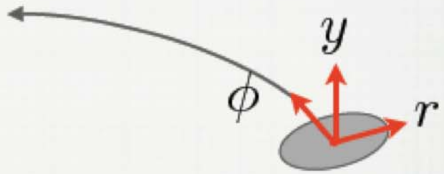
Movable mask

Courtesy by Y. Suetsugu

CSR impedance calculation (1)

- A CSR code was developed by K. Oide in 2008
- Solving Maxwell equations

$$
\begin{aligned}
\frac{1}{r}\frac{\partial r E_\phi}{\partial r} - \frac{1}{r}\frac{\partial E_r}{\partial \phi} &= -\frac{\partial B_y}{\partial t} \\
\frac{1}{r}\frac{\partial E_y}{\partial \phi} - \frac{\partial E_\phi}{\partial y} &= -\frac{\partial B_r}{\partial t} \\
\frac{\partial E_r}{\partial y} - \frac{\partial E_y}{\partial r} &= -\frac{\partial B_\phi}{\partial t} \\
\frac{1}{r}\frac{\partial r B_\phi}{\partial r} - \frac{1}{r}\frac{\partial B_r}{\partial \phi} &= \mu_0 j_y + \frac{1}{c^2}\frac{\partial E_y}{\partial t} \\
\frac{1}{r}\frac{\partial B_y}{\partial \phi} - \frac{\partial B_\phi}{\partial y} &= \mu_0 j_r + \frac{1}{c^2}\frac{\partial E_r}{\partial t} \\
\frac{\partial B_r}{\partial y} - \frac{\partial B_y}{\partial r} &= \mu_0 j_\phi + \frac{1}{c^2}\frac{\partial E_\phi}{\partial t} \\
\frac{1}{r}\frac{\partial r E_r}{\partial r} + \frac{1}{r}\frac{\partial E_\phi}{\partial \phi} + \frac{\partial E_y}{\partial y} &= \frac{\rho}{\varepsilon_0}
\end{aligned}
$$



$$j_r = j_y = 0, \qquad j_\phi = \rho c$$

$$
\begin{aligned}
\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial r E_r}{\partial r} + \frac{1}{r^2}\frac{\partial^2 E_r}{\partial \phi^2} + \frac{\partial^2 E_r}{\partial y^2} - \frac{1}{c^2}\frac{\partial^2 E_r}{\partial t^2} - \frac{2}{r^2}\frac{\partial E_\phi}{\partial \phi} &= \frac{1}{\varepsilon_0}\frac{\partial \rho}{\partial r} \\
\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial r E_\phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 E_\phi}{\partial \phi^2} + \frac{\partial^2 E_\phi}{\partial y^2} - \frac{1}{c^2}\frac{\partial^2 E_\phi}{\partial t^2} + \frac{2}{r^2}\frac{\partial E_r}{\partial \phi} &= \frac{1}{\varepsilon_0}\left(\frac{1}{r}\frac{\partial \rho}{\partial \phi} + \frac{1}{c}\frac{\partial \rho}{\partial t}\right)
\end{aligned}
$$

Courtesy by K. Oide

CSR impedance calculation (2)

- Paraxial approximation

T. Agoh and K. Yokoya, "Calculation of Coherent Synchrotron Radiation Using Mesh", Phys. Rev. ST Accel. Beams 7, 054403 (2004)

$$\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial r E_r}{\partial r} + \frac{1}{r^2}\frac{\partial^2 E_r}{\partial \phi^2} + \frac{\partial^2 E_r}{\partial y^2} - \frac{1}{c^2}\frac{\partial^2 E_r}{\partial t^2} - \frac{2}{r^2}\frac{\partial E_\phi}{\partial \phi} = \frac{1}{\varepsilon_0}\frac{\partial \rho}{\partial r}$$

$$\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial r E_\phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 E_\phi}{\partial \phi^2} + \frac{\partial^2 E_\phi}{\partial y^2} - \frac{1}{c^2}\frac{\partial^2 E_\phi}{\partial t^2} + \frac{2}{r^2}\frac{\partial E_r}{\partial \phi} = \frac{1}{\varepsilon_0}\left(\frac{1}{r}\frac{\partial \rho}{\partial \phi} + \frac{1}{c}\frac{\partial \rho}{\partial t}\right)$$

$$\rho \propto \delta(r-R)\delta(y)\exp\left(ik(R\phi - ct)\right)$$

$$E_{r,\phi} = \overline{E}_{r,\phi}(\phi)\exp\left(ik(R\phi - ct)\right)$$

$$\overline{E}_r = \overline{E}_r + \overline{E}_{r0}\ ,$$

$$\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial r\overline{E}_{r0}}{\partial r} + \frac{\partial^2 \overline{E}_{r0}}{\partial y^2} = \frac{1}{\varepsilon_0}\frac{\partial \rho}{\partial r}$$

Courtesy by K. Oide

CSR impedance calculation (3)

- The problem is reduced to solving first order differential equations

$$
\begin{aligned}
\frac{\partial \bar{E}_r}{\partial \phi} &= \frac{i}{2(k^2R^2-1)}\left[kR\left(\left(k^2(r^2-R^2)+1\right)(\bar{E}_r+\bar{E}_{r0}) + r\frac{\partial}{\partial r}(\bar{E}_r+\bar{E}_{r0}) + r^2\left(\frac{\partial^2 \bar{E}_r}{\partial r^2}+\frac{\partial^2 \bar{E}_r}{\partial y^2}\right)\right)\right. \\
&\quad \left. + \left(k^2(r^2+R^2)-1\right)\bar{E}_\phi + r\frac{\partial \bar{E}_\phi}{\partial r} + r^2\left(\frac{\partial^2 \bar{E}_\phi}{\partial r^2}+\frac{\partial^2 \bar{E}_\phi}{\partial y^2}\right)\right] \\
\frac{\partial \bar{E}_\phi}{\partial \phi} &= \frac{i}{2(k^2R^2-1)}\left[kR\left(\left(k^2(r^2-R^2)+1\right)\bar{E}_\phi + r\frac{\partial \bar{E}_\phi}{\partial r} + r^2\left(\frac{\partial^2 \bar{E}_\phi}{\partial r^2}+\frac{\partial^2 \bar{E}_\phi}{\partial y^2}\right)\right)\right. \\
&\quad \left. + \left(k^2(r^2+R^2)-1\right)(\bar{E}_r+\bar{E}_{r0}) + r\frac{\partial}{\partial r}(\bar{E}_r+\bar{E}_{r0}) + r^2\left(\frac{\partial^2 \bar{E}_r}{\partial r^2}+\frac{\partial^2 \bar{E}_r}{\partial y^2}\right)\right]
\end{aligned}
$$

Derivative matrix w/ boundary conditions

Driving terms

Exponent is evaluated by eigensystem of *A*

$$
\begin{aligned}
\frac{d\boldsymbol{f}}{d\phi} &= A\boldsymbol{f} + \boldsymbol{b}, \qquad \boldsymbol{f} = (\bar{E}_r, \bar{E}_\phi), \\
\boldsymbol{f}(\phi) &= \boldsymbol{f}_0 \exp(A\phi) + \boldsymbol{b}\int_0^\phi \exp\left(A(\phi'-\phi)\right) d\phi'
\end{aligned}
$$

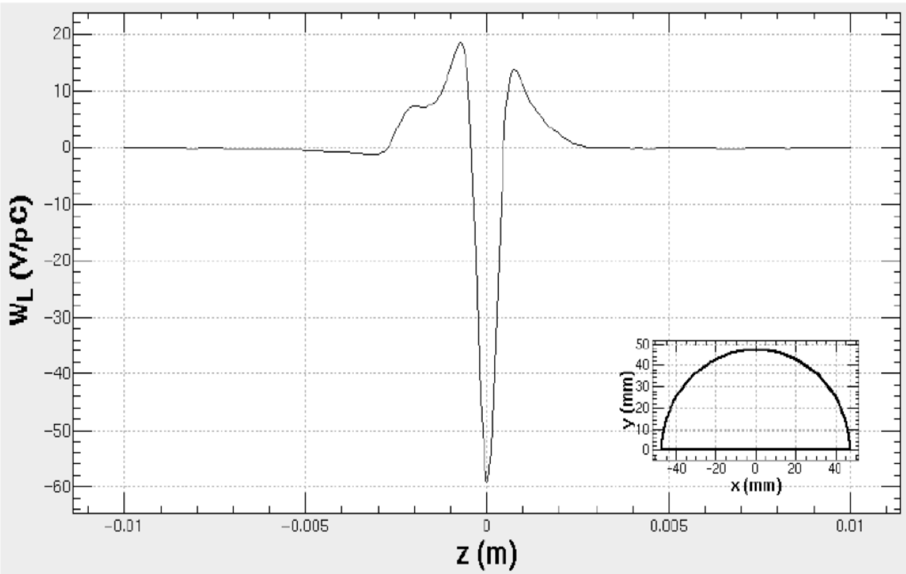
Initial conditions

Courtesy by K. Oide

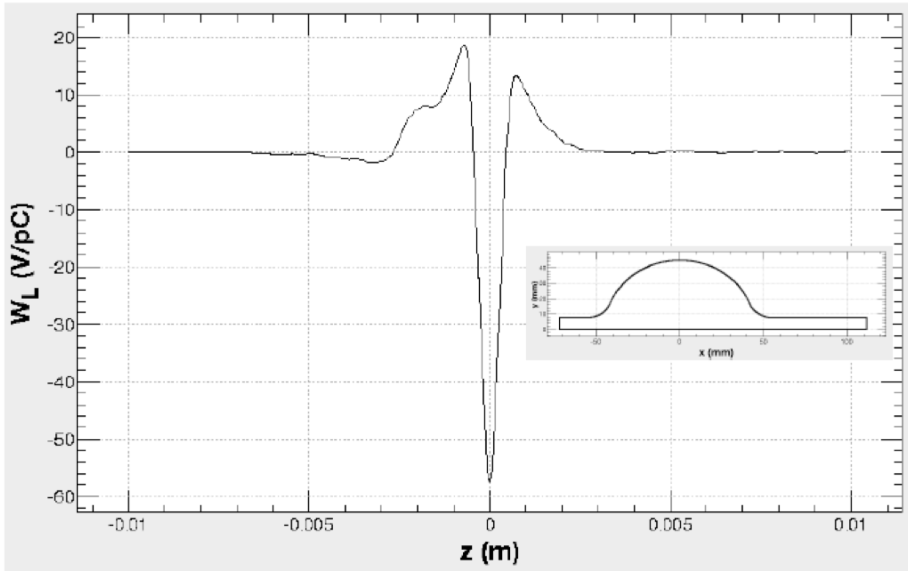
CSR impedance calculation (4)

- Beam pipe is set to be uniform with arbitrary shape

KEKB: Round pipe, R=47 mm



SuperKEKB: Round pipe
w/ antechamber, R=45 mm



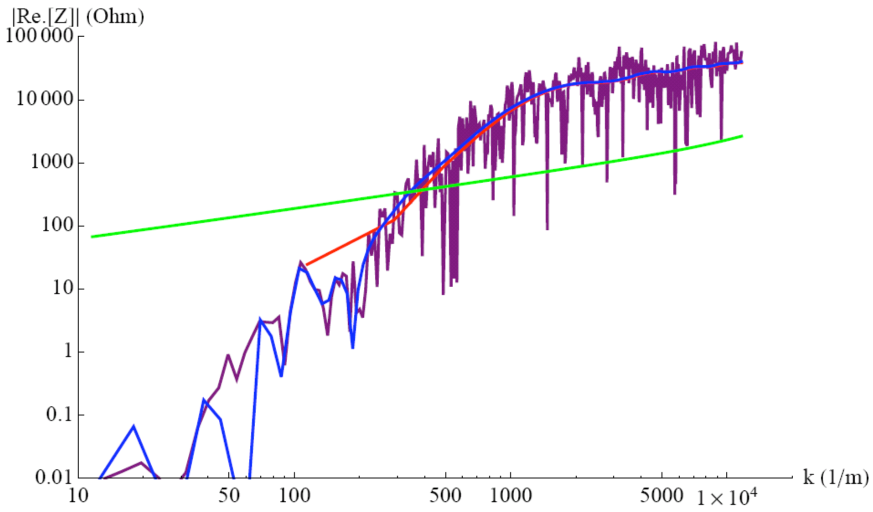
Length=0.89041 m
Bending angle=0.0561 rad

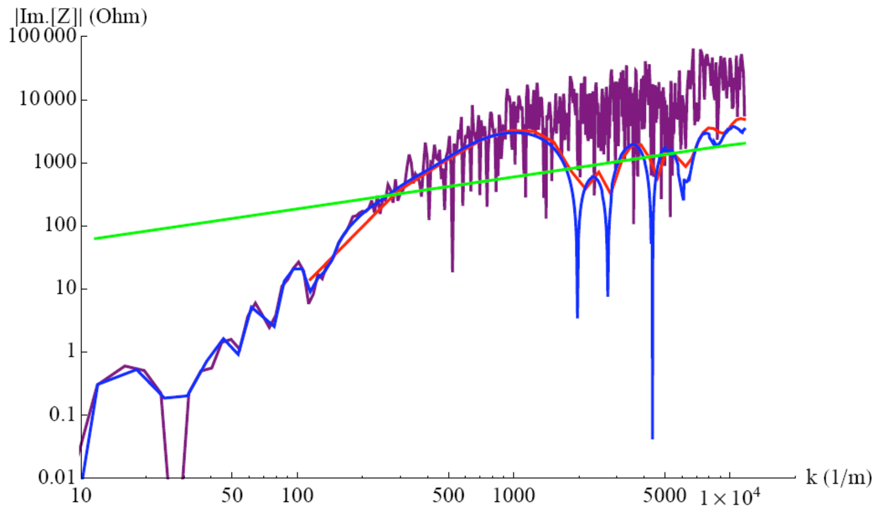
σ_z=0.3mm

CSR impedance calculation (5)

- CSR impedance of dipoles (KEKB-LER)
  - Interference between dipoles is strong

- R=47mm, single dipole with infinite drift x 112
- R=45mm, single dipole with antechamber and infinite drift x 112
- R=47mm, 4 dipoles with finite drifts x 28
- Total RW

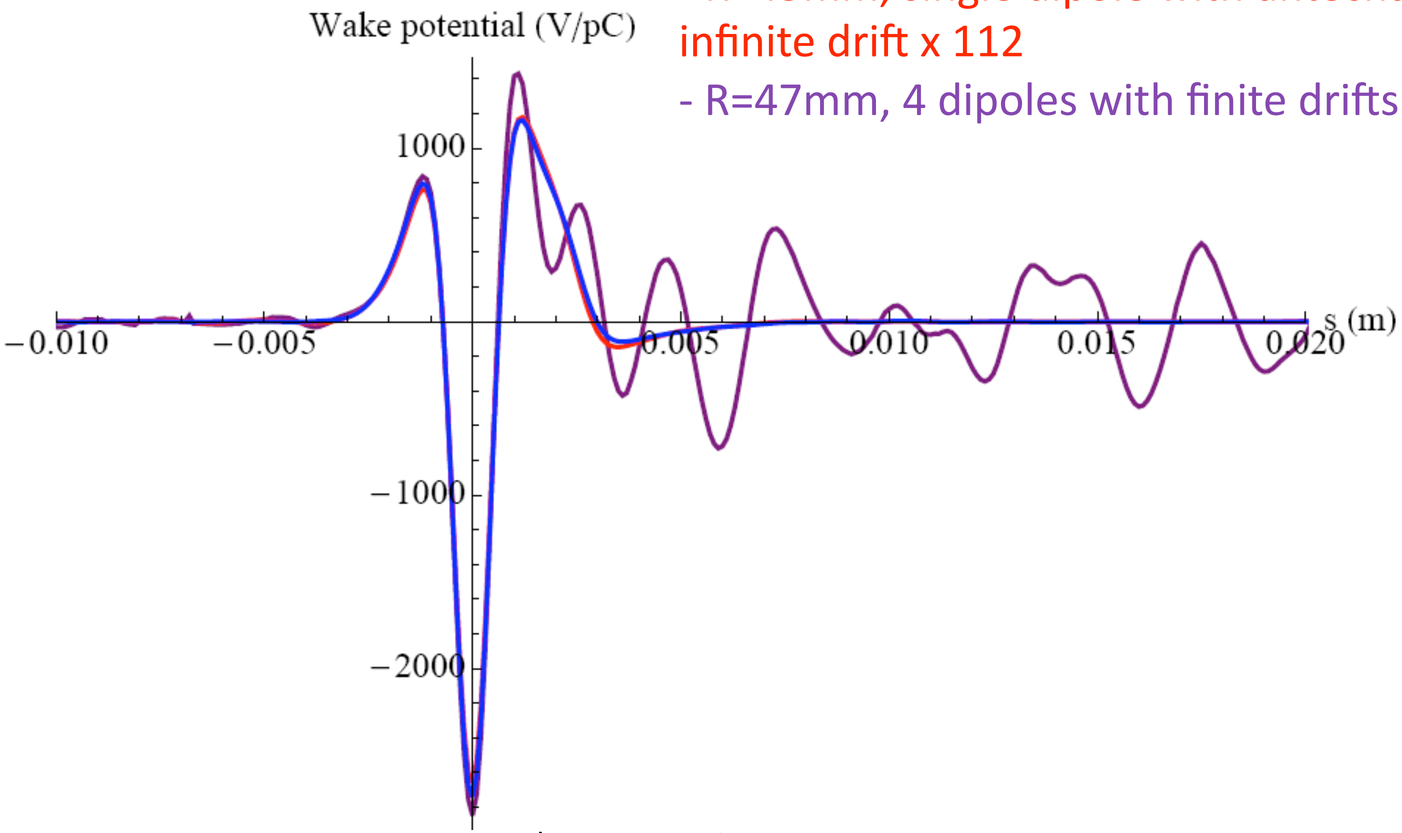




CSR impedance calculation (6)

- CSR wake potential of dipoles (0.5 mm bunch length)

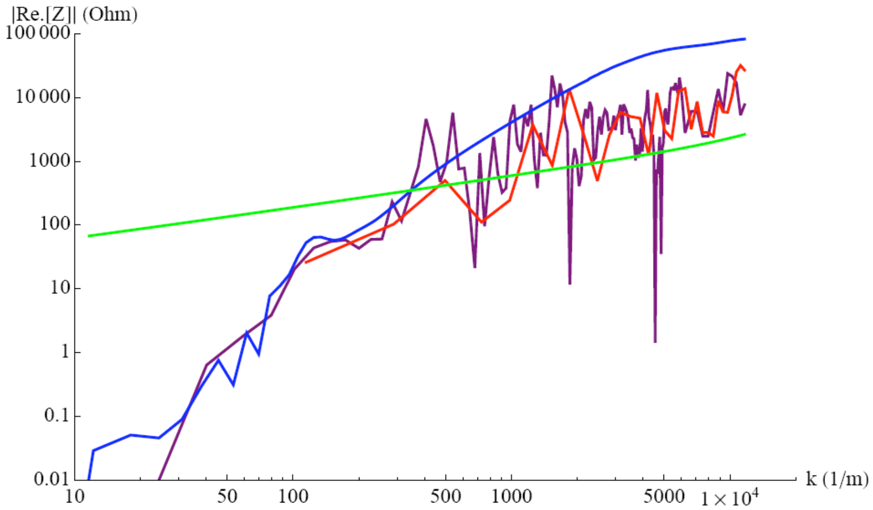
- R=47mm, single dipole with infinite drift x 112
- R=45mm, single dipole with antechamber and infinite drift x 112
- R=47mm, 4 dipoles with finite drifts x 28

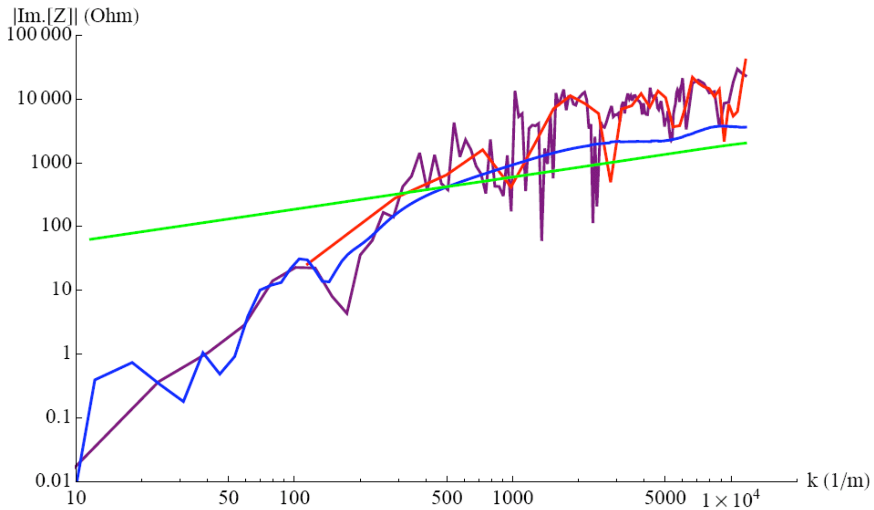


CSR impedance calculation (7)

- CSR impedance of wigglers (KEKB-LER)

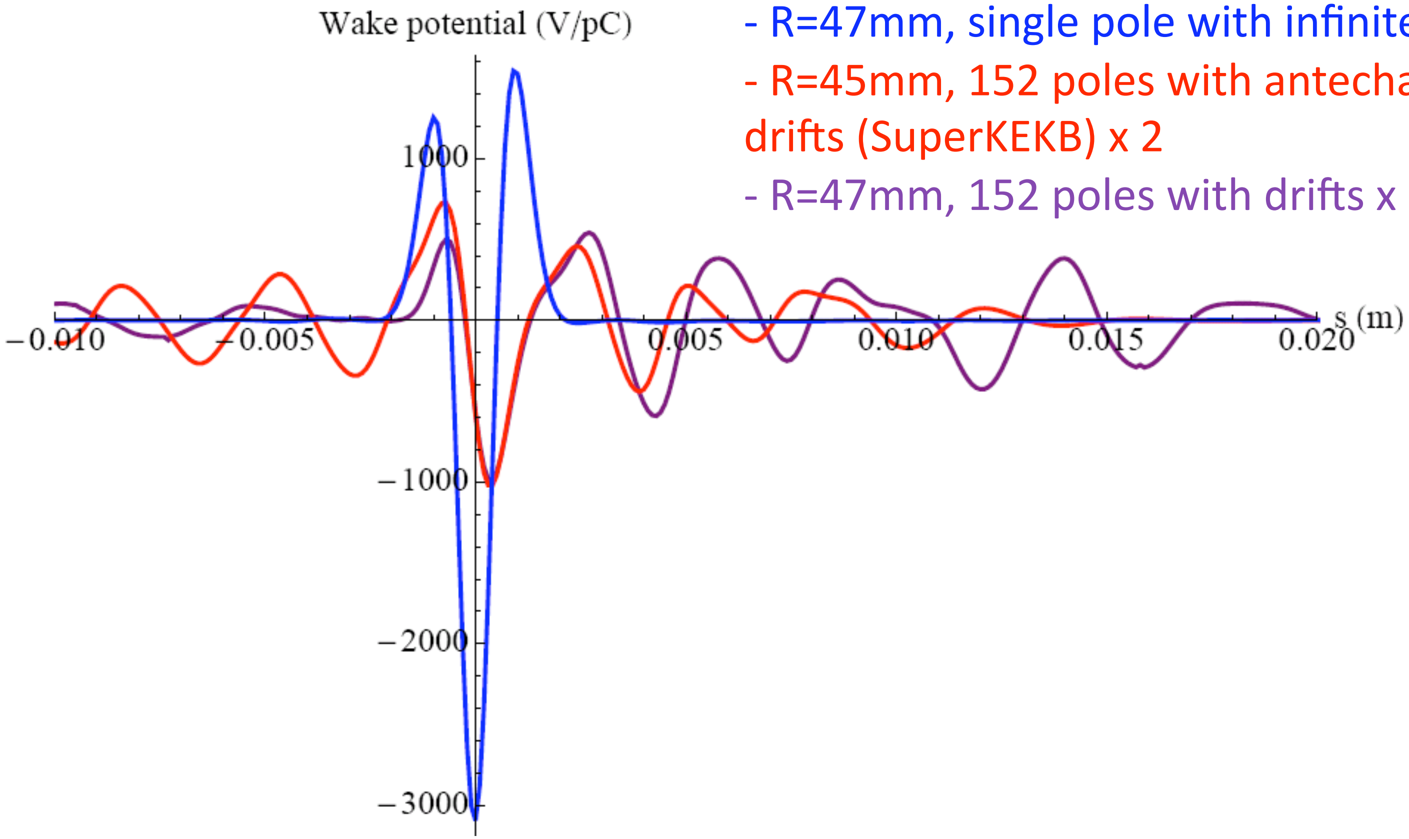
- R=47mm, single pole with infinite drift x 304
- R=45mm, 152 poles with antechamber and drifts (SuperKEKB) x 2
- R=47mm, 152 poles with drifts x 2
- Total RW





CSR impedance calculation (8)

- CSR wake potential of wigglers (0.5 mm bunch length)
  - Drifts between wig. poles relax CSR wake

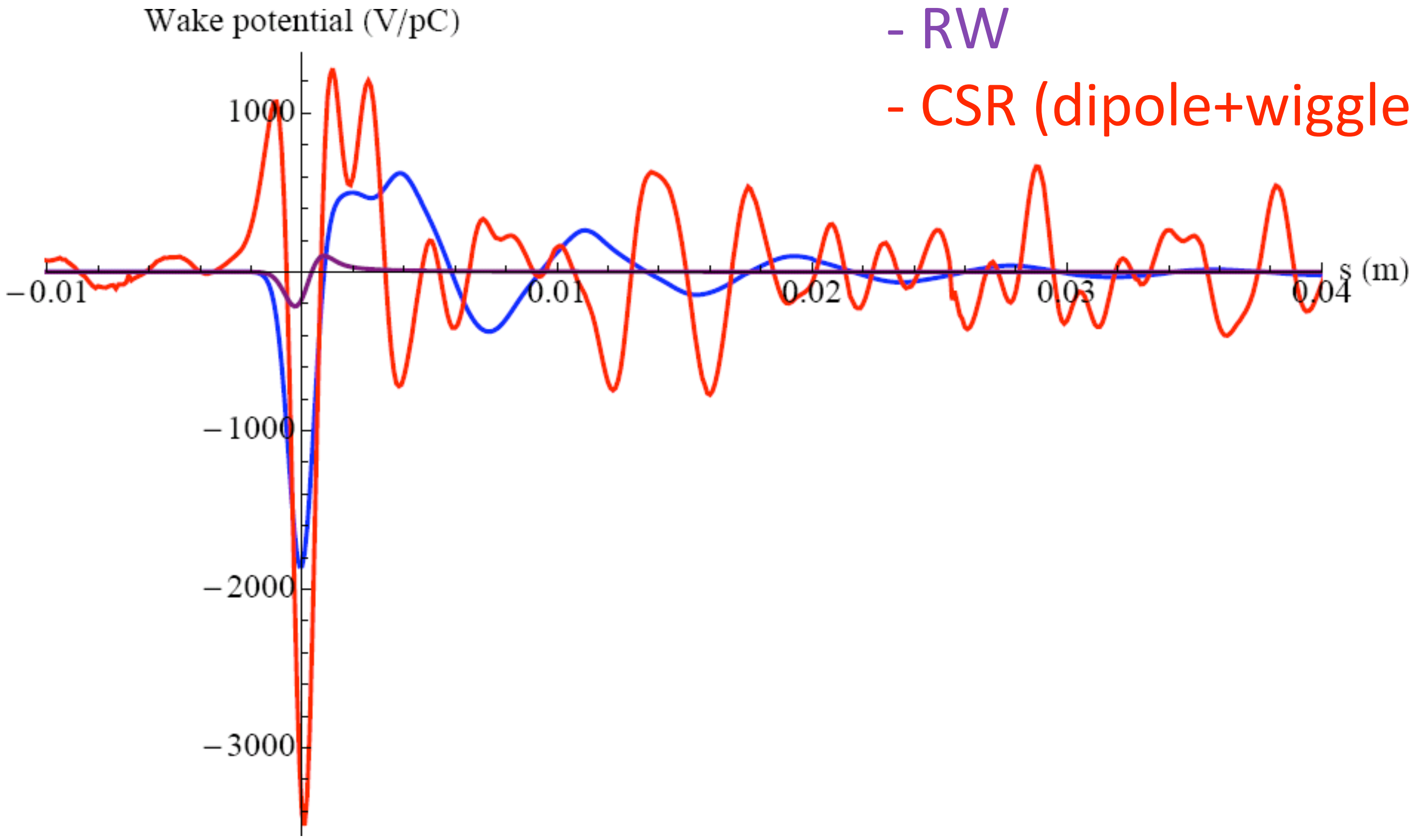


- R=47mm, single pole with infinite drift x304
- R=45mm, 152 poles with antechamber and drifts (SuperKEKB) x 2
- R=47mm, 152 poles with drifts x 2

Total wake potential (1)

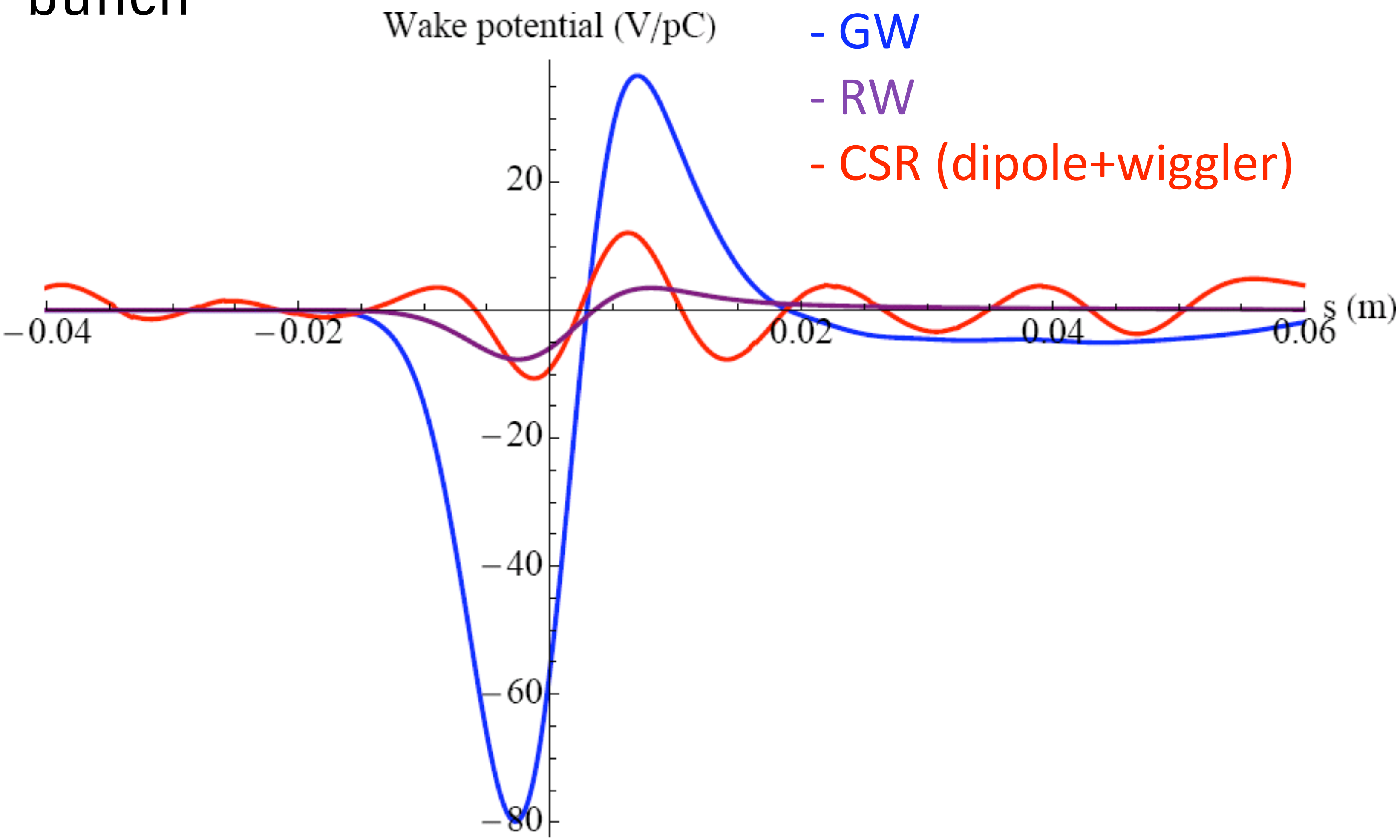
- GW, RW and CSR wake potential of 0.5mm bunch

- GW
- RW
- CSR (dipole+wiggler)



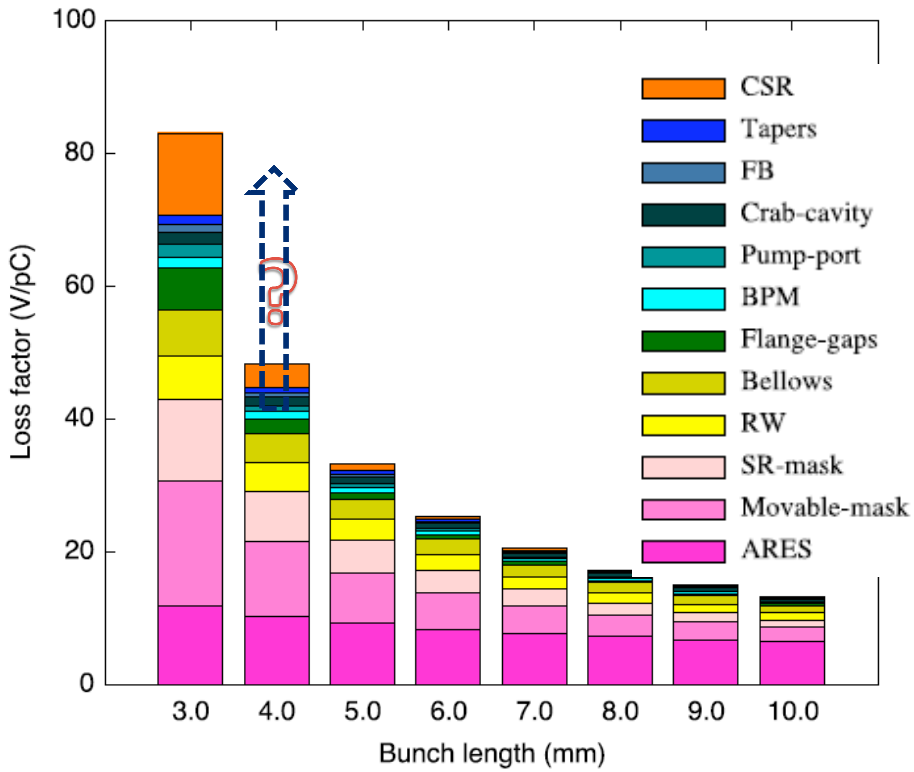
Total wake potential (2)

- GW, RW and CSR wake potential of 4.58mm bunch

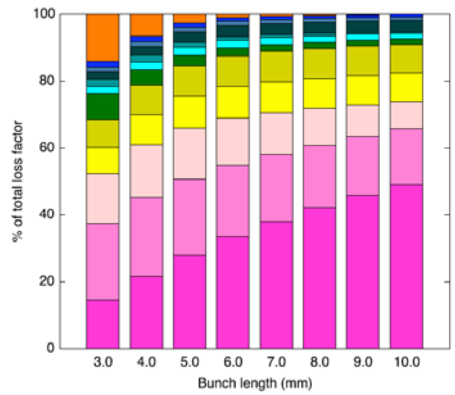


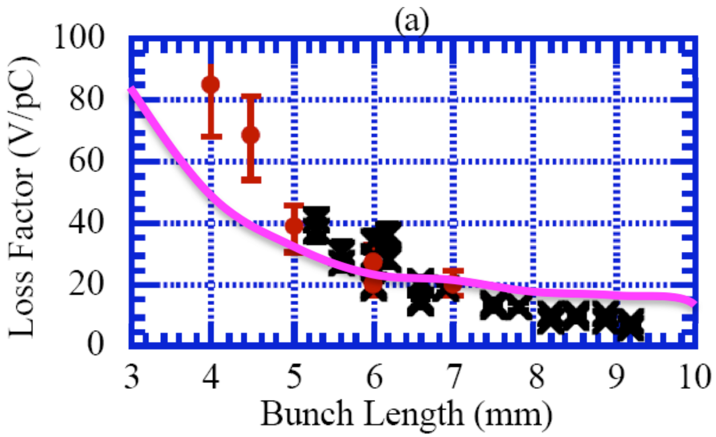
Loss factor from calculated wake potential

- Calculated loss factor is much smaller than measurement when sigma<5mm, but higher when sigma>7mm
- Loss factor due to CSR decays quickly when bunch length increases



Gaussian bunch assumed





Ieiri & Koiso, 2003

Solving VFP equation (1)

- Main equations

$$\frac{\partial\psi}{\partial s}+\frac{\partial q}{\partial s}\cdot\frac{\partial\psi}{\partial q}+\frac{\partial p}{\partial s}\cdot\frac{\partial\psi}{\partial p}=\frac{2\beta}{c}\frac{\partial}{\partial p}[p\psi+\sigma_p^2\frac{\partial\psi}{\partial p}]$$

$$q = z$$

$$p = \Delta p/p_0$$

$$\psi=\psi(q,p,s) \qquad \iint\psi(q,p,s)dpdq = 1$$

$$I_n = \frac{Ne^2}{E_0C}$$

$$\frac{\partial q}{\partial s}=\frac{\omega_s\sigma_z}{c\sigma_p}\cdot p \qquad \frac{\partial p}{\partial s}=-\frac{\omega_s\sigma_p}{c\sigma_z}\cdot q-I_n\cdot F(q,s)$$

$$F(q,s)=\int_{q'=-\infty}^{\infty}W_0(q'-q)\lambda(q')dq'$$

$$\lambda(q,s)=\int\psi(q,p,s)dp$$

Solving VFP equation (2)

- Generally, it is inefficient to apply one integration to the different parts of the whole system. Thus operator splitting is needed.

$$\frac{\partial \psi}{\partial s} = \mathcal{L}\psi = (\sum_{i=1}^{3} \mathcal{L}_i)\psi$$

$$\mathcal{L}_1 = -\frac{\omega_s \sigma_z}{c\sigma_p} \cdot p \cdot \frac{\partial}{\partial q} + \frac{\omega_s \sigma_p}{c\sigma_z} \cdot q \cdot \frac{\partial}{\partial p}$$

$$\mathcal{L}_2 = I_n \cdot F(q, s) \cdot \frac{\partial}{\partial p}$$

Liouville operator: reversible

$$\mathcal{L}_3 = \frac{2\beta}{c} \frac{\partial}{\partial p}[p + \sigma_p^2 \frac{\partial}{\partial p}]$$

→ Fokker-Planck operator: irreversible

Solving VFP equation (3)

- Possible improvement on operator splitting

$$\psi^{n+1} = e^{\Delta s \mathcal{L}} \psi^n$$

First-order splitting:

$$e^{\Delta s \mathcal{L}} \approx e^{\Delta s \mathcal{L}_1} e^{\Delta s \mathcal{L}_2} e^{\Delta s \mathcal{L}_3}$$

Second-order symmetric splitting:

$$e^{\Delta s \mathcal{L}} \approx [e^{\Delta s/2 \mathcal{L}_1} e^{\Delta s \mathcal{L}_2} e^{\Delta s/2 \mathcal{L}_1}] e^{\Delta s \mathcal{L}_3}$$

Refining integration step:

$$e^{\Delta s \mathcal{L}} \approx [e^{\Delta s/k \mathcal{L}_1} e^{\Delta s/k \mathcal{L}_2} e^{\Delta s/k \mathcal{L}_3}]^k$$

Solving VFP equation (4)

- The discrete version of Liouville operator is Frobenius-Perron operator

$$\psi^*(q,p) = \mathcal{F}_1\psi(q,p,n\Delta s) = \psi(R^{-1}(q,p),n\Delta s)$$

$$\psi^{**}(q,p) = \mathcal{F}_2\psi^*(q,p) = \psi^*(K^{-1}(q,p))$$

$$\begin{bmatrix} q' \\ p' \end{bmatrix} = \begin{bmatrix} \cos(\mu_s \Delta s/C) & \beta_z \sin(\mu_s \Delta s/C) \\ -\sin(\mu_s \Delta s/C)/\beta_z & \cos(\mu_s \Delta s/C) \end{bmatrix} \begin{bmatrix} q \\ p \end{bmatrix}$$

$$\begin{bmatrix} q' \\ p' \end{bmatrix} = \begin{bmatrix} q \\ p - I_n F(q,s)\Delta s/C \end{bmatrix}$$

Solving VFP equation (5)

- There are many discretization schemes for Fokker-Planck operators, such as Euler forward/ backward, Crank-Nicolson, Exponentially fitted scheme…

$$\frac{\psi_i^{n+1} - \psi_i^n}{\Delta s} = \frac{2\beta}{c}\psi_i^{n+1} + \frac{\beta p_i(\psi_{i+1}^{n+1} - \psi_{i-1}^{n+1})}{c\Delta p} + \rho_i^{n+1}\frac{2\beta\sigma_p^2}{c}\frac{\psi_{i+1}^{n+1} - 2\psi_i^{n+1} + \psi_{i-1}^{n+1}}{\Delta p^2}$$

Fitting factor: $\rho_i^{n+1} = \frac{p_i \Delta p}{2\sigma_p^2}\coth\frac{p_i \Delta p}{2\sigma_p^2}$

D.J. Duffy, "A Critique of the Crank Nicolson Scheme Strengths and Weaknesses for Financial Instrument Pricing", Wilmott magazine, p. 68-76, July 2004.

Solving VFP equation (6)

- Properties of Exponentially Fitted Scheme
  - It is uniformly stable for all values of
    - Integration step (or time step)
    - Damping coefficient
    - Mesh size
  - No spurious oscillations
  - But computationally expensive

$$AU = F$$

$$U = A^{-1}F$$

$$\mathbf{A} = \begin{pmatrix} \ddots & & \ddots & & 0 \\ & \ddots & & a_{j,j+1} & \\ \ddots & & a_{j,j} & & \ddots \\ & a_{j,j-1} & & \ddots & \\ 0 & & \ddots & & \ddots \end{pmatrix}$$

Simulation results (1)

- Main parameters for KEKB-LER

Parameter	Value	Unit
Circumference	3016.25	m
Beam energy	3.5	GeV
Bunch population	6	10^{10}
Natural bunch legnth	4.58	mm
Synchrotron tune	0.024	
Longitudinal damping time	2000	turn
Energy spread	7.27	10^{-4}

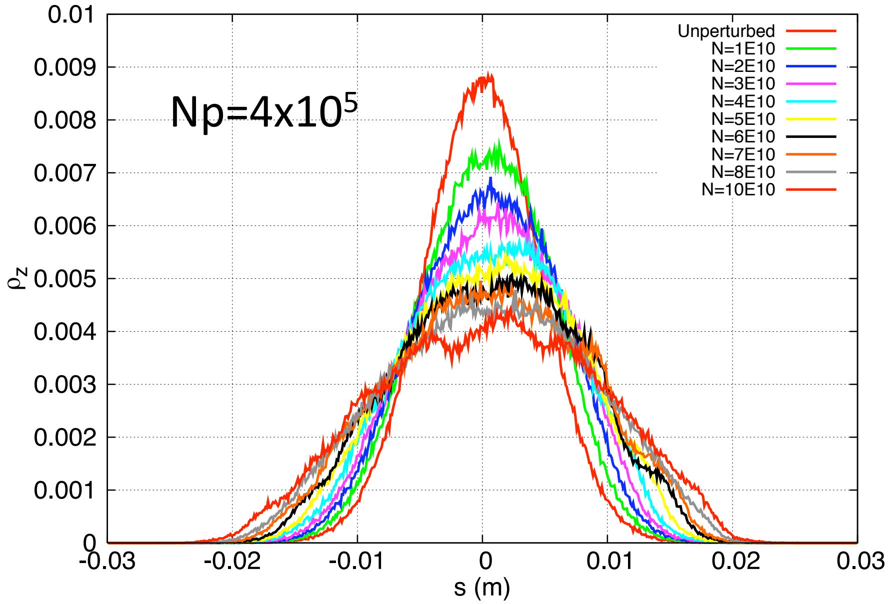
For SuperKEKB LER, only bunch length is changed (3 or 5 mm)

Simulation results (1)

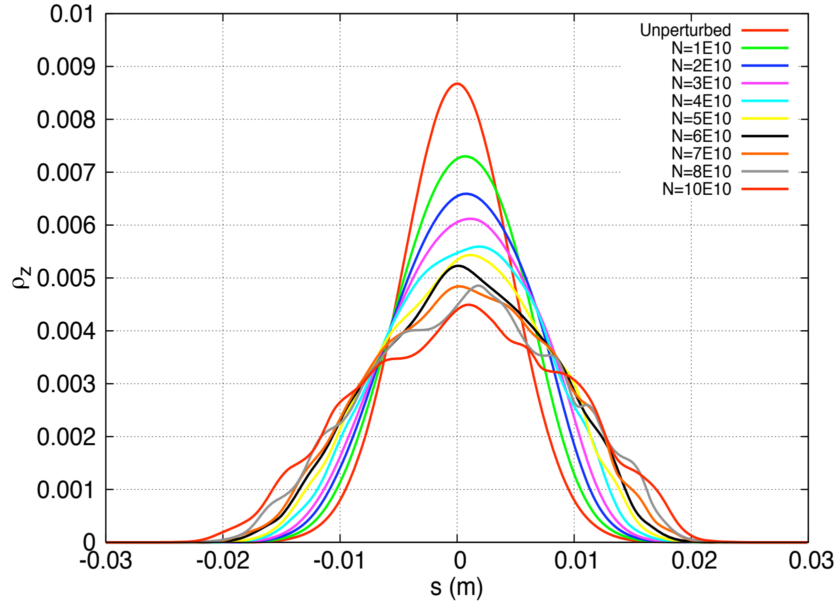
- Tests with resonator impedance model shows that noises are well removed in VFP solver

Parameter	Description	LER
L (nH)	Inductance	116.7
R (KΩ)	Resistance	22.9
C (fF)	Capacitor	0.22

PIC tracking

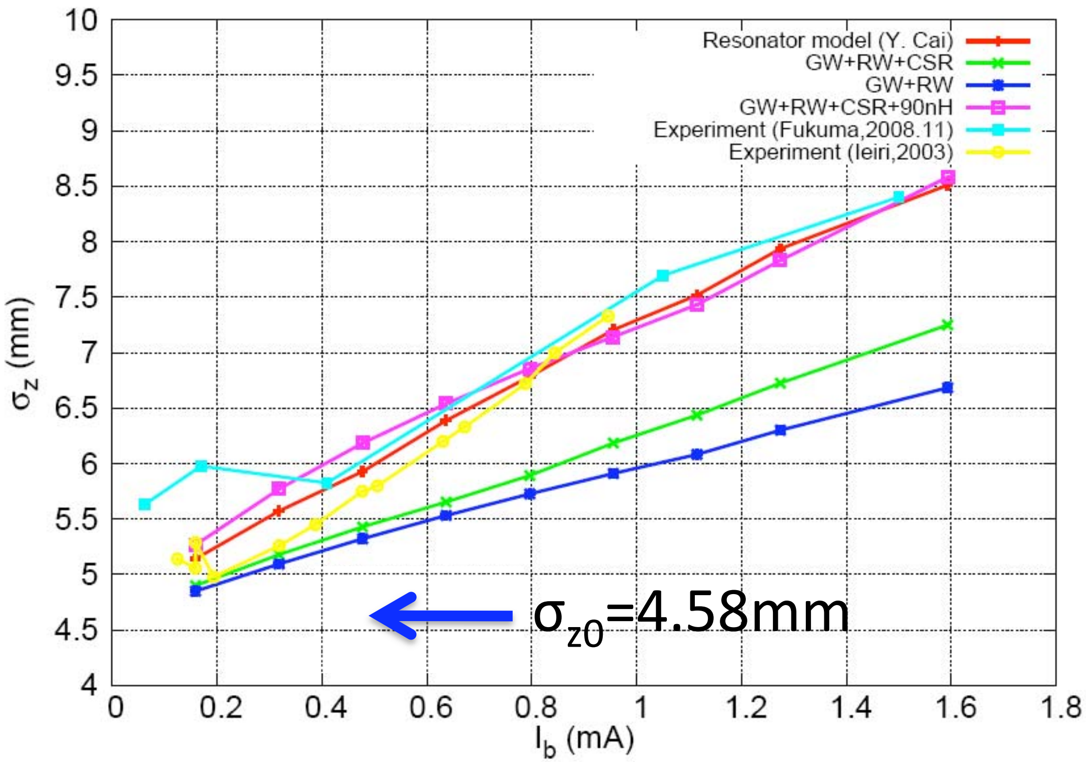


VFP solver



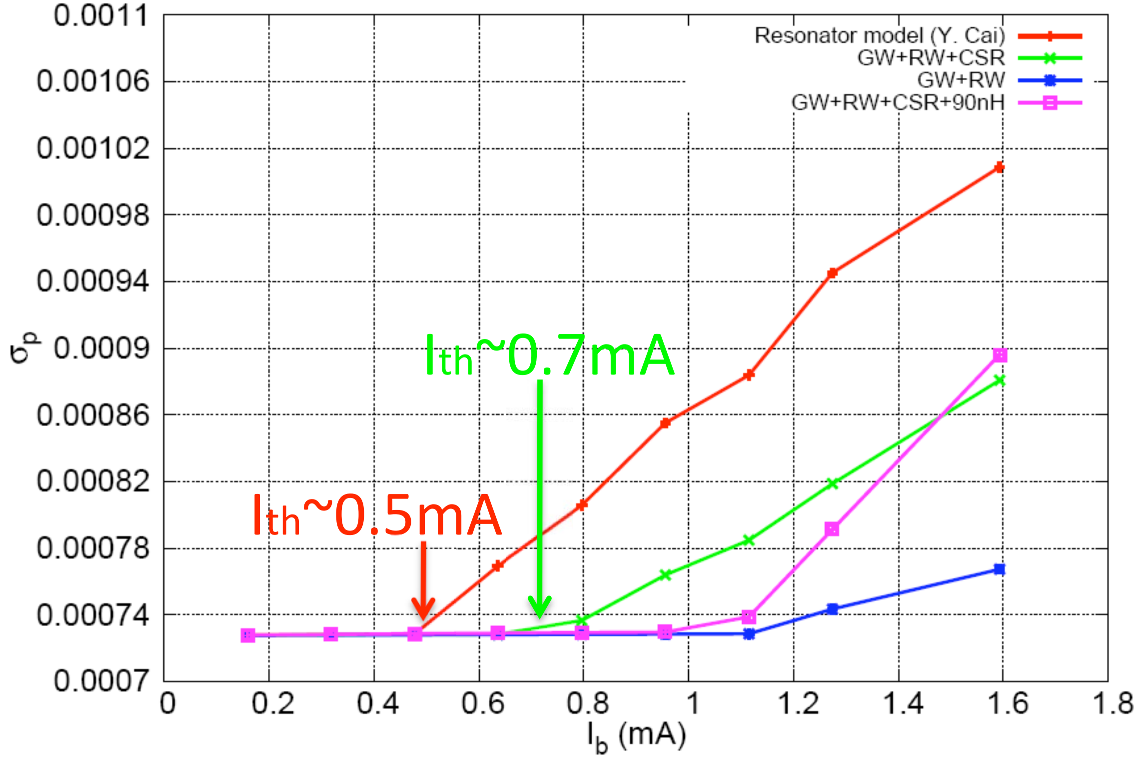
Simulation results (2)

- Pure inductive wake of around 90nH should be added to the numerical impedance model to get the similar bunch lengthening as measurements show!



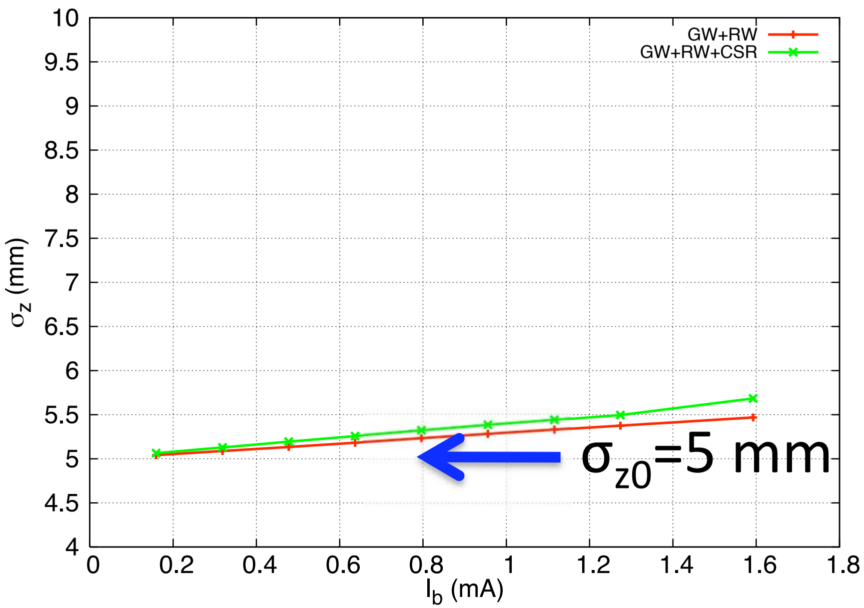
Simulation results (3)

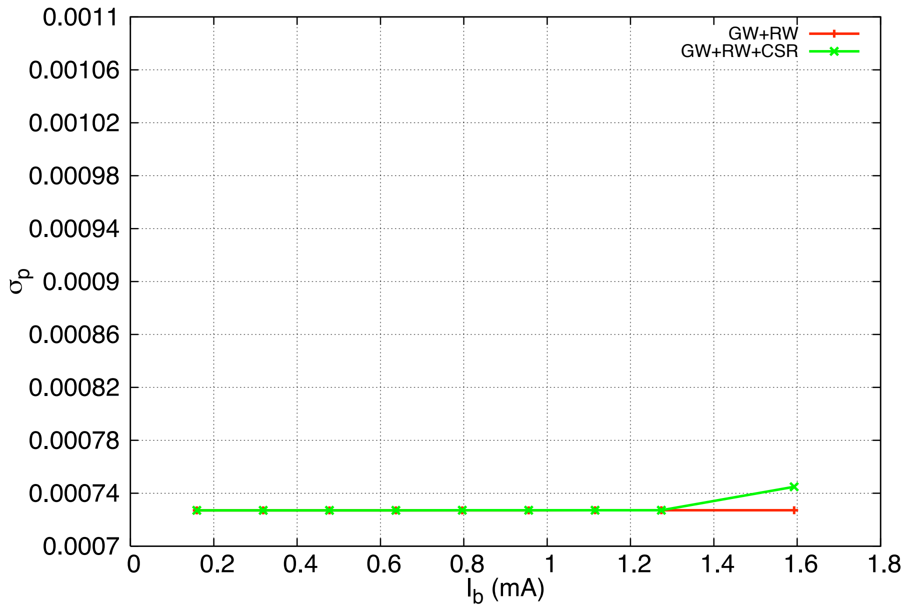
- But when pure inductive wake is added, the Microwave instability threshold gets higher!
- CSR is important for MWI



Simulation results (4)

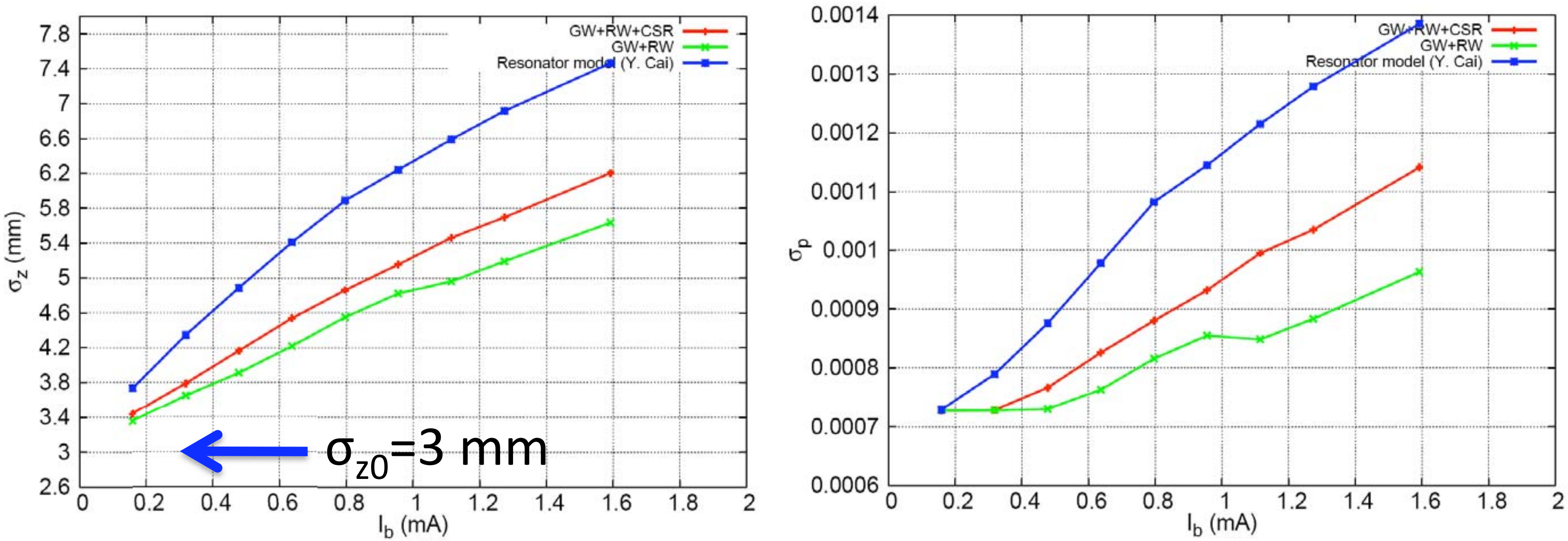
- For nano-beam option of SuperKEKB-LER with 5 mm bunch length, bunch lengthening is not serious.





Simulation results (5)

- But for high-current option of SuperKEKB-LER, MWI is serious and CSR is dominant.



K. Oide reported similar results at KEKB ARC 2009

Summary

- KEKB-LER impedance model
  - The numerical impedance model (GW+RW+CSR) predicts much weaker bunch lengthening than measurements and Y. Cai's resonator model. The discrepancy is around 90nH.
  - CSR may be the last factor to rely on.
- CSR impedance and MWI
  - Lots of work has been done to calculate CSR impedance.
  - Simulations showed that CSR plays essential role in the high-current option of SuperKEKB-LER.
  - CSR impedance can be important source of MWI in KEKB-LER.

Future plans

- Codes development on simulations of MWI
  - Benchmarking
  - Determination of Sawtooth instability threshold
- CSR impedance calculation
  - Optimizing the code
  - Effect of edge field (Suggested by Y. Cai)
  - Shielding of the beam pipe
  - Interference between adjacent bends, especially between wigglers
  - Benchmarking
- Experimental observations of MWI at KEKB-LER
- Integrate the numerical impedance model in beam-beam simulations