MOLECULAR DYNAMICS SIMULATION OF CRYSTALLINE BEAMS EXTRACTED FROM A STORAGE RING

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Abstract

It is well-known that a charged-particle beam is Coulomb crystallized in the low-temperature limit. The feasibility of beam crystallization in a storage ring has been raised by the recent progress in beam cooling techniques and in understanding of the behavior of crystalline beams. To go a step further, we here investigate the extraction and transport process of crystalline ion beams, employing the molecular dynamics simulation technique. The dependence of the stability on the lattice of the extraction beam line is explored to show whether various crystalline beams can be transported stably without collapse of the ordered structure.

INTRODUCTION

Applying a dissipative force to a charged-particle beam circulating in a storage ring, we can reduce the emittance of the beam and even expect the occurrence of a type of phase transition. In fact, the beam finally exhibits an ordered structure at the low-temperature limit if some physical conditions are fulfilled. Such an ultimate state of the beam is known as a crystalline beam [1-3] whose spatial structure is determined by the line density [4]. When the line density is sufficiently low, a one-dimensional (1D) string is formed. By increasing the line density, we can transform the string into a two-dimensional (2D) zigzag crystal and, then, eventually into a three-dimensional (3D) shell crystal. In theory, the emittance of a crystalline beam is zero (except for quantum noise) at the low-temperature limit [5, 6].

In order to form and maintain various kinds of crystalline beams, the following two conditions must be satisfied [7, 8]; the storage ring must be operated below the transition energy for crystal formation, and the average betatron phase advance must be less than 127 degrees per lattice period for crystal maintenance. In an ideal crystalline state, the Coulomb repulsive force perfectly balances with the periodic external focusing force of the ring and random interparticle collisions disappear. The crystalline state is stable and thus lasts long even after the cooling force is removed [9]. As reviewed in Ref. [10], according to many advanced theoretical and numerical works, it is now considered that crystalline beams can exist in a storage ring, at least, in theory.

Here, consider the extraction of a crystalline beam from a storage ring. The crystalline beam is additionally kicked by an extraction device such as an electrostatic deflector or a septum magnet and then transported along the (usually, nonperiodic) beam line. The periodicity of the focusing force is lost, and thus the above-mentioned maintenance condition of crystals is not fulfilled any more. We expect that the emittance of the crystalline beam is increased, or even the crystalline structure of the beam can be destroyed by the extraction process. In order to verify this expectation, molecular dynamics (MD) simulations were carried out. The present MD simulation results show the stability of extracted crystals is consistent with that defined by the maintenance condition of crystals in a ring.

SIMULATION PARAMETERS

For the present simulation study, the MD code "CRYSTAL" is employed [11]. As an extraction device, an electrostatic field can be assumed as well as a normal dipole magnetic field. The equation of motion is integrated by the code in a symplectic manner. In the present MD study, the beam is assumed to be bunched by a longitudinal radio-frequency field in the ring. The Coulomb forces among particles in a bunch are directly calculated. For more detail information on the MD code, see Ref. [10].

As a storage ring lattice, the parameters of S-LSR [12] have been adopted. The operating point of the ring assumed here satisfies the above two conditions. We have also considered several different test extraction lines that have different transverse phase advances from the extraction device to an ideal target position (length: 7.8 m) in order to see how the stability of the extracted beam depends on the lattice design. The main simulation parameters are summarized in Table 1. Here, we show only two cases of high and low phase advances.

MOLECULAR DYNAMICS RESULTS

High-phase-advance lattice

For extraction, a dipole magnetic magnet or

Table 1: Main simulation parameters.

Ring lattice	S-LSR
Ion species	40 keV $^{24}Mg^{+}$
Superperiodicity	6
Circumference	22.56 m
Ring tunes (ν_x, ν_y, ν_z)	(1.44, 1.44, 0.15)
Extraction device	Electrostatic or dipole magnetic field
Beam line length	7.8 m
Phase advances of the beam line [deg]	(110, 81), (161, 85), (174, 101), (310, 167), and, (363, 181)

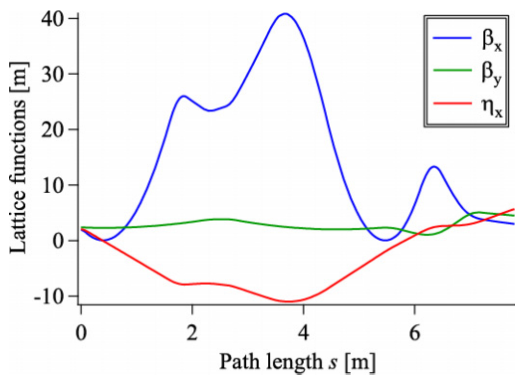

Figure 1: Beta and dispersion functions of the extraction beam line with a high phase advance. The ring betatron tunes have been set at 1.44 in both transverse directions. The horizontal and vertical phase advances from the extraction magnet to the target are, respectively, $\sigma_x = 363$ degrees and $\sigma_y = 181$ degrees.

electrostatic deflector with an extraction angle of 30 degrees and a bending radius of 0.57 m has been assumed in the middle of the straight section in S-LSR. The beta and dispersion functions are shown in Fig. 1. The ring tune has been set at 1.44 in both transverse directions in the present case. The horizontal and vertical phase advances, σ_x and σ_y, from the extraction position to the target are, respectively, 363 degrees and 181 degrees in the case of magnetic extraction. The transverse average phase advance $\sigma_\perp \equiv \sqrt{(\sigma_x^2+\sigma_y^2)/2}$ is 287 degrees.

Figure 2 shows the spatial distributions of a bunched beam composed of 1000 $^{24}Mg^+$ ions before extraction (i.e., in the ring) and after extraction. The whole of the bunch has been extracted in a single turn. As is clearly seen in Fig. 2, the ordered configuration has been completely lost at the end of the beam line. The final transverse normalized rms emittance is 4×10^{-11} m.rad, which is two orders of magnitude higher than that in the ring.

The single-particle orbits of four ions arbitrarily picked up from the beam in Fig. 2 are shown in Fig.3. Because of the strong focusing of the extraction dipole magnet sitting at s = 0 m, the horizontal orbit of each particle crosses the design orbit. This behaviour is different from the fact that single-particle orbits of the crystalline beam in the ring are proportional to each other and oscillate not around the design orbit but around a certain position [5]. We have also confirmed that the emittance abruptly blows up at this position.

At sufficiently low line densities where the corresponding crystalline structure is a 1D string or a 2D zigzag in the ring, the situation has changed; the crystalline structure of a string or a zigzag can be maintained at the end of the beam line after extraction, while an emittance increase is observed. Figure 4 depicts the final spatial distribution of the very-low-intensity 1D string crystalline beam. The final normalized rms emittance of the beam is still very low (3×10^{-19} m.rad).

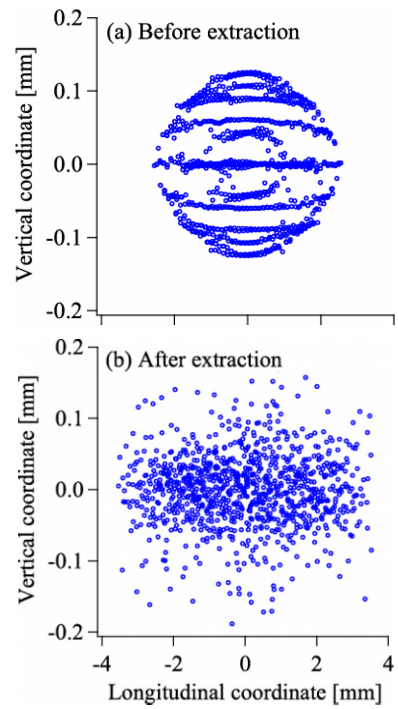

Figure 2: Side views of the bunched beam (a) in the ring before extraction, and (b) at the end of the extraction beam line (s = 7.8 m). The lattice of the beam line in Fig. 1 has been assumed. Each circle corresponds to a single $^{24}Mg^+$ ion with the kinetic energy of 40 keV. The number of ions in a bunch is 1000. The crystalline structure in the upper panel is a two-shell in the middle of the bunch.

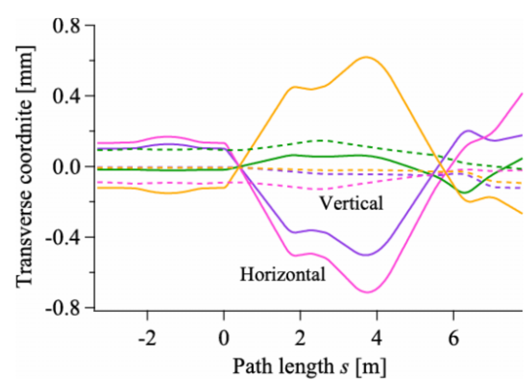

Figure 3: Transverse single-particle orbits of four ions in the beam in Fig. 2. The solid and dashed lines represent the horizontal and vertical orbits, respectively. The origin of the path length s has been set at the entrance of the extraction device. The negative value of s means the path in the ring.

The bunch length of the string becomes larger by about 20% because of no rf focusing in the beam line. The beam exhibits the horizontal head-tail oscillation induced by the momentum dispersion in the beam line [13].

We have replaced the extraction dipole magnet by an electrostatic deflector, and found that the stability of crystals is unchanged.

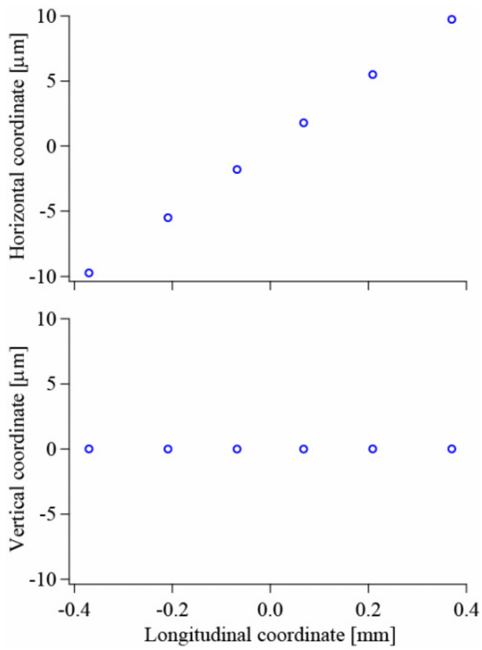

Figure 4: Real-space distribution of a bunched string crystal at the end of the beam line after the extraction. The number of ions in a bunch is six.

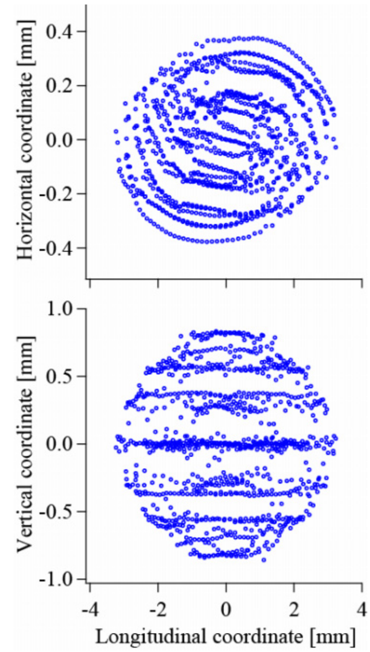

Figure 6: Real-space distribution of the bunched beam at the end of the beam line in the case of low phase advance. The initial distribution in the ring is the same as that in Fig. 2.

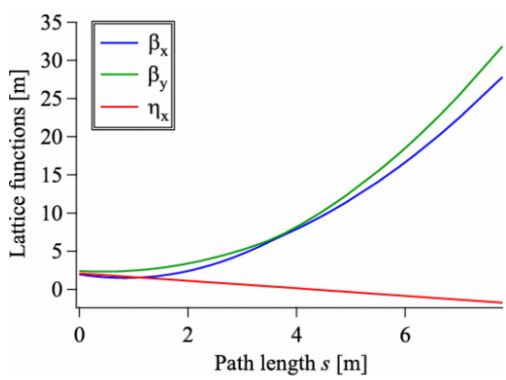

Figure 5: Beta and dispersion functions of the extraction beam line with a low-phase advance. The ring tunes have been set again at 1.44 in both directions. The horizontal and vertical phase advances from the extraction magnet to the target are, respectively, $\sigma_x = 110$ degrees and $\sigma_y = 81$ degrees.

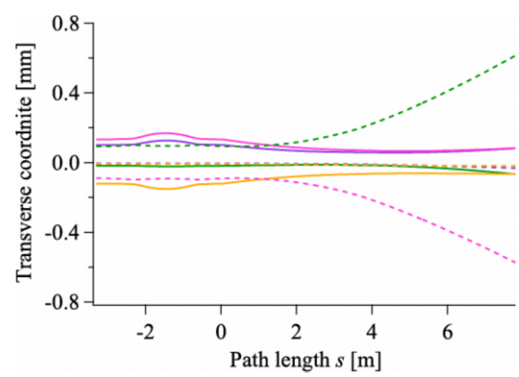

Figure 7: Transverse single-particle orbits of four ions in the beam in Fig. 6. The solid and dashed lines represent the horizontal and vertical orbits, respectively. The orbits of the same four ions as Fig. 3 are plotted. (The orbits of each particle for $s < 0$ are the same as those in Fig. 3.)

Low-phase-advance lattice

The lattice functions of the low-phase-advance case are shown in Fig. 5. In this example, an extraction angle from the ring has been reduced from 30 deg to 7.5 deg to weaken the focusing effect seen in the previous case. In addition, the beam is not tightly focused to keep the phase advance reasonably low. The transverse average phase advance $\sigma_\perp$ is 96 degrees in this case.

The final spatial distribution of the beam after extraction is shown in Fig.6. Unlike the previous case of the high-phase-advance lattice (Fig. 2), the ordered structure of the 3D crystal can be still seen after the extraction. The final emittance of the beam is 3×10^{-12} m.rad. As shown in Fig. 7, the single-particle orbits of the beam do not cross the design orbit (although they are not exactly proportional to each other).

At low line densities, it is also possible to extract 1D strings and 2D zigzag crystals stably.

DISCUSSION AND SUMMARY

We have investigated the extraction and transport of various bunched crystalline beams using the MD simulation technique. Once a crystalline beam is extracted from a storage ring, an increase of the emittance is inevitable. This is due to the non-periodic focusing provided by the extraction device and the beam line. However, the crystalline structures of a string or a zigzag

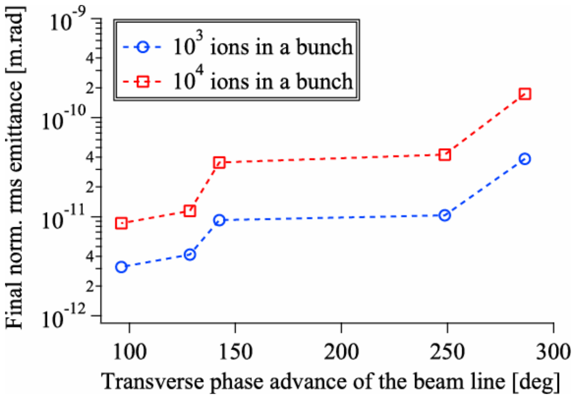

Figure 8: Final transverse rms emittance of the extracted 3D crystalline beam vs. the transverse phase advance of the beam line $\sigma_\perp$. The number of ions in a bunch are 1000 and 10000, respectively. In the lowest-phase-advance case ($\sigma_\perp = 96$ deg), the ordered structure of the beams has been maintained at the end of the beam line.

can be maintained after the extraction in spite of the emittance increase. The ordered configuration of 3D crystals easily melts away when the phase advance of the beam line is large. A reduction of the betatron phase advance of the extraction beam line can suppress the undesirable emittance increase, as shown in Fig. 8. According to the present MD simulation results, it is possible to maintain the 3D ordered configuration when the average phase advance of the beam line is less than 127 degrees, similarly to the maintenance condition in the ring. For long-distance transport of crystalline beams, it may be necessary to make the lattice of the transport line periodic including the matching of dispersion.

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