MODELING SINGLE PARTICLE DYNAMICS IN LOW ENERGY AND SMALL RADIUS ACCELERATORS

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Abstract

This research involves the development of a model of the small circumference (11.5 m) accelerator in which the earth's field has a strong effect, and in which image charge forces are also included. The code used for this simulation was COSY Infinity 9.0 which uses differential algebras to determine high order map elements, as well as quantities such as chromaticity. COSY also uses Normal Form algorithms to determine the betatron tune and any amplitude dependent tune shifts which may result. The power of COSY is that it can derive the required quantities directly form the map without costly integration and tracking. Thus determining the map for both the default elements of the ring, plus the effects of image charge forces, and the earth's magnetic field is both non-trivial, and important. This research uses the Baker Campbell Hausdorf method to determine the map of the ring with the external fields included. Furthermore COSY has the ability to directly implement misalignments within the beamline itself allowing for a study of their effects on beam dynamics. The presentation will include both coding development and applications to the University of Maryland Electron Ring.

INTRODUCTION

With the increase in demand for high current accelerators, methods for determining the effects of space charge become more important. One method for gaining experimental data on the effects of space charge is to use a low energy electron beam to model a high energy heavy ion beam, this is the approach used by the University of Maryland Electron Ring (UMER) [**?**]. UMER sends 10 Kev electrons through a storage ring that is only 11.5 meters in circumference. Currently all acceleration occurs in the electron gun, which sends the beam through a matching section and into the Y-shaped injector. This injector involves offsets on both the injection and recirculation sides, the ring then uses an additional 17 sections comprised of a bending dipole between a pair of quadrupoles, followed by a diagnostic chamber, followed by another set of quadrupoles enclosing a dipole. The arrangement of the elements in the ring are shown in Fig. 1. Due to the small radius and low energy of the beam the effects of the earth's field on the trajectory of the beam is nontrivial. Furthermore, the offsets in the injection and recirculation parts of the Y-shaped section mean that the effects of image charge on the beam should also be taken into account.

First there will be a brief introduction to COSY Infinity and its unique properties, then there will be an overview of how the earth's field, the image charge force, and unique elements contained in this particular beam are implemented. Finally there will be a brief look at some experimental observations.

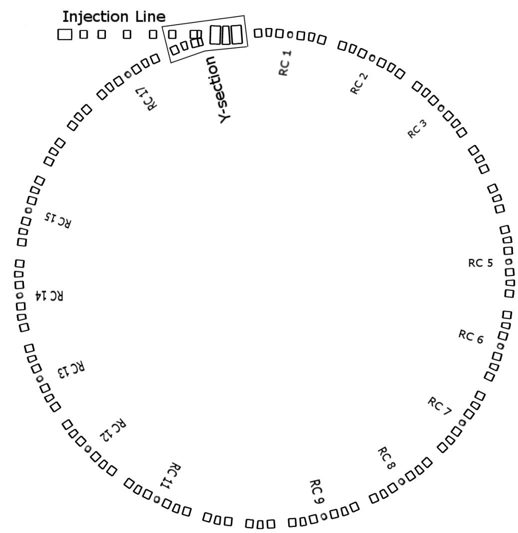

Figure 1: COSY Infinity produced diagram of the University of Maryland Electron Ring. Sections marked RC contain ring chambers which house both non-intercepting Beam Position Monitors, and intercepting Phosphor screens. Sections not marked with an RC number contain glass gaps for current monitors.

CODE DEVELOPMENT

The code used in this study is COSY Infinity 9.0. This code uses differential algebraic vectors which allow not only for an accurate calculation of numerical derivatives, but also carries them through the various mathematical operations. This behavior means that COSY can integrate a test particle through an electromagnetic field and all of the variable dependencies will be preserved, allowing for fast accurate computation of maps [**?**]. COSY also has a large library of default beam elements, so in this study they were used as often as possible.

Short Solenoid

One issue that was dealt with early on was the field profile of a short solenoid in the injection line. The field profile that was measured for the physical element was very different from the kind used in the available COSY solenoids, so the open architecture of COSY allowed us to model the solenoid using a field profile provided by fitted data. The field profile of the short solenoid is shown in Fig 2.

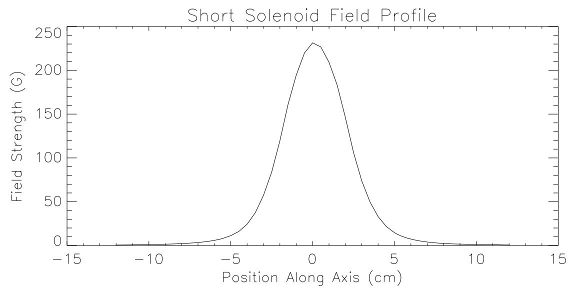

Figure 2: This is a representation of the field of the UMER short solenoid.

Earth's Magnetic Field

The earth's magnetic field accounts for approximately 20% of the bending in the ring, and its direct effects, as well as the methods used to counteract them must be modeled. The earth's field is implemented as a kick using the Baker Campbell Hausdorf (BCH) method. This method shows that if we have a set of two differential equations using the independent variable s, and that they each have a solution that is valid over a given interval in s that is represented by ℓ.

$$\frac{d\vec{z}}{ds} = \vec{g_1}(\vec{z},s) \Longrightarrow \vec{f_1}(\ell), \quad (1)$$
$$\frac{d\vec{z}}{ds} = \vec{g_2}(\vec{z},s) \Longrightarrow \vec{f_2}(\ell), \quad (2)$$

Then the best way to find a solution for the combination of the two equations is to compose the solutions in this manner, with increasing accuracy with decreasing ℓ [3].

$$\frac{d\vec{z}}{ds} = \vec{g_1}(\vec{z},s) + \vec{g_2}(\vec{z},s) \Longrightarrow \vec{f_1}(\frac{\ell}{2}) \circ \vec{f_2}(\ell) \circ \vec{f_1}(\frac{\ell}{2}). \quad (3)$$

In the case of an accelerator element that is being acted on by the earth's magnetic field, the two solutions are expressed as a kick placed between two maps. The number of kicks was determined by increasing the number for the various elements one by one until the answers had converged past the number of significant figures available for the field. The drifts and dipoles each used 6 kicks per element, and the quadrupoles used 15 kicks per element.

The strength of the kick is determined from the equations of motion for the beam, and the strength of the earth's magnetic field is interpolated from a series of measurements previously taken at UMER. The magnetic field kicks are given as:

$$\Delta a(s) = (b_i - \frac{B_y}{B_z})\sin(\frac{B_z}{\chi_{m0}}(1+hx_i)s) + (a_i - \frac{B_x}{B_z})\cos(\frac{B_z}{\chi_{m0}}(1+hx_i)s) + \frac{B_x}{B_z}, \quad (4)$$
$$\Delta b(s) = (\frac{B_x}{B_z} - a_i)\sin(\frac{B_z}{\chi_{m0}}(1+hx_i)s) + (b_i - \frac{B_y}{B_z})\cos(\frac{B_z}{\chi_{m0}}(1+hx_i)s) + \frac{B_y}{B_z}. \quad (5)$$

In this case COSY is using a coordinate system in which x and y are the horizontal and vertical positions respectively, and a and b are analogous to the momentum for each variable. In the case above B_x, B_y, and B_z are the measured field strengths. χ_{m0} is the magnetic rigidity of the beam, and h is the inverse of the radius of curvature, h is zero for a straight element.

In the interests of completeness, and in the unlikely case that there is a zero B_z field, the following equations were also calculated.

$$\Delta a(s) = \frac{B_y}{\chi_{m0}}(1+hx_i)s, \quad (6)$$
$$\Delta b(s) = \frac{B_x}{\chi_{m0}}(1+hx_i)s. \quad (7)$$

Image Charge Kick

This BCH method was also used to implement the image charge force in the injection section. The image charge implementation assumed cylindrical charge symmetry of the beam, which places the image charge at a distance of R^2/ξ where R is the radius and ξ is the offset of the center of the beam. The number of kicks required was analyzed in the same manner as the earth's field, it was found that the number of kicks for the quadrupole and drifts/dipoles used for the earth's field was more than sufficient for the image charge. This then leads to a kick on the beam center of:

$$\Delta a = \frac{\hat{x}}{v_0}\sqrt{-\frac{q\lambda}{2\pi\epsilon_0 m}\ln(\frac{R^2-(x^2+y^2)}{R^2})}, \quad (8)$$
$$\Delta b = \frac{\hat{y}}{v_0}\sqrt{-\frac{q\lambda}{2\pi\epsilon_0 m}\ln(\frac{R^2-(x^2+y^2)}{R^2})}. \quad (9)$$

This allows for the image charge force to be calculated for the center of the beam.

Dipole Modification

As previously stated the earth's magnetic field accounts for approximately 20% of the bending of the beam, in UMER this is counteracted by changing the current to the bending dipoles in the ring. These dipoles have a design bending radius of 10 degrees, but they have their magnetic fields reduced to counteract the earth's magnetic field. This will lead to a small angular and positional offset at the exit of the magnet. COSY Infinity always assumes that if the beam enters at the center of the element then it will exit at the center of the element, so it was necessary to give the bending dipoles in this simulation an offset and a tilt at their exit as a geometric correction. This effect can be seen in Fig 3.

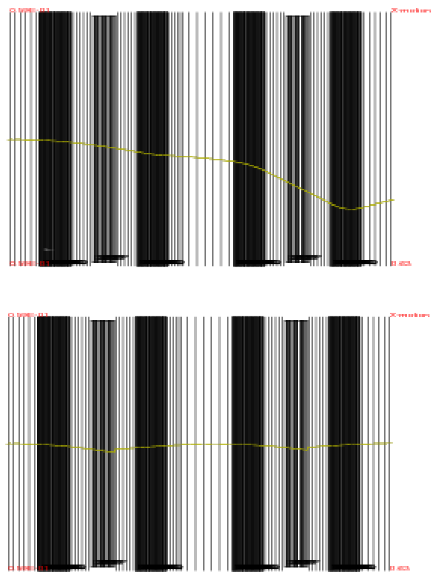

Figure 3: The top shows the COSY generated path of the beam centroid as it moves uncorrected through a ring section. The second shows the same beam with the dipoles adjusted to counteract the earth's magnetic field.

OBSERVATIONS

A series of experiments was recently performed to determine how accurately the COSY model portrayed real life. Fig 4 shows that the agreement between theory and tracking is accurate through RC 9, after this point unresolved issues make accurate simulations difficult. Also a compar-

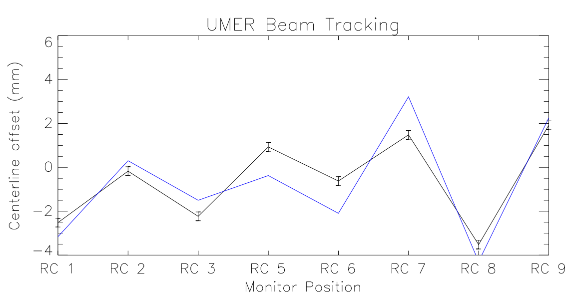

Figure 4: Comparison of horizontal tracking data using the beam position monitors (black) and the predicted location of the beam centroid using COSY Infinity (blue).

ison was made between one set of earth's field compensation values and those calculated using COSY. The previous values were designed to have the bending dipole reduced by an angle equal to the bending caused by the earth's field in that region. The values determined by COSY were found by fitting the bending dipoles such that a beam that entered a ring section with zero angle and zero offset would exit that section with zero angle and zero offset. A comparison of the tracking data is shown in Fig 5.

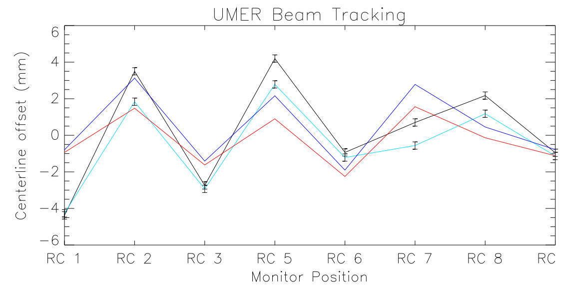

Figure 5: This is a comparison of the measured and predicted trajectories for both methods of earth-field compensation. The black line uses the integrated offset method, while the light blue line uses the COSY calculated values. The dark blue and red lines are the cosy predicted values for the compensation and the COSY calculated settings respectively.

CONCLUSIONS

The adaptability and solid theoretical underpinnings of the code COSY Infinity allow for accurate simulations to be made not only of well behaved large accelerators, but also small radius, low energy accelerators, which have non-trivial complications such as image charge forces and the earth's magnetic field. Using this code on the University of Maryland Electron Ring will allow for a better understanding of high space charge systems without the cost of a heavy ion accelerator.

ACKNOWLEDGEMENTS

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