

# APPLICATION OF MULTI-OBJECTIVE GENETIC ALGORITHM IN ACCELERATOR PHYSICS\*

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## Abstract

The optimization of an accelerator system is important in both design and upgrade stage, and many of them are Multiobjective problems, i.e. searching for a balance between several quantities. A full understanding of this balance could provide the decision maker more information on the final choice. In this paper we present the application of an optimization algorithm called Multiobjective Genetic Algorithm (MOGA) in two problems. One is the lattice of a synchrotron light source (take ALS as an example) and the other is a VHF gun.

## INTRODUCTION

The optimization of an accelerator system is obviously an important problem in both design and upgrade stage. Depending on different system, storage ring or LINAC, collider or light source, this could be minimizing the emittance, optimizing beta functions and bunch length. For a optimization algorithm, the challenges come from the convergence of solutions, constraints on variables and objective functions, conflicting objective functions. In this paper we will introduce an algorithm called multiobjective genetic algorithm (MOGA), show the applications on two problems, one is the lattice optimization for the Advanced Light Source (ALS), a problem with strong constraints in both variable space and objective space. The other is VHF Gun, in which a single simulation cost a couple of hours, therefore in order to get result in a reasonable amount of time, the convergence speed becomes very important.

## GENETIC ALGORITHM AND MULTI-OBJECTIVE OPTIMIZATION

Genetic algorithm (GA) is a search technique in optimization, it was developed in 1970s [8, 7, 4] and now as a class of evolutionary algorithms (EA). The outline of Genetic Algorithm (GA) usually has four steps, first a set of numbers in parameter space are chosen, i.e. the initial population, then they are paired to produce new candidate, we call them parents and children. This is called crossover. The third step is mutation, where children are given a random change according to certain strategy. The last step mimics the nature select process, where the objective functions are evaluated for each child, and the children are sorted according to their corresponding objective

functions. This is a complete generation, and some good children candidate are allowed to continue the evolution.

In the early development, the multiobjective optimization problems (MOP) was converted to a single objective optimization problem by weighted sum method. Later, the truly multiobjective optimization with nondominated sorting was developed based on GA [5]. The detailed mathematical definition of dominance can be found in ref. [4, 2, 10]. It extends the comparison between two scalars to two vectors.

MOGA has been introduced into photoinjector design [2] and accelerator lattice optimization [6, 10]. The comparison of MOGA and GLASS is also shown in [9].

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### Algorithm 1 Multi-Objective Genetic Algorithms

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- 1: Initialize population (first generation, random)
  - 2: **repeat**
  - 3:   select parents to generate children (crossover)
  - 4:   mutation(children)
  - 5:   evaluate(children)
  - 6:   merge(parents, children).
  - 7:   non-dominated sort(rank)
  - 8:   select half of (parents, children)
  - 9: **until** reach a generation with the desired convergence to the PO set
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The structure of our MOGA implementation is shown in algorithm. 1. The first population is initialized with uniformly distributed random numbers, as we will see in storage ring lattice optimizations, most of these random populations at first did not give physical solutions due to transverse stable condition.

Two parents are chosen from the population, and used to generate two children. The newly generated values follows certain probability density function (PDF) as shown in Fig. 1. Following Ref. [4] we are using polynomial PDF with one parameter  $\eta$  to control the shape. This form is convenient to include the boundaries without artificial cuts when the new values are outside of it.

The “new born” children are applied with an operation of mutation, this mimics the effect from nature environment. We also choose a polynomial PDF to describe it. Fig. 2 shows the probability of the old value  $x = -1$  will be mutated to. It has equal probability to go less or greater than  $-1$ .

After the new generation is produced, we then evaluate the objective functions, which are the lattice functions in our case. The results are ranked based on their objective functions and the violation to the constraints. Here we also follow Dr. Deb’s approach [5], where Nondomi-

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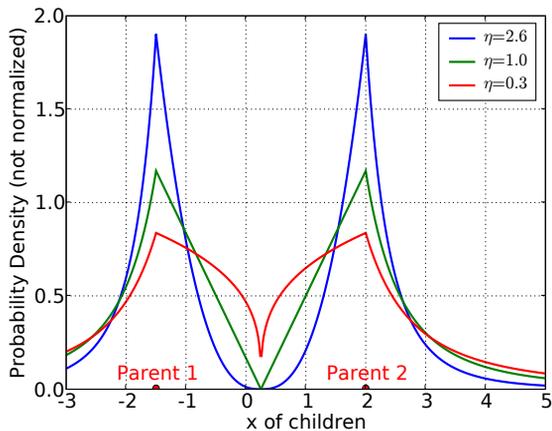


Figure 1: PDF for crossover. This shows the variable within range [-3,5] and two parents are at -1.5 and 2.  $\eta$  controls different shape of PDF.

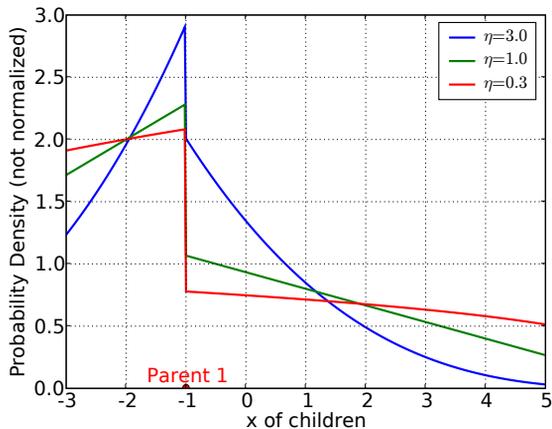


Figure 2: PDF for mutation. Here shows PDF of an example variable defined in range [-3,5] with a value -1.  $\eta$  controls the shape of PDF.

nated sorting is used to find the Pareto optimal. This process is repeated for the non-Pareto set. Those violating the constraints have the lowest rank. A simple way to do this would be building up a table (or matrix):

The table represents “dominance” relations between each pair. Those candidates with empty column are not dominated by any one and are called “nondominated set”. Obviously the nondominated set has a rank 1. The candidates who are violating the constraints are not shown in Table. 1. They can have a similar table, but anyone of them are dominated by anyone in Table. 1.

### LATTICE OPTIMIZATION FOR ALS

We use Advanced Light Source (ALS) as an example to apply MOGA in lattice optimization problems. The ALS is a 3rd generation synchrotron light source located Computer Codes (Design, Simulation, Field Calculation)

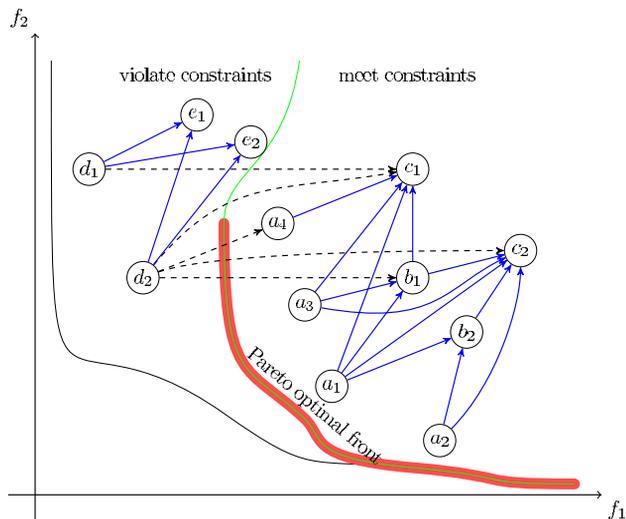


Figure 3: Nondominated Sorting. The candidates are in two group separated by the constraints. The arrows represent the relation of dominance, and the dashed arrow is valid if there are no constraints.

Table 1: Dominance table.

	$a_1$	$a_2$	$a_3$	$a_4$	$b_1$	$b_2$	$c_1$	$c_2$
$a_1$					1	1	1	1
$a_2$						1		1
$a_3$					1		1	1
$a_4$							1	
$b_1$							1	1
$b_2$								1
$c_1$								
$c_2$								

at Lawrence Berkeley National Laboratory optimized for the generation of soft x-ray. The ALS is 200 m in circumference and consists of 12 sectors. The lattice structure of each of the sectors is a triple bend achromat. All sectors are the same with the exception of three sections symmetrically distributed along the ring where the central dipole is superconductive. In this paper we only optimize the sectors with normal conducting dipoles, and the three superconductive bends then can be matched.

The first problem we applied to optimize is the emittance and beta function. In this problem, the emittance as one of the most important quantities of all light sources need to be minimized, while the beta function in this case want to be around 1 meter. The constraints are transverse stability, i.e. the one turn transfer matrix should have a trace in range [-2,2], the maximum beta function less than 30 meters, and the maximum dispersion less than a few centimeters. The optimal results are shown in Fig. 4 and the corresponding brightness change are shown in Fig. 5.

The optimal results are in two isolated regions, and the history of evolution is presented in Fig. 6.

A second optimization on high-low beta is also carried

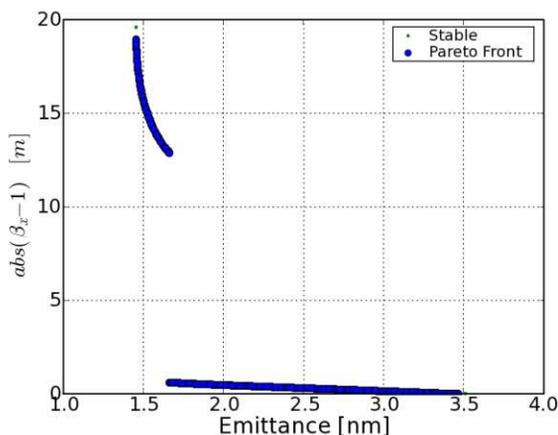


Figure 4: Emittance and beta function of the optimal solutions.

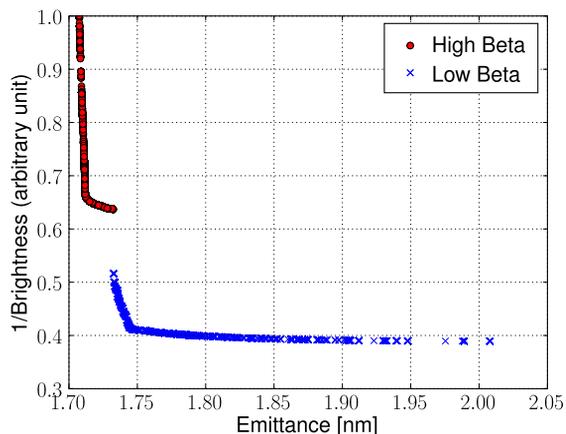


Figure 5: Brightness and emittance. The brightness is normalized by the maximum one in this plot.

out on ALS lattice. Two sectors are treated as one with high and low beta functions in each straight section. The low beta is about 1 meter and the high beta is about 10 meter. The third objective function is still emittance. Fig. 7 shows Pareto optimal set (now is a surface in 3D) projected into  $\beta_{high}-\beta_{low}$  plane.

The lattice optimization problems usually have many constraints, eight in the dynamics or the practical way. As a dynamics system, many of the randomly generated value can not give a physical solution. This could be a serious problem for deterministic algorithms where extra efforts are needed when the predicted solution fails. This effort is first considering the constraints instead of objective functions, therefore is quite different direction from the original setup for objective functions. For MOGA, this kind of problem does not exist, since it is population based and uses the sorting to select/deselect candidates for the next iteration. In this way, it can easily survive the objective-functions-constraints conflict situations mentioned before.

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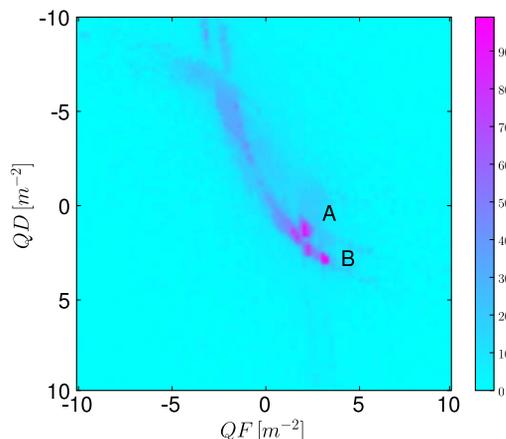


Figure 6: Evolution history in parameter space. The color represents how many generation the candidate in this region can survived without being replaced by better candidates

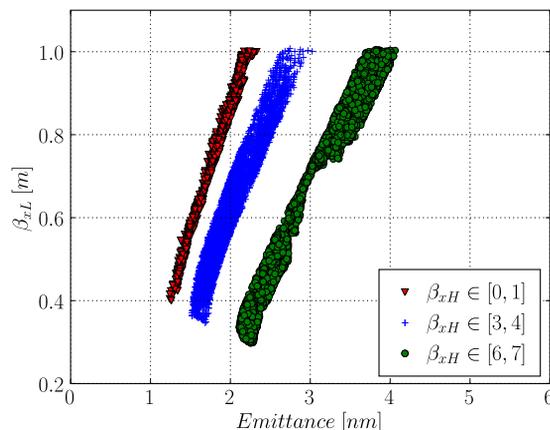


Figure 7: Low beta

In next section we also show a VHF gun optimization problem, which does not have this dynamics stability problem, but the computing time is long and fast convergence is quite important.

### VHF PHOTOINJECTOR OPTIMIZATION

MOGA was also applied on VHF photoinjector optimization at LBNL [12]. The single calculation of beam quantity needs a few hours. This makes the algorithm requiring derivatives (approximation by finite difference method) not practical.

Since MOGA is population based, no interaction between evaluation of each candidate, it is very suitable for parallelization. We used the master-slave model to run MOGA on a cluster with 128 CPUs. All nodes carry out the beam simulation, while the master node do extra MOGA optimizations which is significantly small effort compared

with dynamics simulations.

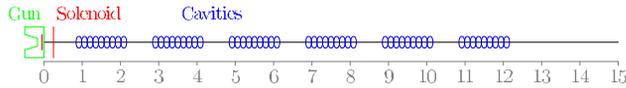


Figure 8: Caption

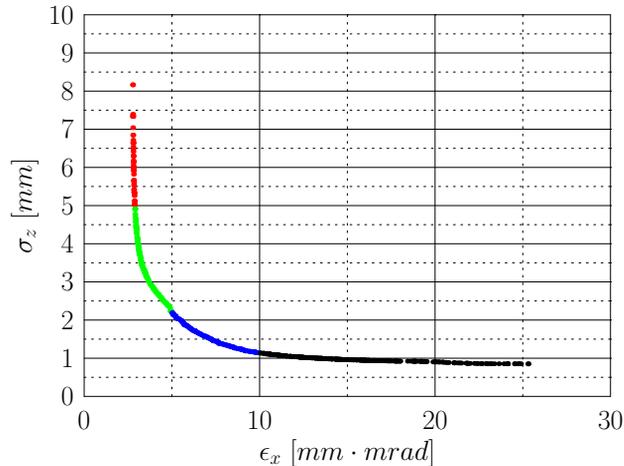


Figure 9: Caption

Fig. 8 shows the layout of a very simple case, to help our understanding of dynamics and various limitations []. Beam is launched from the left cavity, and pass through 6 cavities. The emittance and bunch length at  $s = 15$  meter is obtained from Astra as objective functions. The final optimal solutions is shown in Fig. 9.

## ACKNOWLEDGMENT

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