

## BEAM CHARACTERISATION USING LASER SELF-MIXING\*

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### Abstract

Non-destructive beam diagnostics are highly desirable for essentially any accelerator or storage ring. This concerns the characterization of the primary beam itself, but also for example of atom and molecular jets that are crossed with the primary beam as experimental targets or for diagnostics purposes. A laser feedback interferometer based on the optical self-mixing effect provides a low-cost, robust, compact and non-invasive sensor for velocity, displacement and density measurements of various targets. This contribution presents results from theoretical and experimental studies into the factors influencing the performance and accuracy of this sensor. Parameters that have been assessed include the target velocity, the size of scattering particles, their density, type and scattering properties.

### INTRODUCTION

Gaseous targets and gas jets have many applications in different fields of accelerator physics. A curtain shaped gas-jet is used as a non-destructive beam profile monitor for various types of the particle beams [1-2]. The beam profile monitor is based on the ionisation of the gas jet when it interacts with the beam resulting in 2-dimensional profile picture. Gas jets are used as a source of a laser induced plasma in laser-plasma acceleration experiments [3]. Supersonic gas jets in various configurations are used for the production and spectroscopy of radioactive isotopes [4]. The increasing importance of gas jets means that their characterisation is essential and a sensor is required to obtain information about the velocity, the density and the temperature. The laser self-mixing (SM) sensor is proposed as easy integratable, compact and cheap device for such purposes [5-7].

A laser velocimeter is under development in the QUASAR Group at the Cockcroft Institute/University of Liverpool, UK for the purposes of measuring the velocities of gas jets. The gas jet consists of neutral molecules, such as argon, nitrogen or helium, and forms a curtain of 1-20 mm diameter. Molecules move uniformly with a velocity which can vary from 100 m/s to 2000 m/s. The density of the gas jet depends on the pressure in the vacuum chamber and is expected to be in the range of  $10^9$ – $10^{12}$  particles/cm<sup>3</sup> [1-2]. An SM sensor is expected to measure the flow of gas jets with these parameters. It has been successfully used for characterising velocities of solid targets up to 100 m/s [8], and for some fluids up to 1 m/s [9]. The scattering of light off gas jets and off the seeding particles, which are added to improve the SM signal, directly influences the performance and accuracy of the results.

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### THEORY AND METHODOLOGY

The self-mixing phenomenon is based on the coupling of laser light reflected or scattered off a moving target back into the cavity and interacting with the light inside the laser cavity. The backscattered light's wavelength is shifted due to the Doppler effect and the intensity of the backscattered light depends on the particulars of the target i.e. optical properties etc. The here-presented sensor is based on the SM effect in semiconductor lasers. The detection system includes a photodiode (PD), which is part of the commercially available laser diode. The SM phenomenon influences both the wavelength of the light and its power fluctuation. The interaction within the cavity causes backscattered light to be amplified, so the sensor doesn't require powerful light and there is no need for a complex optical system.

#### *Scattering Theory of Light off Various Targets: Expected Spectrum*

The distribution function of an electro-magnetic field scattering off a target is expected to be Gaussian [10], independent on the properties of the scattering media and the character of its motion. However, if the amount of illuminated scatters is large enough and their movements are correlated, the resulting function depends on the type of motion of the scatters. For example, the spectrum of scattered light is a Lorentzian function when a large amount of particles undergoes Brownian motion [11]. The process of scattering off a target or group of particles is a complicated process which requires different considerations depending on the nature of the particle.

1. If the light scatters from density fluctuations in a medium, there is a finite correlation between different coherent volumes [12].

2. If the light scatters form a rotating target, the motion of the scatters can vary from fully correlated to completely uncorrelated. It can be assumed that different parts of the rotating target are indifferent from each other. At the same time, the motion of the scatters can be characterised by the distribution of the velocities of the delta-function with the velocity of the centre of the light focus point [13].

3. If the light scatters off a moving flow with a specific velocity, the scatters are independent from each other. However, the flow can be characterised with a velocity distribution within an illuminated volume. Hence, the spectrum should be similar to the second case taking into account the velocity distribution.

The spectrum is to be calculated first for the rotating target with the velocity distribution of delta-function, and to be modified in more difficult case. The theoretical spectrum of the light scattered off a rotating ground glass has been analysed [14]. Assuming the size of the light

beam  $\sigma$  was focused by the lens with focus  $f$  on the target where the spot has size  $A$ , and that the target is located at a distance  $z$ , which is very close to the focus plane, then the spectrum of the signal is Gaussian:

$$I(\omega) = I_0 \frac{1}{A} \frac{N}{v/w_0} \exp\left(-\frac{(\omega - \omega_0)^2}{2v^2/w_0^2}\right) \quad (1)$$

Hence, the spectrum has a peak at the frequency centred on around the frequency of the incident light  $\omega_0$  and is broadened proportionally to the velocity  $v$  of the target and inversely proportional to the beam waist radius  $w_0$ . The Gaussian peak and its broadening are independent on the distance to the object. The amplitude of the scattered spectrum is inversely proportional to the velocity of the target and inversely proportional to the illuminated area of the target. The illuminated area  $A$  can be calculated based on the beam radius. Hence, the amplitude of the scattered light increases with a decreasing size of the beam waist, and increases with an increasing amount of scattered centres  $N$ .

Broadening of the spectrum means that there are additional components added to the spectrum, so effectively speaking light is Doppler-shifted over many frequencies. They appear due to the roughness of the surface, i.e. 3D-dimension of the surface.

The discussion above was correct for the case of the light beam perpendicular to the target. Now, taking into the account the non-zero angle between the velocity vector and the wavelength vector, i.e.  $\mathbf{k} \cdot \mathbf{v}$ , a similar equation can be found for this case.

The geometry of a self-mixing experiment for the case of a rotating wheel is shown in Fig. 1a. The wavelength vector  $\mathbf{k}$  is related to the wavelength by  $\mathbf{k} = n2\pi/\lambda$ , where  $\mathbf{n}$  is the unit vector in the direction of the light propagation. In the geometry presented in the figure  $\mathbf{k} \cdot \mathbf{v} = kv \cos \alpha$ . This will lead to a shift in the main frequency  $\omega_0$ . As it was mentioned earlier, broadening of light when incident perpendicular to the target appears due to the roughness of the surface in that plain. Hence in the case of an inclined target, the broadening is proportional not to the whole component of the velocity, but to the component perpendicular to the  $\mathbf{k}$  vector, i.e. to  $v \sin \alpha$  in the geometry in Fig. 1. In the general case, it can be written as  $|\mathbf{n} \times \mathbf{v}|$ . The integral leads to a similar expression to Eq. 1 with a difference in the central frequency of the spectrum and broadening component:

$$I(\omega) = I_0 \frac{1}{A} \frac{N}{|\mathbf{n} \times \mathbf{v}|/w_0} \exp\left(-\frac{(\omega - (\omega_0 + kv))^2}{2(|\mathbf{n} \times \mathbf{v}|)^2/w_0^2}\right) \quad (2)$$

In the case of the self-mixing technique, the beating of the optical power has a frequency proportional to the shift in the wavelength of the scattered/reflected light. Hence the spectrum of the SM signal is described by Eq. 2, so the peak is proportional to the Doppler shift and broadened according to these formulas.

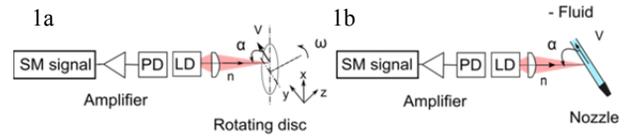


Figure 1: The set-up of SM sensor used for velocity measurements with a fluid as a target. The SM set-up contains a laser diode (LD) with light focused onto a target at an angle  $\alpha$  from the normal of the target surface. A photodiode (PD) takes the role of a detector of the SM modulated light, the signal from which is converted into the voltage signal by a transimpedance amplifier. 1a) A rotating disc target, which was used for the theoretical study into the spectrum which can be obtained from the SM sensor. 1b) A fluid target, the parameters of which are under the scope of this contribution, such as velocity  $v$ , the concentration of seeders.

The described spectrum of the SM signal is based on the interaction of the electric field of the incident light with the surface which has roughness, which causes a distortion of the field scattered/reflected from the surface with some unevenness. The same principle can be applied to liquids since the fluid can be seen as a 3D object for the laser beam.

The amount of backscattered light can be defined based on scattering theory. The solid target is assumed to scatter light evenly in all directions in a solid angle  $(0, \pi)$ . Any type of flow has some centre from which the scattering process can occur. This can be particles, density fluctuations, or temperature fluctuations, which leads to a cross-section distribution of amplitudes over a range of scattering angles.

### Expected Backscattered Light from Gas Jets

An estimation of the expected backscattered light may be found from Rayleigh scattering theory [15]. If laser light with power  $P_0$  is incident on the gas-jets, with constituent particles with diameter  $d$  and particle density  $N$ , will scatter light with optical power  $P_{det}$  within a solid angle  $\Delta\Omega$ , which is defined by the optics of the sensor:

$$P_{det} = P_0 \eta N z_d \cdot \Delta\Omega \cdot \frac{9\pi^2}{\lambda^4} \cdot d^6$$

where  $\eta$  is the optics collection efficiency and  $z_d$  is the depth of the measurement volume. The power attenuation based on this formula for gas-jet parameters, assuming a size of scatter  $0.1 \text{ nm}$  and a gas jet density of  $10^{12} \text{ molecules/cm}^3$  is around  $-240 \text{ dB}$ . The minimum level of feedback is  $-90 \text{ dB}$  for a SM sensor [7]. To bring the attenuation level closer to the required value, seeding with additional particles of at least  $10 \text{ nm}$  diameter is necessary.

### Choice of Seeders

Different types of scattering particles could be used for the self-mixing technique. Since the main interest of this

work is to measure high velocities up to 2000 m/s including gases, there is a need to find the most efficient seeding particles or material to obtain the required level of feedback for efficient operation of the SM sensor.

The geometry of the SM sensor is such that the light, which is coupling back into the cavity, is always backscattered light unless there is a modification of the system.

A number of experiments of a similar nature of light interaction were performed for PIV (Particle Imaging Velocimetry), the initial range/choice of the materials for the SM sensor can be taken directly from there. Moreover, there are a few works [5-7] within the SM field to get an idea of which seeders have already worked successfully for SM velocity measurements.

PIV is based on analysing the net displacement of moving particles. In this technique, the flow is seeded with tracer particles, which scatters the light. A laser light sheet, which is formed by a cylindrical lens, illuminates the region of interest. The picture of scattering light is visualized. If the tracer particles are assumed to follow faithfully to the flow, a statement can be made on the average velocity of the flow. Hence, substantial research has been performed into seeding materials and seeding techniques for PIV application [16]. The main seeding materials and their sizes are presented in Table 1. The range of materials which were used for seeding both liquids and gases, are presented among which both liquid and solid types of seeders were used.

Table 1: Seeding Material for Liquid and Gas Flows in PIV Measurements. [16]

Type	Material	D for liquid flows, $\mu\text{m}$	D for gas flows, $\mu\text{m}$
Solid	Polystyrene	10-100	0.5-10
	Alumina $\text{Al}_2\text{O}_3$	2-7	0.2-5
	Titania $\text{TiO}_2$		0.1-5
	Carbon dioxide $\text{CO}_2$		5-15
	Glass spheres	10-100	0.2-3
	Granules for synthetic coatings	10-500	10-50
	Diocetylphthalate		1-10
Liquid	Different oils	50-500	0.5-10
	Diethylhexylsebacate		0.5-1.5
	Helium-filled soap bubbles		1000-3000
Gases	Oxygen bubbles	50-1000	

Generally, the scattering efficiency function strongly depends on the refractive index of the seeders and surrounding area, the size, shape and orientation of particles and the observation angle. Along with this the

scattering efficiency is a function of the ratio of the refractive index of the seeders and surrounding area [13]. The refractive index of air is considerably less than that of water, so the amount of light scattered off the small particles of the same size in air is at least one order of magnitude more than in the water. As a result, the size of the seeders can be smaller for velocimetry of gases, and vice versa, for water flow measurement, a larger size of particles has to be used for a sufficient amount of scattered light. This can be seen in the Table 1.

The nature of gases and analysis of the experiments using different technique for the mapping the velocities shows that other materials can give promising level of the backscattered light for the SM sensor.

As it was mentioned, the self-mixing technique is used not only for displacement measurements, but also for velocity detection of solid and liquid targets. The SM fluid velocity measurements were mainly aimed at biological applications such as blood characterisation [5] and molecular dynamic [6]. Milk and polystyrene (latex) spheres were mostly used for seeding the water.

The size of polystyrene in SM experiments varies from 110 nm up to 1.23  $\mu\text{m}$ , compared to the size of the polystyrene seeders used in fluid PIV experiments of 10  $\mu\text{m}$  to 100  $\mu\text{m}$ . SM system for velocity detection is more sensitive to scattered light, so smaller particles can be used for the velocimetry. The main and easy-to-use seeders are milk. The velocity measurements demonstrated in recent published papers do not exceed 15 cm/s for milk as seeders, 17 cm/s with polystyrene spheres as seeders of 1.23  $\mu\text{m}$  size (and with lower velocities in case of the smaller sizes of the seeders), and up to 35 cm/s for the blood measurements for very high concentration and without reference verification of the experimental results.

## EXPERIMENTS WITH FLUIDS

To measure the velocity of liquids, an experimental set-up was established in order to compare the influence of different parameter from analytical solutions for spectrum of the SM velocimeter with the experimental results.

The experimental set-up of the SM sensor for the measurement of the velocities of the liquids is presented in Fig. 1b. The light from LD was focused using a lens on to the fluid target at an angle of  $\alpha=75^\circ$ . The backscattered light from the target was coupled back into the cavity using the same lens. The signal was measured by PD and a transimpedance amplifier.

The frequency of each SM signal is expected to be proportional to the projection of the velocity of the laser light axis according to the Doppler shift and broadened according to Eq. 2. The Doppler shift of the periodic optical power fluctuation measured with PD and the amplifier is equal to  $(2vcos\alpha)/\lambda$ . In the here-presented set-up, the LD of 650 nm wavelength was used. The data analysis based on the Fast Fourier Transformation allows the spectrum of the signal to be obtained, which is compared with spectrums theoretically obtained before.

The constant velocity of the fluid was achieved by using a pump with variable flow rate and discharged from the tube. The pump rates were used such that the fluid did not enter the chaotic regime allowing a stable and measurable reference velocity of the water stream. The experimental set-up was constructed in such way that different parameters can be varied. The velocity of the fluid can reach up to 3 m/s without entering the chaotic regime. The seeding material parameters were changed such as the material itself (milk, titanium dioxide) and the size of the seeders (5  $\mu\text{m}$ , 1  $\mu\text{m}$ , 150 nm, 21 nm) without changing the viscosity of the fluid. Based on the analysis above and taking into the account the PIV experiments, the experiments were performed with following seeders:

- Milk: 5  $\mu\text{m}$ ;
- Titanium dioxide  $\text{TiO}_2$ : 1  $\mu\text{m}$ ; 150nm; 21 nm.

### The Effect of Varying the Velocity of the Laminar Flow

The variation of the velocity of the target directly influences on not only the value of the Doppler shift in the wavelength, but also the amplitude of the peak of the signal spectrum and its bandwidth. The spectrum of SM signal obtained in the experiments when the velocity of the flow was varied is presented in Fig. 2. When the velocity was increased, the peak amplitude decreased and its full width at half maximum (FWHM) increased proportionally. It agrees well with theory described earlier by Eq. 2.

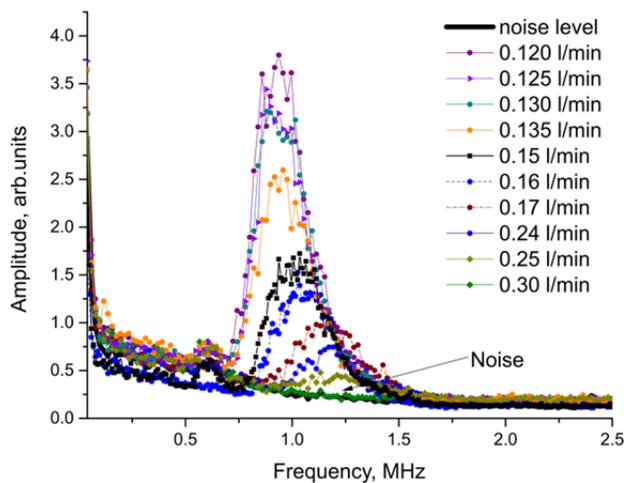


Figure 2: The experimental results of the velocity measurements of the flow of water: the spectrum of the self-mixing signal at different pump rates. The liquid was seeded with titanium dioxide  $\text{TiO}_2$  with a 1  $\mu\text{m}$  diameter. The velocity of liquid was varied from 1.0 m/s to 1.5 m/s. The amplitude of the spectrum peak decreases steadily with increasing velocity.

### Varying the Concentration of Seeding Materials

The concentration of the seeders is responsible for the amount of light being backscattered into the cavity. Varying concentrations of different seeder sizes was

under study to see the influence on both the amplitude of the spectrum peak and its bandwidth.

Figure 3 presents the results of the experiments obtained when the concentration of the seeders  $\text{TiO}_2$  in this case varied from 0.03 wt% to 3.4 wt% (wt% is percentage by weight which can be defined as relation of the weight of the seeders to the weight of the solution multiplied by 100%) in the water. The amplitude of the spectrum peak obtained from the SM signal is averaged over 100 measurements from the signal from the fluid. Figure 3 shows the dependence of the peak amplitude on the concentration of the  $\text{TiO}_2$  particle of 1  $\mu\text{m}$  in diameter by the round white dots, 150 nm in diameter by the square dots, and 21 nm in diameter by the grey triangular dots. The trend of the plots, shown by the blue lines, has exactly the same character for the various sizes of seeders with different amplitudes.

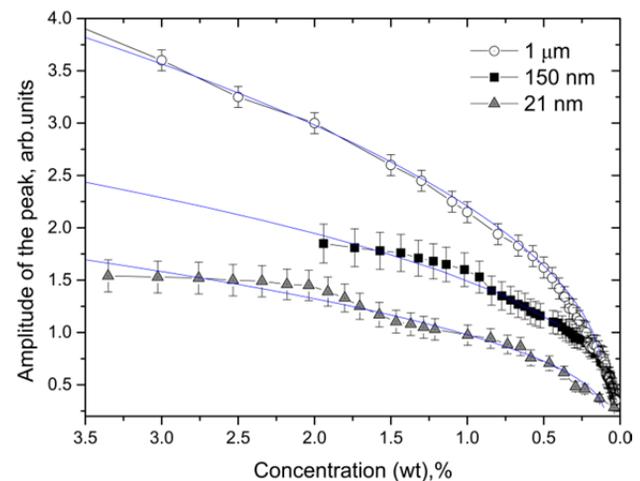


Figure 3: The experimental influence of the concentration of  $\text{TiO}_2$  seeders in the flow of water on the spectrum of the self-mixing signal with a fixed flow velocity (at 1.3 m/s). The obtained result for seeders a 1  $\mu\text{m}$  diameter are shown by round white dots, 150 nm diameter by the black square dots, and 21 nm by the grey triangular dots. The concentration of the seeder was varied in the experiments in the range of 3.4 wt% to 0.03 wt%. The amplitude of the spectrum peak decreases steadily with decreasing concentration.

According to the experimental results for the different size of seeding material, the amplitude of the peak increases for higher concentrations. The FWHM does not depend on the concentration of the seeders and remains constant in the case of fixed flow velocity for all experiments, which is predicted by Eq. 2. The measured velocity of the fluids agreed with the reference velocity with better than 3% precision.

### The Effect of Varying the Size of Seeding Materials

According to theory, the amplitude of the peak of the spectrum should be proportional to the number of seeders

in the flow, i.e. to the concentration. However, Fig. 3 shows the dependence of the concentration on the amplitude of the peak for the same type of seeders ( $\text{TiO}_2$ ) of different size. The peak amplitude of the spectrum is higher for the 1  $\mu\text{m}$  diameter compared to 150 nm and to 21 nm. The amount of backscattered light strongly depends on the size of the particles, when the size of the scatters is of the same order of the wavelength of the scattered light. When the size of the seeding particles was 5  $\mu\text{m}$  it was observed that the amplitude of the peaks was lower as well [9].

### *The Effect of Varying the Optical Properties of Seeding Materials*

During experiments, different types of seeders were used. As it has been shown before [17], the velocity of the fluid directly influences the amplitude of the peak, which decreases with increasing velocity. Hence, after some velocities, it is impossible to detect a signal. Using the same geometry of the set-up, velocities up to 100 m/s were measured when the white paper was used as a target [8]. Switching to fluids, the limit of the detected velocity increased dramatically. In experiments with milk as seeders, a velocity up to 20 cm/s was measured [9]. Using titanium dioxide as a seeding material makes it possible for velocities up to 1.5 m/s to be measured with the same or even lower level of the concentration as milk.

### CONCLUSION

This study has been focussed on the optimisation of a SM sensor to measure the velocity of gas jet based beam profile monitors. A theoretical investigation into the spectrum expected for such a sensor has been presented together with a calculation of the expected level of backscattered signal from a gas jet. A range of different seeding materials added to a water flow was investigated, and such parameters as velocity, reflectivity, and concentration of the seeders in the fluid were under study. The laboratory experiments with  $\text{TiO}_2$  with different diameters (1  $\mu\text{m}$ , 150 nm, 21 nm) showed the dependence of the peak spectrum amplitude from the concentration of seeders with a minimum concentration of 0.03 wt% still being possible to measure velocities with better than 3% accuracy. Using the same type of seeders for the gas jet is currently under investigation since it should improve the SM signal even more than liquids.

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