

# STUDY OF THE TRANSVERSE BEAM EMITTANCE OF THE BERN MEDICAL CYCLOTRON

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## Abstract

The cyclotron laboratory for radioisotope production and multi-disciplinary research at the Bern University Hospital (Inselspital) features an IBA Cyclone 18 MeV proton cyclotron equipped with a Beam Transport Line (BTL), ending in a separate bunker. The horizontal and vertical transverse beam emittances were measured for the first time for this kind of accelerator. Two different techniques were used. A measurement based on quadrupole strength variation and beam width assessment after the last focusing section on the BTL was first performed. A second technique was developed employing 4 beam profilers located at successive positions around a beam waist. These novel beam profile detectors were developed by our group and are based on doped silica and optical fibers. For the data analysis, a statistical approach allowing for estimation of the RMS transverse emittance of a beam with an arbitrary density profile was applied. The results obtained with both methods were found to be in good agreement.

## INTRODUCTION

A cyclotron laboratory for radioisotope production and multi-disciplinary research is in operation at the Bern University Hospital (Inselspital) [1]. The facility is equipped with an IBA Cyclone 18 MeV proton cyclotron shown in Fig. 1. The cyclotron is supplied with two  $H^-$  ion sources, a redundancy aimed at maximizing the efficiency for daily medical radioisotope production. It provides high beam currents up to  $150 \mu A$  in single or dual beam mode. Extraction is realized by stripping  $H^-$  ions in a  $5 \mu m$  thick pyrolytic carbon foil.

The Bern cyclotron laboratory is equipped with a Beam Transport Line (BTL), which is a unique feature for a hospital based facility. It allows to carry out multi-disciplinary research in parallel with daily radioisotope production. A schematic view of the BTL is presented in Fig. 2. Alternate beam focusing and defocusing is realized by two horizontal-vertical (H-V) quadrupole doublets, the former located in the cyclotron bunker and the latter in that of the BTL. A movable cylindrical neutron shutter is located at the entrance of the BTL bunker to minimize the penetration of neutrons during routine radioisotope production. For scientific activities, experimental equipment such as particle detectors or specific target stations are installed at the end of the 6.5 m long BTL.



Figure 1: The Bern cyclotron opened during commissioning.

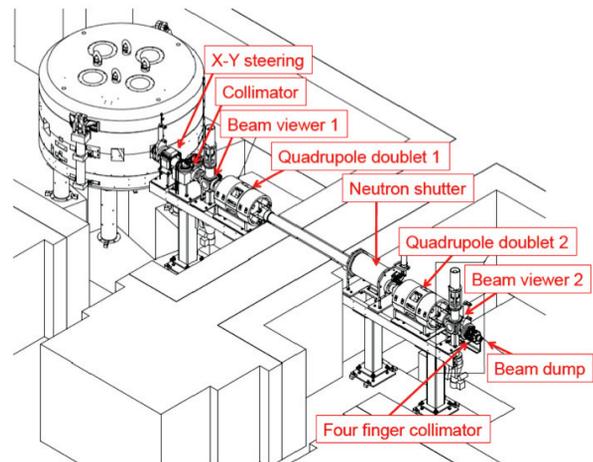


Figure 2: Schematic view of the Bern cyclotron facility, where all the main elements of the BTL are highlighted.

In this paper, we report on the first measurements of the transverse beam emittance of an IBA 18 MeV cyclotron. The measurements were conducted by means of beam profilers developed by our group and named UniBEaM. This detector is a compact device based on doped silica and optical fibers which allows for fully automatized measurements of transverse beam profiles. The first prototype of UniBEaM is described in [2]. For the measurements reported in this paper, a beam current of about  $250 nA$  was used, which is unusual for medical cyclotrons. Such low currents are obtained with the methods described in [3]. This intensity range allows operating the UniBEaM detector in a linear

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regime thus avoiding distortions in the beam profiles. The transverse beam emittance was evaluated by applying two different methods, in which the beam profilers were installed along the BTL. Preliminary results are presented in the next sections.

## THE TRANSVERSE RMS BEAM EMITTANCE OF AN ARBITRARY DENSITY PROFILE

The beam emittance is the main physical quantity used to characterize an accelerated particle beam. It gives an area in the phase space of the particles. There are two phase space variables for each spatial direction - momentum and position, and the beam emittance describes the correlation between them. The transverse beam emittance can be determined for two planes - horizontal and vertical. The phase space is described by position  $x$  and angle  $x'$ , and position  $y$  and angle  $y'$  for the former and the latter plane, respectively. In further considerations, only the horizontal plane is discussed being the vertical plane completely analogous.

Realistic beams are usually far from being Gaussian and an appropriate statistical approach is required for a reliable estimation of the transverse beam emittance. In case of an arbitrary density distribution  $\rho(x, x')$ , the following moments can be defined:

$$\langle x^2 \rangle = \frac{\iint (x - \mu)^2 \rho(x, x') dx' dx}{\iint \rho(x, x') dx' dx} \quad (1)$$

$$\langle x'^2 \rangle = \frac{\iint (x' - \mu')^2 \rho(x, x') dx' dx}{\iint \rho(x, x') dx' dx}, \quad (2)$$

and the covariance:

$$\langle xx' \rangle = \frac{\iint (x - \mu)(x' - \mu') \rho(x, x') dx' dx}{\iint \rho(x, x') dx' dx}, \quad (3)$$

where  $\mu$  and  $\mu'$  are the expectation values for  $x$  and  $x'$ , respectively. The beam matrix  $\sigma(s)$  at the location  $s$  along the beamline is therefore expressed in the following way:

$$\sigma(s) = \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix} = \begin{pmatrix} \langle x^2 \rangle & \langle xx' \rangle \\ \langle xx' \rangle & \langle x'^2 \rangle \end{pmatrix}. \quad (4)$$

The RMS beam emittance  $\varepsilon_{rms}$  is then given by the determinant of the  $\sigma_s$  matrix:

$$\varepsilon_{rms} = \sqrt{\det(\sigma(s))}, \quad (5)$$

and is independent of the location  $s$  according to Liouville's theorem.

## QUADRUPOLE VARIATION METHOD

The quadrupole variation method was first used to measure the transverse beam emittance in both horizontal and vertical planes. The method is depicted in Fig. 3. The last quadrupole magnet of the BTL, located at  $s_0 = 0$ , is defocusing in the horizontal plane and focusing in the vertical. Its

strength was varied and the corresponding beam profiles at the location  $s_1 = 694$  mm were measured with the UniBEaM detector for each magnet setting. The measurements were performed for a beam current of 250 nA, which was monitored throughout the experiment by means of a Faraday cup. The profiler and Faraday cup installed on the BTL are shown in Fig. 4. The UniBEaM monitor was rotated by  $90^\circ$  for the corresponding measurements in the vertical plane.

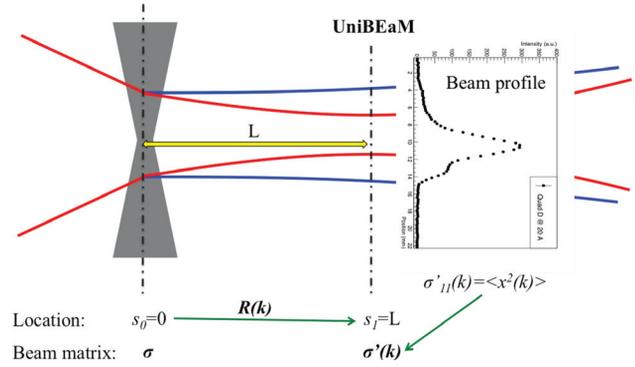


Figure 3: Variation of quadrupole strength for the measurement of the transverse beam emittance.

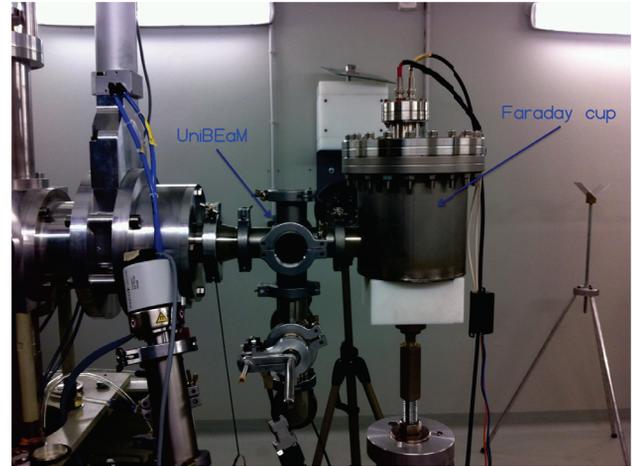


Figure 4: The UniBEaM detector and a Faraday cup installed on the BTL for the quad variation measurement of the transverse beam emittance in the horizontal plane.

Since each profile gives the marginal density distribution, the calculated variance (RMS squared) is an estimate of the  $\sigma'_{11}(k)$  component of the beam matrix  $\sigma'(k)$  at the location  $s_1$  for a given quadrupole strength  $k$ . A defocusing quadrupole magnet of an effective length  $d$  set at the strength  $k$  can be represented by the following matrix:

$$R_{DQ} = \begin{pmatrix} \cosh \sqrt{k}d & \frac{1}{\sqrt{k}} \sinh \sqrt{k}d \\ \sqrt{k} \sinh \sqrt{k}d & \cosh \sqrt{k}d \end{pmatrix}. \quad (6)$$

A drift of length  $L$  follows and leads to the matrix:

$$R_{DR} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}. \quad (7)$$

Eventually, the beam transfer matrix  $R(k)$  is given by the matrix product:

$$R(k) = R_{DR}R_{DQ}, \quad (8)$$

which provides a linear transformation between the beam matrix  $\sigma$  at the entrance to the quadrupole and the beam matrix  $\sigma'(k)$  at the location of the UniBEaM detector. The matrix  $\sigma'(k)$  is obtained by the following algebraic operation:

$$\sigma'(k) = R(k)\sigma R(k)^T, \quad (9)$$

which gives the  $\sigma'_{11}(k)$  component as a function of  $k$ , containing three unknown parameters  $\sigma_{11}$ ,  $\sigma_{22}$ , and  $\sigma_{12} = \sigma_{21}$ . It holds:

$$\sigma'_{11}(k) = f(k; \sigma_{11}, \sigma_{12}, \sigma_{22}). \quad (10)$$

The strength  $k$  is directly related to the quadrupole current  $I$ . During the measurements, the magnet current was varied in the range 17-36 A and 25-63 A for the horizontal and vertical plane, respectively. The factor  $k$  was found on the basis of the quadrupole characteristics studies performed by the manufacturer. The components of the beam matrix  $\sigma$  at the entrance to the quadrupole and the corresponding transverse beam emittance value were obtained by performing a fit of the function  $f(I; \sigma_{11}, \sigma_{12}, \sigma_{22})$  to the data points. The estimated variance values  $\langle x^2 \rangle$  and  $\langle y^2 \rangle$  as a function of the quadrupole current together with the fitted curves are reported in Figs. 5 and 6 for the horizontal and vertical plane, respectively. The fit results for both planes and the corresponding emittance values are reported in Table 1. The transverse RMS emittance in the horizontal plane is 3.6 times bigger than the one in the vertical plane. This can be explained by the fact that particles are accelerated in the horizontal plane and therefore the position spread is significantly larger than in the vertical plane.

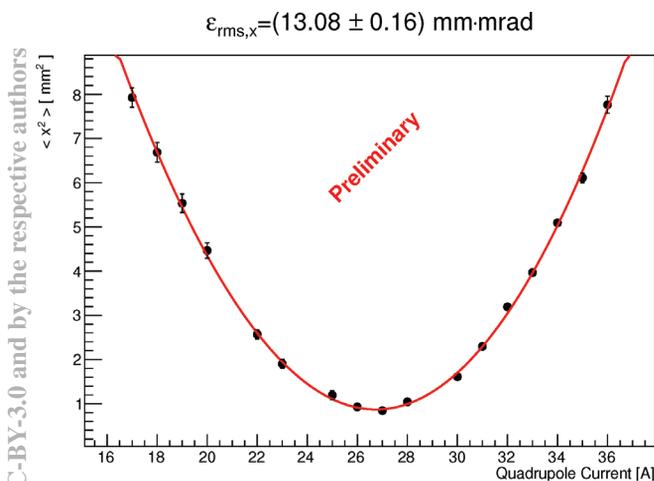


Figure 5: Variance as a function of the quadrupole current obtained in the horizontal plane. The red line corresponds to the best fit.

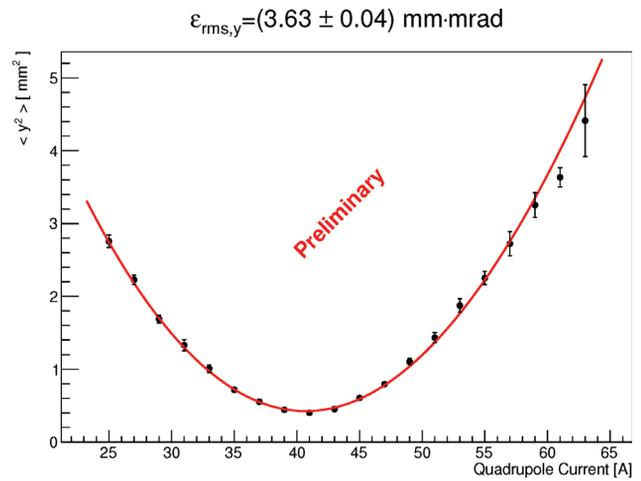


Figure 6: Variance as a function of the quadrupole current obtained in the vertical plane. The red line corresponds to the best fit.

Table 1: Fit Parameters and the RMS Emittance Values Obtained by Quadrupole Variation for Both Horizontal and Vertical Planes

Fit parameter	Horizontal plane	Vertical plane
$\sigma_{11}$ [ $\text{mm}^2$ ]	$200.23 \pm 0.08$	$21.59 \pm 0.36$
$\sigma_{12}$ [ $\text{mm}\cdot\text{mrad}$ ]	$-322.66 \pm 0.08$	$-2.98 \pm 0.07$
$\sigma_{22}$ [ $\text{mrad}^2$ ]	$520.80 \pm 0.22$	$1.02 \pm 0.02$
$\tilde{\chi}^2$	0.98	1.04
$\epsilon_{rms}$ [ $\text{mm}\cdot\text{mrad}$ ]	$13.08 \pm 0.16$	$3.63 \pm 0.04$

## MULTIPLE BEAM PROFILER METHOD

In this method, four UniBEaM detectors were installed on the BTL, as shown in Fig. 7. With respect to quad variation, this method does not require any prior knowledge of the optical elements of the beam line. For a fixed setting of the quadrupole magnets and with a beam current of 250 nA, beam profiles were measured at four successive locations around a beam waist separated by a drift length  $L = 135$  mm, as depicted in Fig. 8. The UniBEaM monitors were rotated by  $90^\circ$  for the corresponding measurements in the vertical plane.

The beam profiles were analyzed in the same way, as in the case of the quadrupole variation method. The variance was calculated for each profile histogram giving an estimate of  $\sigma_{11}(s)$  component of the beam matrix  $\sigma(s)$  at the location  $s$ . The beam transfer matrix  $R(s)$  involves now only a drift:

$$R(s) = \begin{pmatrix} 1 & s \\ 0 & 1 \end{pmatrix}. \quad (11)$$

The beam matrix at any location  $s$  with respect to the location of the first profiler ( $s_0 = 0$ ) is therefore given by the formula:

$$\sigma(s) = R(s)\sigma(0)R(s)^T. \quad (12)$$

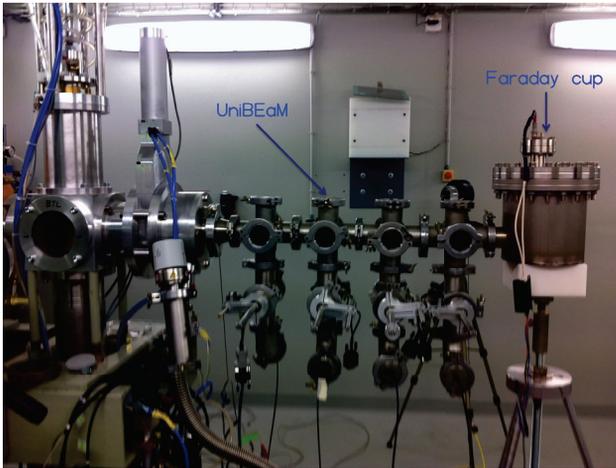


Figure 7: Four UniBEaM detectors and a Faraday cup installed on the BTL for the measurement of the transverse beam emittance in the horizontal plane.

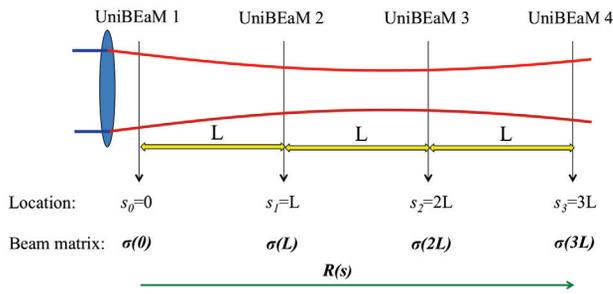


Figure 8: Multiple profilers for the measurement of the transverse beam emittance.

From equation (12) it can be derived that  $\sigma_{11}(s)$  is a quadratic function of  $s$ :

$$\sigma_{11}(s) = \sigma_{22}s^2 + 2s\sigma_{12} + \sigma_{11} = f(s; \sigma_{11}, \sigma_{12}, \sigma_{22}), \quad (13)$$

where  $\sigma_{11}$ ,  $\sigma_{12}$ , and  $\sigma_{22}$  are the components of the  $\sigma(0)$  matrix. These components and consequently the transverse emittance were evaluated by fitting the  $f(s; \sigma_{11}, \sigma_{12}, \sigma_{22})$  function to the four data points representing the estimated variance values as a function of the location  $s$ , as reported in Figs. 9 and 10 for the horizontal and vertical plane, respectively. The fit results for both planes and the corresponding emittance values are reported in Table 2.

### CONCLUSIONS

The transverse beam emittance of the Bern medical cyclotron has been measured for the first time with the use of a novel beam monitor detector developed by our group. The emittance was evaluated with the two different techniques: quadrupole variation and multiple profilers installed along the beamline. The results were found to be in agreement within  $1.65\sigma$  and  $0.71\sigma$  for the horizontal and vertical plane, respectively. The transverse RMS beam emittance in the horizontal plane is almost 4 times bigger than the one in the

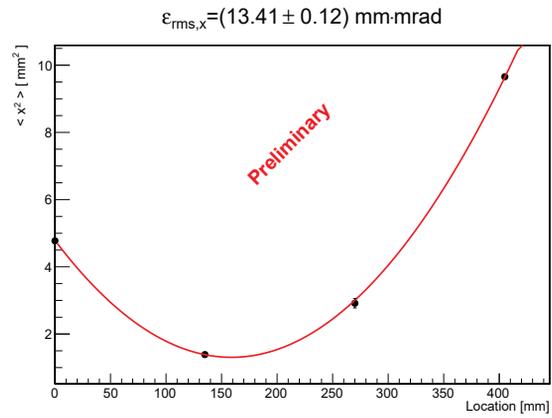


Figure 9: Variance as a function of the location obtained in the horizontal plane. The red line corresponds to the best fit.

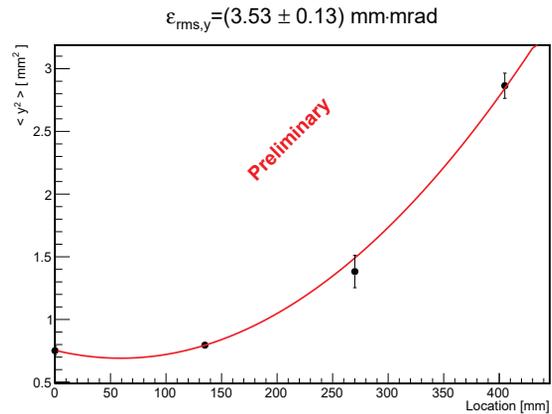


Figure 10: Variance as a function of the location obtained in the vertical plane. The red line corresponds to the best fit.

vertical plane. This is due to acceleration in the horizontal plane, which causes an increase of the particle position spread along the  $x$ -direction. The measured emittance values will be implemented in the simulation of the BTL to provide beams of different shapes and sizes for multi-disciplinary research activities.

Table 2: Fit Parameters and the RMS Emittance Values Obtained by Using Multiple Profilers for Both Horizontal and Vertical Planes

Fit parameter	Horizontal plane	Vertical plane
$\sigma_{11}$ [mm <sup>2</sup> ]	$4.79 \pm 0.09$	$0.75 \pm 0.04$
$\sigma_{12}$ [mm·mrad]	$-21.90 \pm 0.48$	$-1.06 \pm 0.19$
$\sigma_{22}$ [mrad <sup>2</sup> ]	$137.72 \pm 2.06$	$17.99 \pm 1.15$
$\tilde{\chi}^2$	0.47	0.76
$\epsilon_{rms}$ [mm·mrad]	$13.41 \pm 0.12$	$3.53 \pm 0.13$

**REFERENCES**

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