# Algorithm to Improve the Beta-Function Measurement and its Evaluation in Storage Rings Lattices

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## Introduction

The measurement of the Beta Function in accelerators is an important task during the commissioning, because all the properties of the focusing structure are described and calculated using the Twiss functions or Courant-Snyder parameters. In Storage Rings,

1. by taking the tune change obtained when varying the intensity of quadrupoles
2. by using the matrix response to fit the corresponding parameters
3. by shaking the beam to obtain a betatron motion In accelerators like the LHC, the Beta-Function measurement is done from the Phase Advance Measurement using the Transfer Matrix.

## Contribution

During the measurement of any optical quantity in an accelerator, it is expected to have a correspondence between the model and what is measured at the machine. The proposed new algorithm is to correlate both planes. It implies the following:

$$\frac{\cot\Delta\Phi_{1,2}^{x,ide}-\cot\Delta\Phi_{1,3}^{x,ide}}{\cot\Delta\Phi_{1,2}^{y,ide}-\cot\Delta\Phi_{1,3}^{y,ide}}=\frac{\cot\Delta\Phi_{1,2}^{x,mea}-\cot\Delta\Phi_{1,3}^{x,mea}}{\cot\Delta\Phi_{1,2}^{y,mea}-\cot\Delta\Phi_{1,3}^{y,mea}}$$

There exists an optical function that would be called $\rho$ from where the $\beta$ function can be measured.

$$\begin{aligned}\beta^{x,measure} &= \rho_z[\cot\Delta\Phi_{1,2}^{y,ide}-\cot\Delta\Phi_{1,3}^{y,ide}]\\ \beta^{y,measure} &= \rho_z[\cot\Delta\Phi_{1,2}^{x,ide}-\cot\Delta\Phi_{1,3}^{x,ide}]\end{aligned}$$

Additionally an optimization can be added

$$\rho_z=\beta^{z,ide}\frac{\tau(x,2)\pm\cot\Delta\Phi_{1,2}^{x,mea}}{\tau(y,2)\pm\cot\Delta\Phi_{1,2}^{y,mea}}$$

or

$$\rho_z=\beta^{z,ide}\frac{\tau(x,2)\pm\cot\Delta\Phi_{1,3}^{x,mea}}{\tau(y,2)\pm\cot\Delta\Phi_{1,3}^{y,mea}}$$

where $\tau(z,m)=\cot\Delta\Phi_{m-1,m}^{z,mea}-\cot\Delta\Phi_{m-1,m+1}^{z,mea}$ and its depends on how close the measure fraction is close to the ideal fraction.

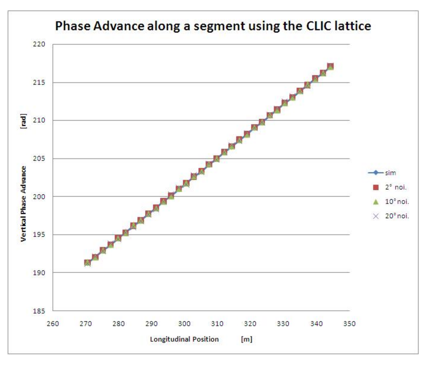
## Transfer Matrix Remark

$$\begin{pmatrix}\sqrt{\frac{\beta_f}{\beta_i}}(\cos\phi_{fi}+\alpha_i\sin\phi_{fi}) & \sqrt{\beta_f\beta_i}\sin\phi_{fi}\\ -\frac{1+\alpha_i\alpha_f}{\sqrt{\beta_f\beta_i}}\sin\phi_{fi}+\frac{\alpha_i-\alpha_f}{\sqrt{\beta_f\beta_i}}\cos\phi_{fi} & \sqrt{\frac{\beta_f}{\beta_f}}(\cos\phi_{fi}-\alpha_f\sin\phi_{fi})\end{pmatrix}$$

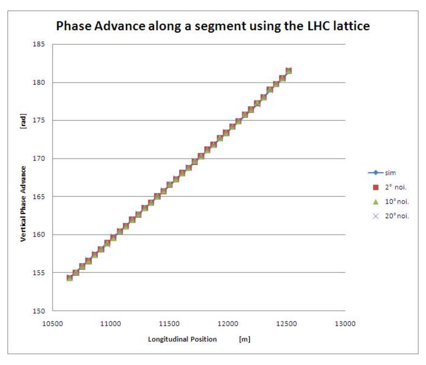
When applying to three adjacent BPMs, the value of the beta function at the first lattice element can be obtained as a function of the matrix elements, on the first row of the matrices that transfer from the element 1 to 2, and from 1 to 3, usually called $m_{11}$, $m_{12}$ ,$n_{11}$, and $n_{12}$, respectively.

## Results

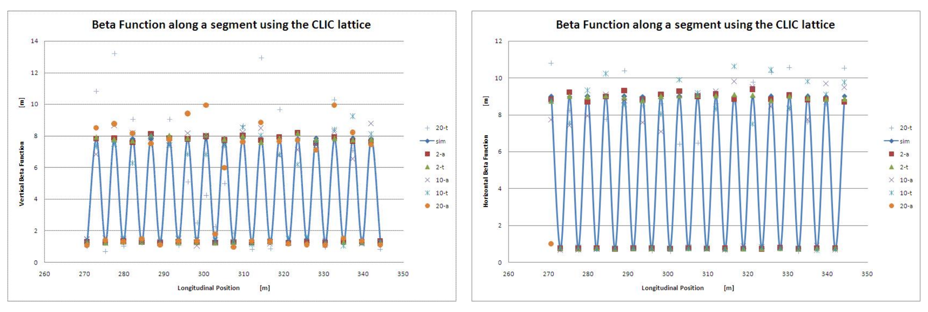
The proposed new algorimth is tested using the standard software MAD-X by a comparison with the traditional way to obtain the beta function measurement. Just a sector of each ring is used. For CLIC lattice with 35 BPMS. For the LHC, 39 BPMs.




Phase Advances, without and with noise for 2, 10 and 20$^o$. Y plane. CLIC.




Phase Advances, without noise and with noise of 2, 10 and 20$^o$ Y Phase. LHC.



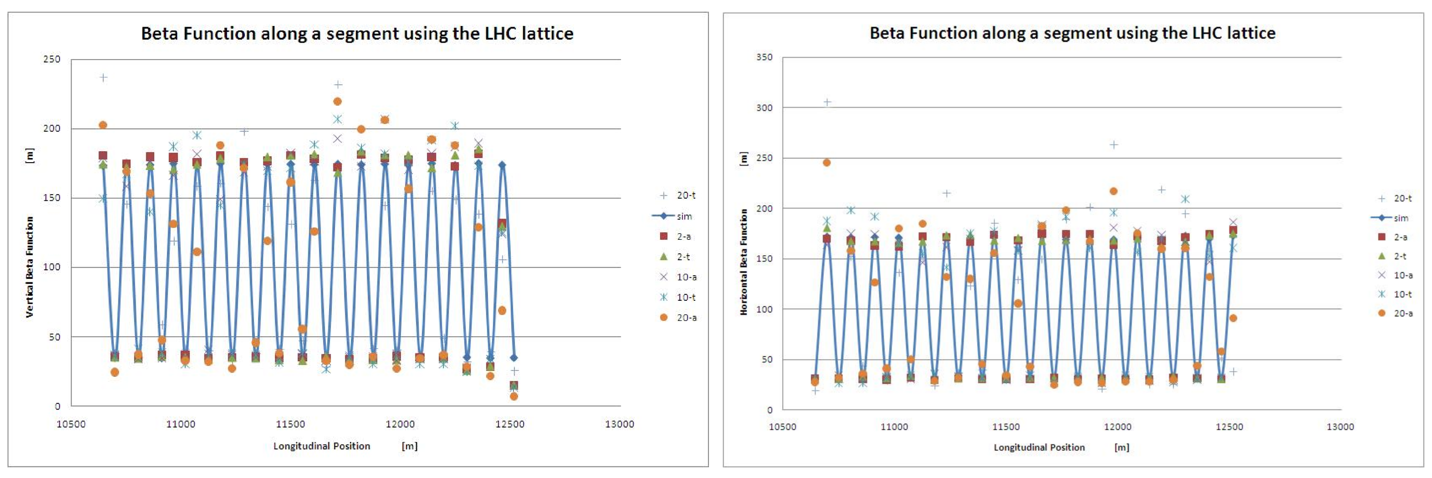

Beta Functions for a sector of the CLIC lattice, wou and w noise for 2, 10 and 20$^o$

For each beta measurement a relative error to the ideal beta is obtained. And for the entire segment a global relative error, denoted by $Err.$, is asociated to be the average of the errors at the different locations.

**Table 1.** Beta Fuction $Err.$ using CLIC lattice, and LHC lattice.

| Case. | $Err. < \beta_x >$ [%] | $Err. < \beta_y >$ [%] | Num. Orb. | $Err. < \beta_x >$ [%] | $Err. < \beta_y >$ [%] |
|---|---|---|---|---|---|
| 20-t | 14.0064 ± 0.47 | 21.5973 ± 1.1 | 10 | 23.9845 ± 1.6 | 21.0685± 0.53 |
| 20-a | 11.1688 ± 0.46 | 11.3702 ± 0.54 | 10 | 16.0208 ± 0.94 | 15.6167± 0.70 |
| 20-t | 14.0898 ± 0.20 | 21.0753 ± 0.37 | 100 | 22.8245± 0.53 | 22.7674 ±0.37 |
| 20-a | 10.4034 ± 0.14 | 10.4941 ± 0.14 | 100 | 16.7904± 0.45 | 16.3095± 0.37 |
| 20-t | 14.4705 ± 0.063 | 20.8487 ± 0.11 | 1000 | 22.3585± 0.16 | 22.7038± 0.12 |
| 20-a | 10.5313 ± 0.046 | 10.6460 ± 0.046 | 1000 | 16.9809± 0.15 | 16.5626± 0.11 |
| 10-t | 6.6708 ± 0.2369 | 10.1476 ± 0.35 | 10 | 10.4422± 0.43 | 11.2105± 0.65 |
| 10-a | 8.6858 ± 0.34 | 8.9647 ± 0.35 | 10 | 7.1806± 0.54 | 7.1872± 0.57 |
| 10-t | 6.9188 ± 0.093 | 9.9884 ± 0.14 | 100 | 10.3884± 0.16 | 11.0943± 0.17 |
| 10-a | 8.8234 ± 0.10 | 8.9866 ± 0.10 | 100 | 7.2590± 0.15 | 7.2989± 0.15 |
| 10-t | 6.8664 ± 0.029 | 10.0190 ± 0.050 | 1000 | 10.4041± 0.055 | 10.9358± 0.054 |
| 10-a | 8.7920 ± 0.031 | 8.9517 ± 0.031 | 1000 | 7.5054± 0.053 | 7.4269± 0.051 |
| 2-t | 1.4010 ± 0.062 | 1.9980 ± 0.11 | 10 | 1.9998± 0.075 | 2.1757± 0.12 |
| 2-a | 2.4296 ± 0.097 | 2.4272 ± 0.098 | 10 | 2.2319± 0.051 | 2.2607± 0.058 |
| 2-t | 1.3594 ± 0.018 | 2.0168 ± 0.031 | 100 | 2.0484± 0.033 | 2.1714± 0.033 |
| 2-a | 2.4009 ± 0.040 | 2.4002 ± 0.040 | 100 | 2.2017± 0.027 | 2.2133± 0.027 |
| 2-t | 1.3584 ± 0.0056 | 1.9917 ± 0.010 | 1000 | 2.0329± 0.010 | 2.1555± 0.010 |
| 2-a | 2.3827 ± 0.012 | 2.3823 ± 0.012 | 1000 | 2.2014± 0.0092 | 2.2127± 0.0090 |

Beta Functions, w. and wou. noise for 2, 10 and 20$^o$. LHC lattice.




With 10 noi. CLIC $Err$ ↑ by 20% compare for X-plane, and ↓ for Y-plane by 10%.
With 10 noi. LHC $Err$, ↓ 30%.
With 20 noi. ↓ % of 26 and 49 for CLIC, and 29 to 27 for LHC.

## A Future Direction

The algorithm introduced in this paper allows reducing the noise presented when performing the Beta Function measurement. In applications to the LHC Mad-X lattice, it is found cases where the improvement is close to 30% compare to the traditional one, when the noise is 10$^o$ or 20$^o$; using the CLIC Mad-X lattice a reduction close to 25% and 50% are observed with a noise of 20$^o$ . Althought further studies are needed to establish the ideal conditions for its application in real machines, this new algorithm could serve as a complement and/or improvement to the traditional technique.

## References

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