# **ELECTRON BEAM DIAGNOSTICS USING RADIATION FROM A FREE ELECTRON LASER**

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## Abstract

In most devices based on a high energy electron beam, which used for electromagnetic radiation production, great efforts are focused on the electron beam quality improvement. This is the case in a Free-Electron Laser (FEL) where electron beam with a low normalized emittance is required. Thus, diagnostic tools are required to investigate e-beam properties, such as beam emittance, longitudinal space charge, energy spread and velocity spread.

In this paper we present analysis of radiation measurements obtained from a pre-bunched e-beam FEL. The measurements were made for a wide range of frequencies and for beam currents from low currents to high currents, where space charge effects can not neglected. We apply a frequency domain formulation to analyze the measured radiation. The spectral signature of the radiation emission obtained from a pre-bunched e-beam can provide vital information on e-beam properties. We show that a rigorous analysis of the measured radiation, allows characterization of the e-beam parameters.

This analysis can provide some insights to the development of e-beam accelerators and radiation sources devices and to help physicists interpreting radiated signals.

# **INTRODUCTION**

In order to maintain a stable high energy electron beam based devices a quality electron beam is required. Devices such as FEL, Optical Klystron (OK), Self Amplified Spontaneous Emission (SASE) and Coherent Synchrotron Radiation (CSR) are based on the electron beam parameters and as a result great efforts are made to maintain and characterize the e-beam emittance, energy spread, velocity spread and brightness [1-4].

In addition, for short wavelengths radiation emission devices an ultra-short bunched electron beam should be applied. To achieve such electron beam, many techniques has been developed [5-7]. However, during the creation of such a bunched electron beam, space-charge effects can change the bunch profile, and can results in energy spread and velocity modulation of the electron beam. This energy spread can affect the profile of the coherent radiation emission and can result in radiation profile different than planed for the device.

In this paper we describe an analysis of radiation emission obtained from a pre-bunched electron beam and we will investigate the velocity modulation impact on the electron beam quality. This analysis can give crucial information about the influence of the velocity modulation on the electron beam quality.

### THE ANALYTICAL BASIS

order to get some insight from the In experimental results we compare the measured radiated power to the analytical model developed by Schnitzer and Gover [8]. The analytical model is based on cold beam fluid plasma and takes into account collective effects and initial conditions of e-beam current density and velocity modulation. The radiated power is characterized in the low and the high gain regimes, for a tenuous and for a dense e-beam.

The total radiated power at the end of the wiggler is given by  $P(L_w) = |C(L_w)|^2 \mathcal{P}_q$  where  $C(L_w)$  is the field amplitude coefficient and  $\mathcal{P}_q$  is the normalization power. The total radiated power at the output as expressed in [8] contains three terms:

$$P(L_w) = P(\theta) F_{FEL}(\theta, \theta_p) + P_B F_{PB}(\theta, \theta_p) + \sqrt{P(\theta)P_B} F_{SPB}(\theta, \theta_p)$$
(1)

vttribution 3.0 (CC-BY-3.0) where P(0) is the electromagnetic power injected into the interaction region.  $P_B$  is the prebunching power parameter [8] and  $F_{\Diamond}(\theta)$  are the detuning functions for different regimes of operation.  $\theta_p$  is

the plasma parameter (space charge parameter) and  $\theta$  is the detuning parameter  $(\theta = \omega / v_z - k_z - k_w)$ .

The first term on the right hand side (r.h.s.) of Eq. (1) describes the stimulated emission radiation in FELs in which an e-beam having randomly distributed energy and density modulation passes the interaction region and interacts with an injected electromagnetic wave. In that case no initial condition are introduced on the e-beam (no current ) density or velocity modulation,  $M_I = M_V = 0$ ). The second term on the r.h.s. of Eq. (1) describes the p additional radiation emission for the case where a current density and/or velocity modulation are introduced on the e-beam, but in the absence of an injected electromagnetic wave into the interaction 🥯 region (i.e. P(0) = 0). This term is called "Prebunched Beam" (PB) radiation or "Super-Radiance" (SR). This term is the main subject of this work. The third term on the r.h.s. of Eq. (1) describes the radiation emission for the case where both current density and/or velocity modulation are introduced on the e-beam and an electromagnetic wave is injected into the interaction region. This term is called "Stimulated Pre-bunched Beam" (SPB) radiation or "Stimulated Super-Radiance" (SSR).

The plasma parameter,  $\theta_p$ , characterizes the collective effects of the e-beam. As will be shown in the following, the plasma parameter describes the different radiation emission regimes for varying sets of e-beam parameters. For a very high-energy e-beam and for a tenuous e-beam the plasma parameter is negligible ( $\theta_p \approx 0$ ), and thus, no collective effects exist in the interaction process. On the other hand, for a dense electron beam or for a low energy e-beam, where the plasma parameter is  $\theta_p > \pi$ , the space-charge (collective) effects in the e-beam cannot be neglected, and  $\theta_p$  is very important in description of the interaction process. In the present work we do not neglect the space charge parameter, as it provides the tool for predicting the behaviour in the collective regime.

The FEL detuning function  $F_{PB}(\theta, \theta_p)$ , defined in Eq. (1), can be expressed in the form [8]:

$$F_{PB}(\theta,\theta_p) = (M_J\theta)^2 \frac{\left(\frac{\theta_p}{\theta}\sin\theta_p - \sin\theta\right)^2 + \left(\cos\theta_p - \cos\theta\right)^2}{\left(\theta^2 - \theta_p^2\right)^2} + M_V^2 \frac{\left(\frac{\theta}{\theta_p}\sin\theta_p - \sin\theta\right)^2 + \left(\cos\theta_p - \cos\theta\right)^2}{\left(\theta^2 - \theta_p^2\right)^2} + M_J M_V \left[2\sin\phi\sinc\theta_p \frac{\left(\cos\theta_p - \cos\theta\right)}{\theta^2 - \theta_p^2} - \cos\phi F_{FEL}(\theta,\theta_p)\right]$$
(2)

where  $\phi \equiv \phi_V - \phi_J$  and  $\phi_J$ ,  $\phi_V$  are the phases of the current density and velocity modulation, respectively, with respect to the ponderomotive phase.

The current modulation index  $M_J$  is defined as  $\widetilde{M}_J = \widetilde{J}_{1z}(0)/J_o = M_J \exp(i\phi_J)$  and the velocity modulation index  $M_V$  is defined as  $\widetilde{M}_V = M_V \exp(i\phi_V) = k L_w \widetilde{V}_{1z}(0)/V_{oz}$ . The phases  $\phi_J, \phi_V$  are the phases of the current density and velocity modulation, respectively, with respect to the ponderomotive wave phase.

In the collective regime synchronism with the fast and slow space charge wave is achieved if the condition  $\theta + \theta_p \cong 0$  and  $\theta - \theta_p \cong 0$  is satisfied respectively. We calculated the prebunched beam detuning function  $F_{PB}(\theta, \theta_p)$ , based on the analytical expression (Eq. (2)), for the TAU FEM experimental parameters (as per table 1), where an e-beam current of 1.5A, an e-beam energy of 70keV were assumed. The plasma parameter is  $\theta_n \cong 6.1$ . The results obtained from the analytical expression are plotted in Fig. 1. Note that the term which is proportional to  $M_i^2$  (solid curve) is  $\theta^2$ times larger than the term which is proportional to  $M_{\rm v}^2$  (dashed curve). As can be seen from the r.h.s. of Eq. (2) the first two terms (proportional to  $M_{i}^{2}$ and  $M_{\rm v}^2$ ) are symmetrical functions around  $\theta = 0$ (corresponds to  $f \approx 5 GHz$  in Fig. 2). The third term on the r.h.s. of Eq. (2) exists only if both the current density and the velocity modulation are introduced on the e-beam (proportional to  $M_J M_V$ ). We also note that only the third term depends on the relative phase  $\phi$ . Note that this term can be asymmetrical with respect to  $\theta$ . Thus, for the case where both current density and velocity modulation are introduced on the e-beam, the detuning function  $F_{PR}$  is not necessarily a symmetrical function of the detuning parameter  $\theta$  and depending on the relative phase  $\phi$ 



may exhibit asymmetry properties.

Figure 1: The calculated prebunched beam detuning function  $F_{PB}(\theta, \theta_p)$  based on the analytical expression (Eq. (2)). The solid curve is for  $M_J=0.2$  and  $M_V=0$ . The dashed curve is for  $M_V=0.2$  and  $M_J=0$ .

We also calculated the prebunched beam radiation power  $P_{PB}$  (or super-radiance power) for the same parameters used for Fig. 1 calculations. The obtained result is plotted in Fig. 2. The difference between the two maxima of the radiated power is due to the frequency dependence of the prebunching power parameter  $P_B$  (decrease with frequency [8]).



Figure 2: The calculated Prebunched Beam power (Super-Radiance)  $P_{PB}$  based on the analytical expression (Eq. (1)), for the TAU FEM experimental parameters (as per table 1 with I=1.5A,  $E_k=70 keV$  and  $\theta_n \approx 6.1$ ).

#### THE FEM EXPERIMENTAL SETUP

The gain and super-radiance power (prebunched e-beam) measurements were carried out on a unique, table-top Free Electron Maser (FEM) developed at Tel-Aviv University (TAU) [9]. A schematic of the experimental setup is shown in Fig. 3. The very wide range of FEM operating parameters which are possible is given in table 1. The pre-modulated electron beam is derived from a modified Travelling Wave Tube (TWT) prebuncher. A Pierce-type electron gun used in the prebuncher allows attainment of a maximal e-beam current of 1.5A. The electron beam modulation frequency and the current density modulation level  $M_J$  are set by adjustment of the r.f. input signal to the TWT prebuncher ( $P_{RFin}$ ).



Figure 3: Schematic illustration of the Travelling Wave prebunched beam FEM.

# EXPERIMENTAL INVESTIGATION OF THE VELOCITY MODULATION PARAMETER AND OF ITS RELATIVE PHASE

The super-radiance power from a prebunched e-beam was measured for a wide range of e-beam currents. In the space charge dominated regime  $(\theta_p \ge \pi)$  it was found that

the maximal radiated power, of the fast and of the slow space charge waves, are not equal. Assuming current density modulation only due to prebunching in analytical and computer simulations  $(M_J \neq 0)$  does not results in a good correspondence between the calculated and the experimental results; (the calculations do not predict unequal maximal radiated power for the fast and for the slow space charge waves in the low gain regime). Good agreement with experimental data is obtained only if one assumes that a velocity modulation  $(M_V)$  exists in the ebeam having a relative phase of  $(\phi)$  with respect to the current density modulation (see Fig. 4).



Figure 4: Comparison of measured (+ sign) prebunched beam power vs. frequency with analytical calculations (line) for parameters as per table 1 and for  $M_J = 0.1$ . The analytical curve is plotted for an e-beam current of 0.87A, an energy of 68.3keV and a phase difference of  $\phi \approx 0.7\pi$ (instead of the measured 0.93A and 70keV).

Table 1: The Parameters of the Prebunched Beam FEM

Electron beam energy	55-70 keV
Electron beam current	0.1-1.2 A
Prebuncher frequency band	$3 \text{GHz} \leq f_m \leq 12 \text{GHz}$
Prebuncher input power	$0 \le P_{RF_{in}} \le 3W$
Wiggler magnetic field	300 Gauss
Wiggler period	4.44 cm
Number of periods	Nw =17
Mode	<i>TE</i> <sub>10</sub>
Waveguide cross-section	2.215 cm×4.755 cm

The current density modulation  $(M_J)$  can be controlled by adjustment of the r.f. power to the input of the TW prebuncher. However, the velocity modulation depth  $(M_V)$ and the relative phase  $(\phi)$  between the current density modulation and the velocity modulation cannot be adjusted and controlled externally. The two parameters  $M_V$  and  $\phi$ , which we input to the numerical calculations, were chosen so as to obtain best correspondence with measured data. From comparisons of measured and calculated results it was found that if a velocity modulation of about 0.05% and a relative phase of  $0.7\pi$ - $1.3\pi$  is used in the calculations best fit with measurements is obtained. In the space charge dominated regime  $(\theta_p \ge \pi)$  the prebunched beam detuning function  $F_{PB}(\theta, \theta_p)$  reaches maximal values for synchronism with the slow plasma wave  $(\theta + \theta_p \approx 0)$  and with the fast plasma wave  $(\theta - \theta_p \approx 0)$ . In the limit  $\theta + \theta_p \approx 0$  the detuning function  $F_{PB}(\theta, \theta_p)$  (Eq. (2)) is reduced to [8]:

$$F_{PB}(\theta \pm \theta_{p} \approx 0) = \left(\frac{M_{J}}{2}\right)^{2} \left[l + sinc^{2}\theta_{p} + 2sinc(2\theta_{p})\right] + \left(\frac{M_{V}}{2\theta_{p}}\right)^{2} \left[l + sinc^{2}\theta_{p} - 2sinc(2\theta_{p})\right] + M_{J}M_{V} \left[sinc^{2}\theta_{p} sin\phi \mp \frac{1}{2\theta_{p}} \left(l - sinc^{2}\theta_{p}\right)cos\phi\right]$$
(3)

In each experiment we set the current density modulation level  $M_J$  and the e-beam current (i.e. the space charge parameter  $\theta_p$ ). Using the chosen values of  $M_J$  and  $\theta_p$  in Eq. (3) gives two equations for the two unknowns  $(M_V, \phi)$ , which can be easily solved. The prebunched beam power  $P_{PB}(\theta, \theta_p) = P_B(\theta) F_{PB}(\theta, \theta_p)$  is obtained from Eq. (1) with  $P(\theta) = \theta$ . The prebunched power parameter  $P_B(\theta)$  can be calculated at the two maximal radiated power levels. Using this calculated  $P_B(\theta)$  value in the last expression gives the value of  $F_{PB}(\theta \pm \theta_p \approx \theta)$ for the fast and for the slow space charge waves.

We calculate the detuning function based on the experimental results as plotted in Fig. 5 (the prebunched beam power vs. detuning parameter  $\theta$ ). Based on the experimental parameters used for this experiment we calculate the plasma frequency parameter  $(\theta_p \cong 3.46)$ . The maximum power corresponds to the slow space charge wave is  $P_{PB} = 4.72 W$  and is obtained at frequency of 4.48GHz. The calculated prebunched power parameter corresponding to this maximum is  $P_B = 976.76 W$ . Using these values in Eq. (1) gives  $F_{PB} = 5.21 \cdot 10^{-3}$ . The maximum power corresponds to the fast space charge wave is  $P_{PB} = 1.67 W$  and is obtained at frequency of 5.12GHz. The calculated prebunched power parameter corresponding to this maximum is  $P_B = 817.63 W$ . Using these values in Eq. (1) gives  $F_{PB} = 2.04 \cdot 10^{-3}$ . Substituting  $F_{PB} = 5.21 \cdot 10^{-3}$  and  $F_{PB} = 2.04 \cdot 10^{-3}$  in Eq. (3) and solving for  $M_V$  and  $\phi$  we found:  $M_V=0.07\%$ and  $\phi \approx 0.75\pi$ . Those values of  $M_V$  and  $\phi$  are closer to those assumed for best fit between measured results and the analytical model.



Figure 5: Measured prebunched beam power vs. detuning parameter  $\theta$  for TAU FEM parameters as per table 1, for an e-beam current of 0.93A, an e-beam energy of 70keV and for a current density modulation  $M_I = 0.1$ .

#### SUMMARY AND CONCLUSIONS

In this work we have shown that e-beam parameters can be obtained from comparison of analytical model and measurements. We showed that the velocity modulation of the e-beam can substantially affects the radiation emission scheme. This experimental investigation can contribute to the analysis of devices based on modulated e-beam in the space charge dominated regime.

#### REFERENCES

- E. L. Saldin, et al., Phys. Rev. ST Accel. Beams 9 (2006) 050702.
- [2] V. Ayvazyan, et al., Phys. Rev. Lett. 88 (2002) 104802.
- [3] E. Chiadroni, et al., Appl. Phys. Lett. 102 (2013) 094101.
- [4] P. Emma, et al., Nature Photonics 4 (2010) 641.
- [5] F. Stulle, et al., Phys. Rev. ST Accel. Beams 10 (2007) 031001.
- [6] R. Ganter, et al., Phys. Rev. Lett. 100 (2008) 064801.
- [7] R. Kuroda, et al., Nucl. Instr. and Meth. A, 593, (2008) 91.
- [8] Schnitzer I., Gover A., Nucl. Instr. and Meth. A, 237, (1985) 124.
- [9] M. Arbel, et al., Phys. Rev. Lett. 86 (2001) 2561.