Recent progress in SR interferometer

-for small beam size measurement-

T. Mitsuhashi, KEK

Agenda

1. Brief introduction of beam size measurement through SR interferometry.

2. Theoretical resolution of interferometry

3. Reflective interferometer for measurement of beam size down to 5μm range.

4. Imbalanced input method for measurement of very small beam size less than 5μm

1. A brief introduction to beam size measurement through SR interferometry

To measure a size of object by means of spatial coherence of light (interferometry) was first proposed by H. Fizeau in 1868!

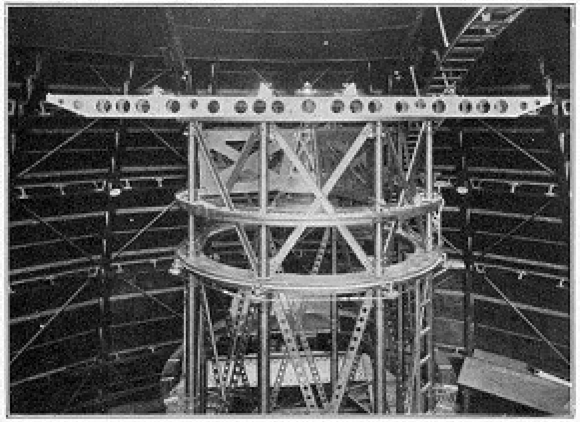
This method was realized by A.A. Michelson as the measurement of apparent diameter of star with his stellar interferometer in 1921.

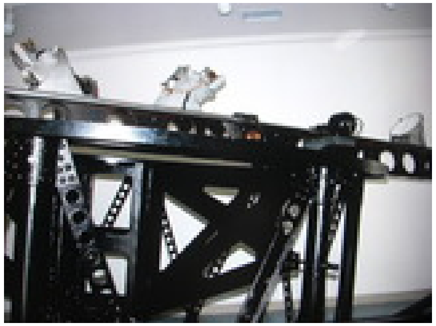
This principle was now known as “ Van Cittert-Zernike theorem” because of their works;

1934 Van Cittert

1938 Zernike.

Michelson's stellar interferometer
Wilson mountain observatory





Spatial coherence and profile of the object
Van Cittert-Zernike theorem

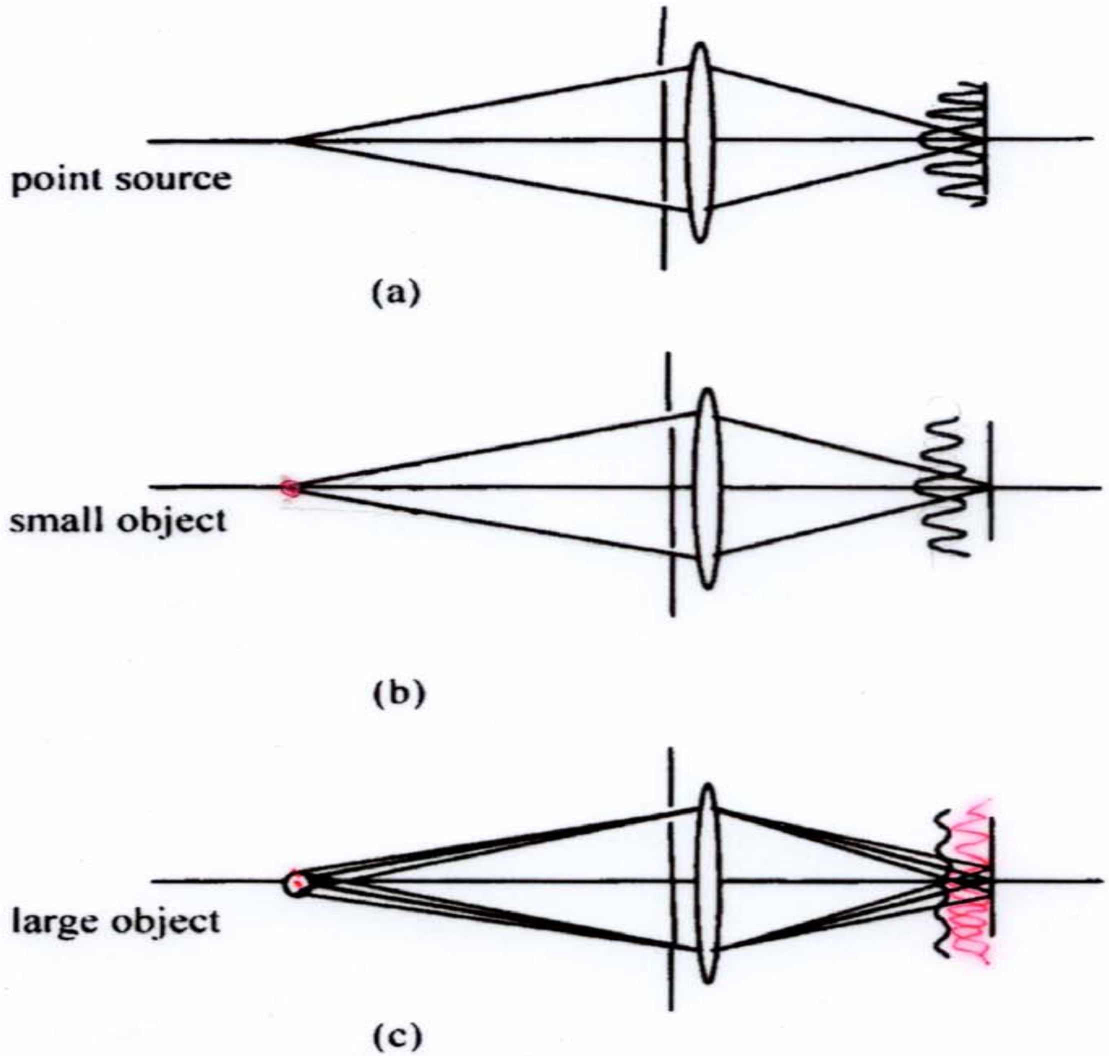
According to van Cittert-Zernike theorem, with the condition of light is 1st order temporal incoherent (no phase correlation), the complex degree of spatial coherence $\gamma(\upsilon_x,\upsilon_y)$ is given by the Fourier Transform of the spatial profile f(x,y) of the object (beam) at longer wavelengths such as visible light.

$$\gamma(\upsilon_x, \upsilon_y) = \int\int f(x,y) \exp\{-i \cdot 2 \cdot \pi(\upsilon_x \cdot x + \upsilon_y \cdot y)\} dxdy$$

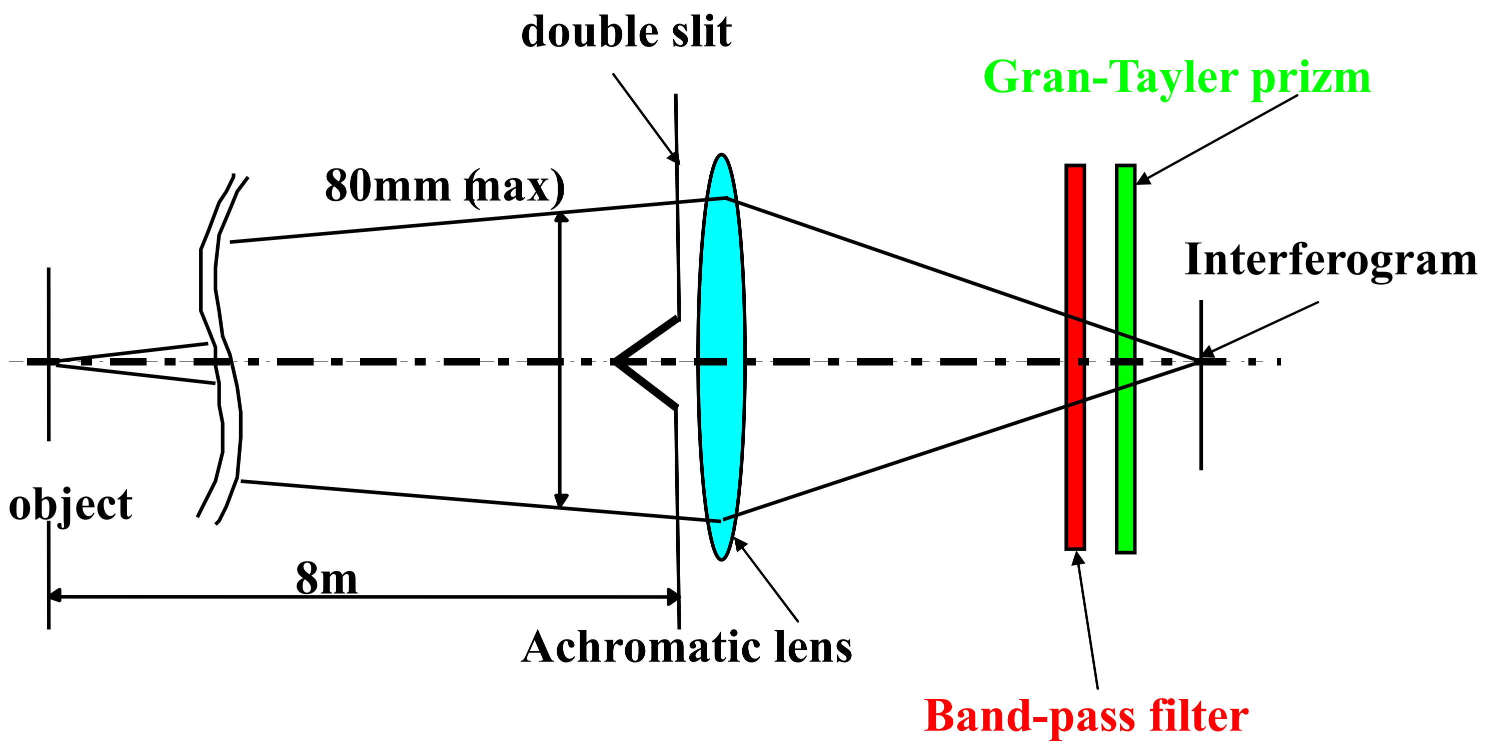
where υ_x,υ_y are spatial frequencies given by;

$$\upsilon_x = \frac{D_x}{\lambda \cdot R_0}, \quad \upsilon_y = \frac{D_y}{\lambda \cdot R_0}$$

Simple understanding of van Cittert-Zernike theorem



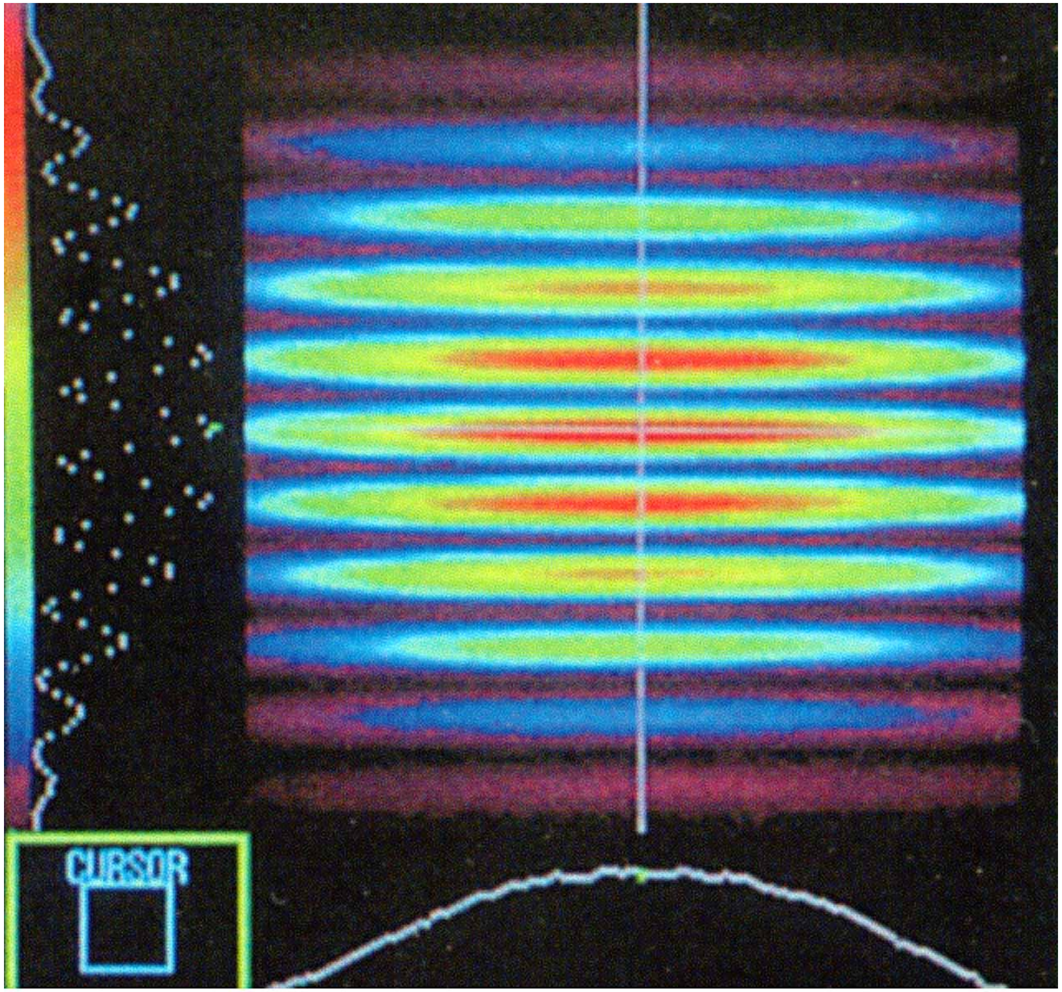
Typical arrangement for refractive interferometer



$$I(y,D)=\int (I_1+I_2)\cdot\left\{\sin c\left(\frac{\pi\cdot a\cdot y\cdot\chi(D)}{\lambda\cdot f}\right)\right\}^2\cdot\left\{1+\gamma\cdot\cos\left(k\cdot D\cdot\left(\frac{y}{f}+\psi\right)\right)\right\}d\lambda$$

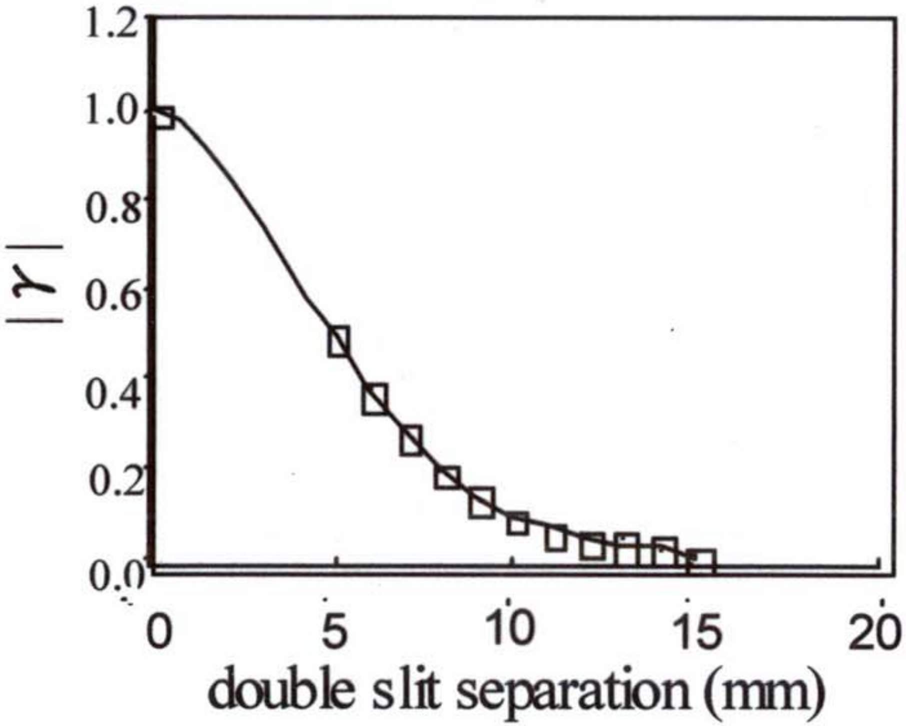
$$\gamma=\left(\frac{2\cdot\sqrt{I_1\cdot I_2}}{I_1+I_2}\right)\cdot\left(\frac{I_{max}-I_{min}}{I_{max}+I_{min}}\right)\quad,\quad \psi=\tan^{-1}\frac{S(D)}{C(D)}$$

Typical interferogram in vertical direction at the Photon Factory (1994).
D=10mm

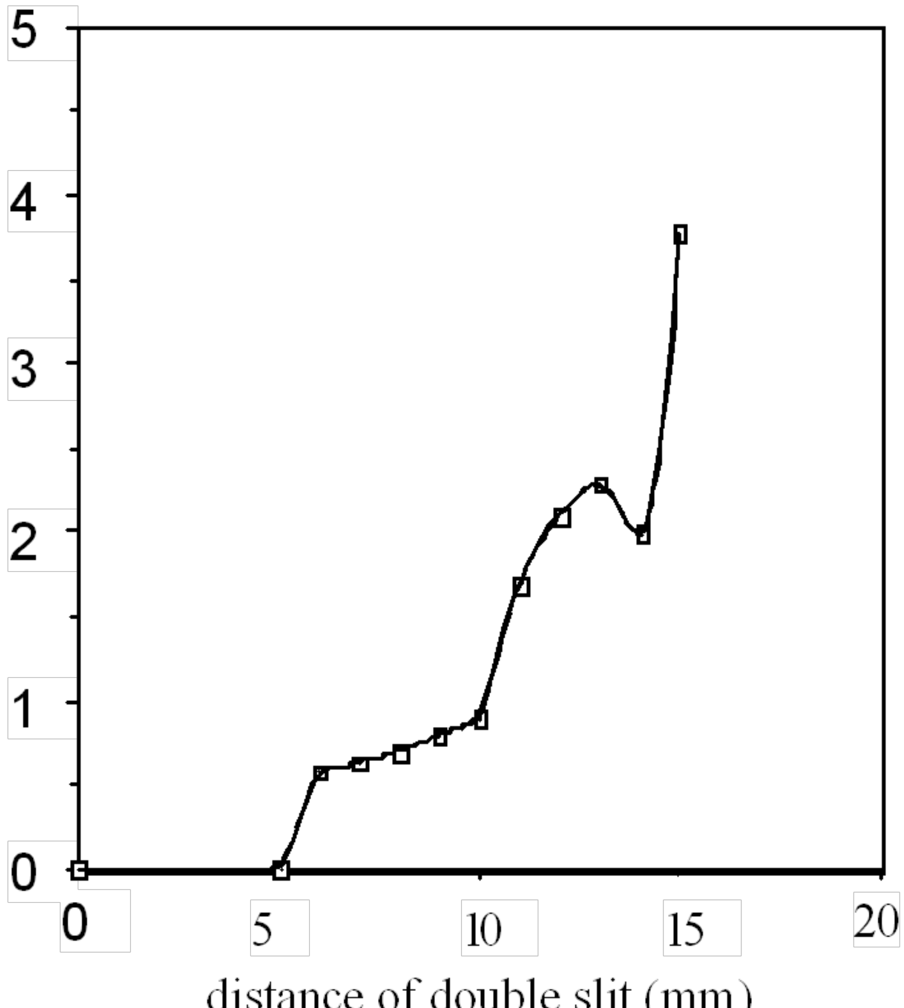


Result of spatial coherence measurement

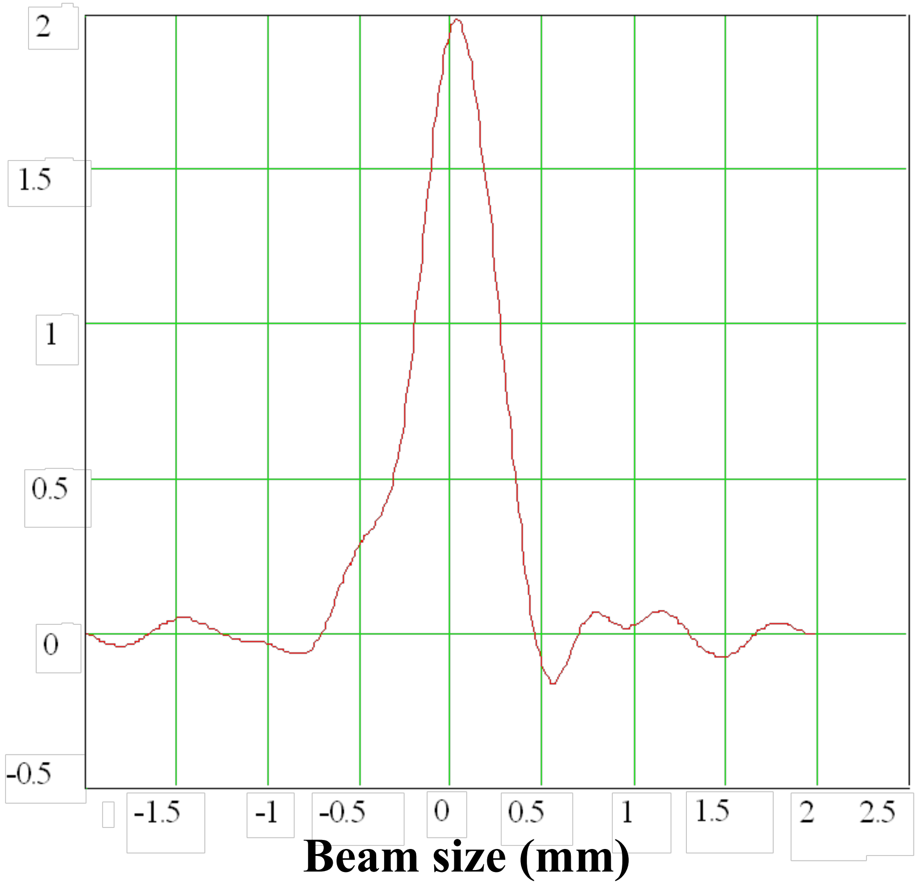
Absolute value of the complex degree of spatial coherence me asured at the Photon Factory KEK. (1994)



Phase of the complex degree of spatial coherence
vertical axis is phase in radian



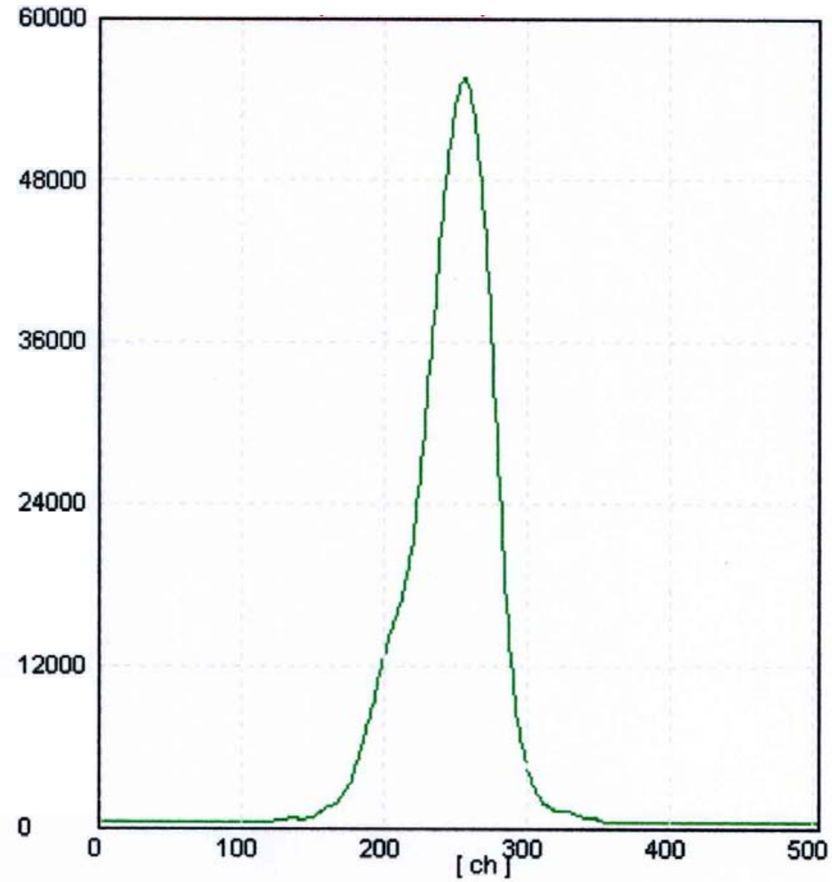
Reconstruction of beam profile by Fourier transform



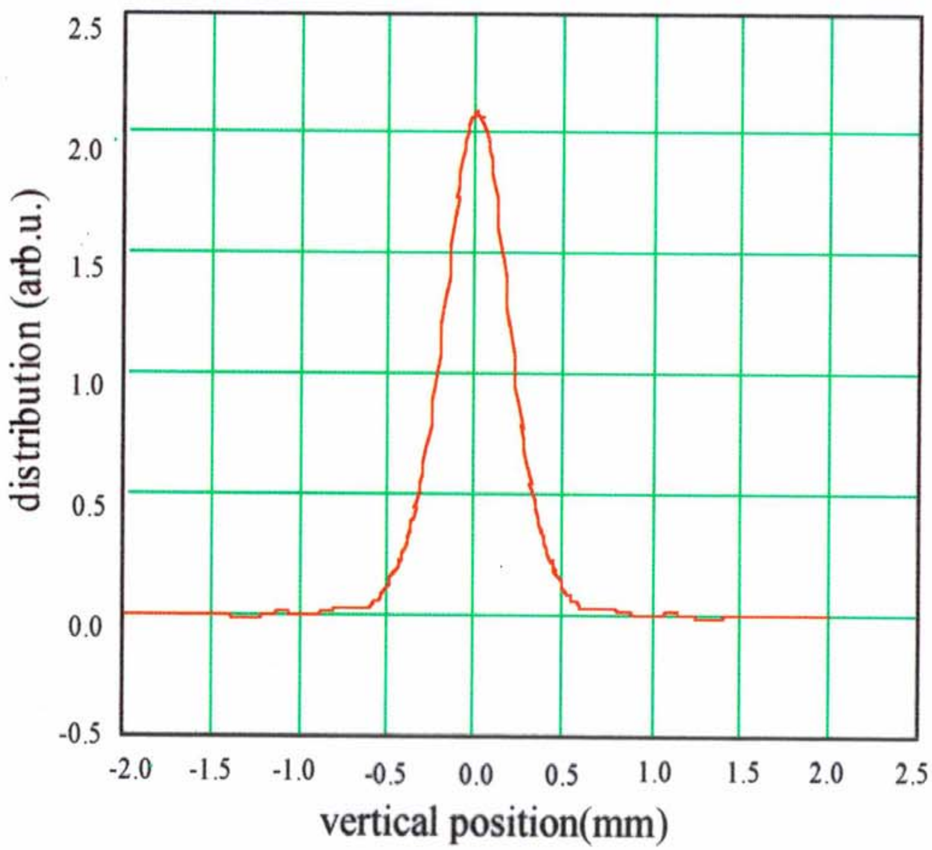
Vertical beam profile obtained by a Fourier transform of the complex degree of coherence.

Comparison between image

Beam profile taken with an imaging system



Vertical beam profile obtained by Fourier Cosine transform



SMALL BEAM SIZE MEASUREMENT

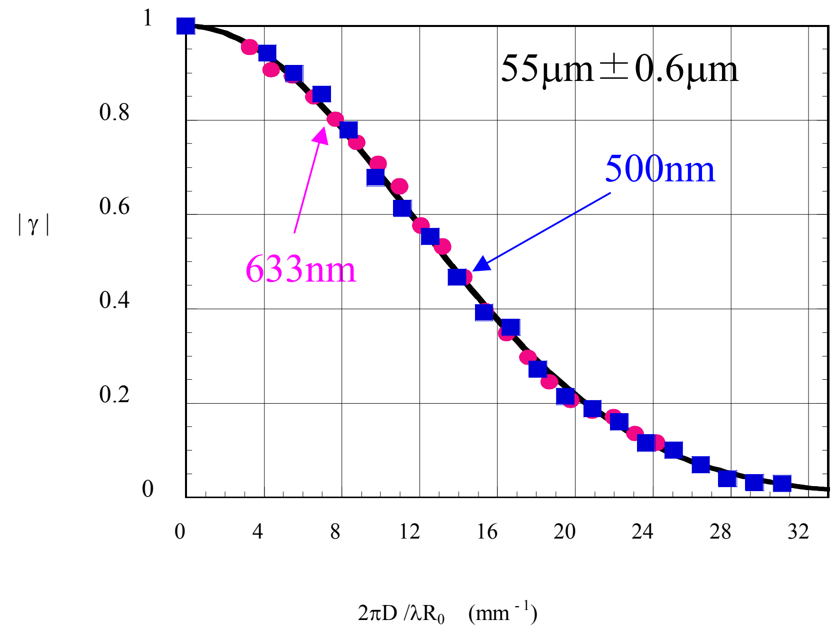
We often approximate the beam profile with a Gaussian shape. A spatial coherence is also given by a Gauss function. We can evaluate a RMS width of spatial coherence by using q least-squares analysis. The RMS beam size σ_{beam} is given by the RMS width of the spatial coherence curve σ_γ as follows:

$$\sigma_{beam} = \frac{\lambda \cdot R}{2 \cdot \pi \cdot \sigma_\gamma}$$

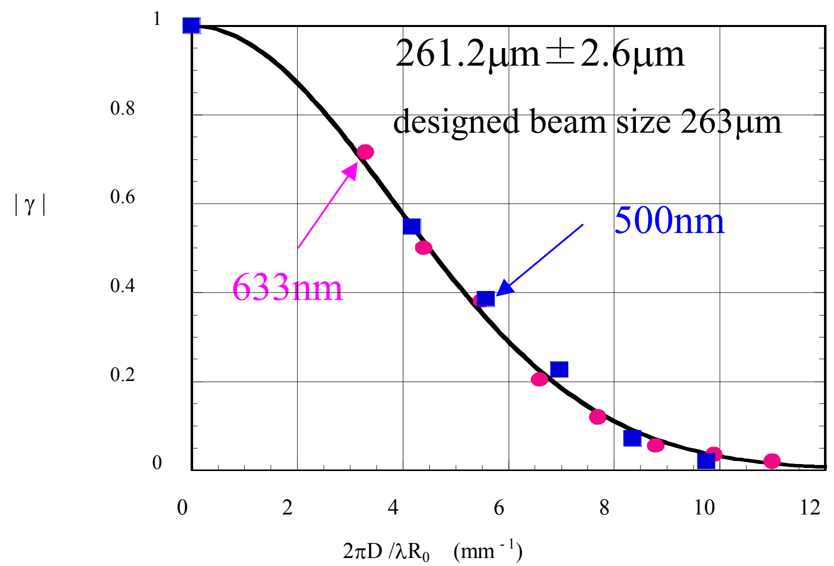
where R : distance between the beam and the double slit.

λ : wave length

Vertical and horizontal beam size at the Photon Factory



(a) vertical



(b) horizontal

We can also evaluate the RMS. beam size from one data of visibility, which is measured at a fixed separation of double slit. The RMS beam size σ_{beam} is given by ,

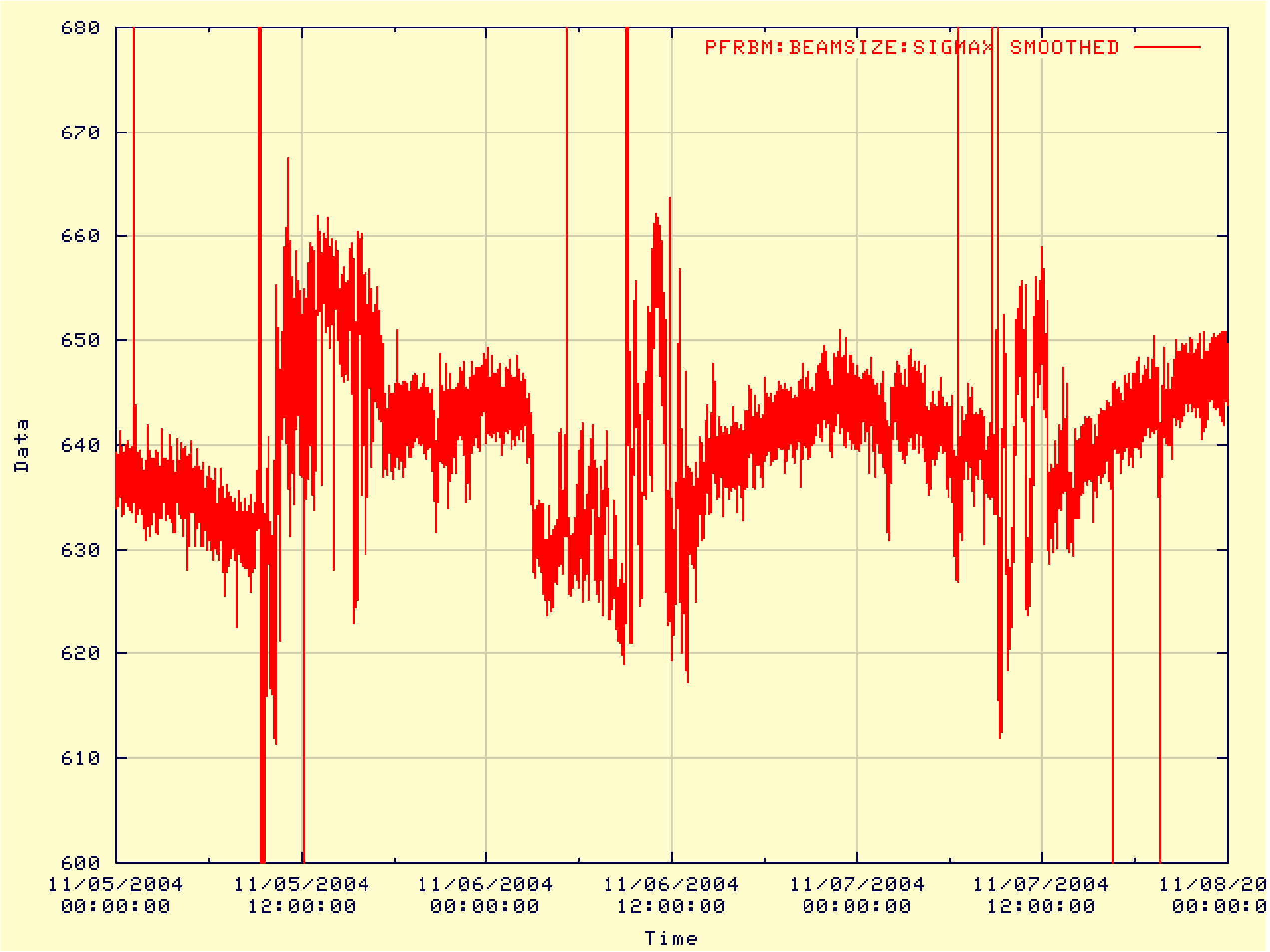
$$\sigma_{beam} = \frac{\lambda \cdot R_0}{\pi \cdot D} \cdot \sqrt{\frac{1}{2} \cdot \ln\left(\frac{1}{\gamma}\right)}$$

where γ denotes the visibility, which is measured at a double slit separation of D.

To consider that in the case to make an image, the resolution is limited by diffraction which is a Fourier transform using a given region of spatial frequency space (measurement in the real space).

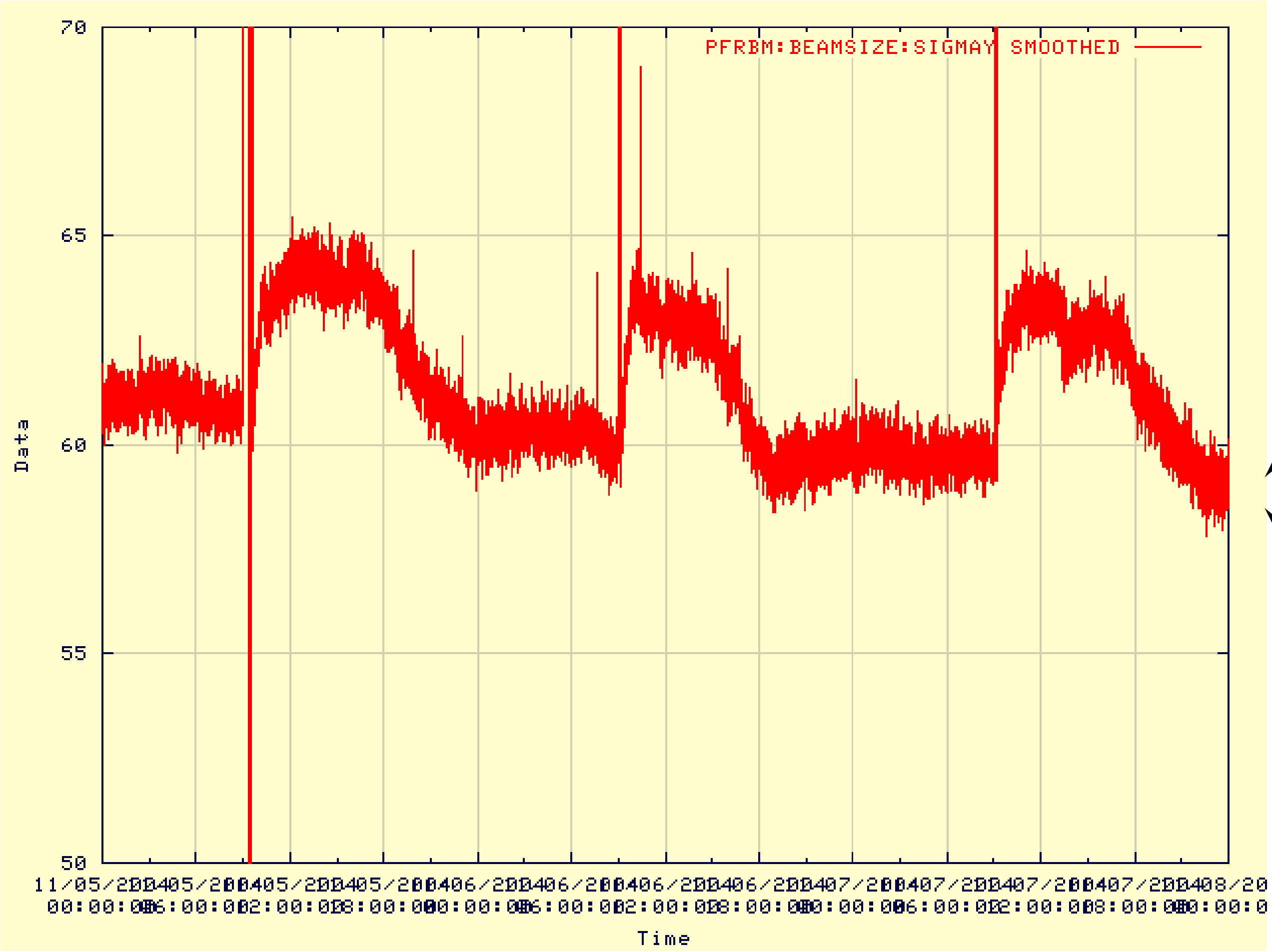
In the case of interferometry, we can measure a small beam size with limited region of spatial frequency space by means of these two methods (measurement in the inverse space).

Horizontal beam size measurement



±3μm

Vertical beam size measurement

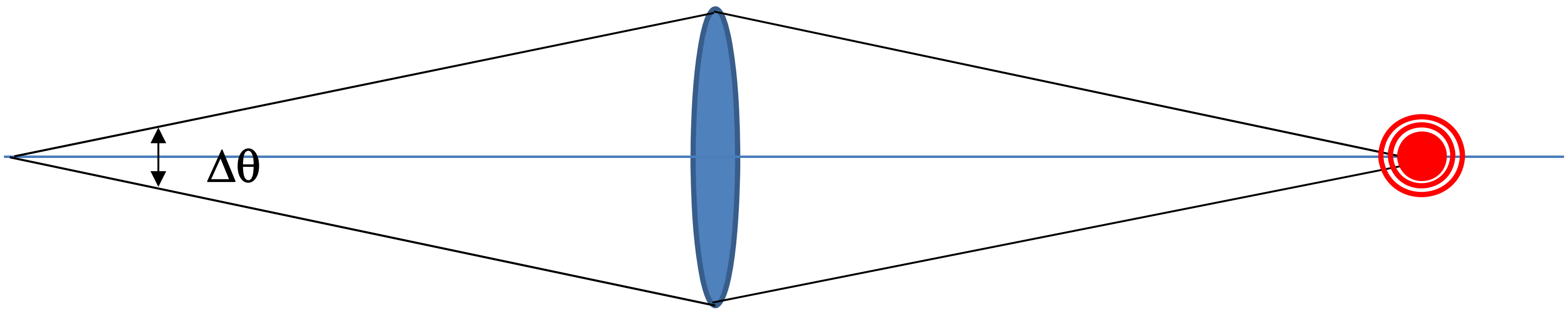


±1μm

2.Theoretical resolution of interferometry

Uncertainty principle in phase of light

Uncertainty principal in imaging.

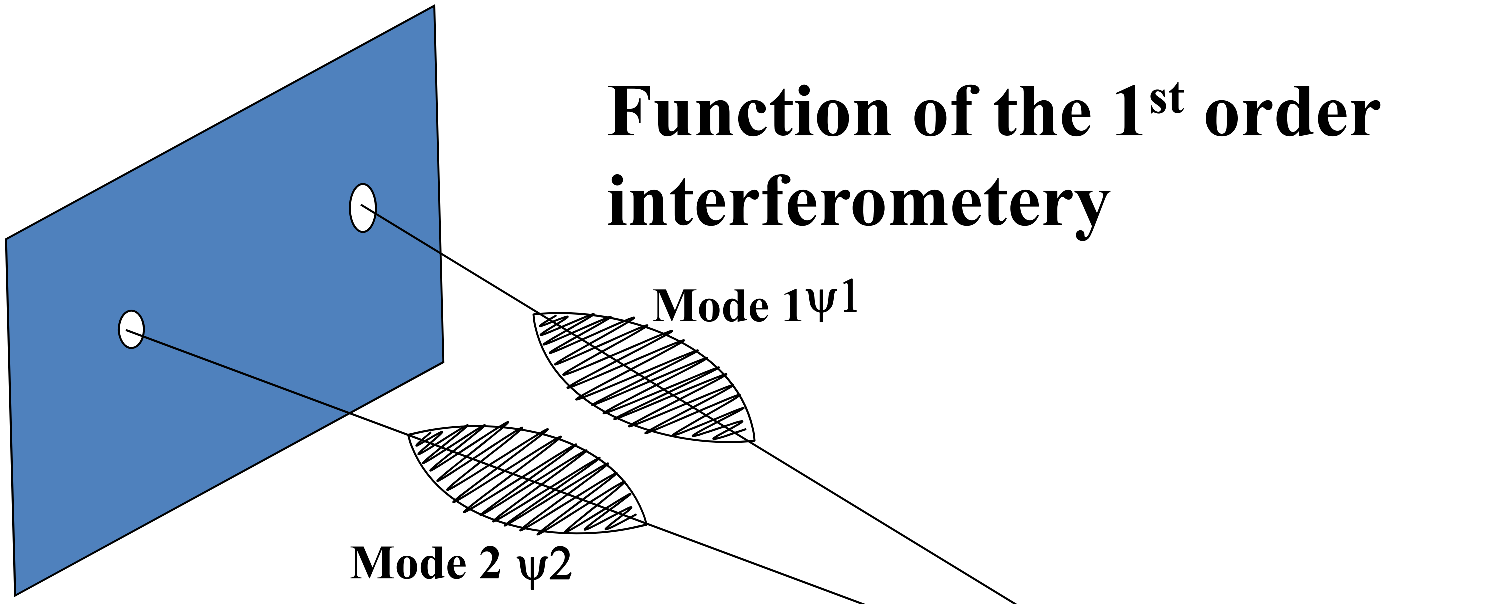


$$\Delta\theta/\lambda\cdot\Delta x\geq 1,$$

So, large opening of light will necessary to obtain a good spatial resolution.

Uncertainty principal in interferometry ?

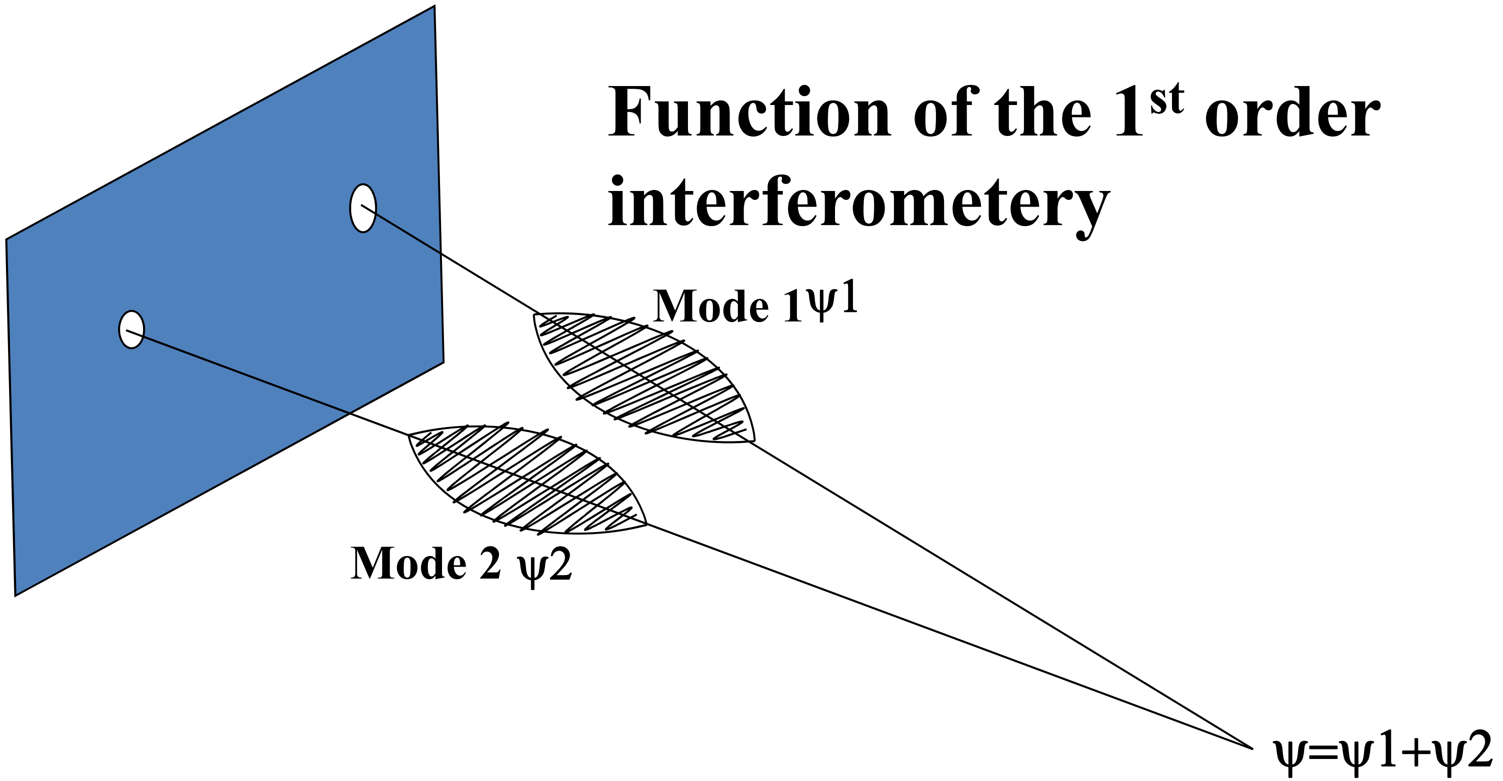
Uncertainty principal in interferometry



Function of the 1st order interferometery

Measure the correlation of light phase in two modes

Uncertainty principal in interferometry



Function of the 1st order interferometery

Measure the correlation of light phase in two modes ⇒ **Uncertainty in Phase $\Delta\phi$**

The interference fringe will be smeared by the uncertainty of phase.

$$I(y,D)=\int_{\Delta\phi}(I_1+I_2)\cdot\left\{\operatorname{sinc}\left(\frac{\pi\cdot a\cdot y}{\lambda\cdot f}\right)\right\}^2\left\{1+\cos\left(k\cdot D\frac{y}{f}+\phi\right)\right\}d\phi$$

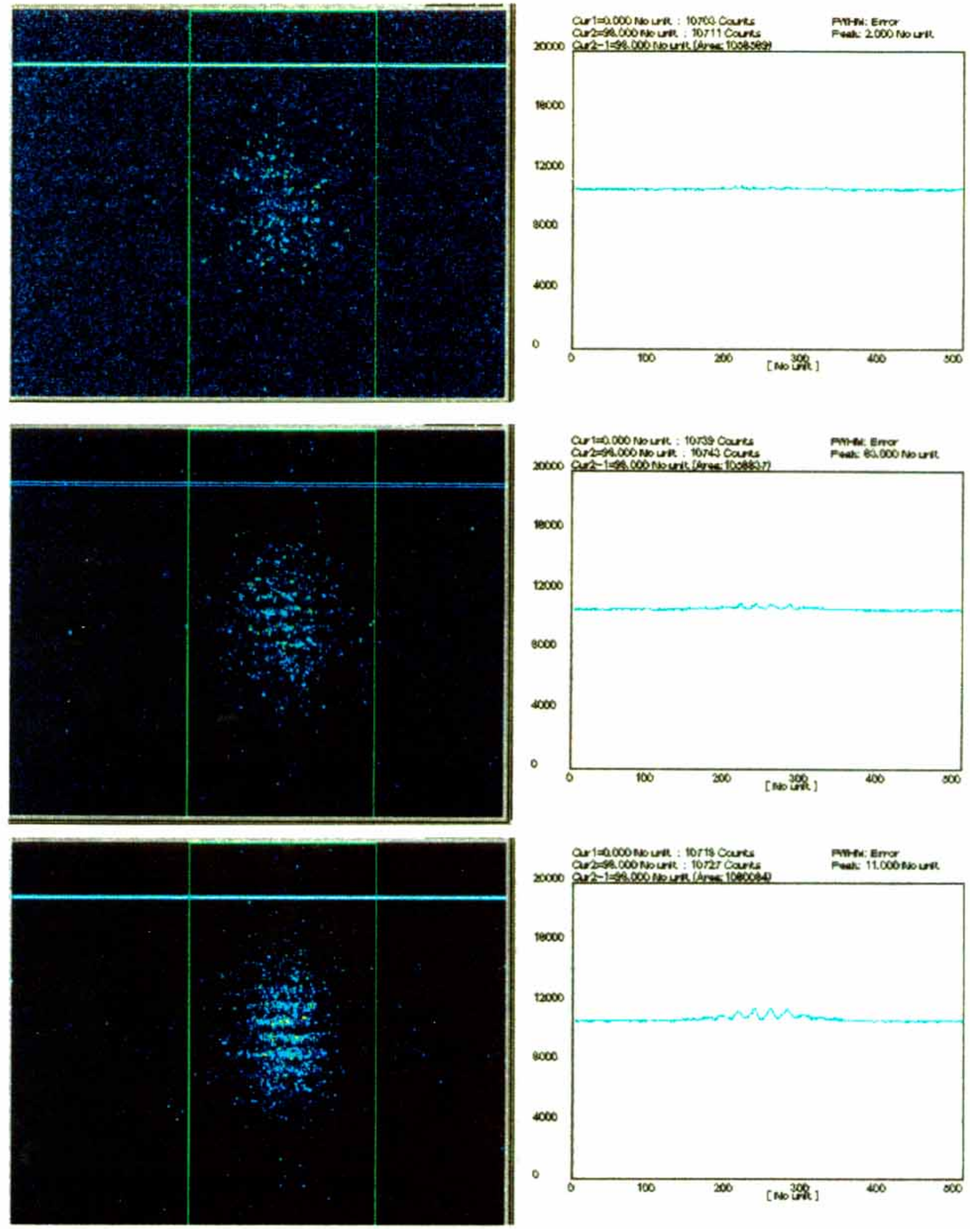
According to quantum optics,

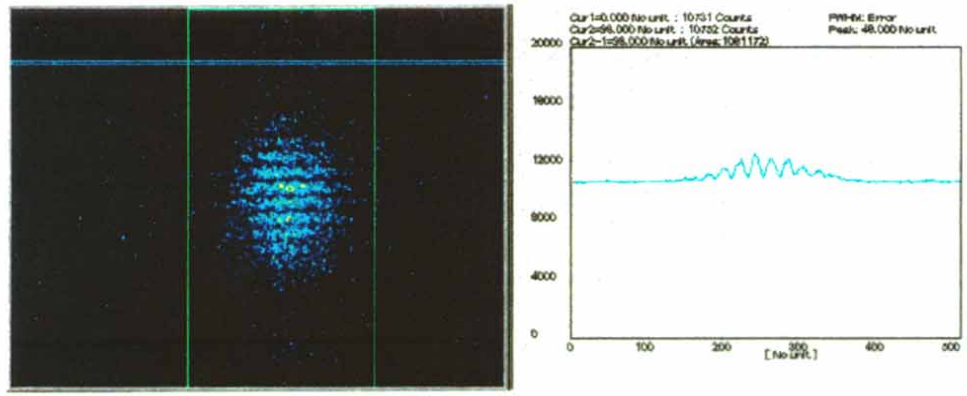
Uncertainty principle concerning to phase is given by

$$\Delta\phi \cdot \Delta N \geq 1/2$$

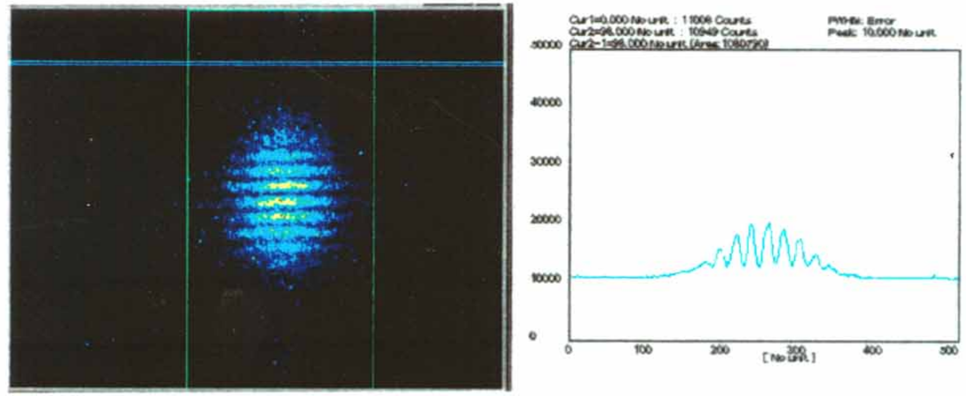
where ΔN is uncertainty of photon number.

We cannot observe interference fringe with small number of photons!

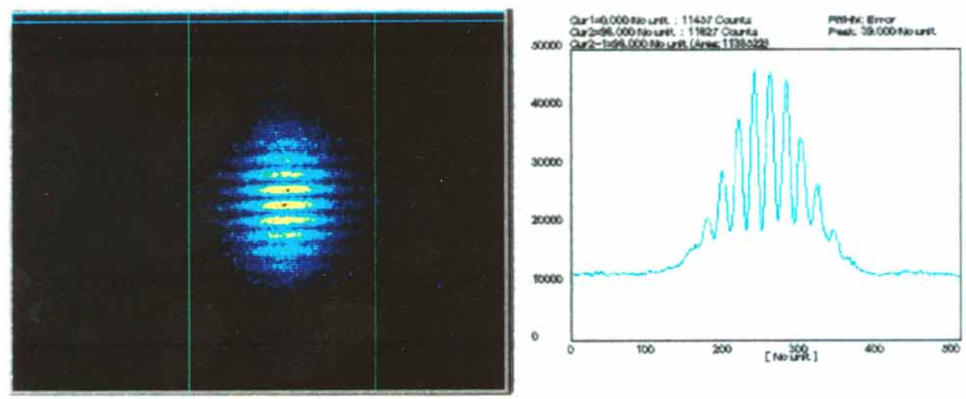




Cur1=0.000 No unit : 10731 Counts
Cur2=96.000 No unit : 10732 Counts
FWHM: Error
Peak: 48.000 No unit
20000
16000
12000
8000
4000
0
0
100
200
300
400
500
[No unit]



Cur1=0.000 No unit : 11006 Counts
Cur2=96.000 No unit : 10949 Counts
FWHM: Error
Peak: 10.000 No unit
50000
40000
30000
20000
10000
0
0
100
200
300
400
500
[No unit]



Cur1=0.000 No unit : 11437 Counts
Cur2=96.000 No unit : 11827 Counts
FWHM: Error
Peak: 39.000 No unit
50000
40000
30000
20000
10000
0
0
100
200
300
400
500
[No unit]

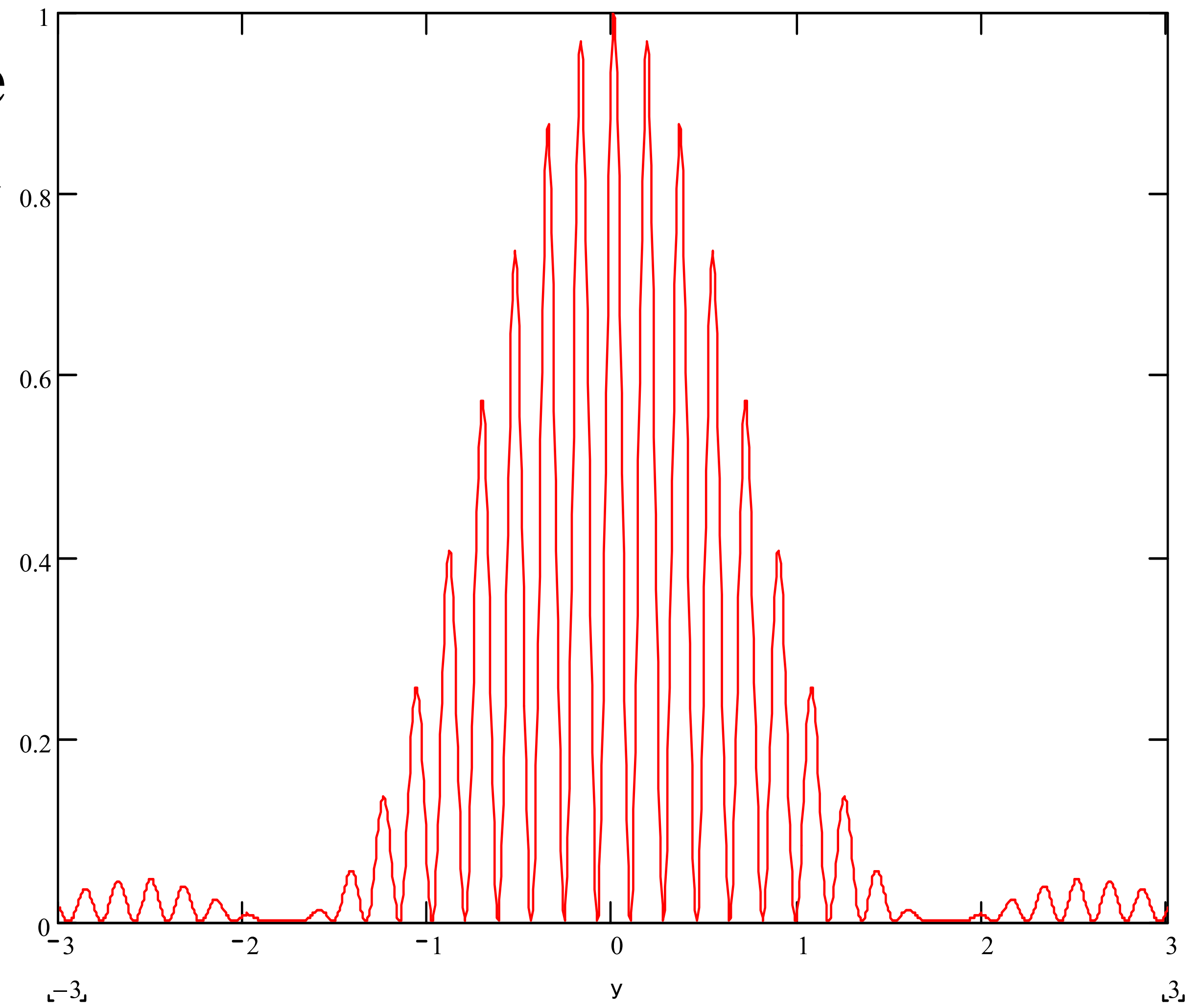
Actually, different from imaging, we can use large number of photons (intensity), so uncertainty in phase is very small (this is the reason light seems wave)

A comparison between imaging, we can use large number of photons (intensity), so uncertainty in phase is very small (this is the reason light seems wave)

As a result, theoretical resolution is very high, and practically resolution will be limited by measurement error such as baseline noise in detector.

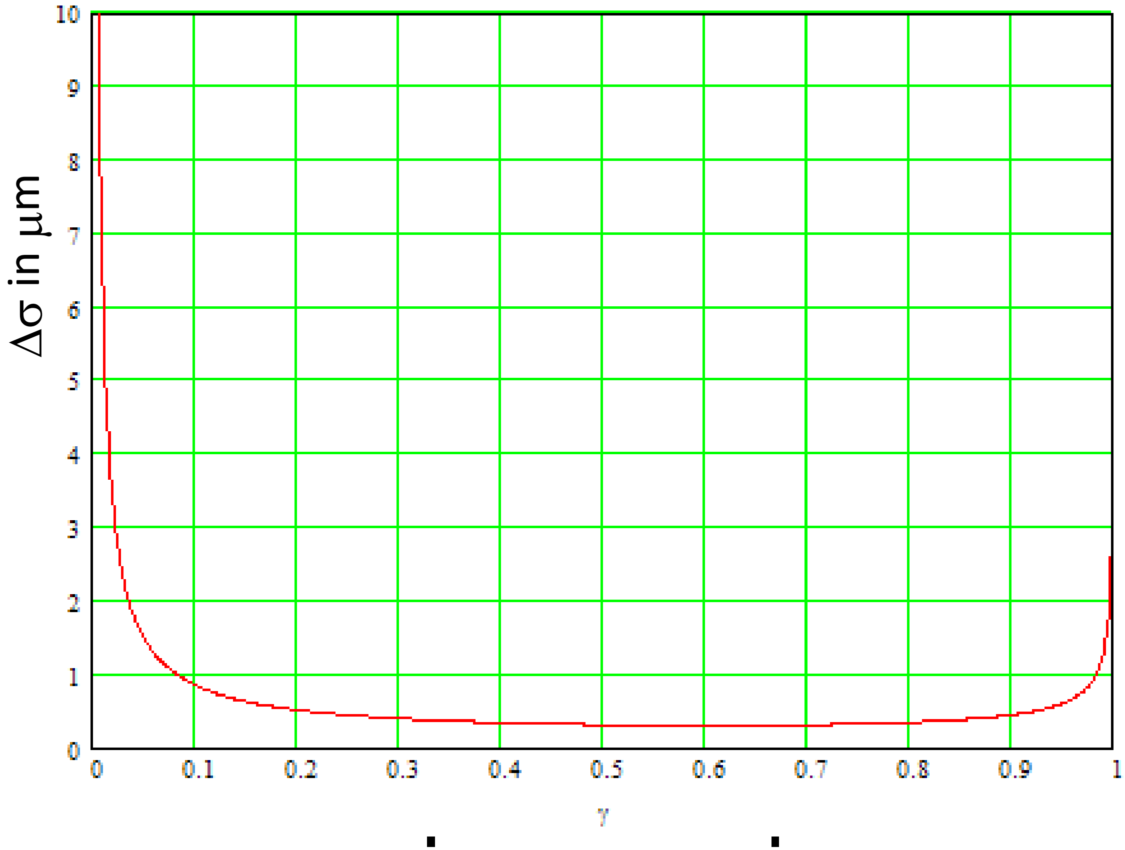
Small size of the beam will give a good visibility

Strongly influenced by baseline noise!



Error transfer from $\Delta\gamma$ to $\Delta\sigma$ with constant $\Delta\gamma$

$$\Delta\sigma \propto \frac{1}{\gamma \cdot \ln\left(\frac{1}{\gamma}\right)} \Delta\gamma$$



So, important point in small beam size measurement

is

How to escape from noise in visibility measurement

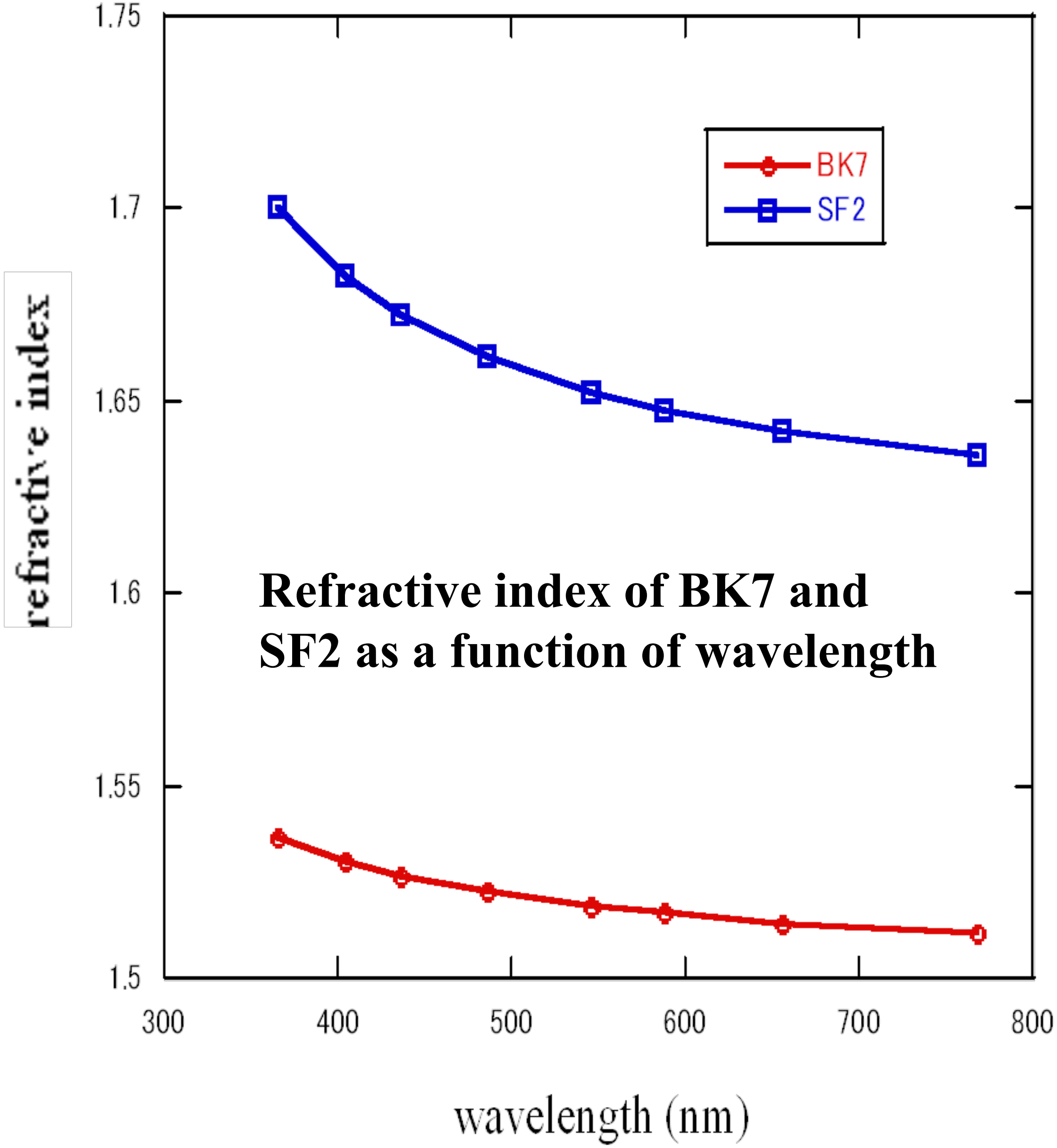
1. **Use larger separation of double slit**

2. **Use shorter wavelength**

Both of this will reduce visibility of interferogram

1. **Use larger separation of double slit**
   limited by opening angle of SR

2. **Use shorter wavelength**
   mainly limited by chromatic aberrations in focusing optics.

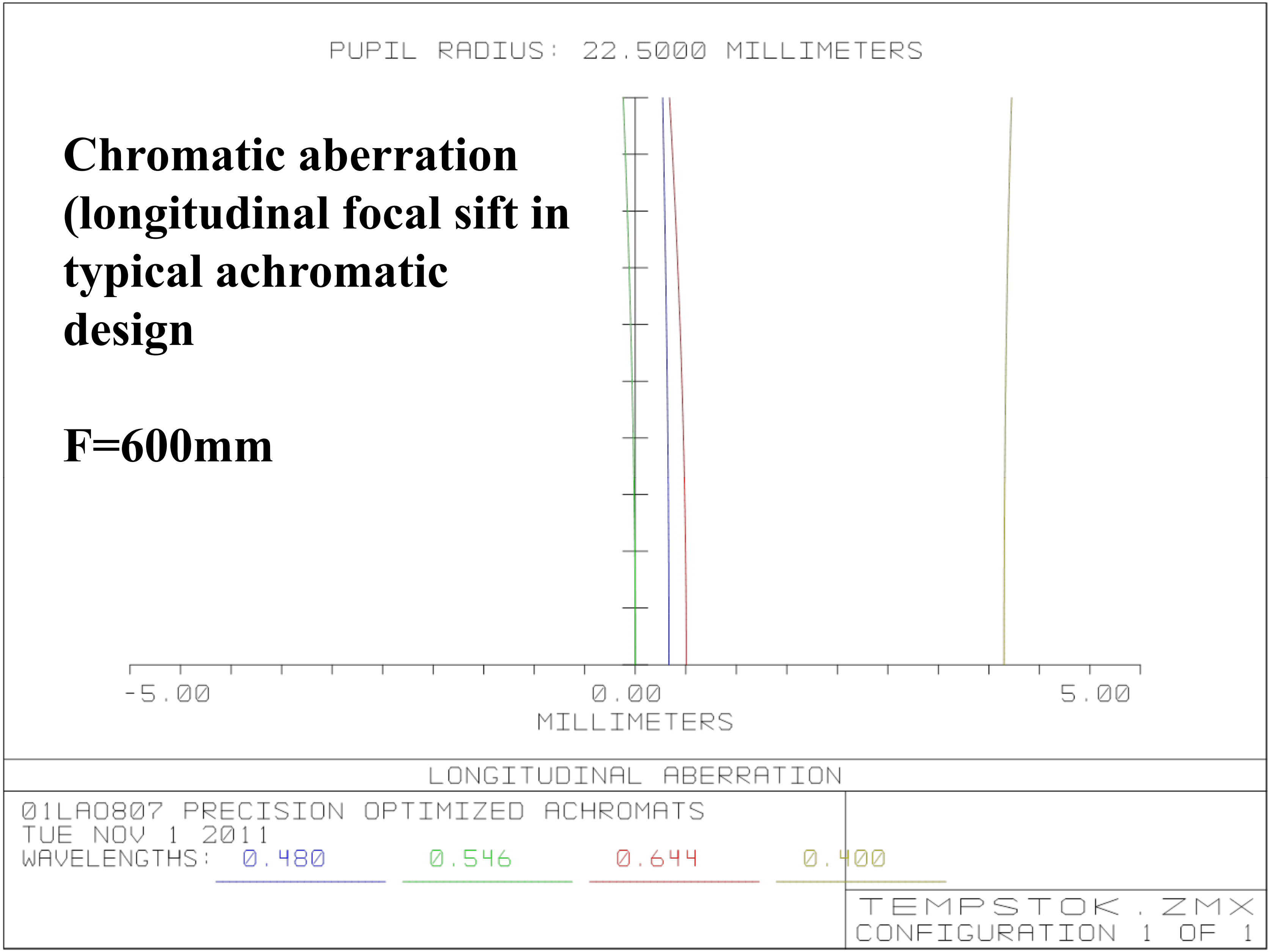


Refractive index of BK7 and SF2 as a function of wavelength

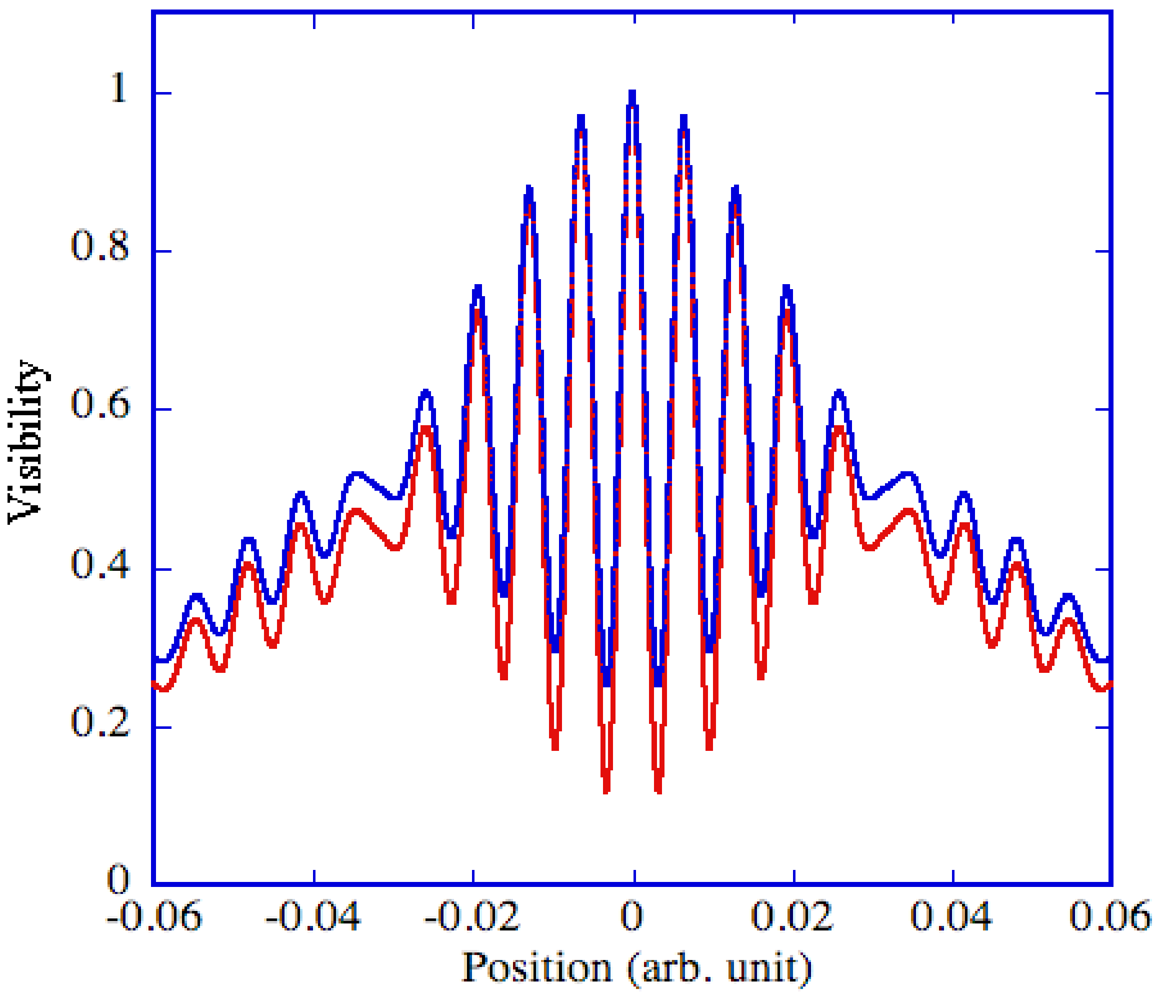
Elimination of chromatic aberration at <u>400nm is very difficult</u> due to large partial dispersion ratio of glass

Chromatic aberration (longitudinal focal sift in typical achromatic design

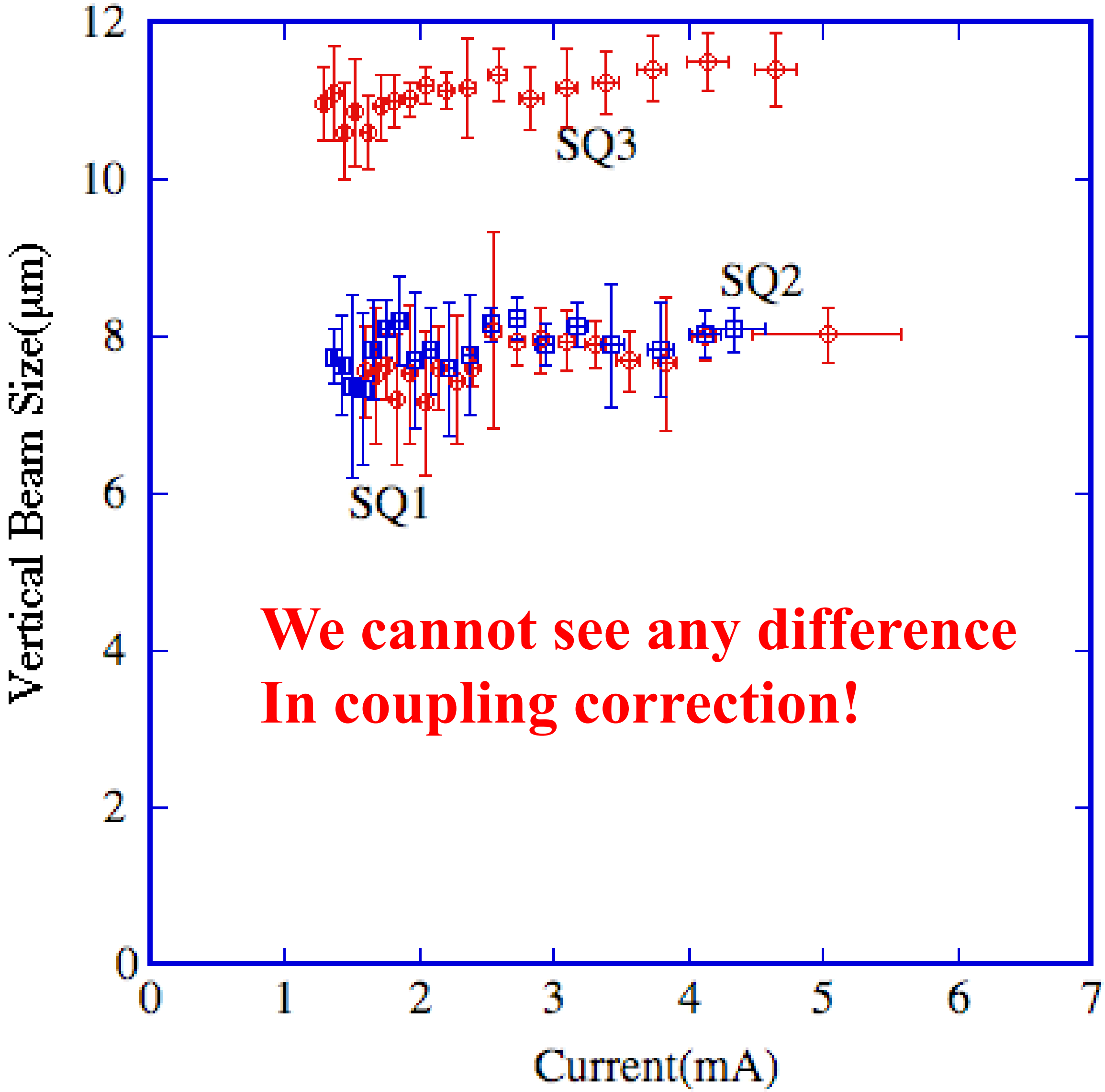
F=600mm



Interferogram with chromatic aberration and without chromatic aberration.
λ=400nm, Δλ=80nm
Lens:achromat
D=45mm f=600mm



Results by normal refractive interferometer using λ=400nm



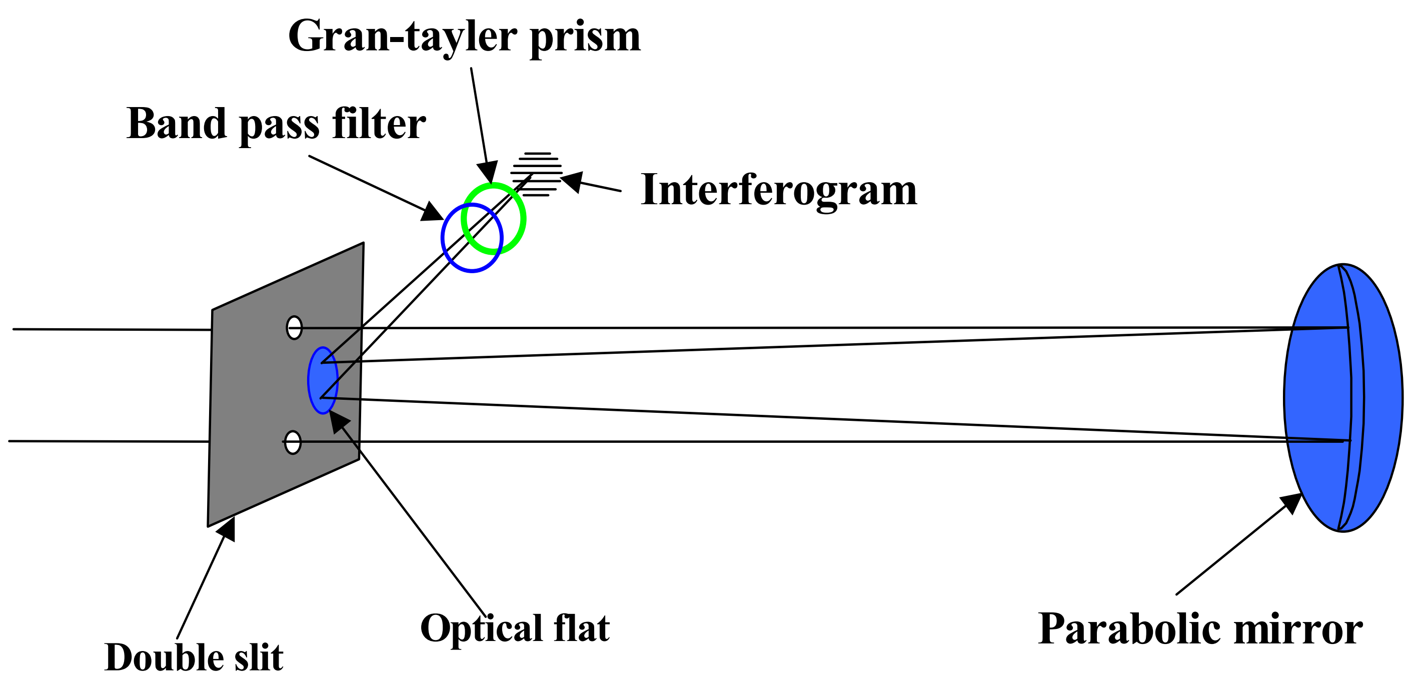
If the chromatic aberration at 400nm is measure source of error in 5μm range beam size measurement,

Use reflective optics!
Reflective system has no chromatic aberration.

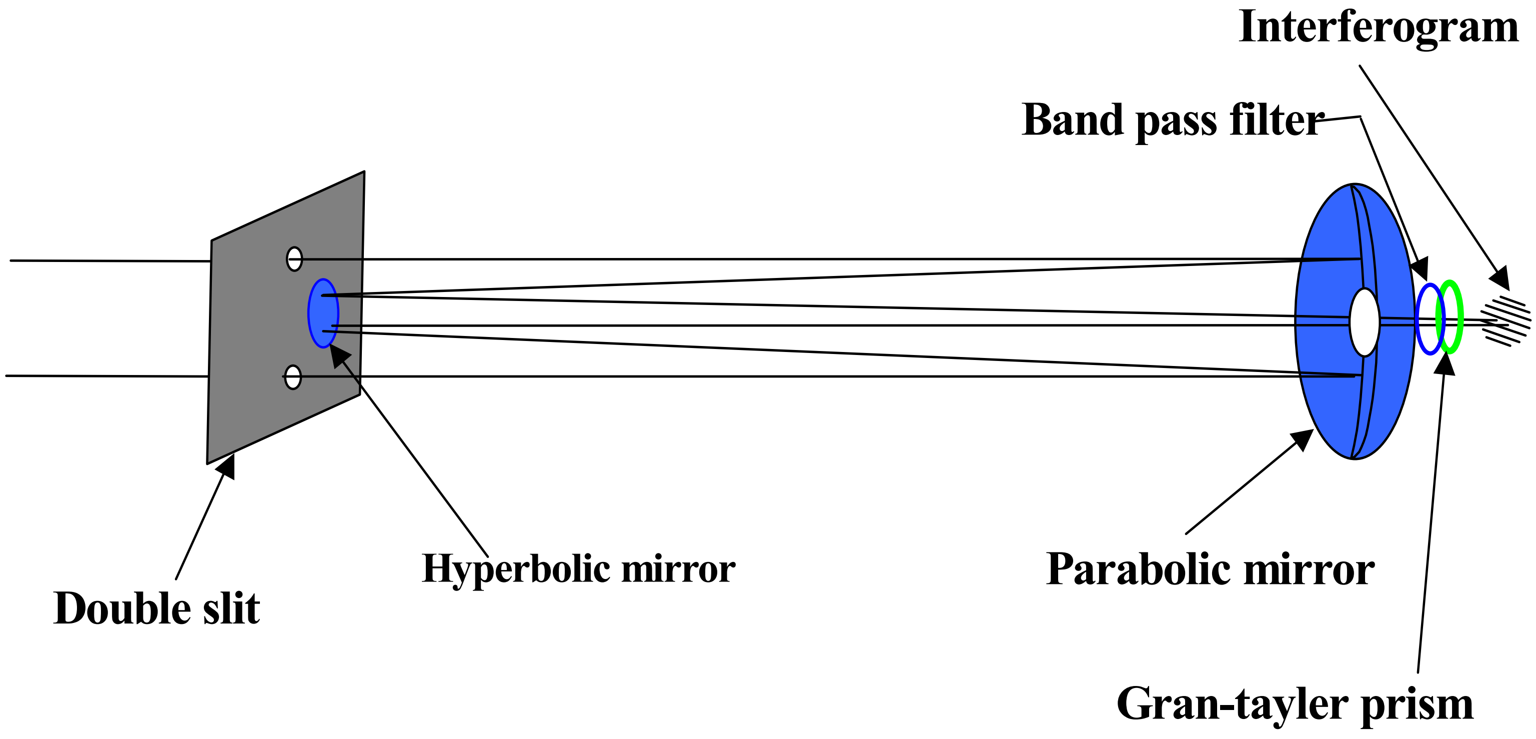
3. Reflective interferometer

Possible arrangement for reflective optics for interferometer
1. On axis arrangement

Newtonian arrengement of optics

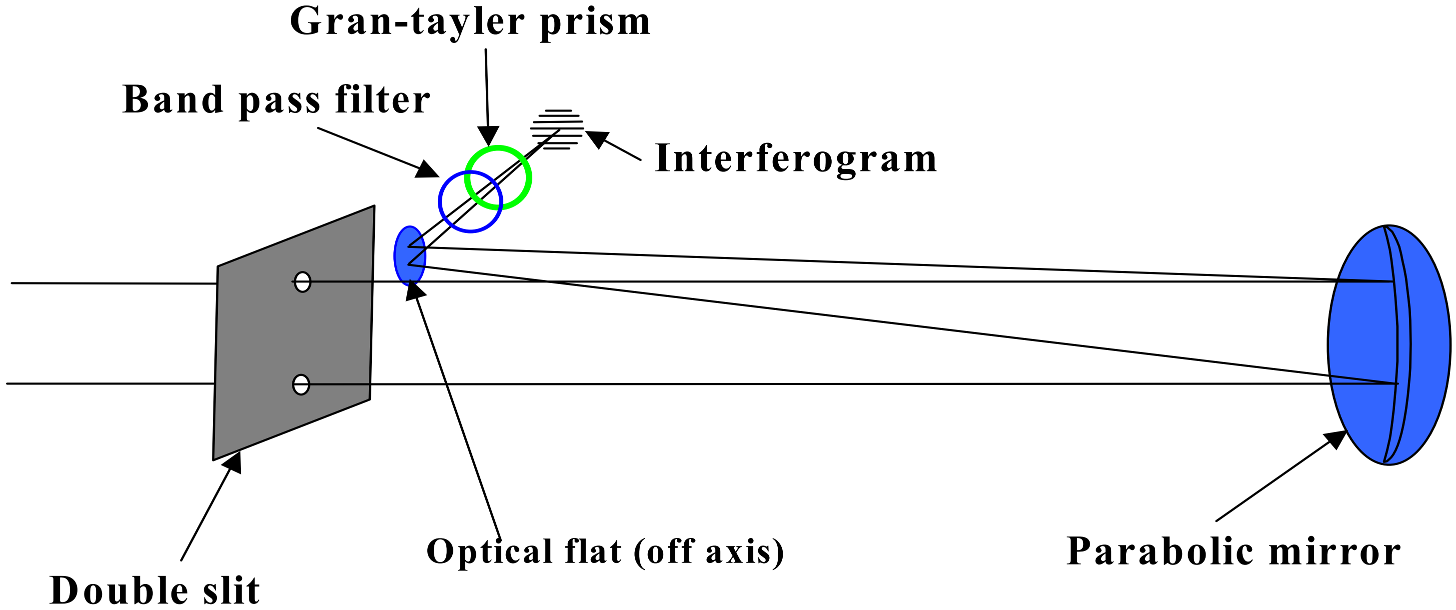


Cassegrainian arrangement of optics



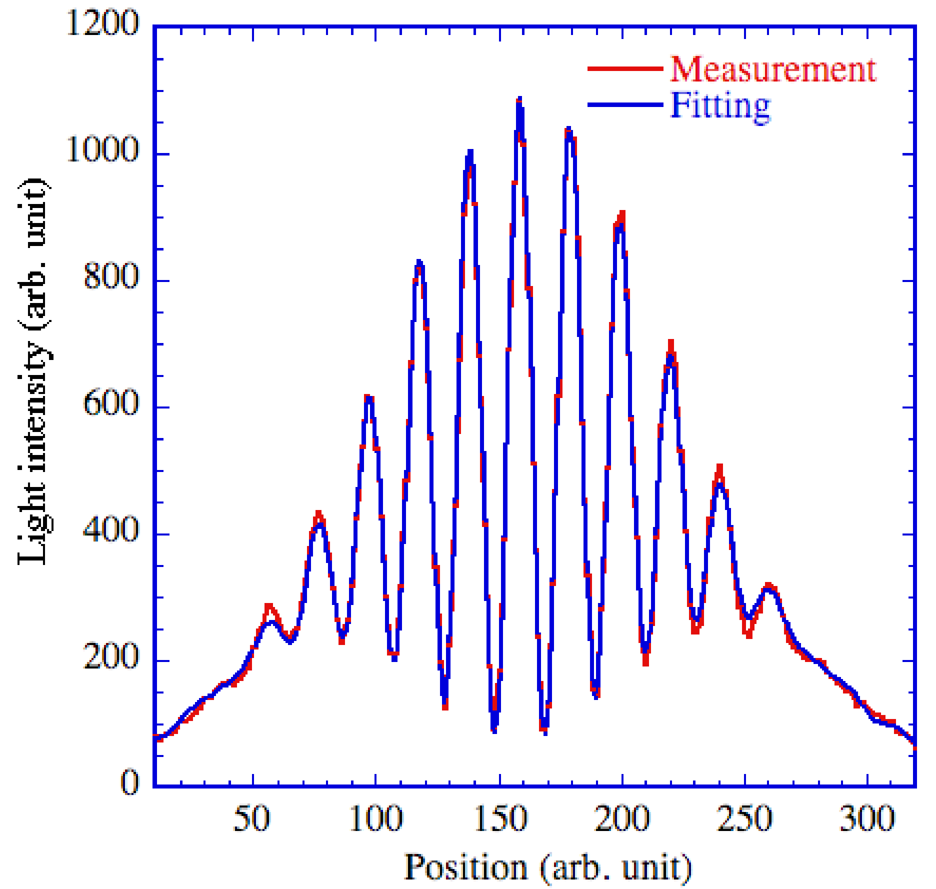
2. Off axis arrangement

Herschelian arrengement of optics

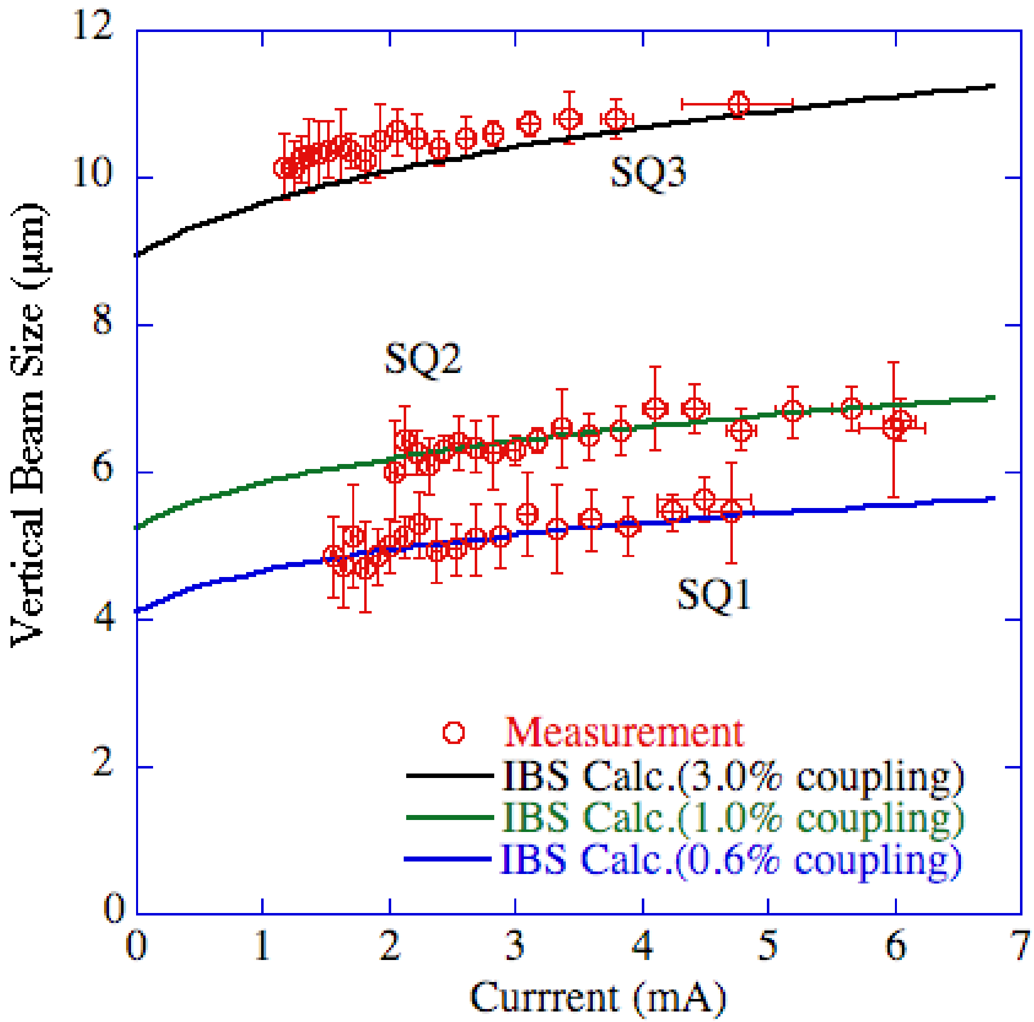


Measured interferogram At ATF, KEK

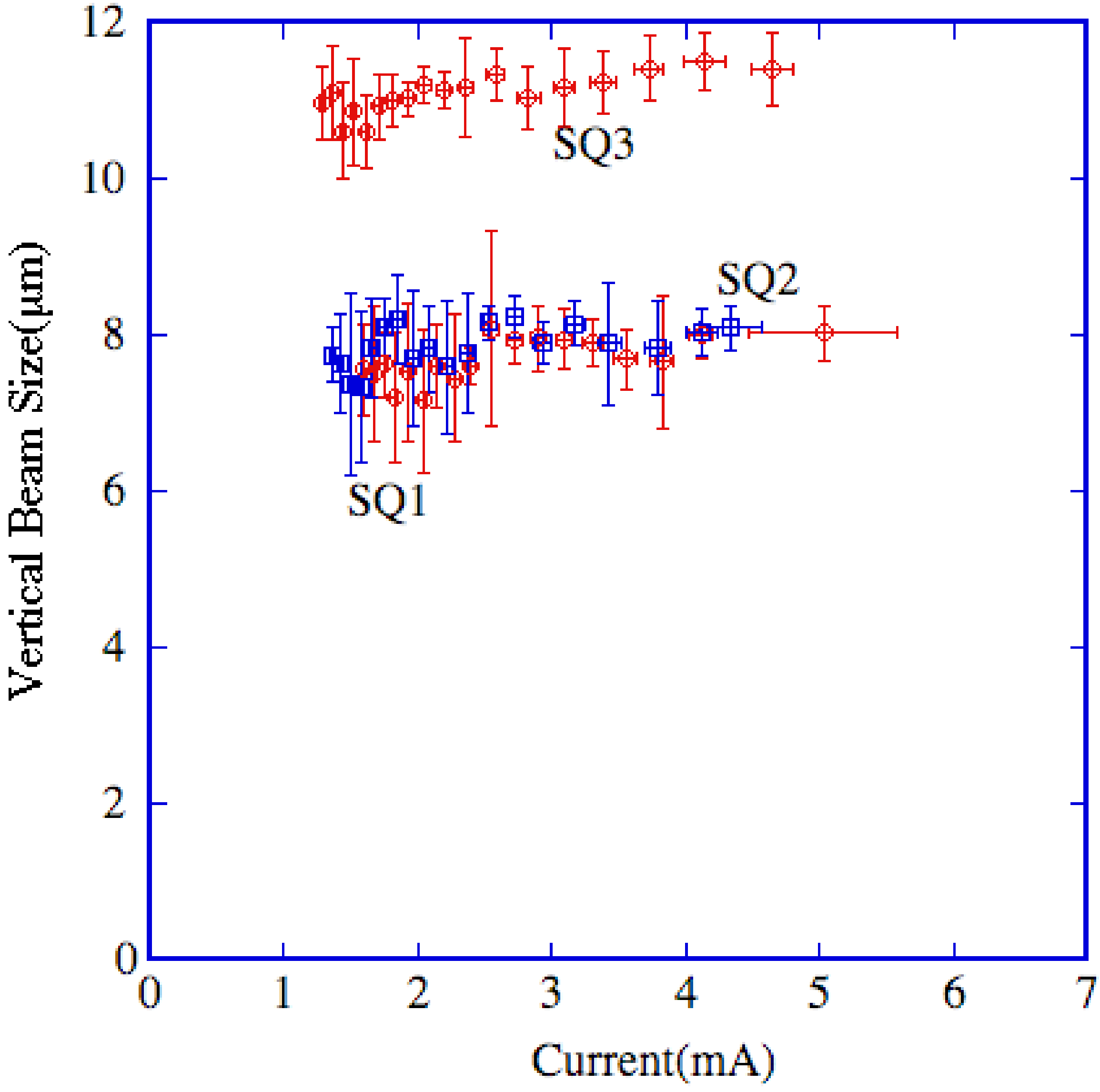
Result of beam size is 4.73μm±0.55μm



The x-y coupling is controlled by the strength of the skew Q at ATF



Remember same results by normal refractive interferometer using λ=400nm

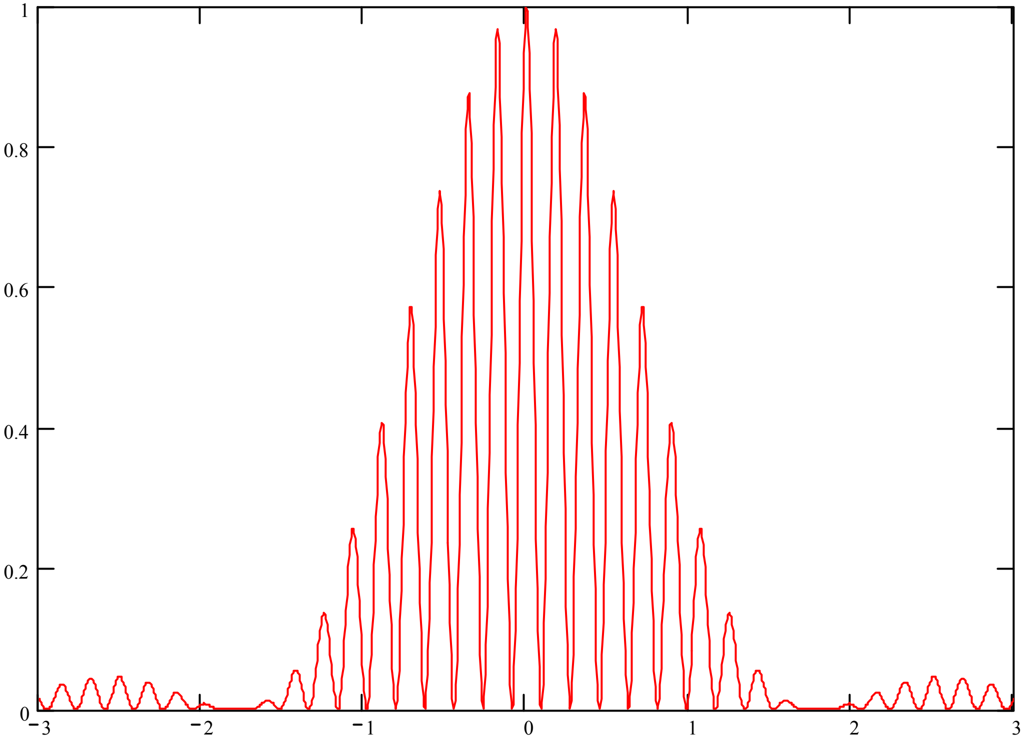


The reflective interferometer is more useful than refractive interferometer especially for shorter wavelength range.

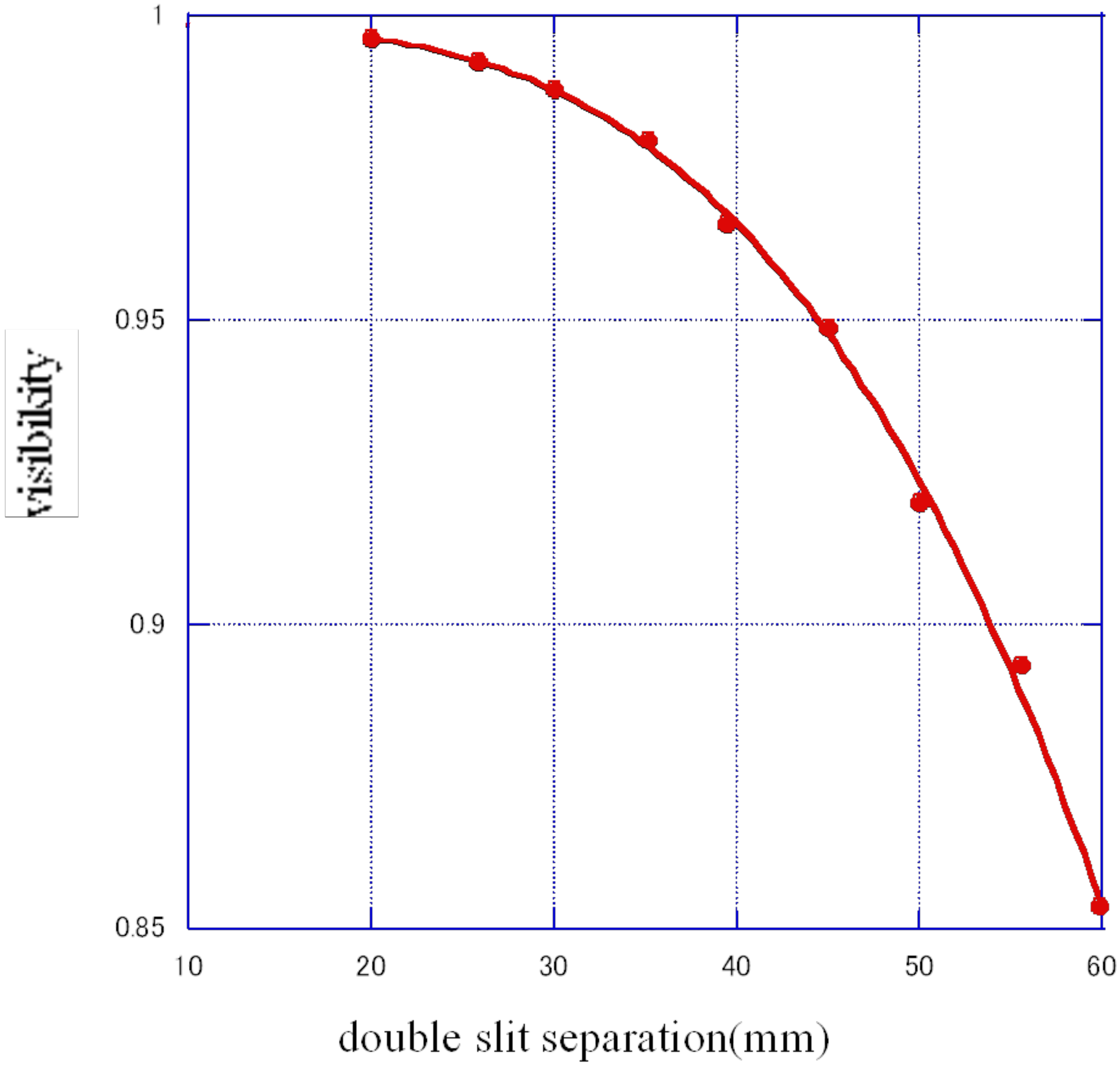
Actually, it is chromatic aberration-free, and reflectors are cheaper than lenses in large aperture.

If we can not use more shorter wave length,

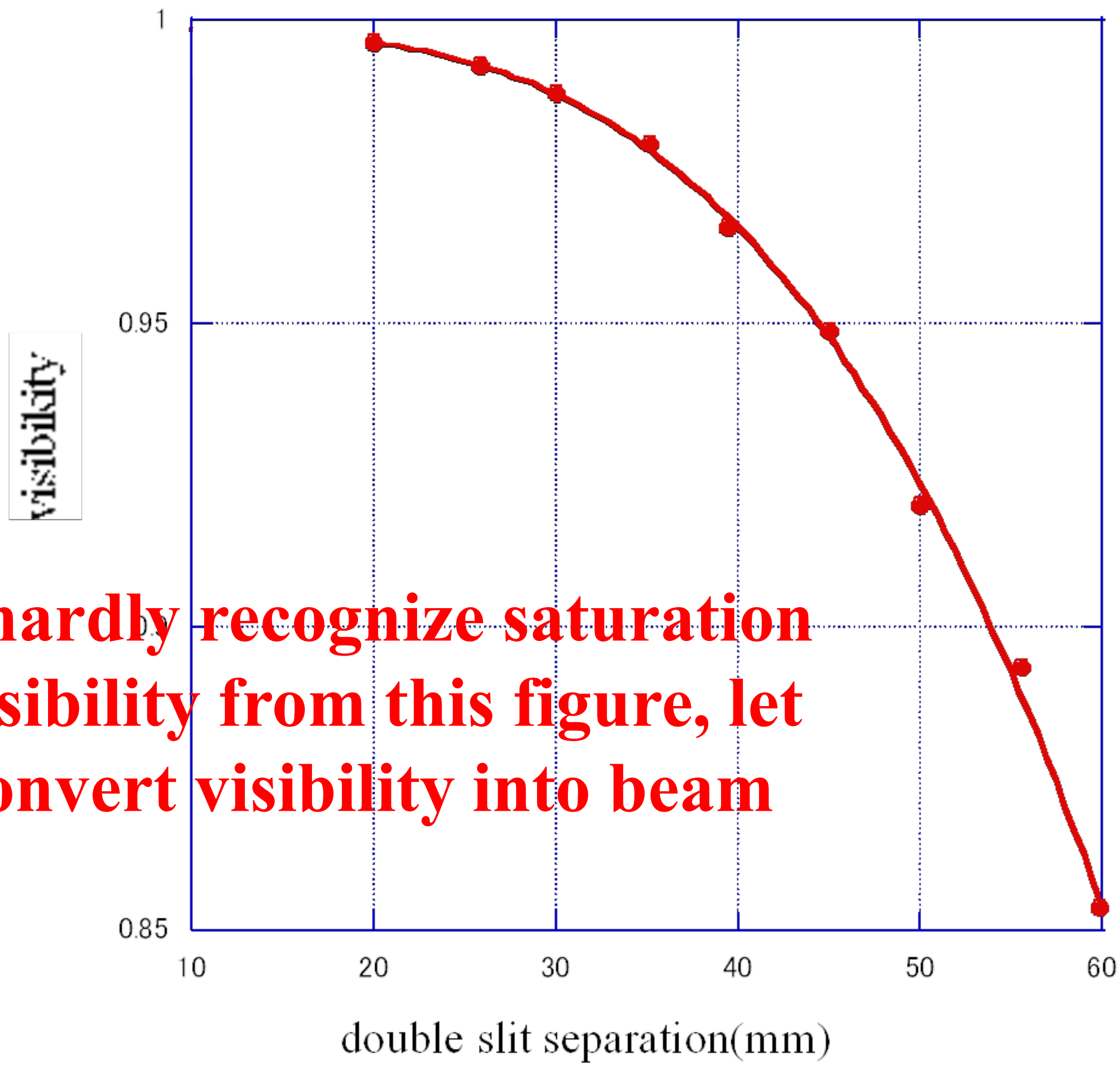
How we can do for more smaller beam size measurement?



Result of visibility for beam size 5.8μm (l=550nm) with several separation of double slit.

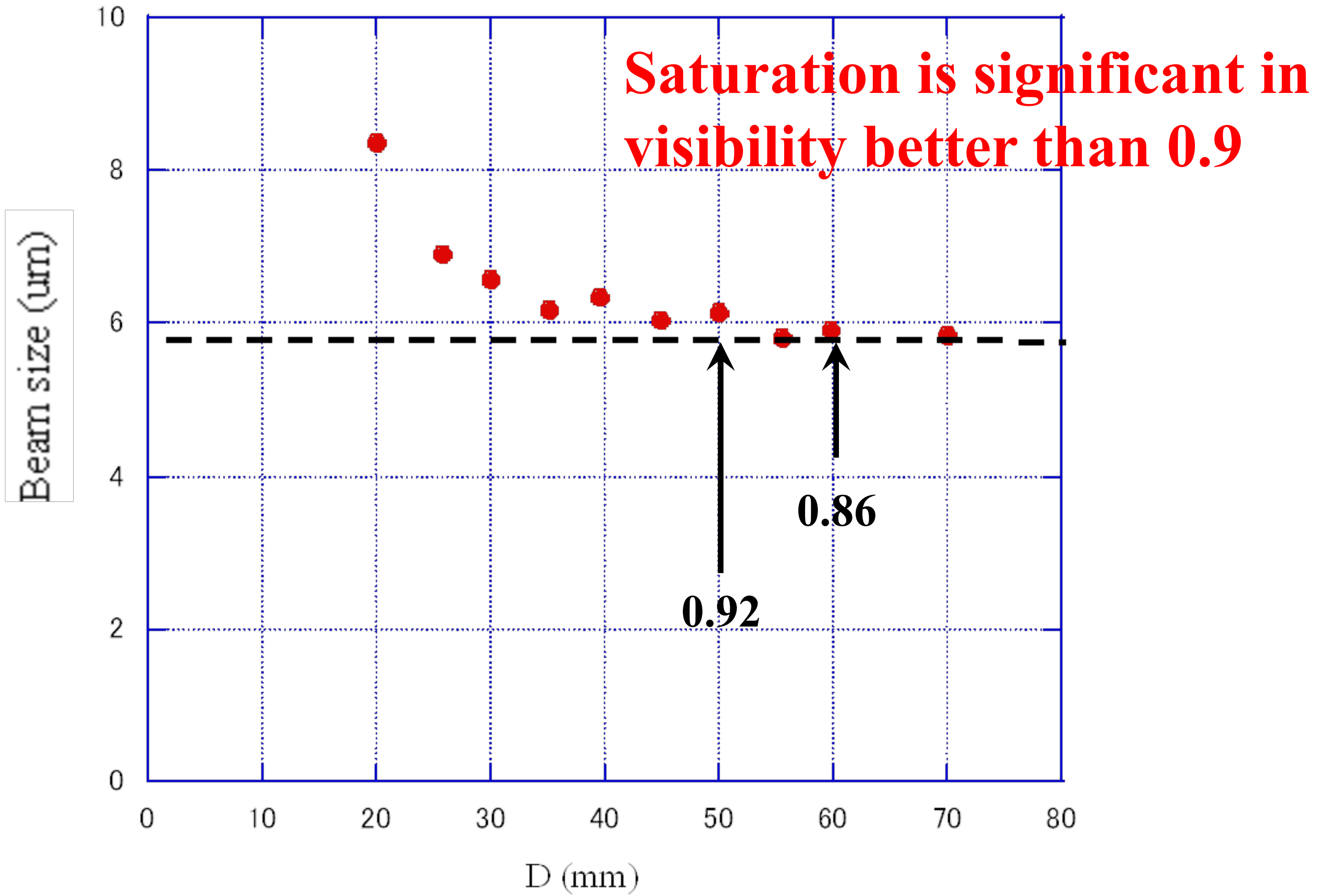


Result of visibility for beam size 5.8μm (l=550nm) with several separation of double slit.



We hardly recognize saturation in visibility from this figure, let us convert visibility into beam size!

Convert visibility into beam size. We can see clear saturation in smaller double slit range which has visibility near 1.



4. Imbalanced input method

Another method to escape from noise
for
more small beamsize measurement

Let's us consider equation for interferogram.

$$I(y,D)=\int (I_1+I_2)\cdot\left\{\sin c\left(\frac{\pi\cdot a\cdot y\cdot\chi(D)}{\lambda\cdot f}\right)\right\}^2\cdot\left\{1+\gamma\cdot\cos\left(k\cdot D\cdot\left(\frac{y}{f}+\psi\right)\right)\right\}d\lambda$$

$$\gamma=\left(\frac{2\cdot\sqrt{I_1\cdot I_2}}{I_1+I_2}\right)\cdot\left(\frac{I_{max}-I_{min}}{I_{max}+I_{min}}\right) \quad , \quad \psi=\tan^{-1}\frac{S(D)}{C(D)}$$

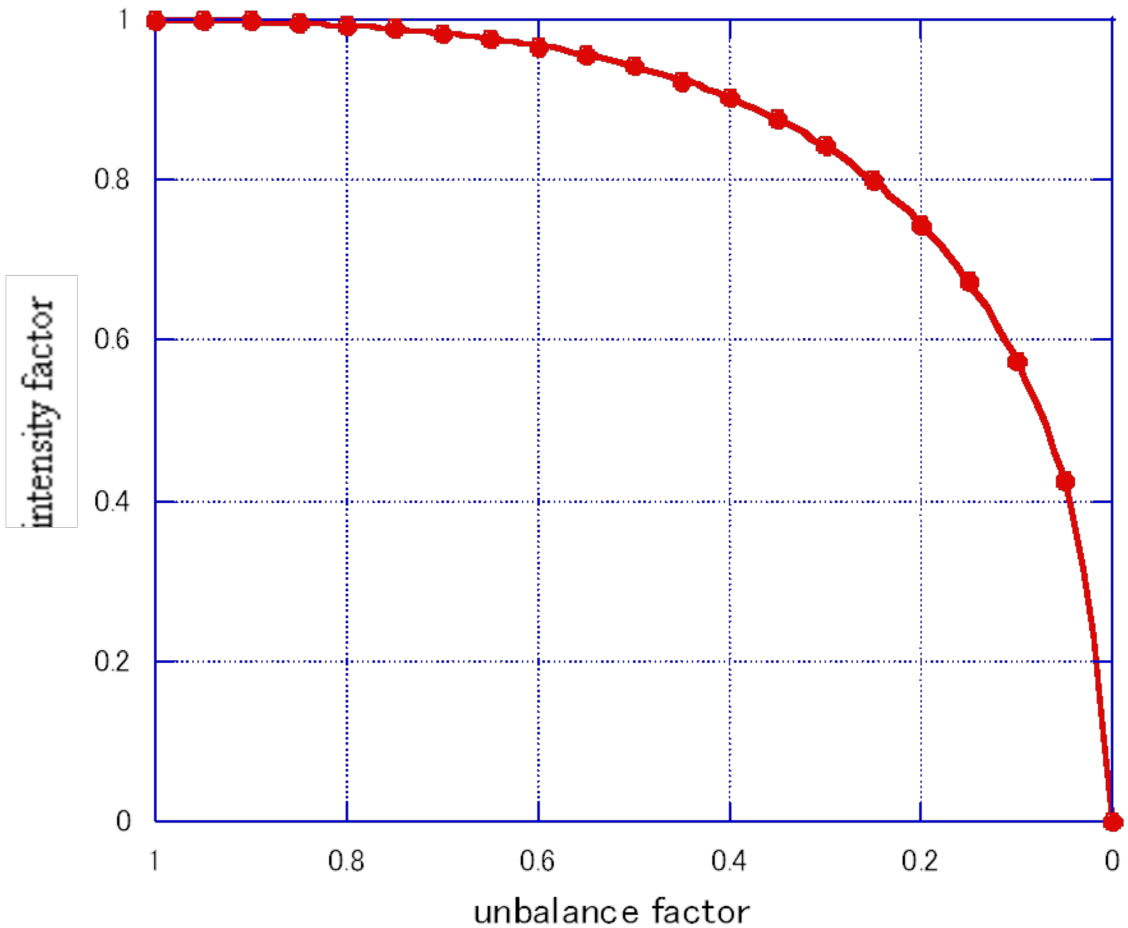
In this equation, the term "γ" has not only real part of complex degree of spatial coherence but also intensity factor!

$$\gamma=\left(\frac{2\cdot\sqrt{I_1\cdot I_2}}{I_1+I_2}\right)\cdot\left(\frac{I_{max}-I_{min}}{I_{max}+I_{min}}\right)$$

If $I_1 = I_2$, γ is just equal to real part of complex degree of spatial coherence , but if $I_1 \neq I_2$, we must take into account of intensity factor;

$$\frac{2 \cdot \sqrt{I_1 \cdot I_2}}{I_1 + I_2}$$

This intensity factor is always smaller than 1 for $I_1 \neq I_2$.



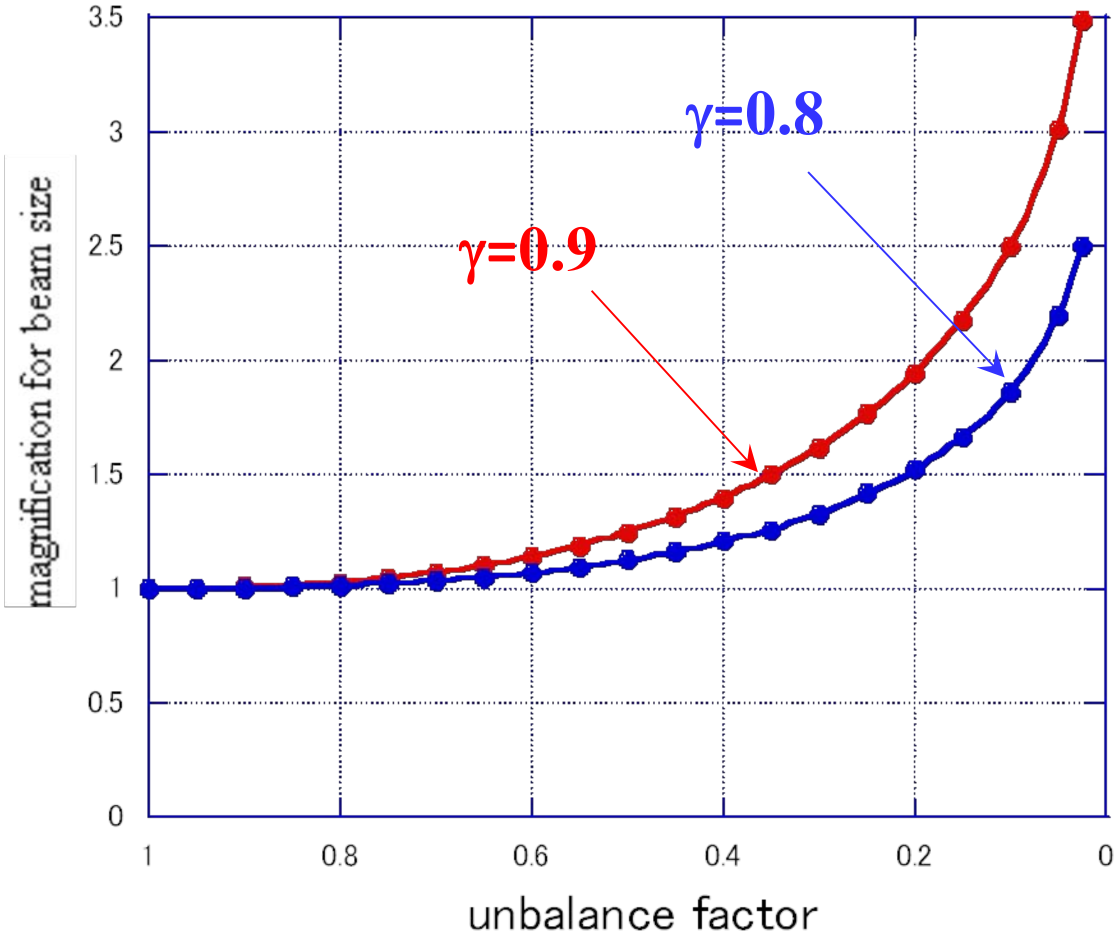
1
0.8
0.6
0.4
0.2
0
intensity factor
1
0.8
0.6
0.4
0.2
0
unbalance factor

$$\gamma = \left(\frac{2 \cdot \sqrt{I_1 \cdot I_2}}{I_1 + I_2} \right) \cdot \left(\frac{I_{max} - I_{min}}{I_{max} + I_{min}} \right)$$

Since intensity factor is smaller than 1 for $I_1 \neq I_2$, the "γ" will observed smaller than real part of complex degree of spatial coherence.

This means beam size will observed larger than primary size and <u>we know ratio between observed size and primary size.</u>

This is magnification!

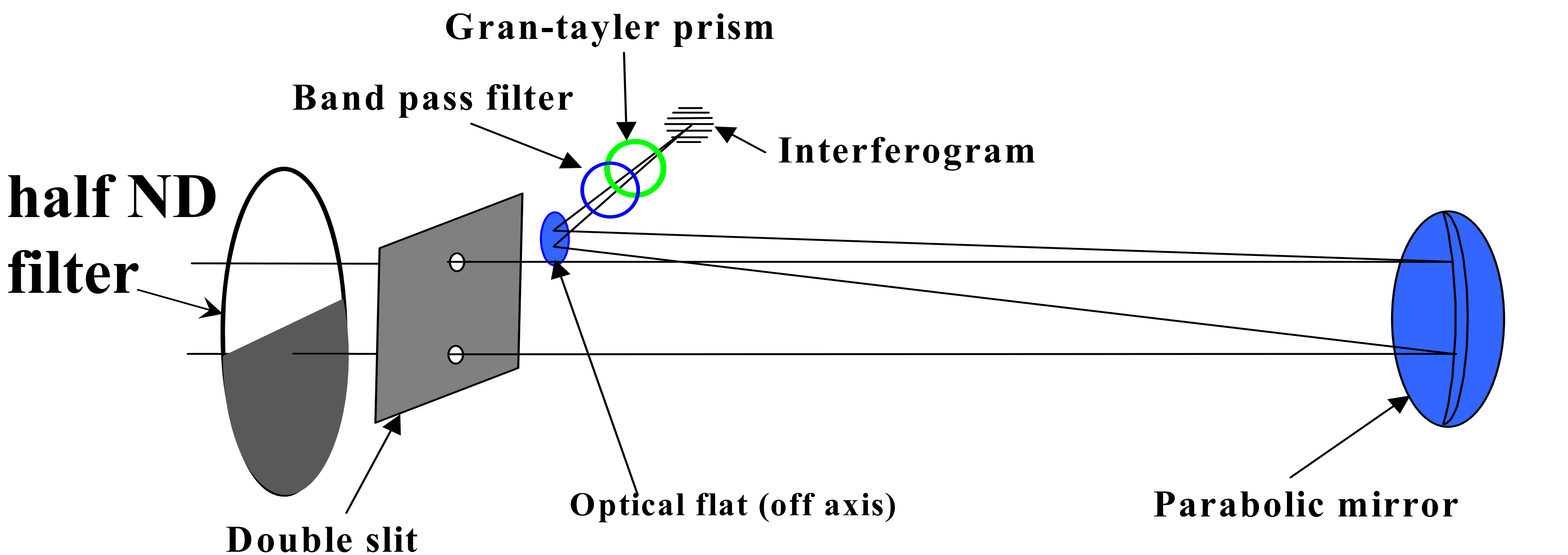


We can use magnification range up to 2 for $I_1 : I_2 = 1 : 0.2$ or 3 for 1 : 0.05.

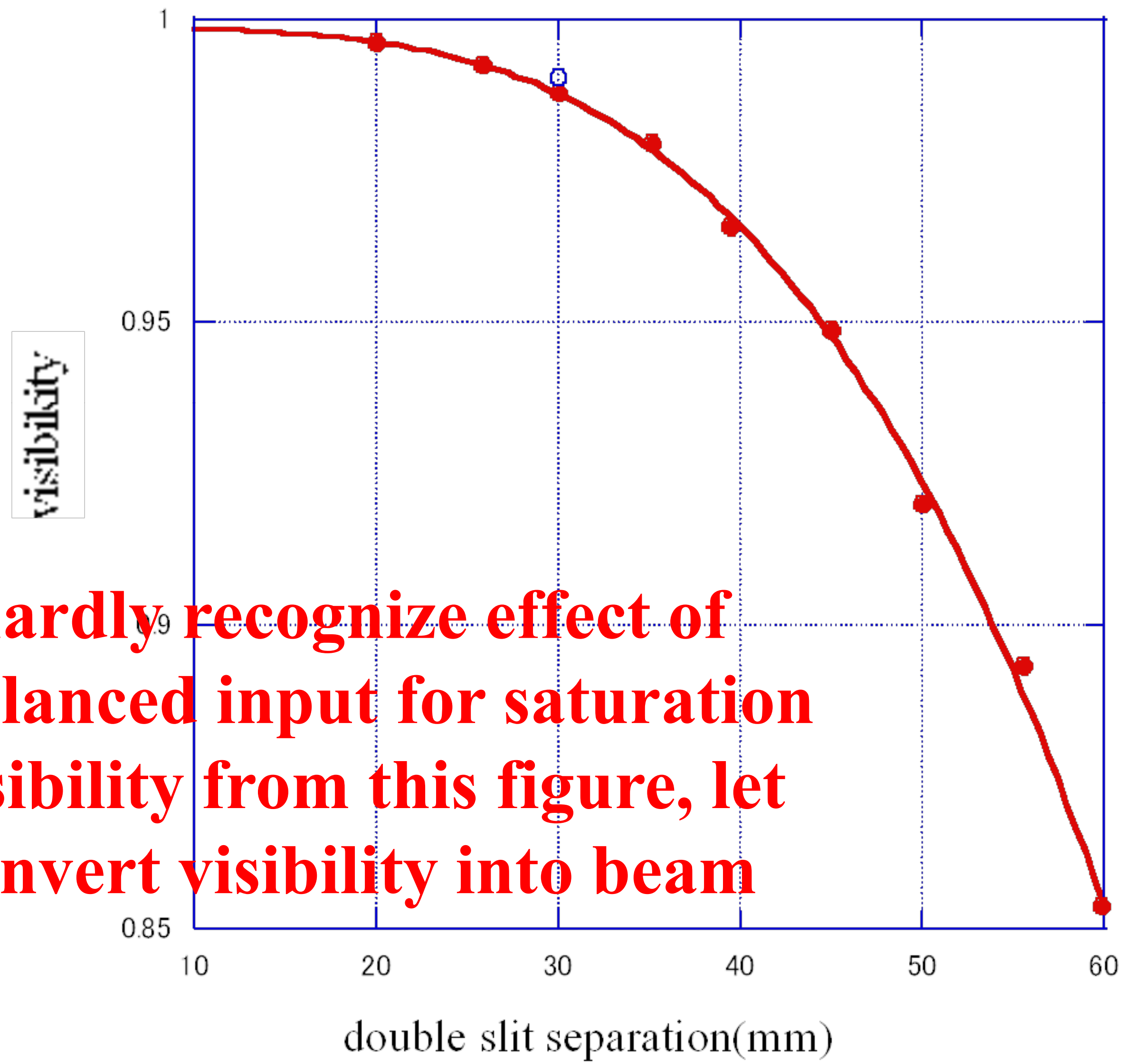
In interferometry, we can magnify beam size by very simple way;

applying imbalance input for double slit!

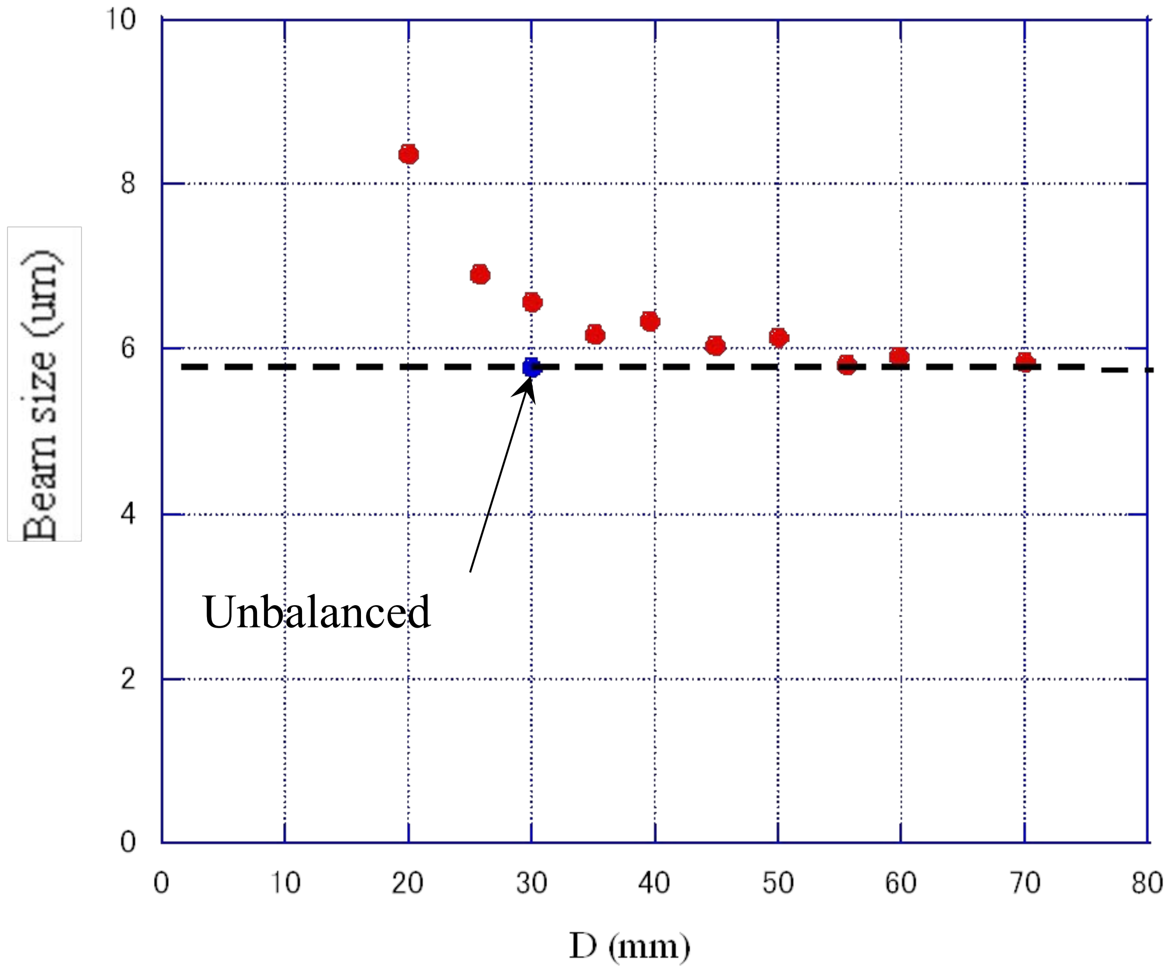
Setup for imbalanced input by half ND filter



Appling unbalance method for D=30mm.
$I_1 : I_2 = 0.853:0.249$



We hardly recognize effect of unbalanced input for saturation in visibility from this figure, let us convert visibility into beam size!



Beam size (um)
10
8
6
4
2
0
Unbalanced
0
10
20
30
40
50
60
70
80
D (mm)

Further result of unbalanced technique, please hear presentation of Dr. Mark Boland

Conclusion

Smallest result of beam size at ATF is 4.7μm with reflective SR interferometer using double slit separation of 45-55mm, λ=400nm. This size is almost small limit with equal input method.

When we will apply imbalanced method;

With magnification factor 2 ⇒ 2.4μm

With magnification factor 3 ⇒ 1.6μm

We are waiting beam size in this range!

Thank you very much for your attention.