RECENT PROGRESS IN SR INTERFEROMETER

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Abstract

Beam size measurement in accelerator is very important to evaluate beam emittance. SR interferometer has been used as one of powerful tools for measurement of small beam size through special coherence of visible SR. Recent progresses in this technique improve measurable range for smaller beam size less than 10μm. An application of reflective optics to eliminate chromatic aberration in focus system in SR interferometer makes it possible to measure the beam size down to 3μm range. In recent few years an imbalance input technique is developed to introducing magnification into the interferometer.

INTRODUCTION

The synchrotron radiation (SR) monitor based on visible optics is one of the most fundamental diagnostic tools in the high energy accelerators. The monitor gives a static and dynamic observation for visible beam profile, beam size, longitudinal profile, etc. These greatly improve the efficiency of commissioning and operation of the accelerator. In this monitor, the visible SR is extracted by a mirror from the SR source such as bending magnet in accelerator, then the SR guided into the optical diagnostics systems. During these years, the development of the SR interferometer has been the most significant topic. The idea of the SR interferometer for measurement of beam profile and size struck me while I was performing experiments to investigate the coherence of synchrotron radiation in 1997 [1]. Nowadays, the SR interferometer is recognized as a powerful tool for easily measuring small beam sizes [2]. Recent progresses in improve measurable range for smaller beam size. An application of reflective optics to eliminate chromatic aberration in focus system makes it possible to measure the beam size down to 3μm range [3]. In recent few years, an imbalance input technique is developed to introducing magnification into the interferometer [4]. A simple introduction for interferometry, and results are introduced in the first half, and in the second half, recent progresses on SR interferometer are introduced in this paper.

BEAM PROFILE AND SIZE MEASUREMENT WITH INTERFEROMETRY

The measurements of beam profile and size are most important issues in optical monitor. The most conventional method to observe the beam profile is making an optical image of the beam. The resolution of this method is generally limited by diffraction phenomena. In the usual configuration of the imaging system, the RMS size of diffraction (1σ of the point spread function) is not smaller than 50 μm. Since, research and development in electron storage rings (especially in reducing the beam emittance) has been very remarkable in last few ten years, the above-mentioned profile monitor via imaging system becomes ineffective in precise quantitative measurements of the beam profile and size due to the diffraction resolution limit.

In visible optics, the interferometry is one of the standard methods for measuring the profile or size of very small objects such as angular dimension of the star. The principle to measure the profile of an object by means of spatial coherency was first proposed by H. Fizeau in 1898 [5], and is now known as the Van Cittert-Zernike theorem [6] with their work in 1932 (Van Cittert) and 1933 (Zernike). It is well known that A. A. Michelson measured the angular dimension (extent) of a star with this concept [11] with his stellar interferometer in 1935. The SR interferometry for the measurement of the spatial coherence of visible region of the SR beam was first performed by author in 1997 [1]. And in the same time, the author demonstrated that this method is applicable to measure the beam profile and size. Since the SR beam from a small electron beam has better spatial coherence, this method is suitable for measuring a small beam size. The characteristics of this method are: 1) one can measure beam sizes as small as 10 μm range with 1-μm resolution in a non-destructive manner using visible light (typically 500 nm); 2) the measurement time is a 1-2 seconds for size measurement; 3) due to self-consistency in interferometry, this method is classified in an absolute measurement. For the point 3), the absolute measurement means all the free parameters in interferometry such as wavelength are measured by interferometry and a ruler. In this meaning, interferometry is classified into one of most fundamental measurement method. Otherwise, other methods, such as imaging always use information from interferometry, such as wavelength.

THEORETICAL BACKGROUND OF THE INTERFEROMETRY

According to the work of H. Fizeau in 1868 [5], the visibility (contrast) of the interference fringe taken by an interferometer, is higher for a small light source and lower for a large light source. As the smallest limit of light source, a point source gives an interferogram with visibility 1. We now interpret the point source as the single mode of a photon which gives an interferogram of visibility 1. We can represent the general light source by an ensemble of point sources. Let us assume each point source has no 1st order temporal coherence. The interferogram given by such an ensemble of point sources are superimposed and the interference fringe is smeared. Then, the visibility of the obtained

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interferogram in large object will have reduced the visibility, as shown in Figure 1.

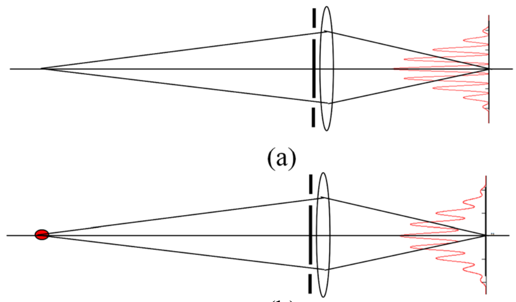

Figure 1: A simple representation of the visibility of an interferogram and light source size. (a) A point source yieldss an interferogram with a visibility of 1; (b) an ensemble of point source yields a reduced visibility.

In the reverse manner, we can measure the size of the light source by measuring the visibility of the interferogram. This concept is known as the van Cittert-Zernike theorem

According to van Cittert-Zernike theorem, under the assuming each point source has no 1st order temporal coherence, the profile of an object is given by the Fourier transform of the complex degree of 1st order spatial coherence at longer wavelengths, as in the visible light [6]. Let f (y) denote the beam profile as a function of position y, R denote the distance between the source beam and the double slit, and γ denote the 1st order complex degree of spatial coherence as a function of spatial frequency υ. Then γ is given by the Fourier transform of f(y) as follows:

$$\gamma(\upsilon)=\int f(y)\exp(-2\pi i\upsilon\cdot y)dy\,,\qquad \upsilon=\frac{2\pi D}{\lambda R}\quad (1).$$

We can obtain the beam profile and the beam size via Fourier inverse transform of complex degree of 1st order spatial coherence measurement with the interferometer.

SR INTERFEROMETER

To measure the spatial coherence of SR beams, a wavefront-division type of two-beam interferometer using polarized quasi-monochromatic rays was designed as shown in Fig. 2 [1][2].

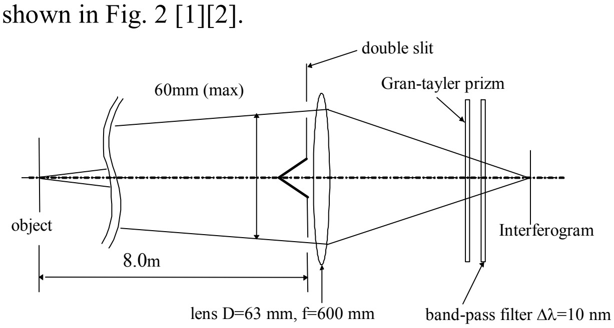

Figure 2: Outline of the SR interferometer. The wavefront-division type of two-beam interferometer.

In the vertical plane, the elliptical polarity of the synchrotron radiation is opposite to that in the median plane of the electron beam orbit. Therefore, there exists a π phase difference between the phases of the interference fringes corresponding to the σ- polarized and π-polarized components [1]. By this reason, we must eliminate interferogram produced π-polarized component. For this purpose, we used a Glan-Tayler polarizer (extinction ratio is 10^{-6}, and please note extinction ratio in dichroic film polarizer is very often not sufficient for interferometry). With this interferometer, the intensity of the interferogram is given by,

$$I(y,D)=\int\varsigma(\lambda)\cdot(I_1+I_2)\cdot\left\{sinc\left(\frac{\pi\cdot\alpha\cdot y\cdot\chi(D)}{\lambda\cdot f}\right)\right\}^2\cdot\left\{1+\gamma\cdot cos\left(k\cdot D\cdot\left(\frac{y}{f}+\psi\right)\right)\right\}d\lambda$$

$$\gamma=\left(\frac{2\sqrt{I_1\cdot I_2}}{I_1+I_2}\right)\cdot\left(\frac{I_{max}-I_{min}}{I_{max}+I_{min}}\right),\quad \psi=tan^{-1}\frac{S(D)}{C(D)}\qquad (2)$$

where y denotes position in the interferogram, a denotes the half-height of a slit, $\zeta(\lambda)$ denotes the rocking curve of the band-pass filter, and f denotes the distance between secondary principal point of the lens and the interferogram. γ is the real part of the complex degree of spatial coherence (visibility of the interferogram). $S(D)$ is the sine component and $C(D)$ the cosine component of the Fourier transformation of the distribution function of the SR source. $\chi(D)$ in this equation represents an instrumental function of the interferometer; this term has a cosine-like dependence, and it comes mainly from two sources: 1) a cosine term in the Fresnel-Kirchhoff diffraction formula [7]; 2) reduction of the effective slit height as the double slit separation D increases. This term χ is normally neglected in diffraction theory under the paraxial approximation, but neglect this term is not sufficiently precise in the interferometer. A typical interferogram observed with the SR interferometer at the Photon Factory (PF), KEK [1] is shown in Fig. 3.

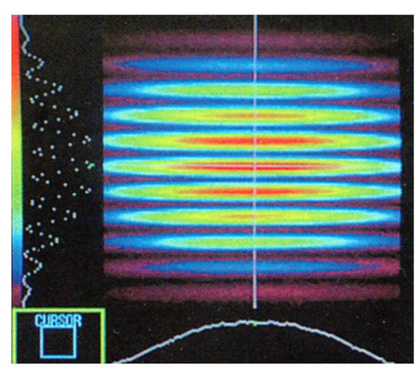

Figure 3: A typical interferogram observed with the SR interferometer.

BEAM PROFILE MEASUREMENT

We can obtain the beam profile by Fourier transform of the complex degree of 1st order spatial coherence. Figure 4 shows the absolute value of the complex degree of 1st order spatial coherence (|γ|, visibility) as measured while changing the double slit separation from 5 mm to 15 mm at the PF [1].

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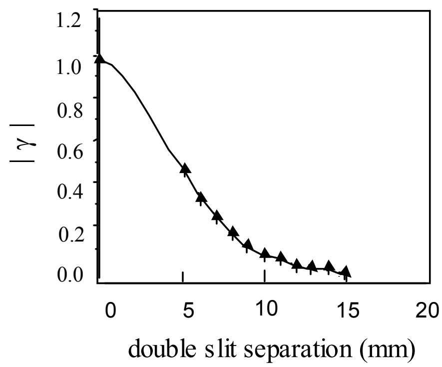

Figure 4: Result of |γ| at the Photon Factory.

Imaginary part (phase term) of 1st order complex degree of spatial coherence is shown in Fig. 5.

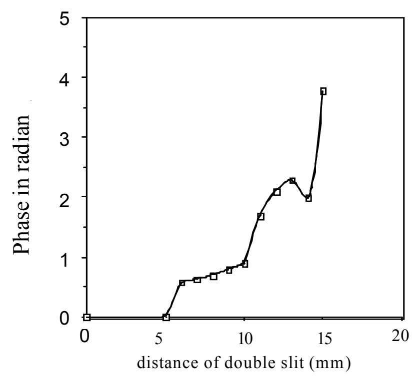

Figure 5: Result of phase term of γ at the Photon Factory Vertical axis is phase in radian.

The result of beam profile through the Fourier transform of 1st order special coherence is shown in Fig. 6. To see Fig. 6, the obtained vertical beam profile has a strange shoulder in left side of peak. To investigate this shoulder, we compare this profile with profile from the optical image. Figure 7 shows a result of optical image of the beam and vertical beam profile taken from this image. The vertical profile taken from optical image also has a strange shoulder, and it is caused by deformation of the first mirror in SR monitor beamline.

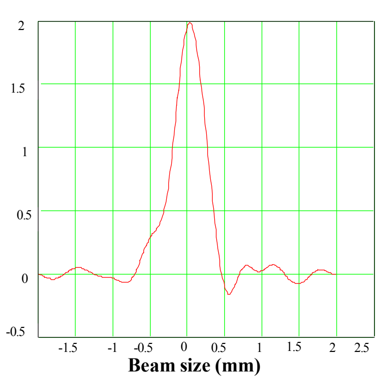

Figure 6: Reconstructed vertical beam profile by Fourier inverse transform of complex degree of 1st order spatial coherence.

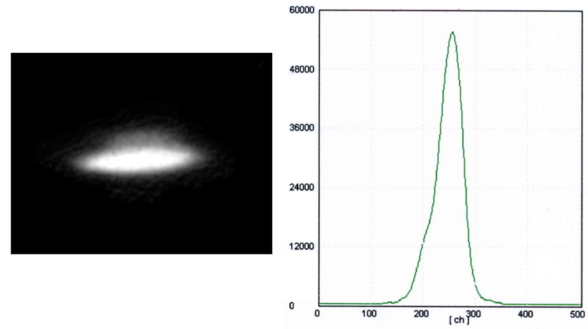

Figure 7: Vertical beam profile taken from optical beam image. (a): optical image of the beam, (b): vertical beam profile from optical image.

The strange shoulder in left side of peak is also appear in the optical image and this asymmetric distribution is caused by thermal deformation of extraction mirror. Since this asymmetric component is come from Fourier sine transform (imaginary part), if we don't include Fourier sine transform, the asymmetric component will automatically eliminate from the profile. The result of Fourier cosine transfer (real part only) is shown in Fig. 8.

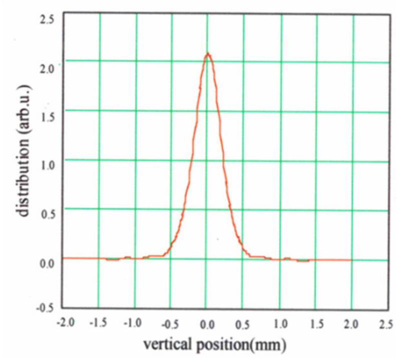

Figure 8: Reconstructed vertical beam profile by Fourier cosine transform.

To see Fig. 8, asymmetric part is completely eliminated from reconstructed beam profile. This technique is one of nice advantage of interferometry to escape from asymmetric aberrations, otherwise we should perform complicated deconvolution processing in imaging.

SMALL BEAM SIZE MEASUREMENT BY MEANS OF GAUSSIAN APPROXIMATION OF BEAM PROFILE

We often approximate the beam profile with a Gaussian shape. Under this approximation, 1st order spatial coherence is given by a it's Fourier transform, a Gaussian function. We can evaluate RMS beam size σ_{beam} from the RMS width of the spatial coherence curve σ_γ as follows:

$$\sigma_{beam} = \frac{\lambda \cdot R}{2 \cdot \pi \cdot \sigma_\gamma} \qquad (3)$$

where R denotes the distance between the beam and the double slit[2]. Results of beam size measurement at PF is shown in Fig. 9 [8]. The incoherent field depth due to

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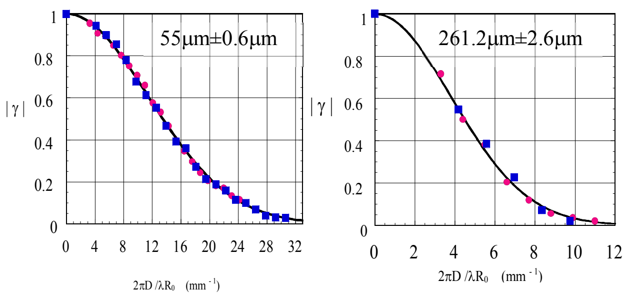
light sources in bunch is sometimes not negligible, and certain correction is necessary. Detail for this correction, please see reference [2].



(a) Vertical (b) horizontal

Figure 9: Results of vertical and horizontal beam size measurement at Photon Factory, KEK.

Under the Gaussian profile approximation, more simply, we can obtain the RMS beam size from one measurement of visibility at certain fixed separation of the double slits. In this scheme, The RMS beam size σ_{beam} is given by,

$$\sigma_{beam} = \frac{\lambda \cdot F}{\pi \cdot D} \cdot \sqrt{\frac{1}{2} \cdot \ln\left(\frac{1}{\gamma}\right)} \qquad (4)$$

where γ denotes the visibility, which is measured at a double slit separation of D [2]. This method is more suitable for daily monitoring of beam size. An example of beam size trend graph taken at Photon Factory is shown in Fig. 10.

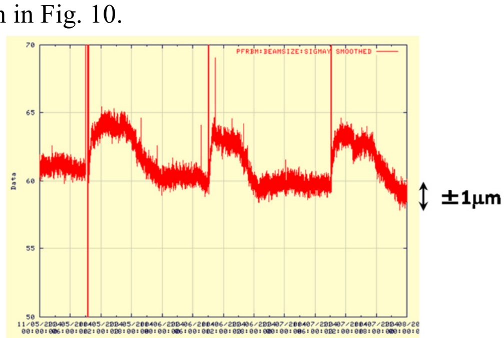


Figure 10: An example of Vertical beam size trend graph at Photon Factory, KEK.

To see Fig. 10, the observed beam size is changing within 60-64μm ± 1μm.

THEORETICAL RESOLUTION OF THE INTERFEROMETER

Let us discuss for theoretical resolution of interferometry from the view point of uncertainty principal in here. Since uncertainty principle in photon statistics is given by using n photons are exited in single state (system has n photons, and observe one time), if we discuss uncertainty principal in n times observation of one photon (system has only one photon in single state at the moment, and observe n times), we need to introduce statistical property of photon assembly for the incoherent ensemble of single states of photons [9]. If we assume the most restrictive class of random processes of photon statistics to be the class of ergodic random process (one time observation for ensemble average over the n photons is equal to temporal average over the n times observation of one photon), the theoretical resolutions due to uncertainty principle are as follows.

Imaging

First of all, we shall discuss theoretical resolution due to uncertainty principal in imaging system, because it is more familiar for readers. The image is a convolution between the geometrical image and the diffraction pattern PSF as shown in Fig. 11.

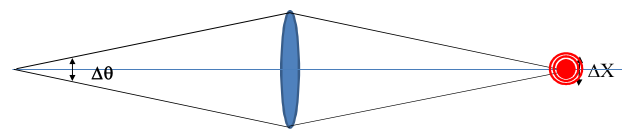


Figure 11: Image is a convolution between the geometrical image and the diffraction pattern PSF.

It is well known that the resolution due to diffraction is limited by the uncertainty principle;

$$\frac{\Delta\theta}{\lambda} \cdot \Delta x \geq 1 \qquad (5)$$

where $\Delta\theta$ and Δx are the uncertainties in the angle and position of the photons in the ensemble, and in here $\Delta\theta$ and Δx are corresponding to divergence of SR beam and width of the PSF. According to this principal, we need a large uncertainty $\Delta\theta$ (large entrance pupil for imaging system) to obtain good spatial resolution. In the case of synchrotron radiation, since the opening angle of the light is strongly limited by relativistic effects, the resolution is fully limited by opening of light due to this uncertainty principle. Off course, instrumental errors such as wavefront error (more generally, aberrations in imaging system) such as a deformation of the mirror are sometimes more significant.

Interferometry

Actually, 1st order interferometry is measuring the coherence of the field of light. Interferometry is a method for determining the axis of photon propagation by means of triangulation using the photon phase coherence of the electric field. According to quantum optics theory, we have following uncertainty principle between phase and number of photons [10]:

$$\Delta\phi \cdot \Delta n \geq \frac{1}{2} \qquad (6)$$

where $\Delta\phi$ and Δn are the uncertainties of the photon phase and the photon number. According to this principal, a large phase uncertainty accompanies a small number of photons or a small phase uncertainty accompanies a large number of photons. The large number of photons serves as like as the "wave of light". When the system has small number of photons, phase of the expectation of electric

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field is uncertain, and serves like as the particles. Due to uncertainty in the phase, interferogram will smear as following manner; .

$$I(y,D) = (I_1 + I_2)\left\{\sin c\left(\frac{\pi a y}{\lambda f}\right)\right\}^2 \left\{1 + \int_{\Delta\phi} \cos(kD\frac{y}{f} + \phi)\xi(\phi)d\phi\right\} \quad (7)$$

where $\xi(\phi)$ denotes probability distribution of phase. Actually, we can have sufficient photons for an interferogram, and theoretical limit due to the phase uncertainty is negligible small. The instrumental error of the phase due to the deformation of the mirror, aberrations in focusing components etc. in actual interferometer is much important. In Actual optical component such as lens normally has a peak to valley wavefront error is better than $\lambda/10$, and if the surface will be smooth, this error corresponds to $\lambda/50$ p-v (0.126rad) over 2mm x 2mm area (this is approximately corresponding to opening area of double slit). If the phase error distribution is Gaussian distribution, this systematic error in the phase can introduce a reducing of the visibility by 0.9994. This visibility corresponds to the object size of 0.26μm with setup of interferometer In the ATF

RECENT PROGRESS IN SMALL BEAM SIZE MEASUREMENT

Recent progresses in small size measurement are introduced in here.

Chromatic Aberration of Lens in Interferometer

In the practical interferometer, we have one bigger problem, the chromatic aberration of lens glass. The Chromatic aberration is caused by dispersion of the glass, and it will introduce phase propagation difference in different wavelength of input light. In actual use of interferometer, certain monochrometer, such as band-pass filter is applied to obtain quasi-monochromatic input lays. According to certain band width of monochrometer, the phase propagation in the glass for different wavelength will not same. Especially in the case of the measurement of the beam size by natural emittance in the weak intensity at the small beam current, we often increase the bandwidth of band-pass filter to obtain sufficient intensity for detector such as CCD camera. Under using a common doublet lens, more than $\lambda/5$ phase error can come from a bandwidth of $\Delta\lambda$=80nm at wavelength of 400nm. A simulated interferograms with the Phase error of $\lambda/5$ due to dispersion effect is indicated in Fig. 12 [3]. To see Fig. 12, the visibility of interferogram is reduced from 0.8 to 0.6.

To escape from this decrease of visibility in the interferogram, we should use the reflective objective mirror instead of the glass objective lens in the interferometer. An on-axis system, the Harsherian is also shown in Fig 13.

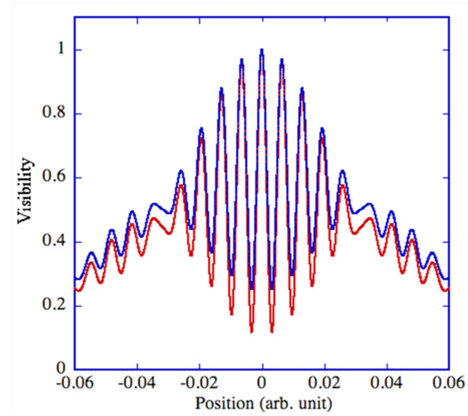

Figure 12: Simulation result of the interferogram with dispersion effect of glass (blue line), and without dispersion effect (red line).

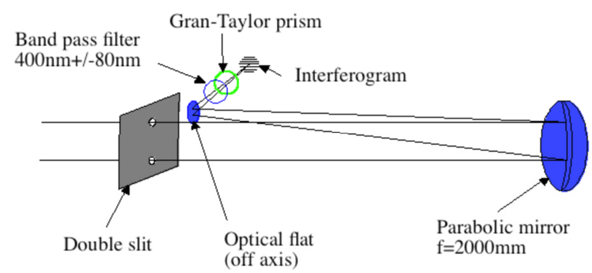

Figure 13: An off axis Herschelian arrangement applied for double slit interferometer.

A very small astigmatism will introduce by a small tilting angle (less than 1 degree) of focusing mirror, but this astigmatism is negligible small (result of a simulation of wavefront error is smaller than $10^{-3}\lambda$) due to the small openings of double slit. According to these reason, we chose the off-axis Herschelian arrangement. A measurement result of interferogram at KEK ATF Dumping ring with an interferometer with the off-axis Herschelian optics is shown in Fig. 14 [3].

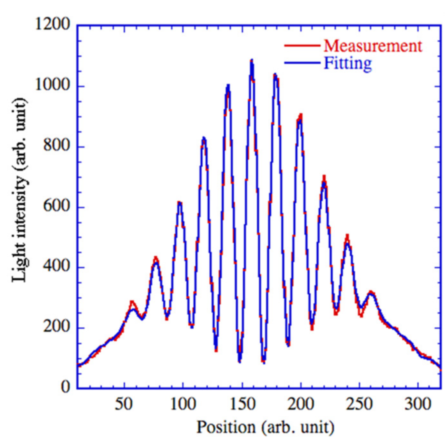

Figure 14: Measurement result of interferogram at KEK ATF Dumping ring with an interferometer having an off-axis Herschelian optics. Red line is measurement result, and blue line is result of best fitting.

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In this measurement, conditions of double slit separation is 60mm at 7.4m from the source point, the wavelength is 400nm and bandwidth is 80nm (rocking curvature of band-pass filter is included in fitting), respectively. Obtained beam size from this interferogram was 4.73μm±0.55μm. The beam current is 1.5mA for this measurement. At the same time, a beam size measurement with a conventional refractive interferometer using a achromatic lens with same conditions was performed. The result of measured vertical beam size was 7.2±0.8 μm.

As an example of application of off-axis Herschelian SR interferometer, a result of X-Y coupling tuning with the skew Q at ATF is shown in Fig. 15 [3]. An clear dependence of vertical beam size with the skew Q change is seen in Fig. 15.

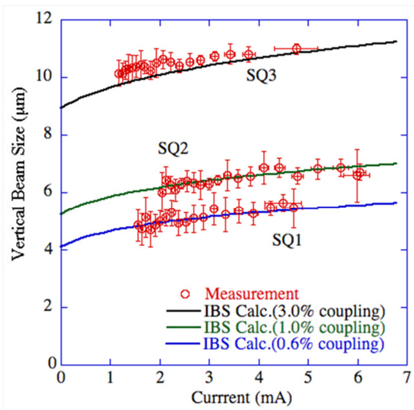

Figure 15: Result of X-Y coupling tuning by using the off-axis Herschelian SR interferometer with the skew Q at ATF.

The same experiment was performed with normal refractive interferometer. A result is shown in Fig. 16 [3].

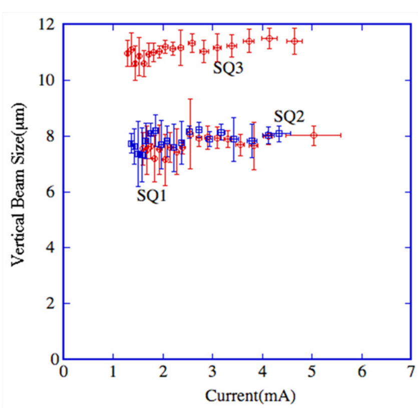

Figure 16: Results of X-Y coupling tuning by using the conventional refractive SR interferometer with the skew Q at ATF.

From Fig. 16, we can see some difference between SQ3 and SQ2, SQ1, but we cannot see difference between SQ1 and SQ2. The current dependence of the beam size is not clear in Fig. 16.

Upper mentioned example of off-axis Herschelian SR interferometer is rather special example for very small beamsize measurement of 5μm in the small beam current range such as 1-2mA. But the reflective system is basically dispersion free, and it is suitable for shorter wavelength. This has an advantage for small beam size measurement. The demerit to use off-axis Herschelian SR interferometer is collimation of the interferometer should be more difficult than collimation in the on axis refractive interferometer. Please note the error in collimation can easily introduce more aberrations.

Further Discussion for Measurement Limit

In previous subsection, we discussed how to escape from dispersion effect in objective lens of SR interferometer at short wavelength range. But even using the reflective interferometer, it is very difficult to use wavelength rage shorter than 400nm. Measurement for small beam size is limited by a systematic and a statistical error of detector system such as CCD in the very good visibility. Actually, certain increase of result of the beam size is observed in γ range very close to 1. An experimental result of measurement of the beam size 5.8μm by changing double slit separation at ATF is shown in Fig. 17 [4].

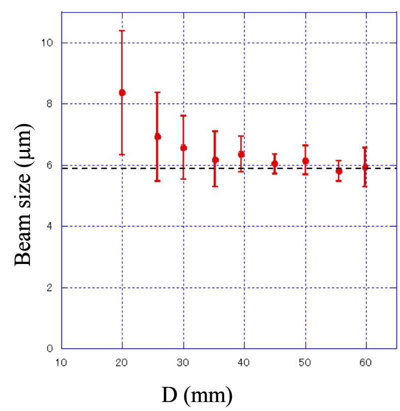

Figure 17: Increase of the beam size in small separation range of double slit. D=40mm corresponds to γ=0.924.

To see Fig. 17, a systematic increase of obtained beam size is observed in the double slit separation range smaller than 40mm in which the visibility of the interferogram exceeded 0.924, due to non-linearity near the baseline of CCD camera. Statistical error is also increased in smaller D range.

Let us discus about error transfer from intensity measurement of interferogram to visibility. Actually, visibility γ is evaluated from the intensity data of interferogram using some non-linear least square retrogression analysis, such as the Lebenbarg-Markart algorism [11]. Since we evaluate error $\Delta\gamma$ in visibility from mean square residual in this process, analytical representation of error transfer is not easy. In here, let us discuss simple error transfer from error ΔI to $\Delta\gamma$ in here. The visibility is given by,

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$$\gamma = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} \qquad (8).$$

Using error transfer relationship,

$$\Delta\gamma^2 = \left(\frac{\partial\gamma}{\partial I_{max}}\right)^2 \Delta I^2{}_{max} + \left(\frac{\partial\gamma}{\partial I_{min}}\right)^2 \Delta I^2{}_{min} \qquad (9)$$

then, $\Delta\gamma$ is given by,

$$\Delta\gamma = \frac{\sqrt{I^2{}_{max} + I^2{}_{max}}}{(I_{max} + I_{min})^2}\Delta I \qquad (10).$$

in here we simply put the errors in intensity at maximum pint and minimum point are same error denoted by ΔI (this putting seems little bit over estimation, since ΔI should inverse proportional to square root of I). Rewritten the equation (10) by using γ,

$$\Delta\gamma = \frac{1}{I_{max}} \frac{\sqrt{1 + \left(\frac{1-\gamma}{1+\gamma}\right)^2}}{\left(1 + \left(\frac{1-\gamma}{1+\gamma}\right)\right)^2}\Delta I \qquad (11)$$

This error transfer relation for $\Delta I/I_{max}$=1% is shown in Fig. 18.

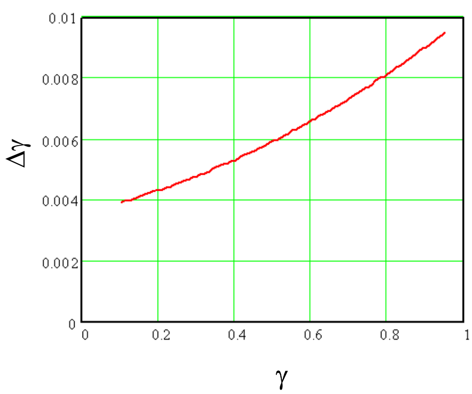

Figure 18: Error transfer relation as a function of γ.

This result indicates we can obtain smaller error when the γ becomes smaller.

In the next, we discuss error transfer from $\Delta\gamma$ to error in beam size $\Delta\sigma$. The error $\Delta\gamma$ in visibility measurement will transfer onto the beam size error $\Delta\sigma$ is given by;

$$\Delta\sigma = \frac{\lambda \cdot F}{\pi \cdot D} \cdot \frac{1}{\gamma \cdot \sqrt{8 \cdot \ln\left(\frac{1}{\gamma}\right)}} \cdot \Delta\gamma \qquad (12)$$

Error in λ, F and D is not included in equation (12). A γ dependence of error transfer from $\Delta\gamma$ to $\Delta\sigma$ under assuming $\Delta\gamma$=0.01 is shown in Fig. 19. The same parameter of ATF condition is applied in this calculation. To see Fig.19, we will have significant error enhancement in γ range larger than 0.9. As a conclusion, for the measurement of very small beam size, we should measure the beam size at the smaller γ. It means, we should use shorter wavelength as short as possible and double slit separation as large as possible. But in actually, useable wavelength is limited by dispersion of optical component glass (aberrations and absorption), and shorter limit should be 400nm as mentioned in before.

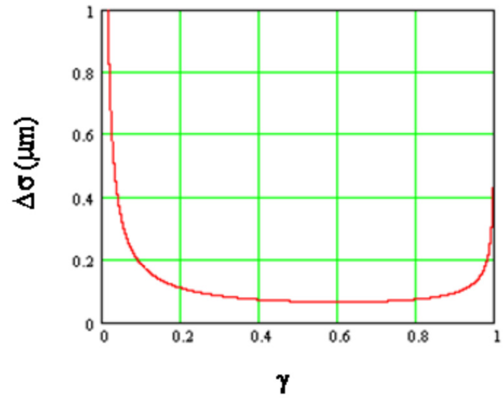

Figure 19: Error transfer from $\Delta\gamma$ to $\Delta\sigma$ under assuming $\Delta\gamma$=0.01 as a function of γ.

To see Fig. 19, we will have significant error enhancement in γ range larger than 0.9. As a conclusion, for the measurement of very small beam size, we should measure the beam size at the smaller γ. It means, we should use shorter wavelength as short as possible and double slit separation as large as possible. But in actually, useable wavelength is limited by dispersion of optical component glass (aberrations and absorption), and shorter limit should be 400nm as mentioned in before. The available double slit separation is limited by opening angle of SR. Using ATF conditions, slit separation is 60mm. In this condition, the γ=0.9 is corresponding to the beam size 3.6μm. This Value should be the smaller limit of measurement range in the SR interferometer. A 1% intensity error ΔI=1% will transfer to beam size error 0.5μm.

Please note, we almost always have more error sources such as air turbulence, floor vibration, etc in actual measurement. So, very careful measurement to escape from such errors is very important for the measurement of very small beam size, too.

Imbalance Input Interferometer

From upper discussion for measurement limit, actual limit in the SR interferometer is still very larger than theoretical limit from uncertainty principal. So, we can introduce a concept of "magnification" as in imaging system for the convenience of observation for interferogram to escape from systematic error such as non-linearity near the baseline of CCD camera.

In the imaging system, we often get into the same problem. When we use only the objective lens, the magnification is often too small, and image size is not sufficient large for the spatial resolution of image detector, such as the CCD camera. In this case, the resolution is actually limited by detector's resolution, not limited by the theoretical resolution from uncertainty principal (diffraction by objective lens). For the purpose of escape

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from such a problem, we apply magnification lens to magnify the image size to obtain convenient image size for detector. We can apply the appropriate magnification under the condition of do not exceed the Rayleigh's criterion of resolution. As same manner, we can introduce a concept of magnification into the interferometry, when the resolution is not limited by theoretical manner.

To consider magnification of the objective size in the interferometry, let us return to the beam size as a function of γ,

$$\sigma_{beam} = \frac{\lambda \cdot F}{\pi \cdot D} \cdot \sqrt{\frac{1}{2} \cdot \ln\left(\frac{1}{\gamma}\right)} \qquad (13)$$

Replacing the γ by using intensities I_1 and I_2, the beam size is given by,

$$\sigma_{beam} = \frac{\lambda \cdot F}{\pi \cdot D} \cdot \sqrt{\frac{1}{2} \cdot \ln\left(\frac{I_1 + I_2}{2 \cdot \sqrt{I_1 \cdot I_2}} \cdot \frac{1}{V}\right)} \qquad (14)$$

in here V denotes visibility of interferogram. Introducing an intensity imbalance factor M by,

$$M = \frac{2 \cdot \sqrt{I_1 \cdot I_2}}{I_1 + I_2} \qquad (15)$$

then, the beam size is represented by,

$$\sigma_{beam} = \frac{\lambda \cdot F}{\pi \cdot D} \cdot \sqrt{\frac{1}{2} \cdot \ln\left(\frac{1}{M} \cdot \frac{1}{V}\right)} \qquad (16).$$

Let's define the transverse magnification β by,

$$\beta = \frac{\sigma_{imbalance}}{\sigma} = \sqrt{\frac{\ln(M)}{\ln(V)} + 1} \qquad (17).$$

The transverse magnification β at V=0.9 as a function of imbalance factor is shown in Fig.20.

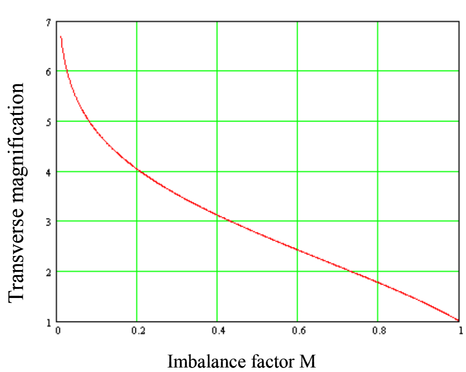

Figure 20: Transverse magnification as a function of imbalance factor.

As a conclusion, we can introduce a magnification into the SR interferometer by using an imbalanced input configuration. For example, intensity imbalance factor 0.2 will give the magnification of about 4. The intensity imbalance factor by changing the intensity in one slit of double slit is shown in Fig. 21. From this figure, the intensity imbalance factor 0.5 is corresponding to intensity ratio of 1:0.072 and 0.2 is corresponding to 1:0.011.

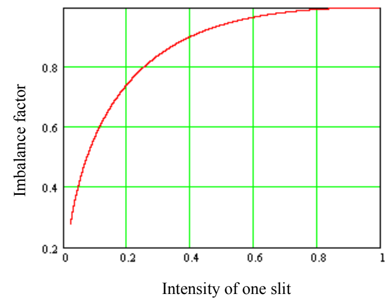

Figure 21: The intensity imbalance factor as a function of changing the intensity in one slit of double slit.

Roughly speaking, we can obtain intensity imbalance factor 0.2 by using neutral density filter ND2 in front of one slit. This will give a magnification about 4.

As well as the appropriate magnification in the imaging system, appropriate magnification must be applied to the interferometry. From view point of uncertainty principle, imbalance input introduce a decrease of intensity at one slit of double slit, and it oppositely increases the phase fluctuation. In the practical condition, the SR interferometer has, an intensity at double slit of order of 10^8 photons/sec in 10nm bandwidth at 400nm (in the setup in ATF). When we will apply 100 times decrease in intensity in one slit, the fluctuation of light phase will be order of 10^{-6} rad. It is still negligible small compare with that from wavefront error in the optical components (about 0.126rad) as described in before. The statistical error in the beam size is enhanced by intensity imbalance factor M. This error enhancement due to $\Delta I/I_{max}$=1% at M=0.2 (corresponding magnification is 4) is estimated by using Eq. (11) and Eq.(12). A result is shown in Fig. 22.

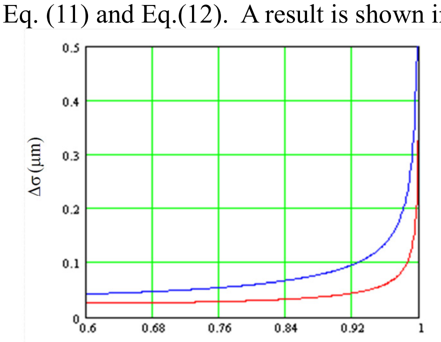

Figure 22: Error enhancement due to $\Delta I/I_{max}$=1% at M=0.2. Blue line indicates with error enhancement in imbalanced input. Red line indicates error transfer relation in balanced input.

To see Fig. 22, error enhancement due to imbalance input is still range of 0.1-0.3μm in the case of at M=0.2 (corresponding magnification is about 4). Error in beam size due to wavefront error is estimated to 0.24μm. As a conclusion, magnification by the use of imbalance input, we can escape from a difficulty of the measurement of the very good visibility, due to non-linearity near the baseline

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of CCD camera and we can make it possible to measure submicron range beam size with visible SR.

The magnification given by eq. 17 can also represented as ratio between two separations D1 and D2, as follows,

$$\beta = \frac{D_2}{D_1} = \sqrt{\frac{\ln(M)}{\ln(V)} + 1} \qquad (18)$$

In here, one visibility measures at D_1, and other visibility measures at D_2 with imbalance ratio M. The eq.17 represents magnification in beam size, and the eq.18 represents magnification in spatial frequency (double slit separation), respectively. The same value of visibility will observe in different D with imbalance input as shown in Fig. 23.

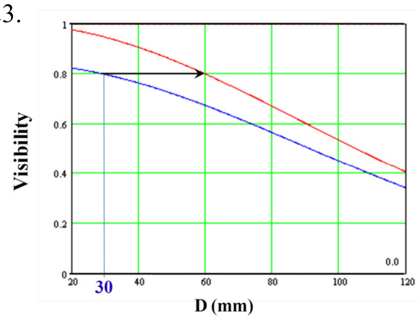

Figure 23: The same value of visibility will observe in different D with imbalance input. Red line denotes visibility without imbalance input, blue line denotes visibility with imbalance input M=0.843.

We can measure the same beamsize with smaller double slit separation by imbalance input.

In the last, please note imbalance input method is only the method to magnify existing information to escape from noises. Using it in appropriate range is important.

Experiment of Imbalance Input Interferometer

An experimental set up of SR interferometer using an imbalanced input configuration is shown in Fig. 24 [3]. A special ND filter which lower half of glass flat is covered with an Aluminium coating to reduce the intensity is used for obtain imbalanced input for double slit. This filter is set in front of the double slit of interferometer.

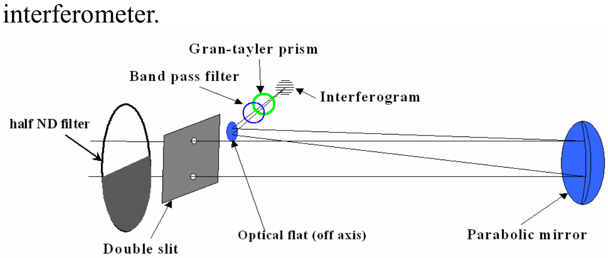

Figure 24: Setup of imbalance input interferometer. A special ND filter which lower half of glass flat is covered with an Aluminium coating is set in front of double slit.

Other setup is just same setup as in the reflective interferometer. A measurement result of imbalance input interferometer for same 5.8μm beam size measurement is shown in Fig. 25 [3]. The filter which has imbalanced ratio 0.842 was used for this experiment. In the Fig. 25, imbalance input interferometer result is indicated blue dot.

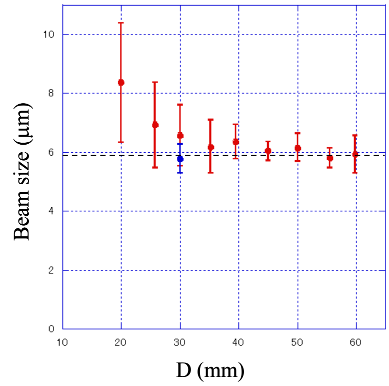

Figure 25: A measurement result of imbalance input interferometer for same 5.8μm beam size measurement (indicated by blue dot).

From this figure, the imbalance input interferometer result is clearly escape from systematic increase in the obtained beam size as in the result from the balance input interferometer at D=30mm. Further experiment on imbalanced input SR interferometer, reader can find paper written by M.J. Boland and the author in somewhere in this proceedings.

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