# RF quadrupole for Landau damping in accelerators. Analytical and numerical studies

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# Outline

- Introduction
  - LHC octupoles for Landau damping
  - Stability diagram for Landau damping
- Frequency spread induced by RF quadrupole
- Parameters of Landau damping scheme in LHC based on RF quadrupole
- Numerical investigation of stability in the presence of RF quadrupole
- Conclusions

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# LHC octupoles for Landau damping

*Landau damping, dynamic aperture and octupoles in LHC*, J. Gareyte, J.P. Koutchouk and F. Ruggiero, LHC project report 91, 1997

For given actions $J_x$, $J_y$, the gradient perturbation is constant; the tune shifts may easily be computed from the classical tune-shift formula, with the proper sign inversion for the vertical plane (negative gradients give positive vertical tune shifts):

$$\Delta Q_x = \left[\frac{3}{8\pi}\int \beta_x^2 \frac{O_3}{B\rho}\,ds\right] J_x - \left[\frac{3}{8\pi}\int 2\beta_x\beta_y \frac{O_3}{B\rho}\,ds\right] J_y, \quad (8)$$

$$\Delta Q_y = \left[\frac{3}{8\pi}\int \beta_y^2 \frac{O_3}{B\rho}\,ds\right] J_y - \left[\frac{3}{8\pi}\int 2\beta_x\beta_y \frac{O_3}{B\rho}\,ds\right] J_x. \quad (9)$$

$$\sum_i (O_3 l)_i = B\rho\,\frac{8\pi}{3}\,\frac{1}{\beta_F^2+\beta_D^2}\,\frac{|\Delta Q_{coh}|}{4\epsilon}, \quad (21)$$

For LHC version 4.3, the most demanding requirements are observed at 7 TeV where

| $\beta_F$ m | $\beta_D$ m | $\epsilon$ nm | $\lvert\Delta Q_{coh}\rvert$ |
|---|---|---|---|
| 175.5 | 32.5 | 0.5 | $0.223 \times 10^{-3}$ |

which yield

$$\sum_i (O_3 l)_i = 0.685 \times 10^6 \text{ Tm}^{-2}. \quad (22)$$

This integrated octupole strength requires about 5 octupoles per arc and per family, i.e., a total of 10 octupoles per arc, with the new characteristics $O_3 = 62000$ Tm$^{-3}$ and $l = 0.328$ m [11]. There are presently 23 positions available for octupoles in each arc [12],

**80 octupoles of 0.328m each are nesessary to Landau damp the most unstable mode at 7 TeV with $\Delta Q_{coh}$=0.223e-3**

**In LHC, 144 of these octupoles (total active length: 47 m) are installed in order to have 80% margin and avoid relying completely on 2D damping**

# Stability diagrams for Landau damping (1)

Berg, J.S.; Ruggiero, F., **LHC Project Report 121, 1997**

Distribution function:

$$\Psi_0(\boldsymbol{J}) = \frac{(2\pi)^3}{\varepsilon_x \varepsilon_y \varepsilon_z} S(J_x/\varepsilon_x, J_y/\varepsilon_y)\lambda(J_z/\varepsilon_z)$$

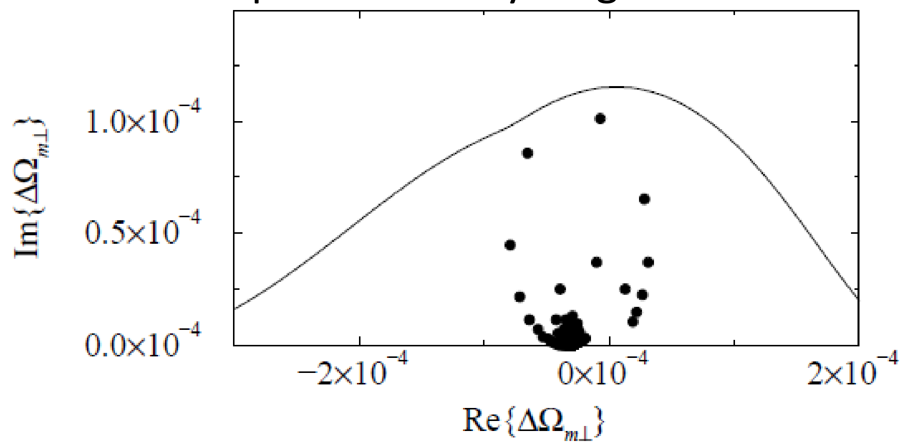
Dispersion equations:

(1) $$\frac{1}{\Delta\Omega^{coh}_{x,y}} = -\int \frac{J_{x,y}\,\partial\Psi_0/\partial J_{x,y}}{\Omega^T_{x,y} - \omega_{x,y}(J_x, J_y) - m\omega_z(J_x, J_y)} d^3\boldsymbol{J}$$

(2) $$\frac{1}{\Delta\Omega^{coh}_{x,y}} \int J_z^{|m|}\Psi_0\, d^3\boldsymbol{J} = \int \frac{J_z^{|m|}\Psi_0}{\Omega^Z_{x,y} - \omega_{x,y}(J_z) - m\omega_z(J_z)} d^3\boldsymbol{J}$$

(3) $$\frac{1}{\Delta\Omega^{coh}_z} \int J_z^{|m|}\,\partial\Psi_0/\partial J_z d^3\boldsymbol{J} = \int \frac{J_z^{|m|}\,\partial\Psi_0/\partial J_z}{\Omega^Z_z - m\omega_z(J_z)} d^3\boldsymbol{J}$$

An example of stability diagram:




3D vector of tune linearized in terms of action:

Octupole tune spread: $Q_x = Q_0 + aJ_x + bJ_y$

$$\begin{pmatrix}\omega_x\\ \omega_y\\ \omega_z\end{pmatrix} = \begin{pmatrix}\omega_{Lx}\\ \omega_{Ly}\\ \omega_{Lz}\end{pmatrix} + \begin{bmatrix} a_{xx} & a_{xy} & a_{xz}\\ a_{yx} & a_{yy} & a_{yz}\\ a_{zx} & a_{zy} & a_{zz}\end{bmatrix} \begin{pmatrix} J_x/\epsilon_x\\ J_y/\epsilon_y\\ J_z/\epsilon_z\end{pmatrix}$$

Potential well distortion, actually non-linear !

# Stability diagrams for Landau damping (2)

Berg, J.S.; B Ruggiero, F., **LHC Project Report 121, 1997**

### 3 LONGITUDINAL TUNE SPREAD

Next, consider Landau damping of either transverse or longitudinal oscillations due to longitudinal tune spread. In this case, one computes

$$\left[\int_0^\infty u^{|m|} f(u)\,du\right]^{-1} \int_0^\infty \frac{u^{|m|} f(u)\,du}{\Delta\Omega_m - v_z u}, \qquad (12)$$

where the symbols are defined as:

| Symbol | Transverse | Longitudinal |
|---|---|---|
| $f(u)$ | $\lambda(u)$ | $d\lambda/du$ |
| $v_z$ | $a_{yz} + m a_{zz}$ | $m a_{zz}$ |
| $\Delta\Omega_m$ | $\Omega - \omega_{Ly} - m\omega_{Lz}$ | $\Omega - m\omega_{Lz}$ |

and compares the results to $1/\Delta\Omega_m$ for the linear lattice.

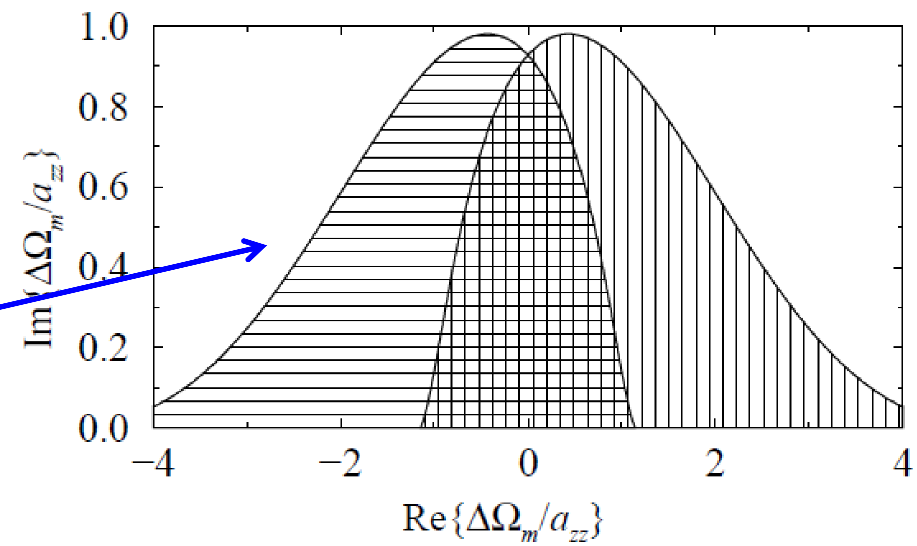



Figure 3: Stability curves for transverse oscillations when $a_{yz} = 0$. Vertical lines give the stable region for $m = +1$, and horizontal lines give the stable region for $m = -1$.

- Make longitudinal tune spread larger than the coherent tune shift -> Landau damping

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# Effect of RF quadrupole

Transverse kick: $\Delta \boldsymbol{p}_\perp = p k_2 (-x\boldsymbol{u}_x + y\boldsymbol{u}_y)\cos(\omega t + \varphi_0)$

Panofsky-Wenzel theorem

where $k_2 = \frac{b^{(2)}}{\rho B_0} = \frac{q}{pc}\frac{1}{\pi r}\int_0^{2\pi}\left\|\int_0^L \left(E_x - cB_y\right)e^{j\omega z/c}\,dz\right\|\cos\varphi\,d\varphi$

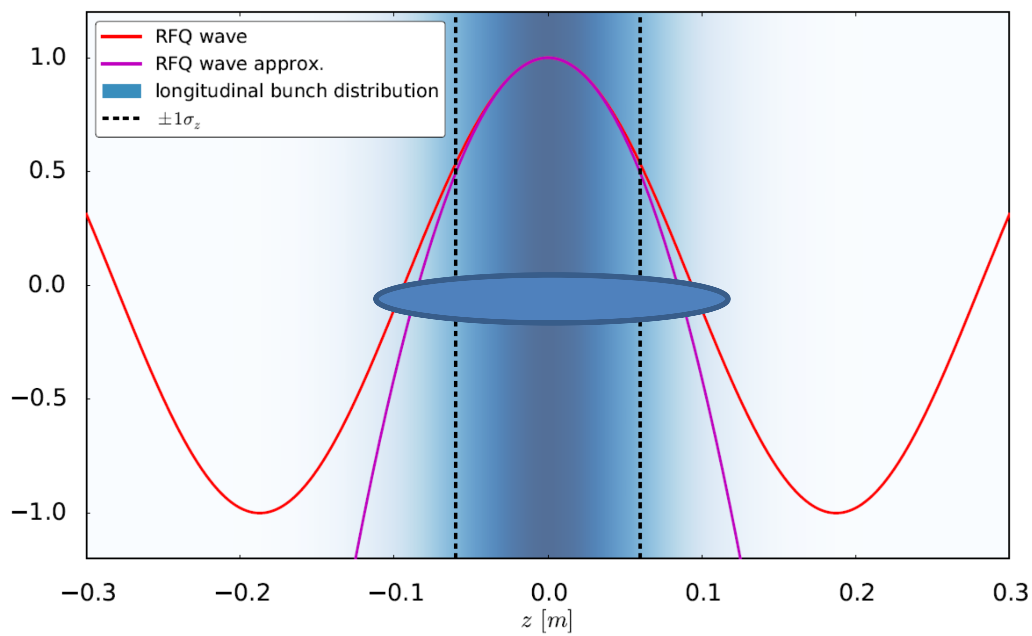
Accelerating voltage: $V = \frac{\omega b^{(2)}}{2}(x^2 - y^2)\sin(\omega t + \varphi_0)$

On crest of RFQ wave:

$\varphi_0 = 0$

$\cos(\omega t + \varphi_0) = \cos(\omega z/c)$

$[B'L](z) = b^{(2)}\cos(\omega z/c)$



# Longitudinal spread of betatron tune and Transverse spread of synchrotron tune induced by RF quadrupole

3D vector of tune linearized in terms of action:

$$\begin{pmatrix}\omega_x\\ \omega_y\\ \omega_z\end{pmatrix} = \begin{pmatrix}\omega_{Lx}\\ \omega_{Ly}\\ \omega_{Lz}\end{pmatrix} + \begin{bmatrix}a_{xx} & a_{xy} & a_{xz}\\ a_{yx} & a_{yy} & a_{yz}\\ a_{zx} & a_{zy} & a_{zz}\end{bmatrix}\begin{pmatrix}J_x/\epsilon_x\\ J_y/\epsilon_y\\ J_z/\epsilon_z\end{pmatrix}$$

**Longitudinal spread of both horizontal and vertical betatron tunes is non-zero for RF quadrupole**

**Both horizontal and vertical spreads of synchrotron tune are non-zero for RF quadrupole**

$$\omega_s = \omega_{s0} + \frac{1}{2}\frac{\omega_0^2|\eta|h_2V_2}{\omega_{s0}2\pi c\rho B_0} = \omega_{s0} + \frac{\omega_0}{8\pi}\frac{b^{(2)}}{\rho B_0}\left(\frac{\omega}{c}\right)^2\frac{|\eta|c}{\omega_{s0}}(y^2 - x^2)$$

$$x, y = \sqrt{2J_{x,y}\beta_{x,y}}\cos(\Theta_{x,y}); \quad \left\langle x^2, y^2\right\rangle_{\Theta_{x,y}} = J_{x,y}\beta_{x,y}$$

$$\Delta\omega_s = \frac{\omega_0}{8\pi}\frac{b^{(2)}}{\rho B_0}\left(\frac{\omega}{c}\right)^2\frac{|\eta|c}{\omega_{s0}}(J_y\beta_y - J_x\beta_x)$$

$$a_{zx} = -\beta_x\varepsilon_x\frac{\omega_0}{8\pi}\frac{b^{(2)}}{\rho B_0}\left(\frac{\omega}{c}\right)^2\frac{|\eta|c}{\omega_{s0}}; a_{zy} = +\beta_y\varepsilon_y\frac{\omega_0}{8\pi}\frac{b^{(2)}}{\rho B_0}\left(\frac{\omega}{c}\right)^2\frac{|\eta|c}{\omega_{s0}}$$

If $\pi\sigma_z\sigma_E / E = \varepsilon_{x,y}$ matrix is symmetric: $a_{zx} = a_{xz}; a_{zy} = a_{yz}$

$$B'L(z) = b^{(2)}\cos\left(\frac{\omega}{c}z\right) = b^{(2)}\left[1 - \left(\frac{\omega}{c}\right)^2\frac{z^2}{2} + o(z^4)\right]$$

$$z = \sqrt{2J_z\beta_z}\cos(\Theta_z); \quad \Rightarrow \left\langle z^2\right\rangle_{\Theta_z} = J_z\beta_z = \sigma_z^2\frac{J_z}{\varepsilon_z}$$

$$\Delta Q_{x,y} = \frac{\beta_{x,y}KL}{4\pi} = \frac{\beta_{x,y}(s)}{4\pi}\frac{B'L}{\rho B_0}$$

$$\Delta\omega_{x,y} = \pm\beta_{x,y}\frac{\omega_0}{4\pi}\frac{b^{(2)}}{B_0\rho}\left[1 - \left(\frac{\omega}{c}\right)^2\frac{\beta_z J_z}{2}\right]$$

$$a_{xz} = -\beta_x\frac{\omega_0}{8\pi}\frac{b^{(2)}}{\rho B_0}\left(\frac{\omega}{c}\right)^2\sigma_z^2 \quad \text{focusing}$$

$$a_{yz} = +\beta_y\frac{\omega_0}{8\pi}\frac{b^{(2)}}{\rho B_0}\left(\frac{\omega}{c}\right)^2\sigma_z^2 \quad \text{de-focusing}$$

**7um >> 0.5nm, for LHC 7TeV => $a_{zx} \ll a_{xz}$; $a_{zy} \ll a_{yz}$ => Longitudinal spread is much more effective**

↑TDR longitudinal $\varepsilon_z(4\sigma)$ =2.5eVs and transverse normalized $\varepsilon^N_{x,y}(1\sigma)$=3.75um emittances are used

# Longitudinal spread of the betatron tune

### 3 LONGITUDINAL TUNE SPREAD

Next, consider Landau damping of either transverse or longitudinal oscillations due to longitudinal tune spread. In this case, one computes

$$\left[\int_0^\infty u^{|m|} f(u)\, du\right]^{-1} \int_0^\infty \frac{u^{|m|} f(u)\, du}{\Delta\Omega_m - v_z u}, \qquad (12)$$

where the symbols are defined as:

| Symbol | Transverse | Longitudinal |
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and compares the results to $1/\Delta\Omega_m$ for the linear lattice.

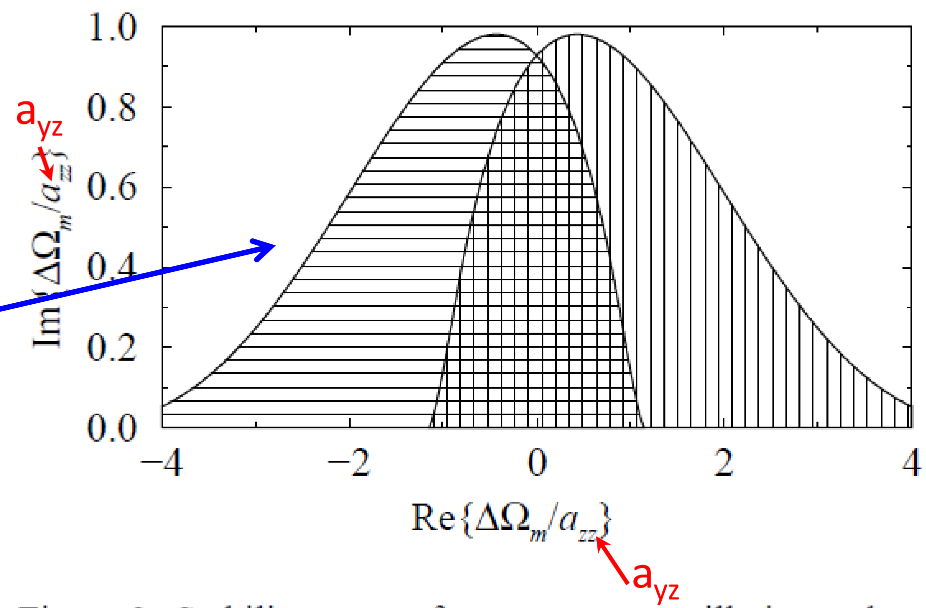


Figure 3: Stability curves for transverse oscillations when $a_{yz} \not= 0$. Vertical lines give the stable region for $m = +1$, and horizontal lines give the stable region for $m = -1$.

- In the case of RF quadrupole $\mathbf{a_{yz}}$ is not zero. One can just substitute $\mathbf{a_{yz}}$ instead of $\mathbf{ma_{zz}}$
- Make longitudinal tune spread $\mathbf{a_{yz}}$ larger than the coherent tune shift ($\mathbf{\Delta\Omega_m}$) -> Landau damping.
- This gives the RF quadrupole strength required for Landau damping:

$$a_{yz} = \beta_y \frac{\omega_0}{8\pi} \frac{b^{(2)}}{\rho B_0} \left(\frac{\omega}{c}\right)^2 \sigma_z^2 \equiv \Delta\Omega = \left|\Delta Q_{\mathrm{coh}}^{y}\right| \omega_0 \quad \Rightarrow \quad b^{(2)} = \rho B_0 \frac{2}{\pi} \frac{\lambda^2}{\sigma_z^2} \frac{\left|\Delta Q_{\mathrm{coh}}^{x,y}\right|}{\beta_{x,y}}$$

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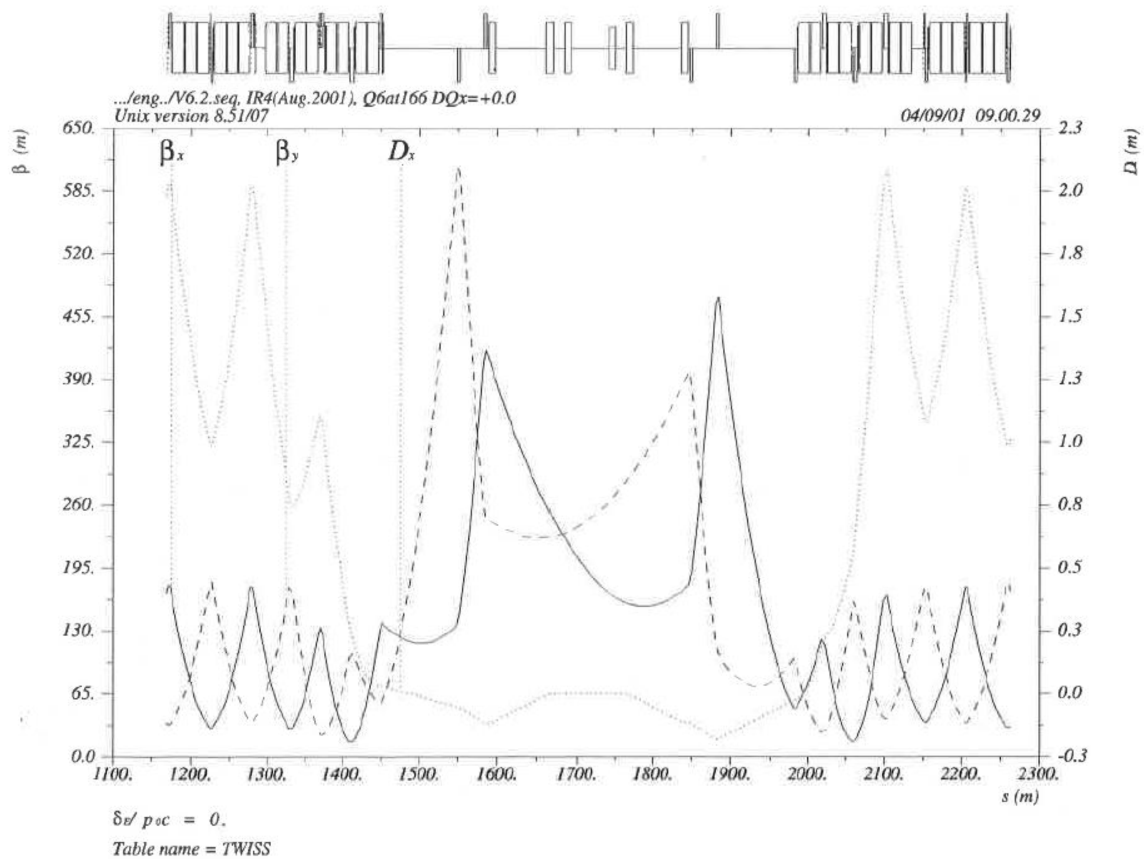
# RF quadrupole in IR4 of LHC




Form LHC TDR

Figure 4.16: The optic functions for Beam 1 in IR4.

$$b^{(2)} = \rho B_0 \frac{2}{\pi} \frac{\lambda^2}{\sigma_z^2} \frac{\left|\Delta Q_{\mathrm{coh}}^{x,y}\right|}{\beta_{x,y}}$$

- $|\Delta Q_{coh}| \approx 2e\text{-}4$
- $\beta \approx 200m$
- $\sigma_z = 0.08m$
- $\lambda = 3/8m$ (800MHz)
- $\rho = 2804m$
- $B_0 = 8.33T$

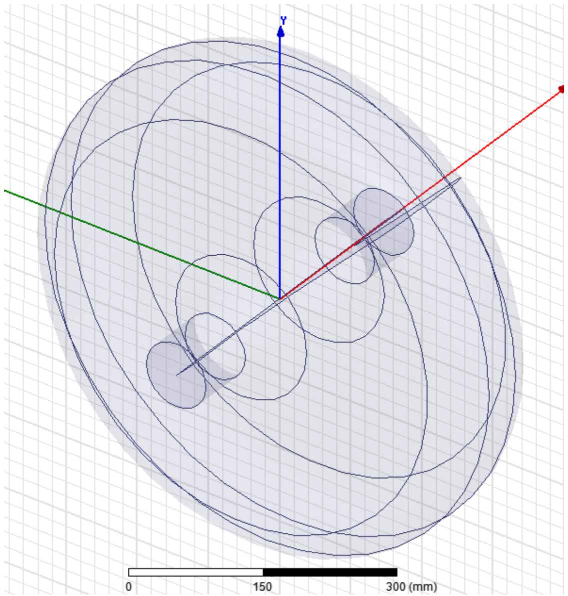
-----------------------

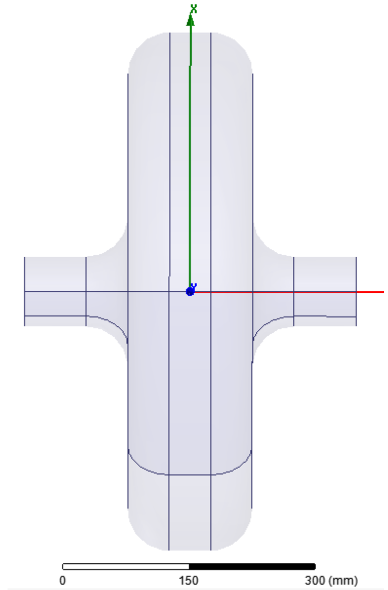
- $b^{(2)} = 0.33Tm/m$

OR

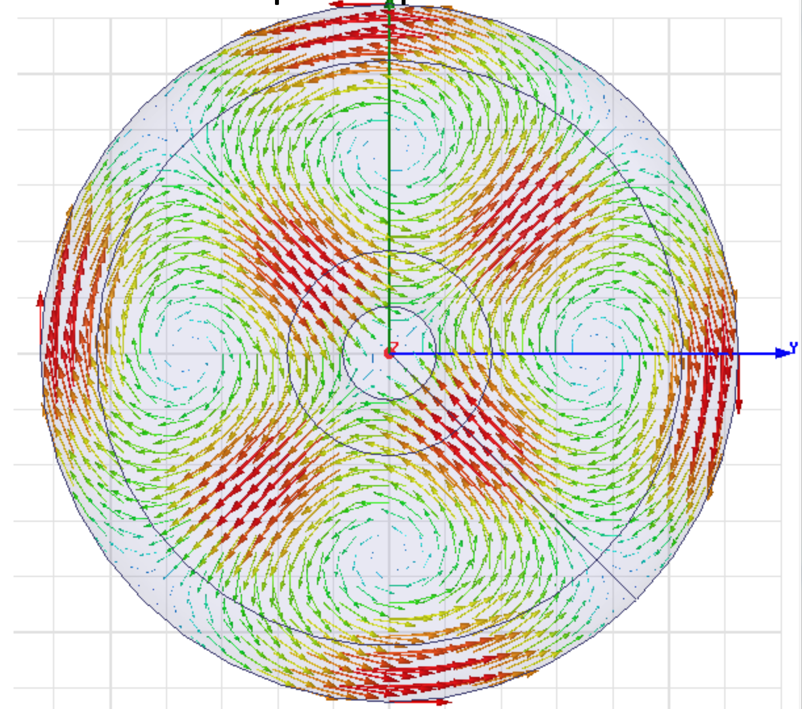
- $k_2 = b^{(2)}/\rho B_0 = 1.4e\text{-}5/m$

# 800 MHz Pillbox cavity RF quadrupole




H-field of TM quadrupolar mode



- For a SC cavity, Max(Bsurf) < 100 mT
- => One cavity can do: $b^{(2)} < 0.12$ Tm/m
- ----------------------
- 3 cavity is enough to provide $b^{(2)} = 0.33$ Tm/m
- Taking the same margin of 80% as for LHC Landau octupoles we arrive to 6 cavities
- **One few meters long cryo-module**

| Stored energy [J] | 1 |
|---|---|
| $b^{(2)}$ [Tm/m] | 0.0143 |
| Max(Bsurf) [mT] | 12 |
| Max(Esurf) [MV/m] | 4.6 |

A. Grudiev, PRST-AB 17, 011001 (2014)

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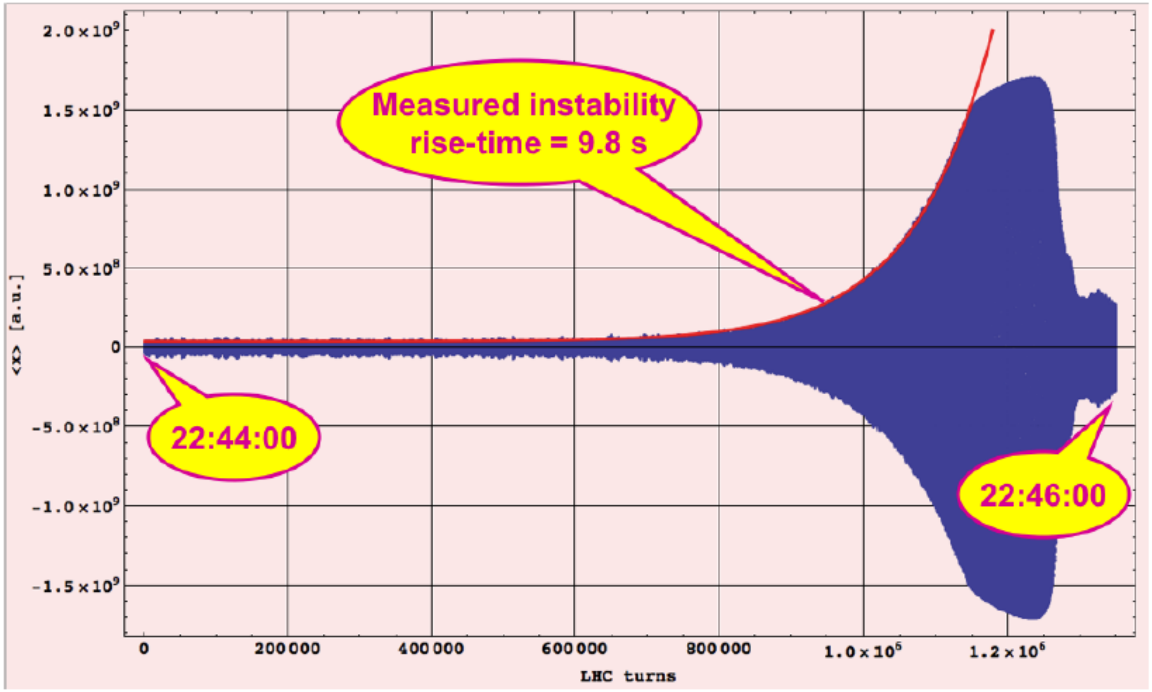
# Results

## LHC MD session at 3.5 TeV – 17.05.2010

- LHC MD at 3.5 TeV, single bunch.
- $1.05 \cdot 10^{11}$ particles, $\epsilon_x^{norm} \approx 5\,\mu m$, $Q'_x \approx 6$ (with large uncertainties).
- Sign of octupole currents $I_f = -I_d < 0$.
- Instability rises after setting octupole currents to $I_f = -10\,\text{A}$.
- Beam is stable when $I_f \leq -20\,\text{A}$.

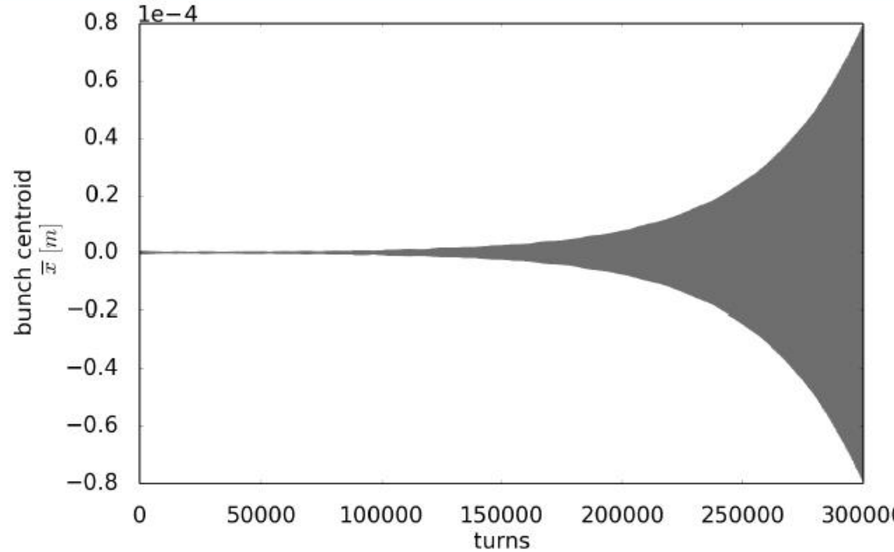


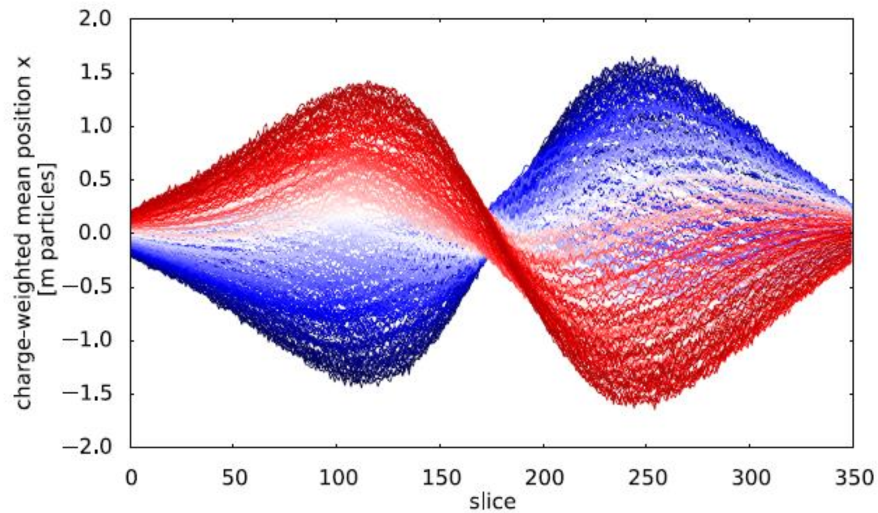

LHC MD data at 3.5 TeV. Beam centroid position $\overline{x}$ vs. number of turns. Landau octupoles are at $I_f = -10\,\text{A}$. Spectral analysis shows that it is a $m = -1$ instability [2, 3].

E. Metral, B. Salvant and N. Mounet, "Stabilization of the LHC Single-Bunch Transverse Instability at High-Energy by Landau Damping", CERN-ATS-2011-102, 2011

**PyHEADTAIL parameters**

- LHC at 3.5 TeV, single bunch.
- $1.05 \cdot 10^{11}$ particles, $\epsilon_x^{\text{norm}} = 3.75\,\mu m$, $Q'_x = 6$.
- Wake tables generated from collimator data (impedance model V2, N. Mounet).
- Dipolar one-turn wakes only.
- $10^6$ macroparticles, 500 slices, $3 \cdot 10^5$ turns.
- $-Im\,(\Delta Q_{\text{coh}}) = 3.6 \cdot 10^{-6}$.
- $Re\,(\Delta Q_{\text{coh}}) = -9.2 \cdot 10^{-5}$.

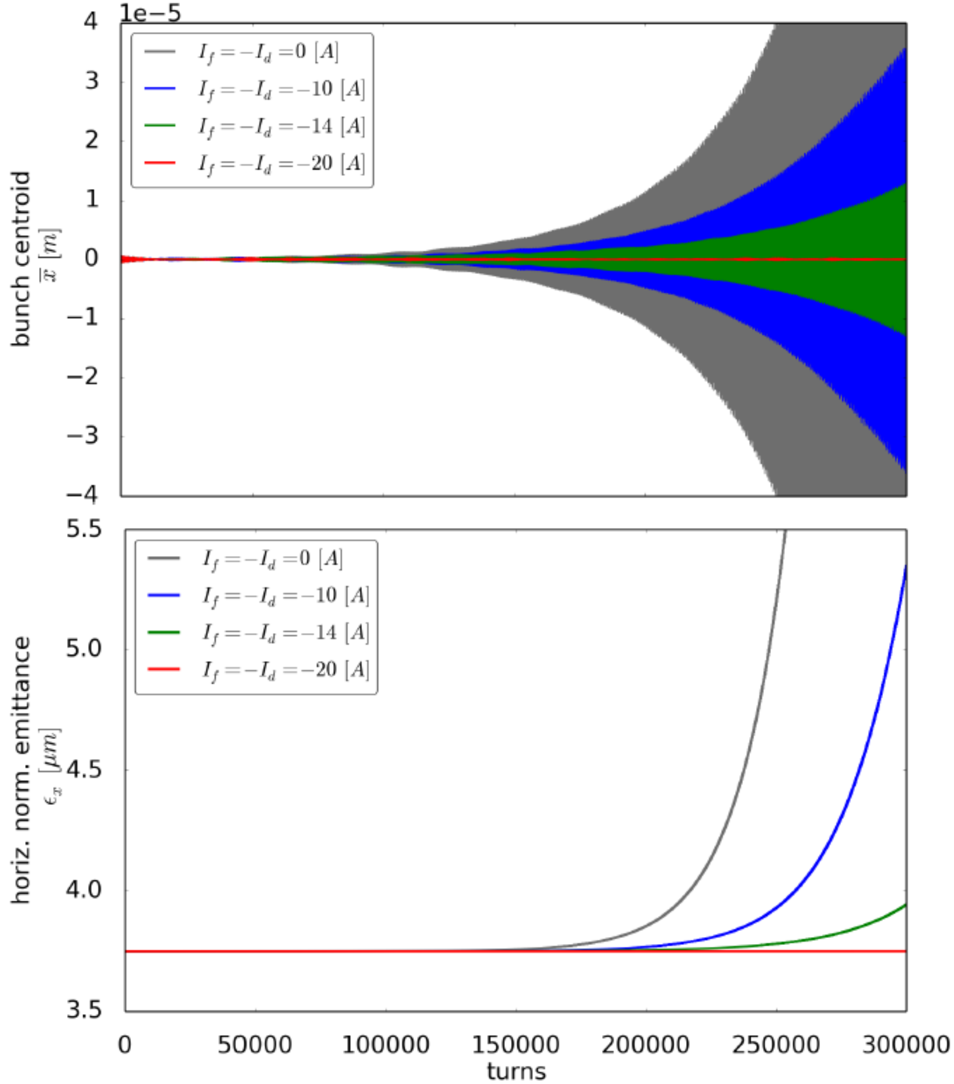


# Results

## Stabilization by means of Landau octupoles

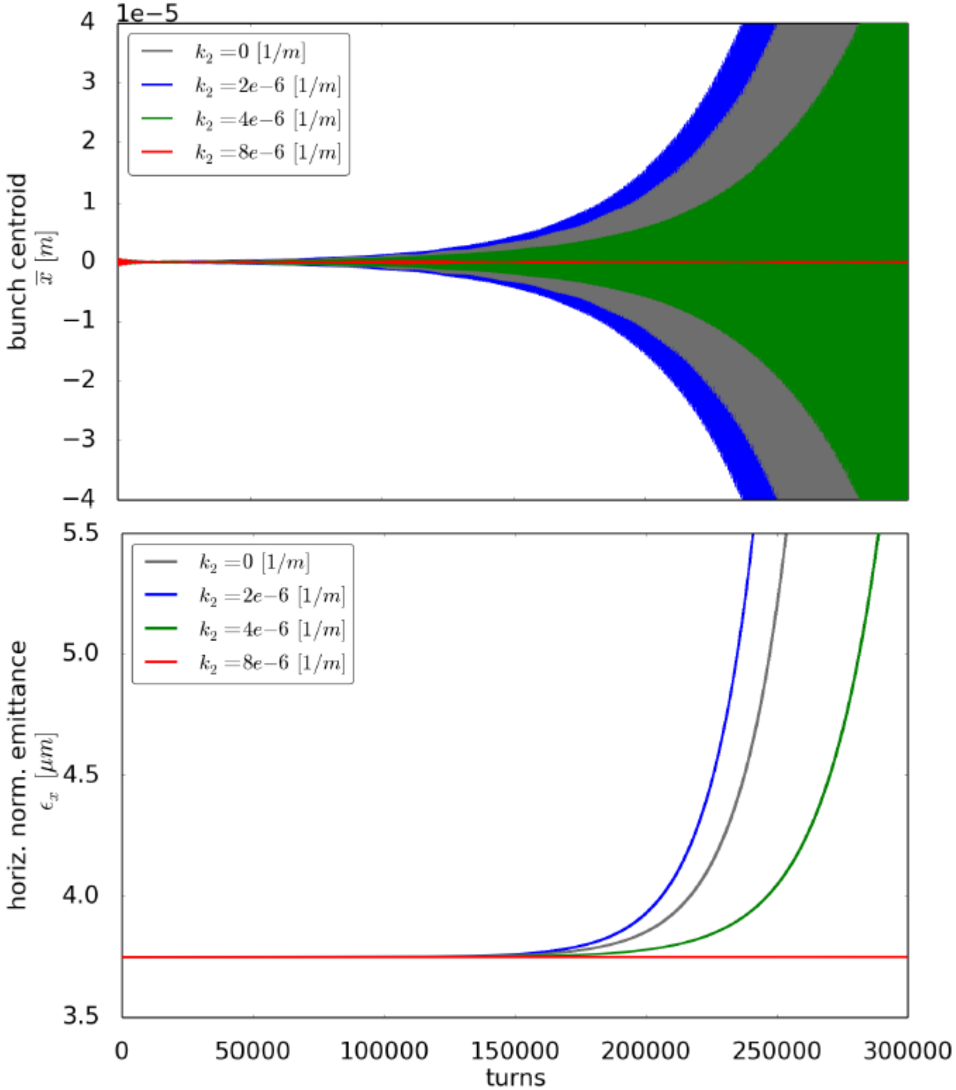
- PyHEADTAIL simulations.
- Beam centroid (top) and normalized emittance (bottom) vs. number of turns.
- PyHT simulations agree that octupoles are able to cure the instability.
- Stabilization threshold $I_f^{\mathrm{PyHT}} = -18 \ldots -20\,\mathrm{A}$ (MD data $I_f^{\mathrm{MD}} = -10 \ldots -20\,\mathrm{A}$).



# Results

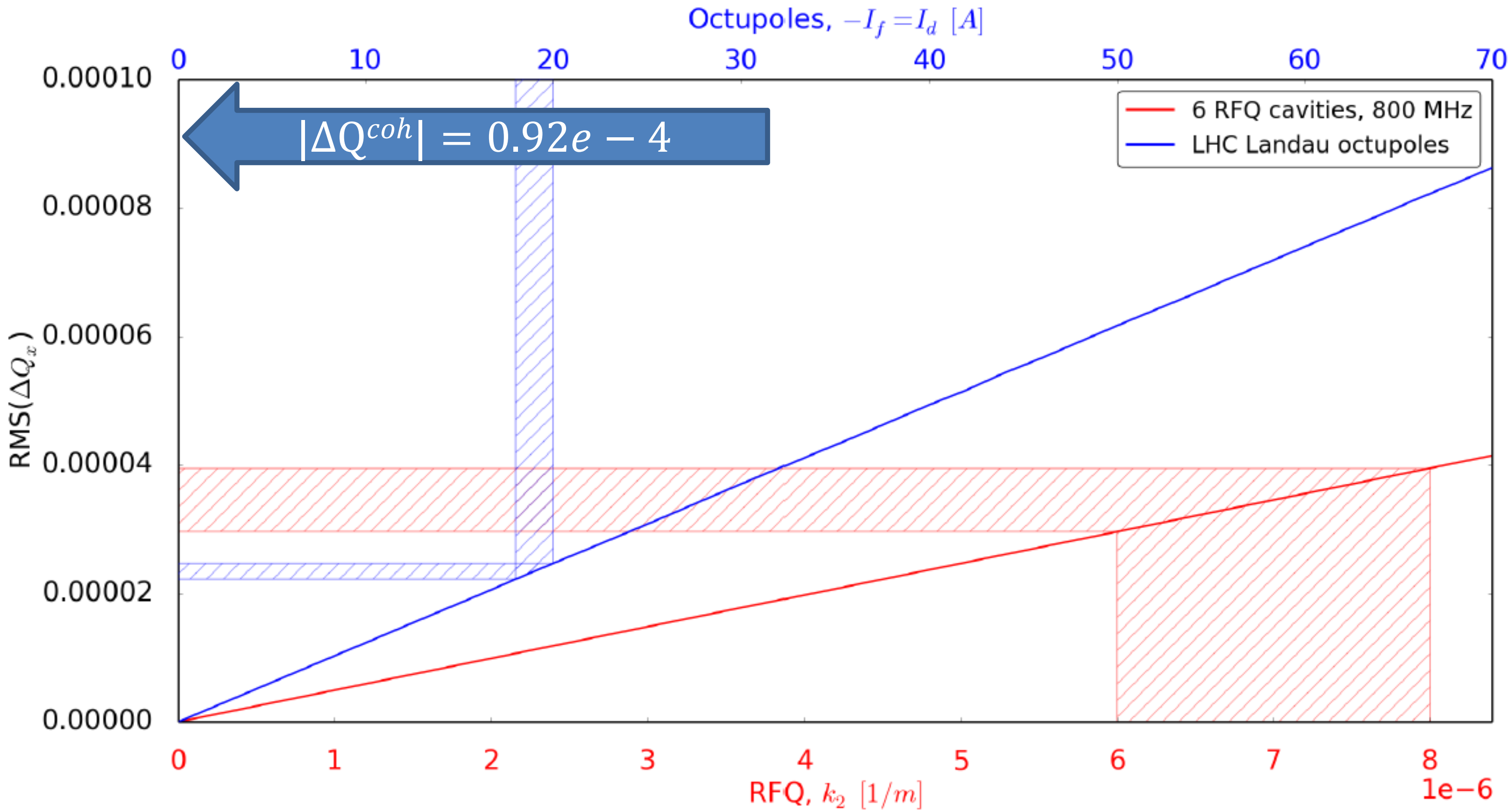
## Stabilization by means of the RFQ (detuner model)

- RFQ detuner model.
- PyHEADTAIL simulations.
- Beam centroid (top) and normalized emittance (bottom) vs. number of turns.
- **The RFQ is equally able to cure the instability.**
- Stabilization threshold $k_2 = 6 \dots 8 \cdot 10^{-6}\ 1/\mathrm{m}$ (1-2 SC cavities, $\approx 0.3\,\mathrm{m}$ [1]).



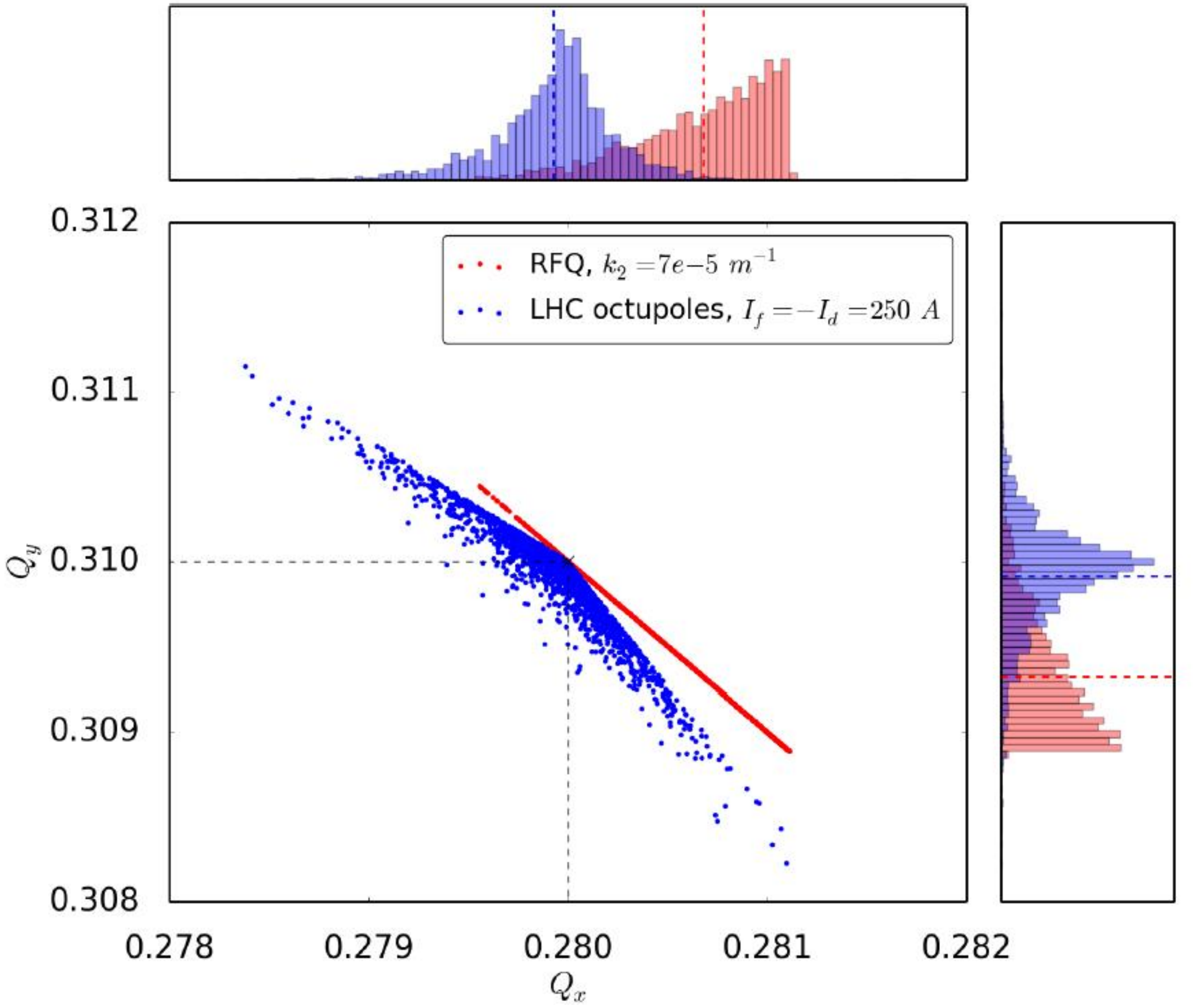
# Results

## Octupole and RFQ tune spreads needed for stabilization



# Background
## Tune footprints of octupoles and RFQ

- **RFQ avg. tune shift**
  $$\propto 1 - \frac{1}{2}\left(\frac{\omega}{\beta c}\right)^2 \beta_z J_z^i.$$
- $\Delta Q_x^i$ and $\Delta Q_y^i$ fully correlated for RFQ.
- **Octupole tune shift**
  $$\Delta Q_x^i = a_{xx} J_x^i + a_{xy} J_y^i$$
  $$\Delta Q_y^i = a_{yy} J_y^i + a_{xy} J_x^i$$
- Comparable detuning strengths, i.e. same RMS spread.



# Summary

- RF quadrupole provides longitudinal spread of betatron tune for Landau damping
- This has been confirmed by the numerical simulations using PyHEADTAIL code
- Longitudinal spread is more efficient for Landau damping than transverse spread since typically longitudinal emittance is much larger than the transverse one
- This advantage becomes more apparent for higher energy and higher brightness beams

# Spare slides

# Synchrotron frequency in the presence of an RF quadrupole

Main RF ($\phi_s$ = 0 at zero crossing):

$$V_{acc}(z) = V_0 \sin\left(\frac{h\omega_0}{c} z + \phi_{s0}\right); \quad \omega_{s0}^2 = \omega_0^2 \frac{|\eta| h V_0 \cos(\phi_{s0})}{2\pi c \rho B_0}$$

RF quad voltage, if $b^{(0)}$ is real.
The centre of the bunch is on crest for quadrupolar focusing but it is on zero crossing for quadrupolar acceleration ($\phi_{s2}$ = 0 ):

$$V_{acc}^{Q}(r,\varphi,z) = \Re\left\{V_{acc}^{(2)} e^{j\frac{\omega}{c}z}\right\} \cdot r^2 \cos(2\varphi)$$

$$= jV_{acc}^{(2)} \sin\left(\frac{\omega}{c} z\right) \cdot (y^2 - x^2) = V_2 \sin\left(\frac{\omega}{c} z\right)$$

Synchrotron frequency for Main RF + RF quad voltage:

$$\omega_s^2 = \omega_0^2 \frac{|\eta| h V_0 \cos(\phi_{s0})}{2\pi c \rho B_0}\left[1 + \frac{h_2 V_2 \cos(\phi_{s2} = 0)}{h V_0 \cos(\phi_{s0})}\right]; \qquad V_2 = \frac{\omega b^{(2)}}{2}(y^2 - x^2)$$

$$\omega_s = \omega_{s0}\sqrt{1 + \frac{h_2 V_2}{h V_0 \cos(\phi_{s0})}} \cong \omega_{s0}\left[1 + \frac{1}{2}\frac{h_2 V_2}{h V_0 \cos(\phi_{s0})}\right] = \omega_{s0}\left[1 + \frac{1}{2}\frac{\omega_0^2 |\eta| h_2 V_2}{\omega_{s0}^2 2\pi c \rho B_0}\right]$$

Useful relation for stationary bucket:

$$\omega_{s0}^2 = \frac{|\eta| h \omega_0 e V_0 c}{2\pi \rho E}; \quad \Delta\hat{E}^2 = \frac{2EeV_0}{\pi |\eta| h} \text{ - bucket hight}$$

$$\frac{\omega_{s0}}{\Delta\hat{E}} = \frac{|\eta| h \omega_0}{2E}; \qquad \sigma_E = \sigma_z \frac{\Delta\hat{E}}{\lambda} = \sigma_z \frac{\omega_{s0} E}{|\eta| h c}$$