Emittance Transfer in Linacs

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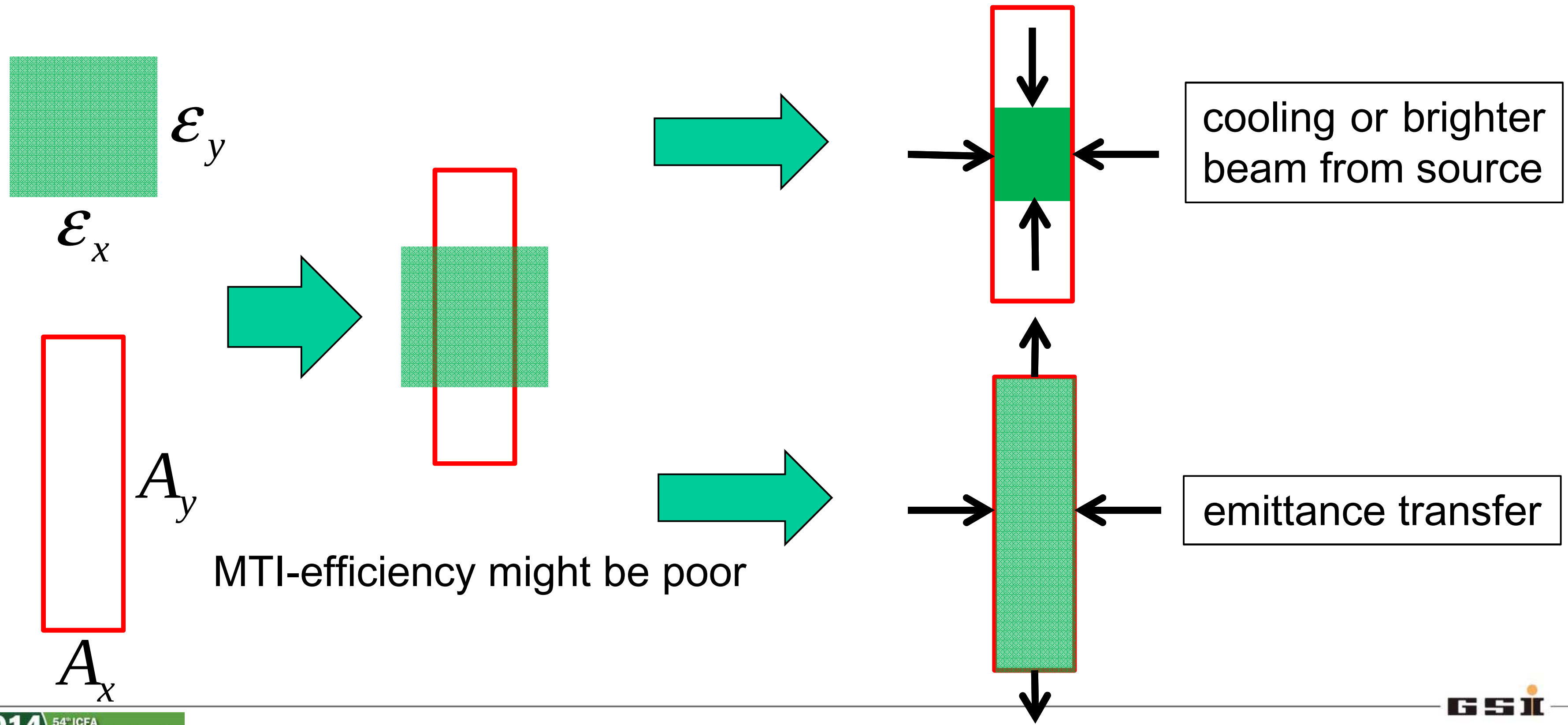
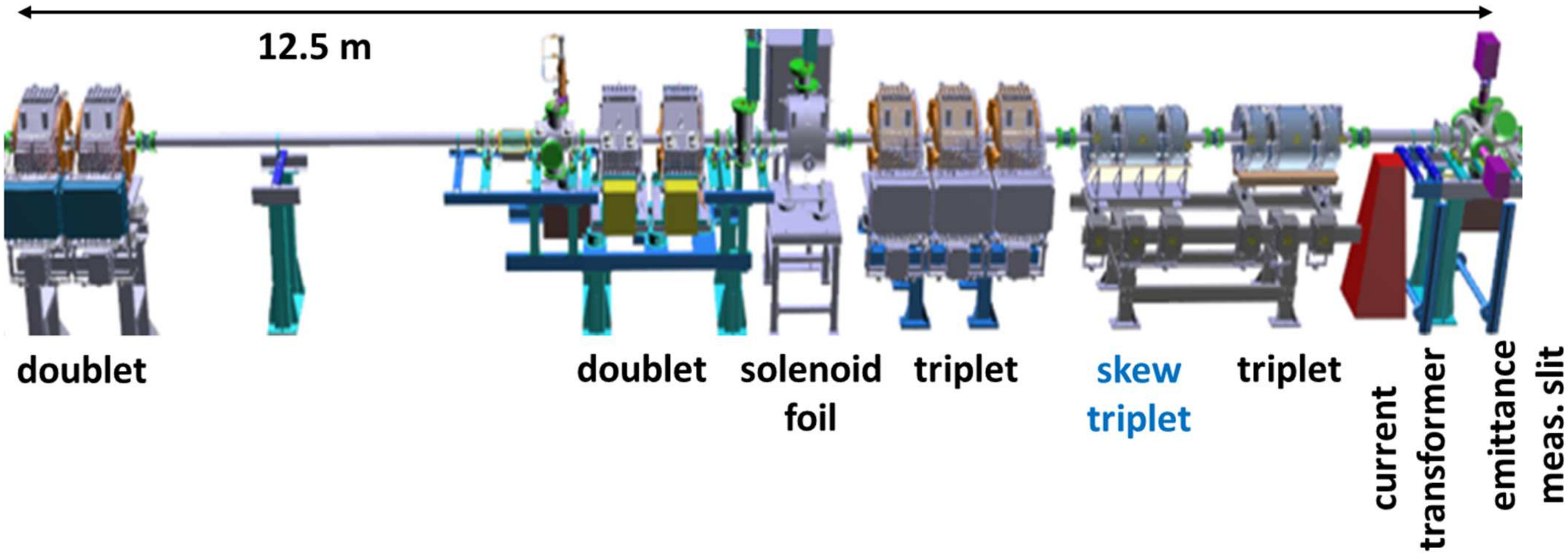
- Motivation
- Beam line for transfer (brief)
- rms & eigen-emittances
- Decoupling features
- Experiment
- Summary

Emittances vs Acceptances

- Round beam is provided by injector linac $\varepsilon_x \approx \varepsilon_y$
- Flat beams may be required by rings (multi-turn-injection) $A_x < A_y$



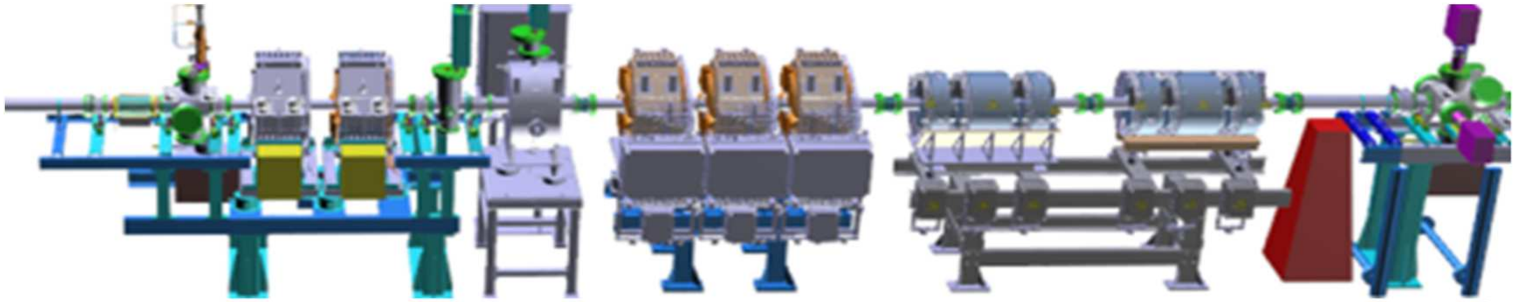
Beam Line for Emittance Transfer



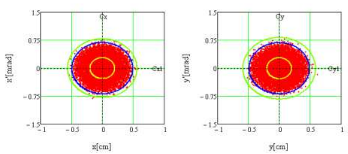
<u>key components:</u>

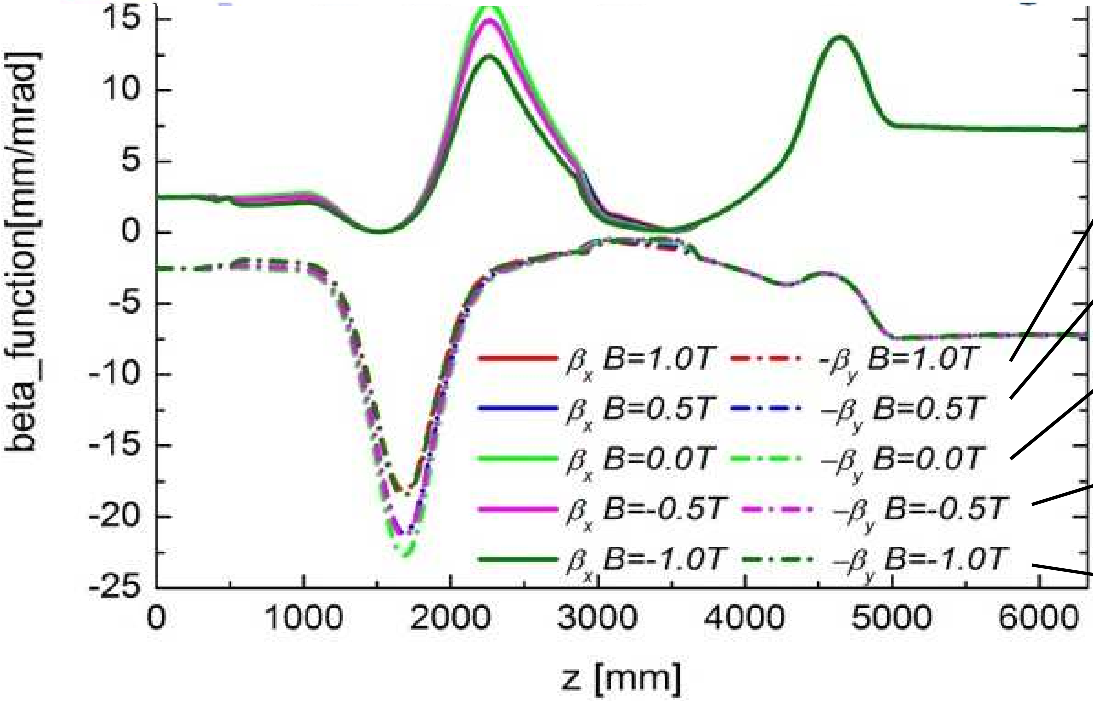
- charge stripper placed at center of a solenoid
- skew triplet to remove inter-plane correlations

Beam Line for Emittance Transfer

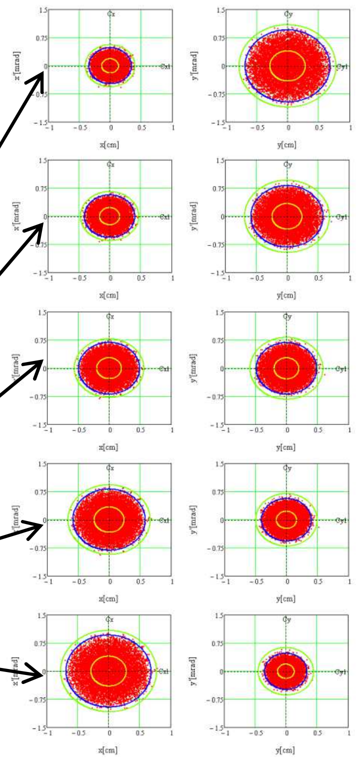


initial phase space
hor. ver.





final phase space
hor. ver.



just the solenoid field is varied !

4d Linear Beam Dynamics

$$E_{4d}^2 = det \begin{bmatrix} <xx> & <xx'> & <xy> & <xy'> \\ <x'x> & <x'x'> & <x'y> & <x'y'> \\ <yx> & <yx'> & <yy> & <yy'> \\ <y'x> & <y'x'> & <y'y> & <y'y'> \end{bmatrix}$$

transport of moments from 1 → 2 as usual :

$$\begin{bmatrix} x \\ x' \\ y \\ y' \end{bmatrix}_2 = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{bmatrix} \begin{bmatrix} x \\ x' \\ y \\ y' \end{bmatrix}_1 , \quad det\, M = 1$$

$$C_2 = M\, C_1\, M^T$$

<u>if x & y planes are not coupled</u>

$$E_{4d}^2 = det \begin{bmatrix} <xx> & <xx'> & 0 & 0 \\ <x'x> & <x'x'> & 0 & 0 \\ 0 & 0 & <yy> & <yy'> \\ 0 & 0 & <y'y> & <y'y'> \end{bmatrix} = (E_x \cdot E_y)^2$$

transport of moments from 1 → 2 as usual :

$$M = \begin{bmatrix} m_{11} & m_{12} & 0 & 0 \\ m_{21} & m_{22} & 0 & 0 \\ 0 & 0 & m_{33} & m_{34} \\ 0 & 0 & m_{43} & m_{44} \end{bmatrix} , \quad det\, M = det\, M_x \cdot det\, M_y = 1 \cdot 1 = 1$$

$$C_2 = M\, C_1\, M^T$$

eigen-Emittances

- linear (4d), Hamiltonian elements preserve :
  - rms emittance $E_{4d}^2 = det \begin{bmatrix} <xx> & <xx'> & <xy> & <xy'> \\ <x'x> & <x'x'> & <x'y> & <x'y'> \\ <yx> & <yx'> & <yy> & <yy'> \\ <y'x> & <y'x'> & <y'y> & <y'y'> \end{bmatrix}$
  - the two eigen-emittances

$$\varepsilon_1 = \frac{1}{2}\sqrt{-tr[(CJ)^2] + \sqrt{tr^2[(CJ)^2] - 16det(C)}}$$

$$\varepsilon_2 = \frac{1}{2}\sqrt{-tr[(CJ)^2] - \sqrt{tr^2[(CJ)^2] - 16det(C)}}$$

$$E_{4d} = \varepsilon_1 \cdot \varepsilon_2$$

$$C = \begin{bmatrix} <xx> & <xx'> & <xy> & <xy'> \\ <x'x> & <x'x'> & <x'y> & <x'y'> \\ <yx> & <yx'> & <yy> & <yy'> \\ <y'x> & <y'x'> & <y'y> & <y'y'> \end{bmatrix} \qquad J := \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

eigen-Emittances

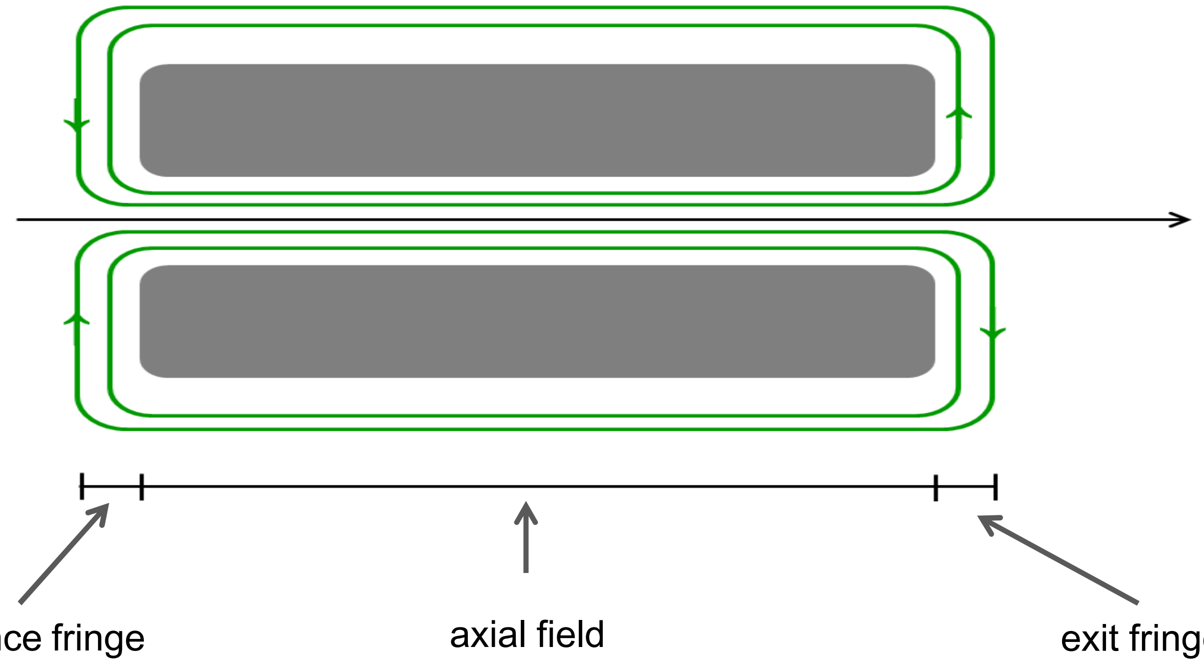
- if, and only if there is no x↔y coupling, i.e. $C = \begin{bmatrix} <xx> & <xx'> & 0 & 0 \\ <x'x> & <x'x'> & 0 & 0 \\ 0 & 0 & <yy> & <yy'> \\ 0 & 0 & <y'y> & <y'y'> \end{bmatrix}$
  - rms emittances = eigen-emittances
- if there is any coupling
  - rms emittances ≠ eigen-emittances
  - coupling parameter $t = \frac{\varepsilon_x \varepsilon_y}{\varepsilon_1 \varepsilon_2} - 1 \geq 0$

term „eigen-emittance“ is quite unknown, since one generally assumes uncoupled beams

Solenoid Transport Matrix

$$\kappa := \frac{B}{2(B\rho)}$$

$$\alpha(L) = -2\kappa L$$



$$M_{fi} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & \kappa & 0 \\ 0 & 0 & 1 & 0 \\ -\kappa & 0 & 0 & 1 \end{bmatrix}$$

$$M_{||} = \begin{bmatrix} 1 & -\frac{1}{2\kappa} sin(\alpha) & 0 & \frac{1-cos(\alpha)}{2\kappa} \\ 0 & cos(\alpha) & 0 & -sin(\alpha) \\ 0 & \frac{cos(\alpha)-1}{2\kappa} & 1 & -\frac{sin\alpha)}{2\kappa} \\ 0 & sin(\alpha) & 0 & cos(\alpha) \end{bmatrix}$$

$$M_{fo} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -\kappa & 0 \\ 0 & 0 & 1 & 0 \\ \kappa & 0 & 0 & 1 \end{bmatrix}$$

complete solenoid matrix $M_{sol} = M_{fo} \cdot M_{||} \cdot M_{fi}$

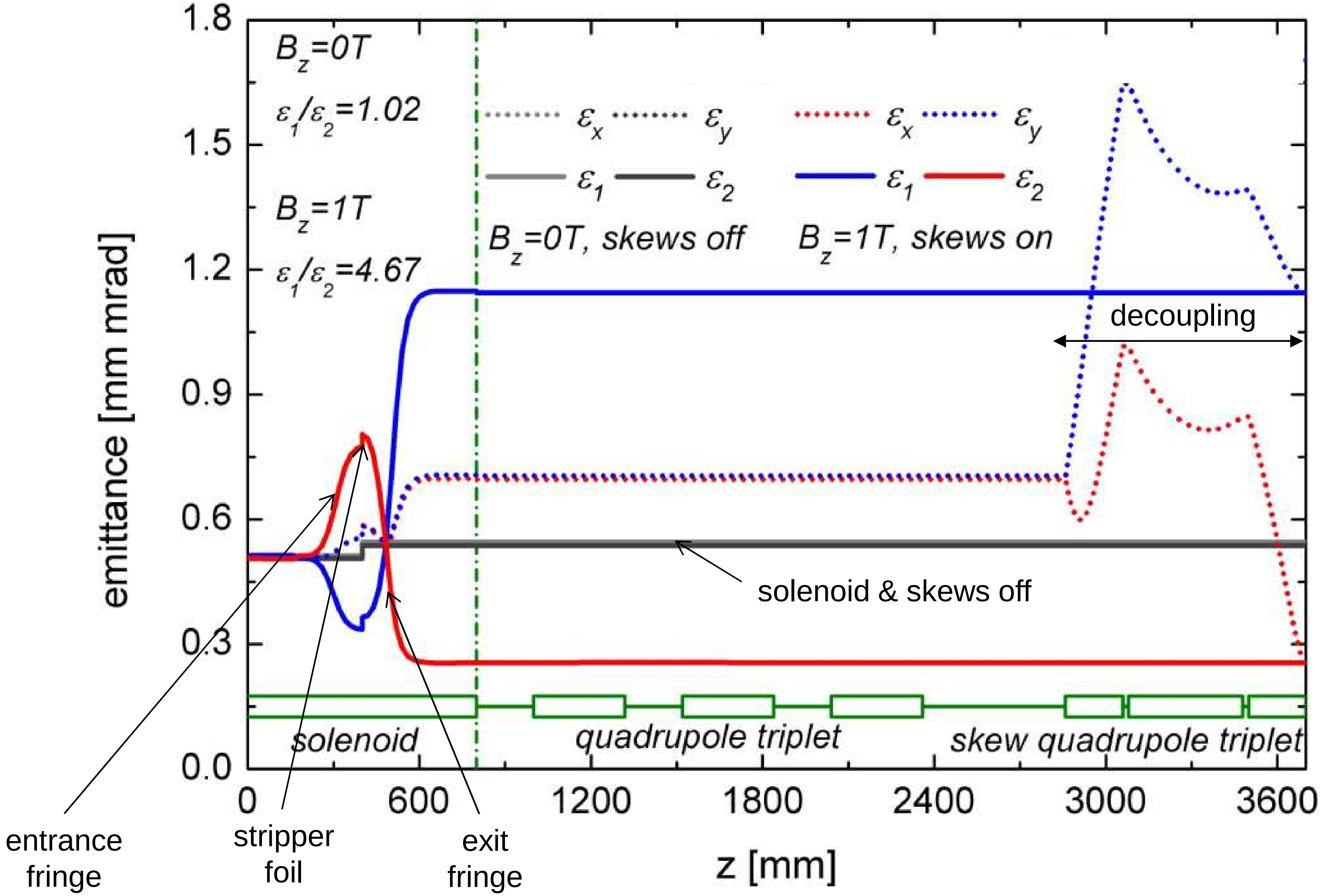
How Elements do Change Emittances

Element	$rms_{x,y}$	4d-rms	$Eigen_{1,2}$
drift	no	no	no
quadrupole	no	no	no
tilted quadrupole	yes	no	no
dipole	no	no	no
tilted dipole	yes	no	no
solenoid	yes	no	no
solenoid fringe	yes	no	yes
solenoid axial field	yes	no	yes

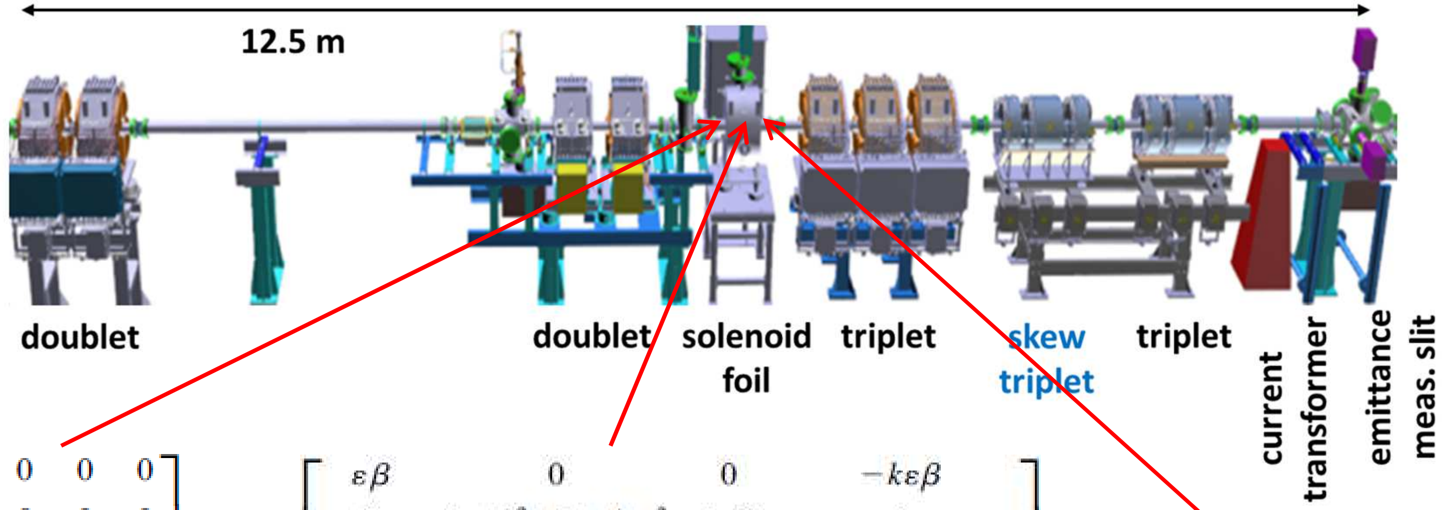
How Elements do Change Emittances

- solenoid axial field is not Hamiltonian
- solenoid fringe field is not Hamiltonian
- they are not Hamiltonian, because the do not exist „stand-alone“
- $\vec{\nabla}\vec{B} = \frac{\delta B_x}{\delta x} + \frac{\delta B_y}{\delta y} + \frac{\delta B_z}{\delta z} = 0$ forbids existence of such stand-alone fields !!
- infact: complete solenoids exist, are Hamiltonian, and preserve eigen-emittances
- change of charge charge state inside solenoid → creates effective „stand-alone“ fringe:
    - ECR sources
    - emittance transfer beam line presented here

Evolution of Emittances along Beam Line



Coupling & Decoupling



$$\begin{bmatrix} \varepsilon\beta & 0 & 0 & 0 \\ 0 & \frac{\varepsilon}{\beta} & 0 & 0 \\ 0 & 0 & \varepsilon\beta & 0 \\ 0 & 0 & 0 & \frac{\varepsilon}{\beta} \end{bmatrix}$$

$$\begin{bmatrix} \varepsilon\beta & 0 & 0 & -k\varepsilon\beta \\ 0 & \frac{\varepsilon}{\beta} + k^2\varepsilon\beta + \Delta\varphi^2 & k\varepsilon\beta & 0 \\ 0 & k\varepsilon\beta & \varepsilon\beta & 0 \\ -k\varepsilon\beta & 0 & 0 & \frac{\varepsilon}{\beta} + k^2\varepsilon\beta + \Delta\varphi^2 \end{bmatrix}$$

center of solenoid,
right behind foil

$$\begin{bmatrix} \varepsilon_n R_n & ak\varepsilon_n\beta_n J_n \\ -ak\varepsilon_n\beta_n J_n & \varepsilon_n R_n \end{bmatrix}$$

$$\delta q := \frac{(B\rho)_{\text{in}}}{(B\rho)_{\text{out}}},$$

$$a := \delta q - 1$$

$$\varepsilon_n = \sqrt{\varepsilon\beta\left(\frac{\varepsilon}{\beta} + a^2k^2\varepsilon\beta + \Delta\varphi^2\right)}, \qquad \beta_n = \frac{\beta\varepsilon}{\varepsilon_n}$$

$$R_n = \begin{bmatrix} \beta_n & 0 \\ 0 & \frac{1}{\beta_n} \end{bmatrix}, \qquad J_n = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

Decoupling (Special Case)

moment matrix

$$\begin{bmatrix} \varepsilon_n R_n & ak\varepsilon_n \beta_n J_n \\ -ak\varepsilon_n \beta_n J_n & \varepsilon_n R_n \end{bmatrix}$$

$$\delta q := \frac{(B\rho)_{\text{in}}}{(B\rho)_{\text{out}}}$$

$$a := \delta q - 1$$

$$\varepsilon_n = \sqrt{\varepsilon\beta\left(\frac{\varepsilon}{\beta} + a^2k^2\varepsilon\beta + \Delta\varphi^2\right)}, \qquad \beta_n = \frac{\beta\varepsilon}{\varepsilon_n}$$

$$R_n = \begin{bmatrix} \beta_n & 0 \\ 0 & \frac{1}{\beta_n} \end{bmatrix}, \qquad J_n = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

can be decoupled by

- triplet with Identity in x-plane and 90°-rotation in y-plane,

$$R_q = \begin{bmatrix} I_n & O_n \\ O_n & T_n \end{bmatrix}$$

$$T_n = \begin{bmatrix} 0 & u \\ -\frac{1}{u} & 0 \end{bmatrix}, \qquad I_n = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad u = \pm\beta_n$$

- which is tilted by 45°
- we call this line $\overline{R}$ „special case“

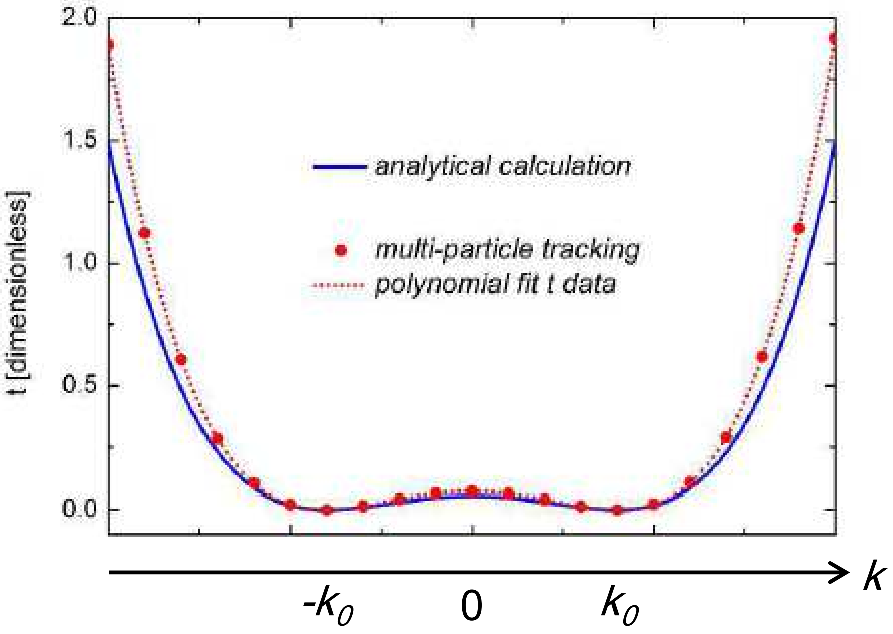
Decoupling (Special Case)

coupling parameter

$$t = \frac{\varepsilon_x \varepsilon_y}{\varepsilon_1 \varepsilon_2} - 1 \geq 0$$

- suppose, decoupling quads are set for initial coupling solenoid strength k_0
- but real coupling is done with k
- decoupling works excellent for $|k| \leq |k_0|$
- additionally, after decoupling:

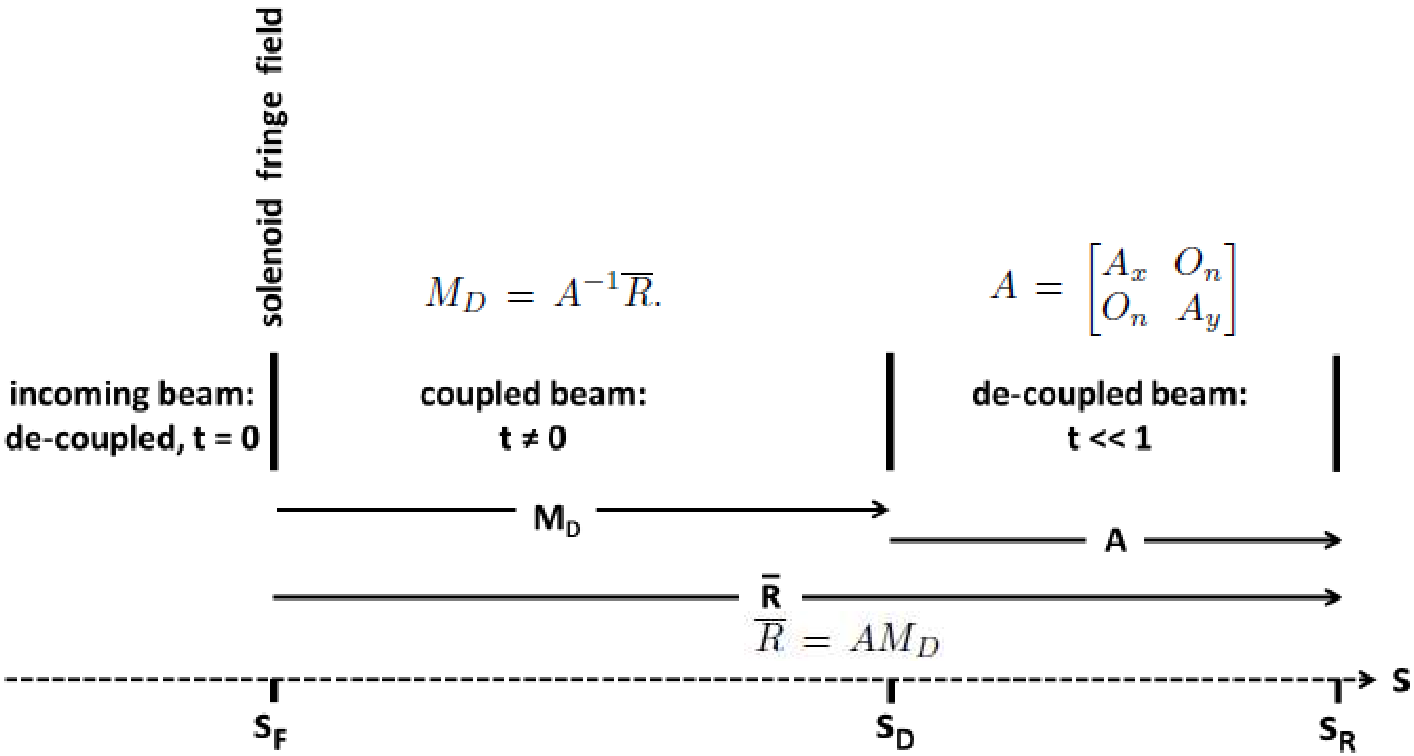
$$\tilde{\alpha}_x = \tilde{\alpha}_y = 0, \quad \tilde{\beta}_x = \tilde{\beta}_y = \beta_n(k_0)$$



- all settings constant except k
- k just changes rms emittance ratio
- rms emittance product is constant
- beam always decoupled for $|k| < |k_0|$
- final Twiss parameters independent from k
- <u>this is really convenient</u> !!

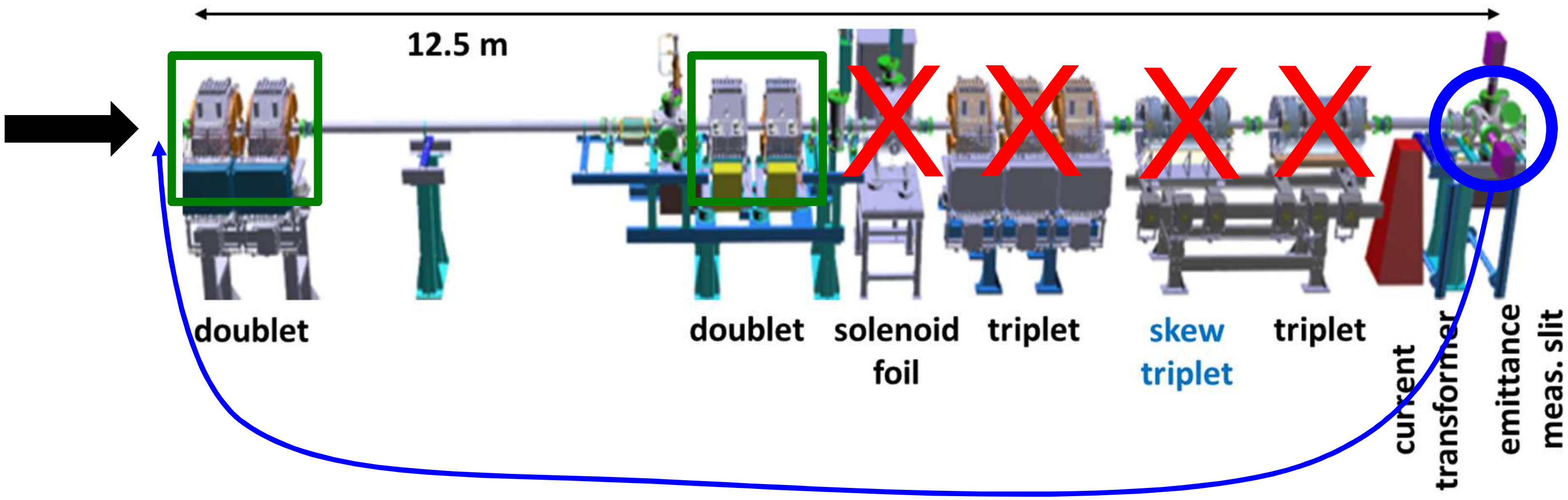
Decoupling (General)

features of $\overline{R}$ hold for any decoupling line M_D:



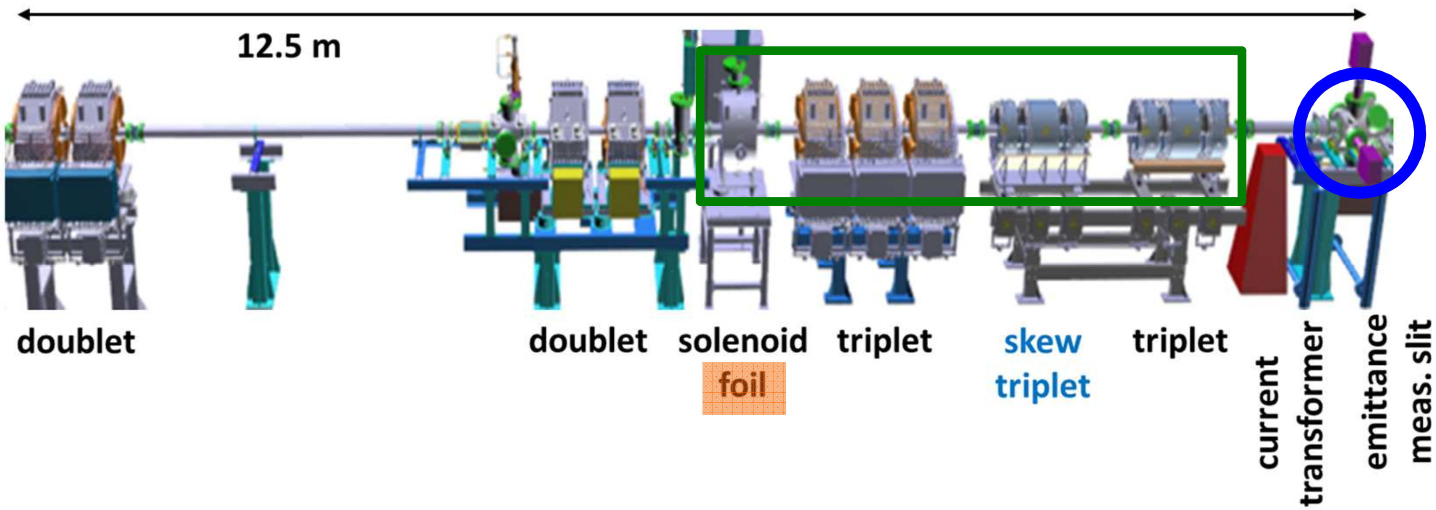
- $\overline{R}$ has the decoupling features
- A^{-1} does not change them, since it does not couple
- accordingly, $M_D = A^{-1}\overline{R}$ has the same features

Experiment at GSI UNILAC



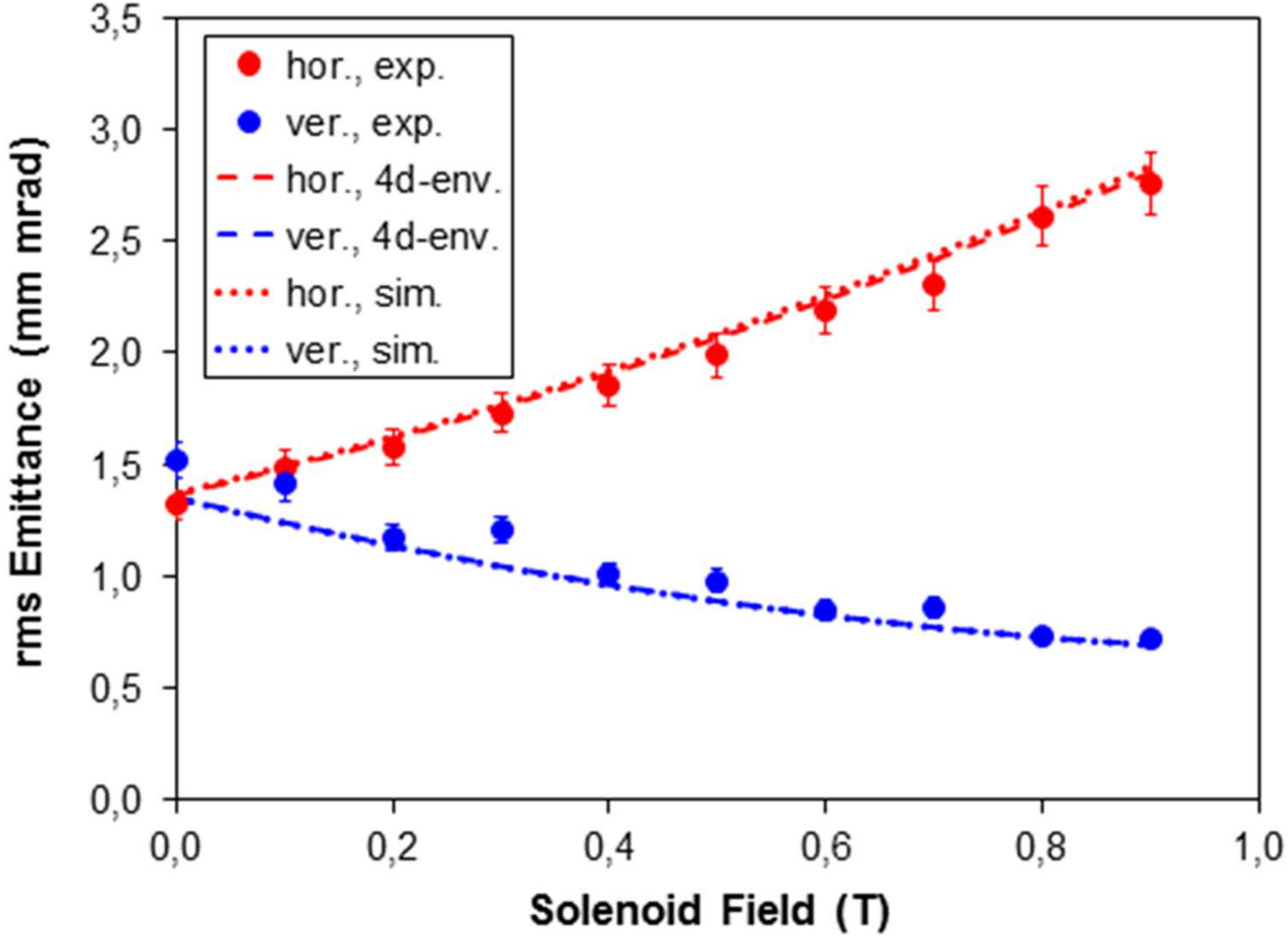
- initial beam: $^{14}N^{3+}$ at 11.4 MeV/u, low intensity
- remove stripping foil, solenoid off, all magnets behind solenoid off
- measure Twiss parameters, track them back to beam line entrance
- set two doublets to optain round, double-waist beam at foil

Experiment at GSI UNILAC



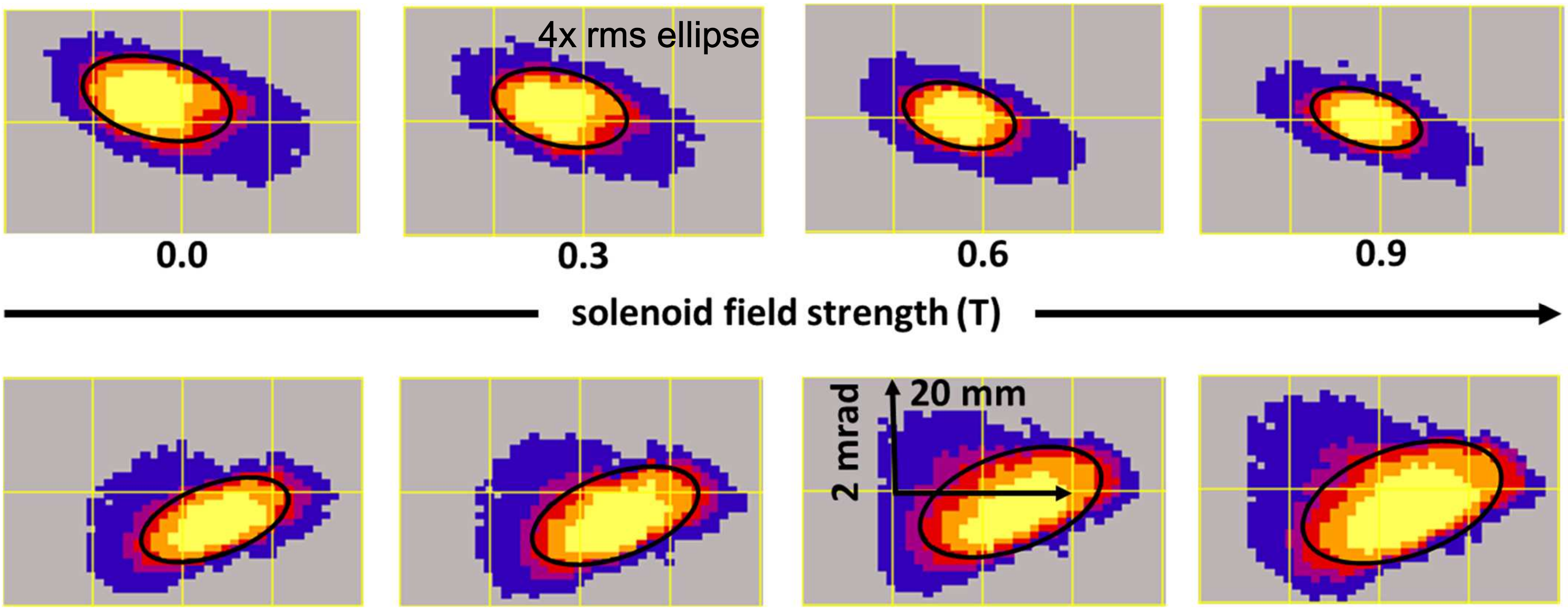
- move in stripping foil → stripping from $^{14}N^{3+}$ to $^{14}N^{7+}$ occurs
- verify that transmission increased from 100% to 233% (7/3)
- switch on solenoid and all magnets behind solenoid
- re-measure emittances
- measure emittances for different solenoid strengths *k*, touch nothing else, plot result

Experiment at GSI UNILAC



product of rms emittances remains constant within resolution of measurement

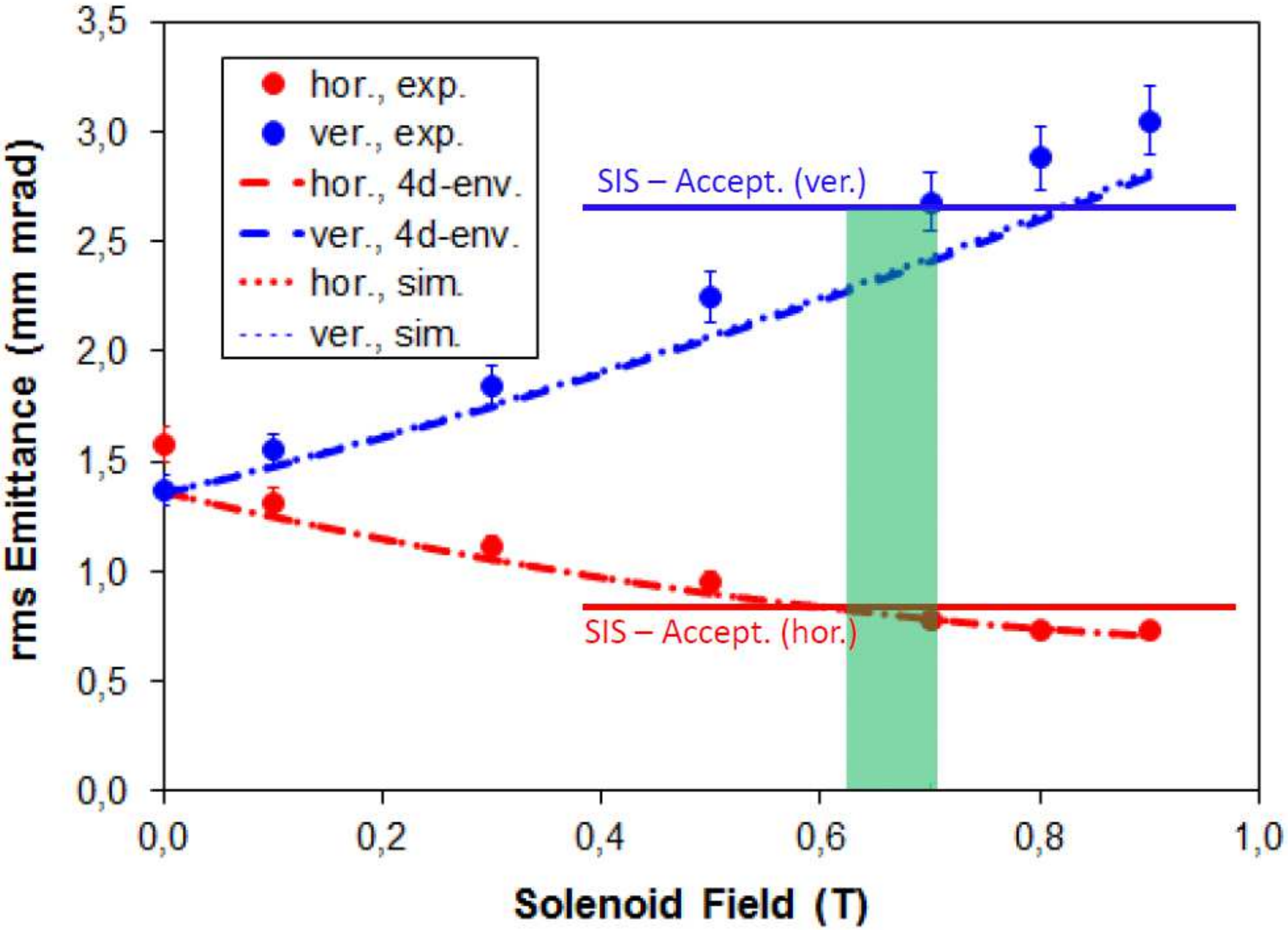
Experiment at GSI UNILAC



Twiss parameters beta & alpha in both planes remain constant within resolution of measurement

Experiment at GSI UNILAC

- repeat solenoid scan with inverted skew-quad gradients
- this is equivalent to scan solenoid field with negative values
- as expected, the hor & ver planes are swapped



Summary

- Emittance transfer can be performed by doing the stripping inside a solenoid
- Demonstrator beam line EMTEX was designed, built, and commissioned at GSI
- EMTEX has very convenient features
  - adjustable transverse emittance partitioning, preserving product of emittances
  - partitioning by one single knob
  - partitioning preserves exit Twiss parameters beta & alpha in all planes
  - facilitating enormously operation of EMTEX
- Simulations indicate that method is robust wrt
  - space charge
  - stripping into many charge states
- Literature:
  - Phys. Rev. ST Accel. Beams 14, 064201 (2011)
  - arXiv 1212.2034 (2012)
  - HB2012, MOP207 (2012)
  - Phys. Rev. ST Accel. Beams 16, 044201 (2013)
  - arXiv 1403.6962 (2014)
  - arXiv 1410.7127 (2014)