Science & Technology Facilities Council
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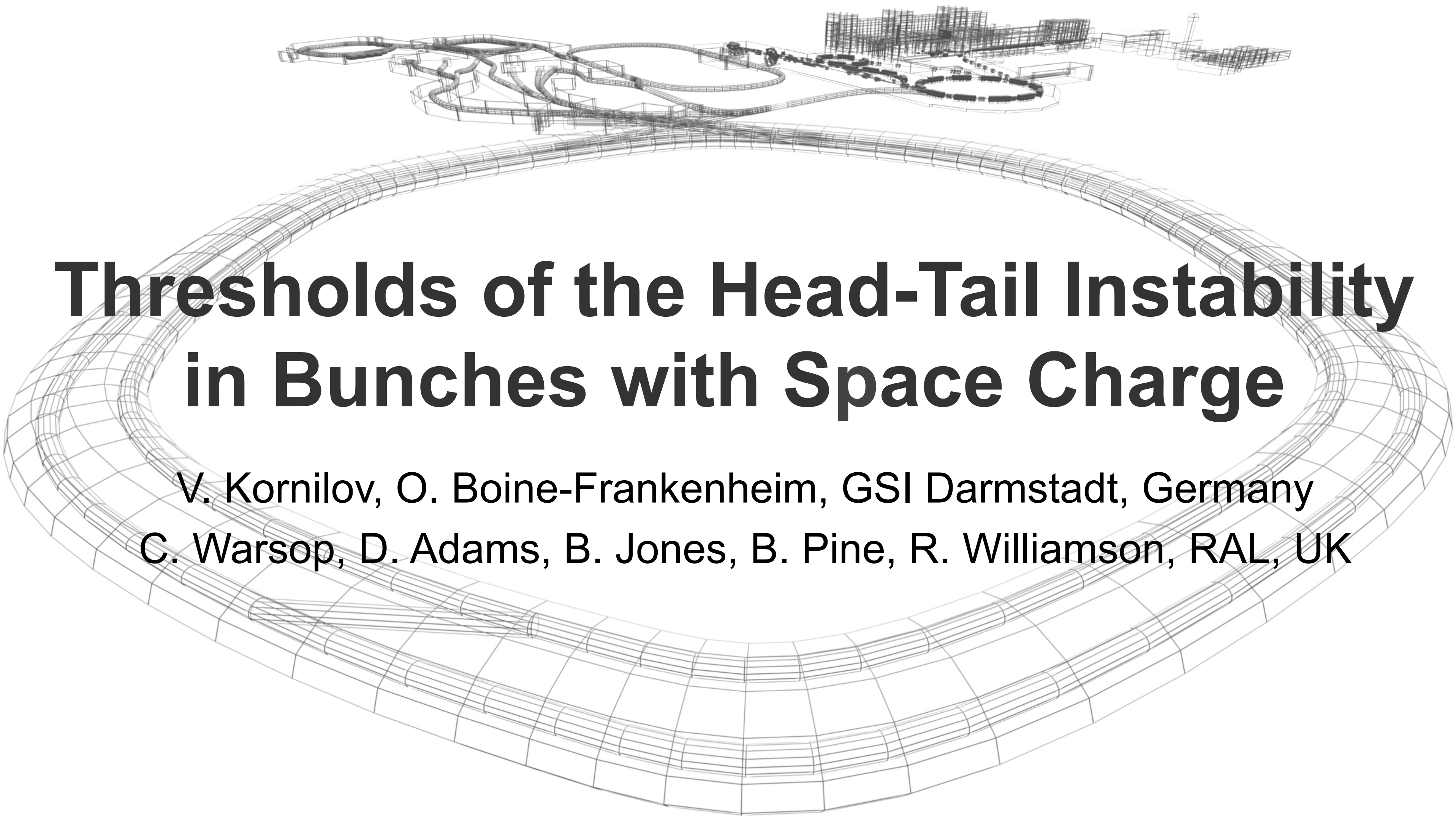


# Thresholds of the Head-Tail Instability in Bunches with Space Charge

V. Kornilov, O. Boine-Frankenheim, GSI Darmstadt, Germany
C. Warsop, D. Adams, B. Jones, B. Pine, R. Williamson, RAL, UK

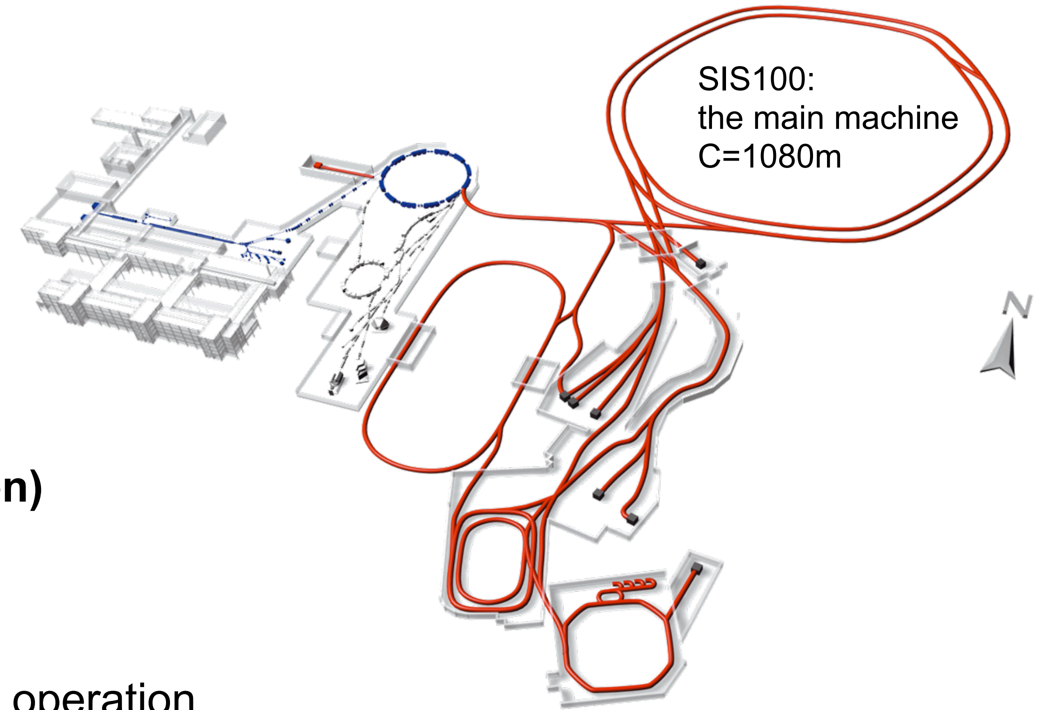
# INTRODUCTION

FAIR Project at GS Darmstadt

SIS100:

| | | |
|---|---|---|
| protons | 4-29 GeV | $N_p$=2x10$^{13}$ |
| $^{238}U^{28+}$ | 0.2-1.5 GeV/u | $N_p$=5x10$^{11}$ |

**One of the main concerns:**
**$U^{28+}$ bunches can be head-tail unstable.**
**(High-Intensity long-time 1s accumulation)**




ISIS Synchrotron of RAL, UK:
unstable head-tail modes produce losses in operation.
One of the concerns, especially for the Upgrade.

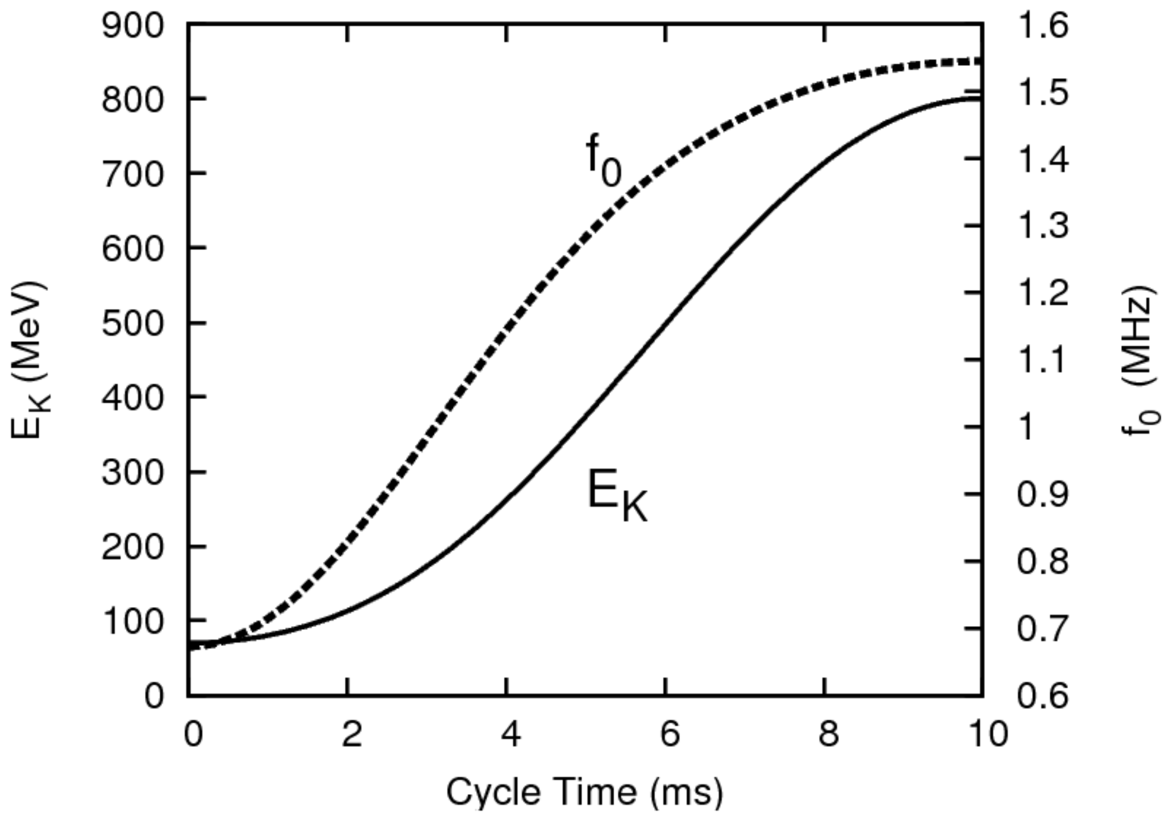
Theoretically, head-tail modes have no thresholds.

In reality, things are different and more complicated:
Transverse Space-Charge (nonlinear own-field), Machine Nonlinearities,
Beam pipe, different 2RF buckets, nonlinearities in 1RF buckets.

# ISIS

Spallation Neutron Source
RAL, near Oxford, UK

C=163m
50 Hz RCS operation
up to $3x10^{13}$ ppp
$Q_h$ = 4.31−4.18
$Q_v$ = 3.85−3.7
$\gamma_{tr}$ = 5.034
2rf, basic *h*=2, 160kV
$t_b$ = 400ns−100ns
at inj ε ≈ 200 mm mrad




Bunch parameters (especially normalized)
similar to the heavy-ion bunches in SIS100 of FAIR

# HEAD-TAIL INSTABILITY IN ISIS

Dedicated Experiment Campaign, 3 Shifts

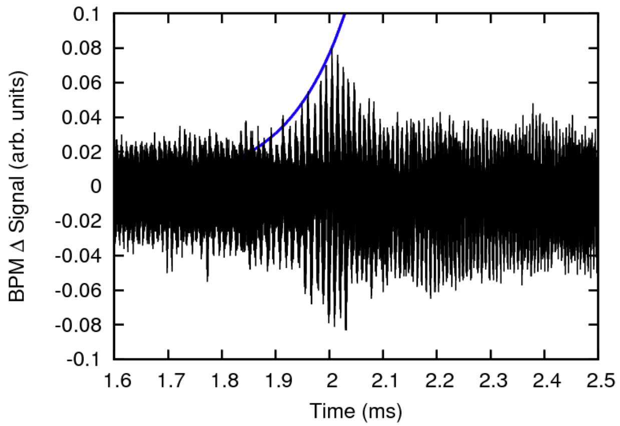
**$Q_v$ above 3.86:**
Strong vertical oscillations and Losses

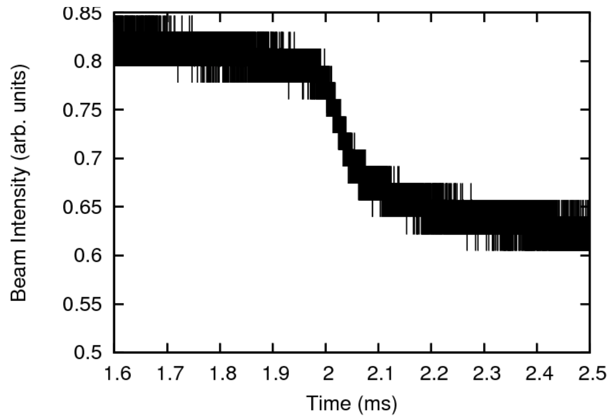
1rf example
$N_{beam}$=8.2e12

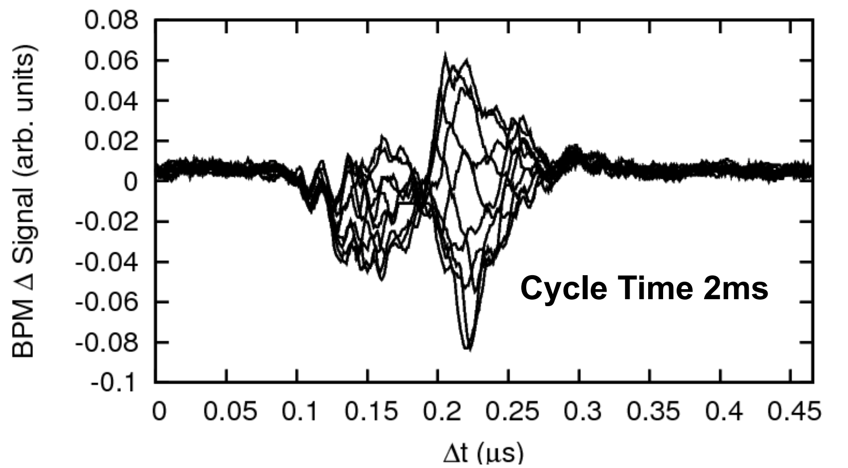
Growth rate generally difficult to determine

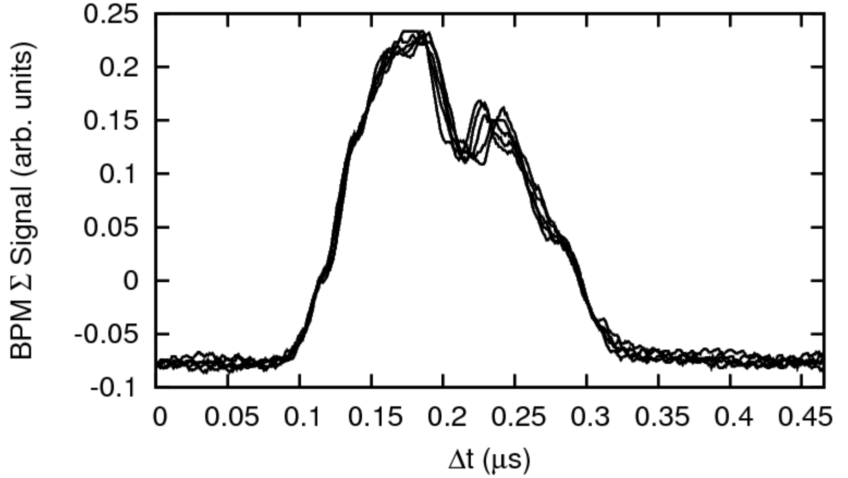
this example:
growth time τ=0.1ms

standing-wave pattern, exponential growth, wiggles due to ξ, dQ/Qs=0.2
➔ k=1 head-tail mode



# HEAD-TAIL INSTABILITY IN ISIS

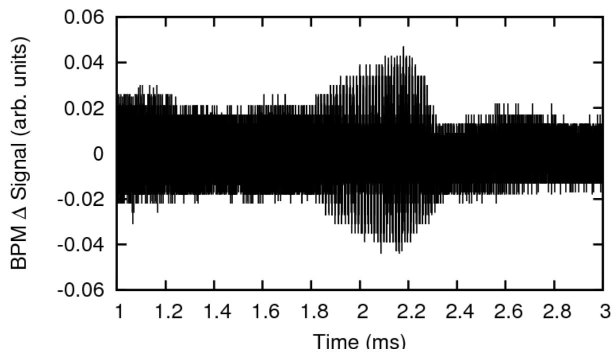
2rf example, flat bunch (length. mode)

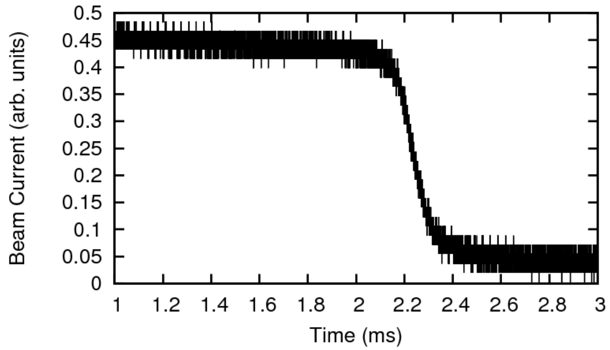
$N_{beam}$=4e12

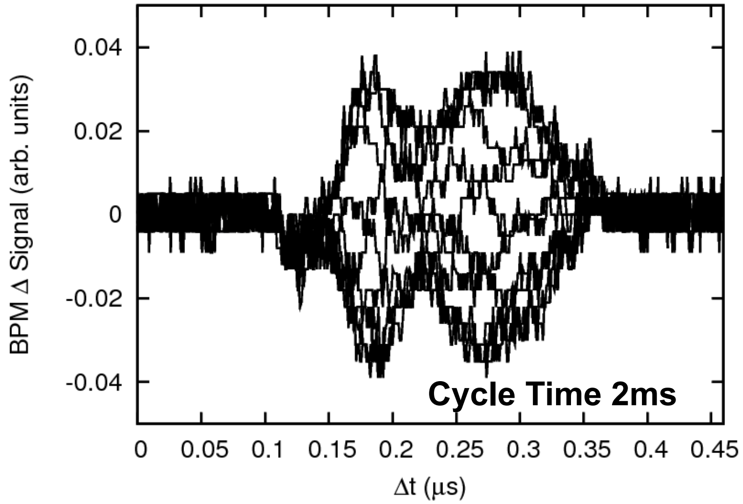
Generally,
long development
difficult to observe,
probably due to
the small pipe/beam:
early losses

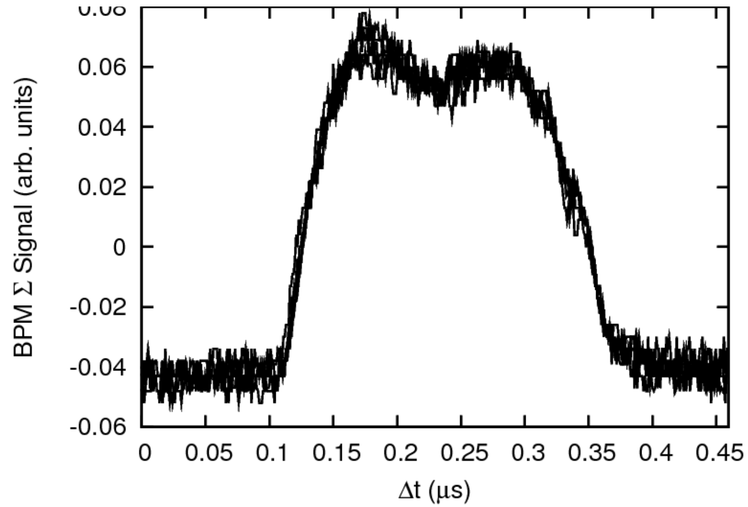
2rf: usually
complex mode
structure

Here the
k=1 mode as well?



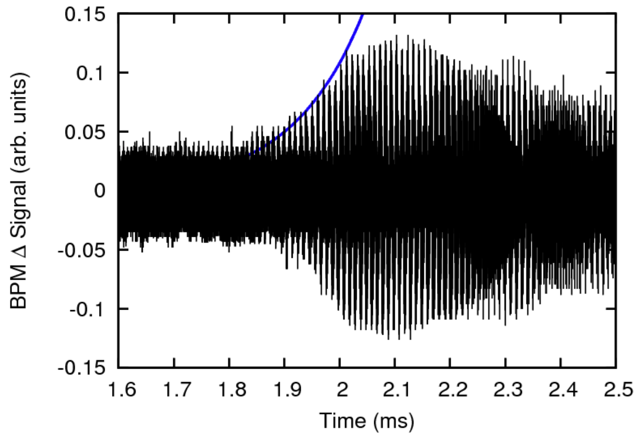
# HEAD-TAIL INSTABILITY IN ISIS

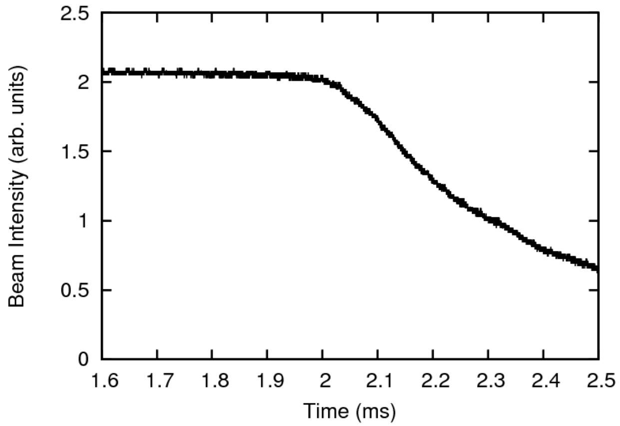
2rf example, flat bunch (length. mode)
high intensity

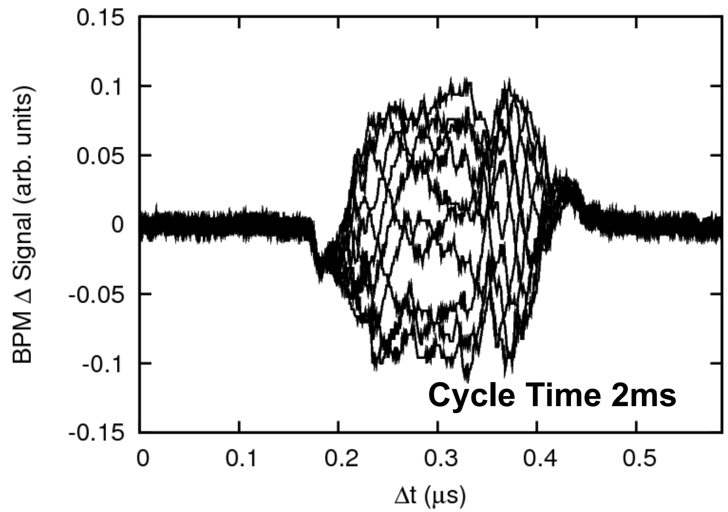
$N_{beam}$=17e12

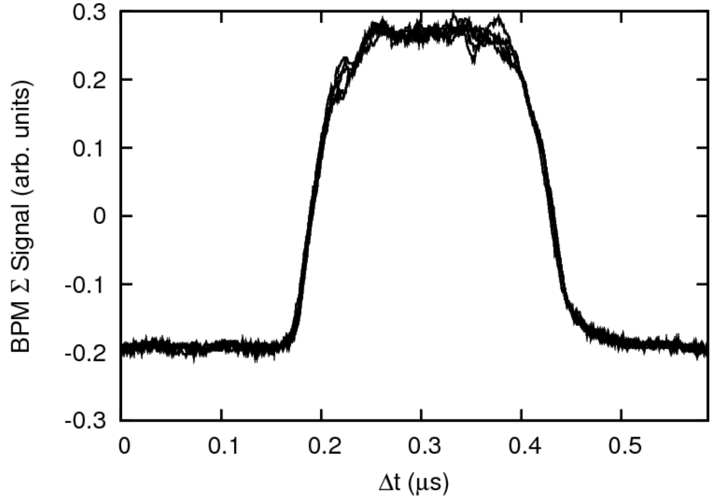
complex mode
structure

growth time
τ=0.13ms



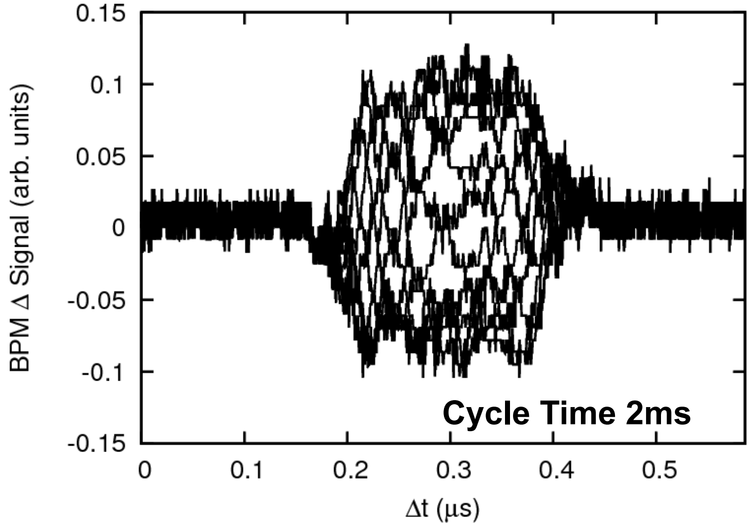
# HEAD-TAIL INSTABILITY IN ISIS

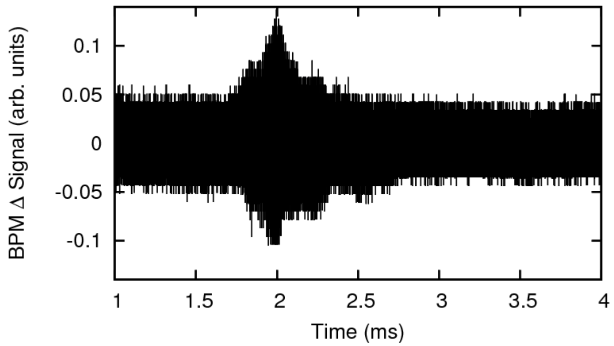
2rf example, Operational rf settings
(note that $Q_v$ not operational)

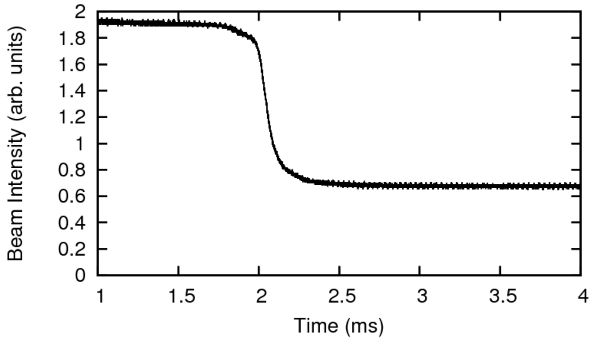
$N_{beam}$=14e12

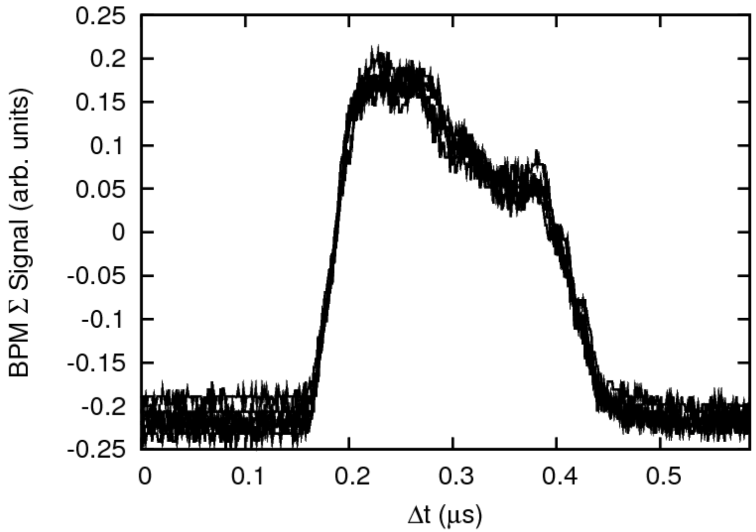
complex mode
structure

here not discussed:
during high-intensity
operation at ISIS
many different
(and very interesting)
modes observed
at different Cycle Times



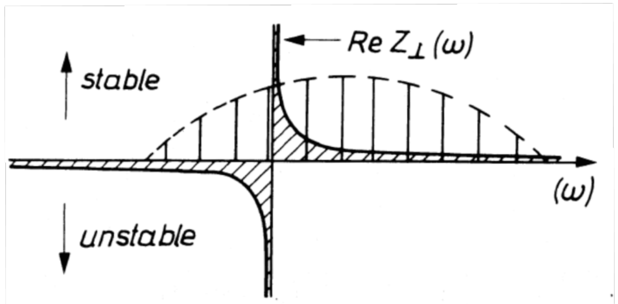
# HEAD-TAIL INSTABILITY IN ISIS: THE DRIVE

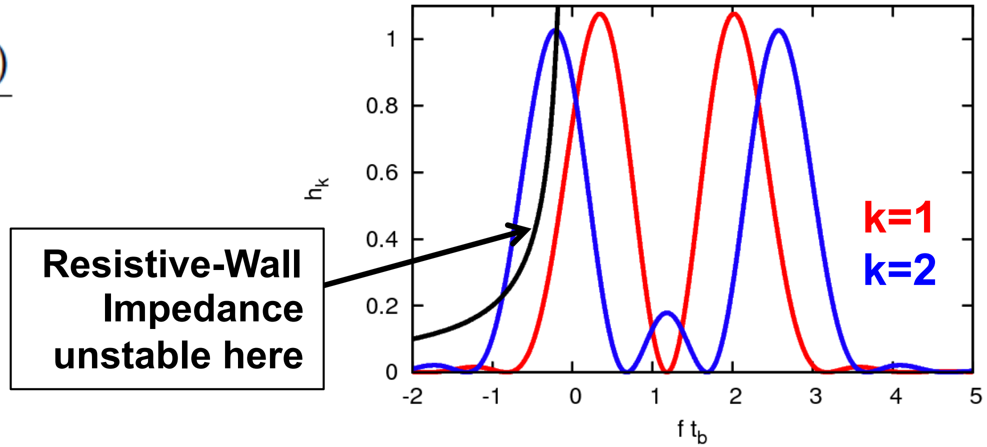
Try to use the approach of F. Sacherer 1974

$$\Delta Q_k = \frac{\Upsilon}{1+k} \frac{\sum(-i) Z_\perp(\omega_p) h_k(\omega_p - \omega_\xi)}{\sum h_k(\omega_p - \omega_\xi)}$$

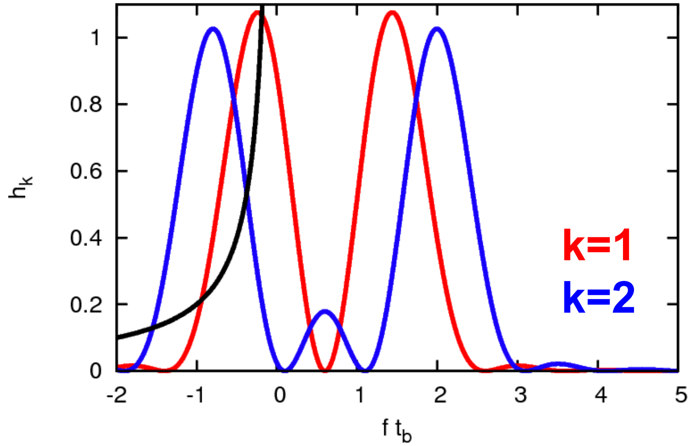
$$\omega_p = (p + Q_0)\omega_0 + k\omega_s$$

$$\Upsilon = \frac{I_0 q_{\text{ion}}}{4\pi\gamma m c Q_0 \omega_0} \qquad \omega_\xi = \frac{Q_0 \xi}{\eta}\omega_0$$




need to reduce the effective bunch length to 0.5 $t_b$ in order to explain the k=1 observation:




Sacherer theory explains:

- Resistive-Wall Impedance as the drive: strong Z at small ($1-Q_{frac}$) [$Q_v$ above 3.86]
- Higher *k* at later Cycle Times: higher $\omega_\xi$

# HEAD-TAIL INSTABILITY IN ISIS: THE DRIVE

$$\Delta Q_k = \frac{\Upsilon}{1+k} \frac{\sum(-i) Z_\perp(\omega_p) h_k(\omega_p - \omega_\xi)}{\sum h_k(\omega_p - \omega_\xi)}$$

$$\omega_p = (p + Q_0)\omega_0 + k\omega_s$$

$$\mathrm{Re}\left(Z_{\mathrm{rw}}^{\perp}\right) = \frac{L_{\mathrm{rw}}}{\pi b^3} \sqrt{\frac{c Z_0}{2\sigma_{\mathrm{rw}} \Omega}}$$

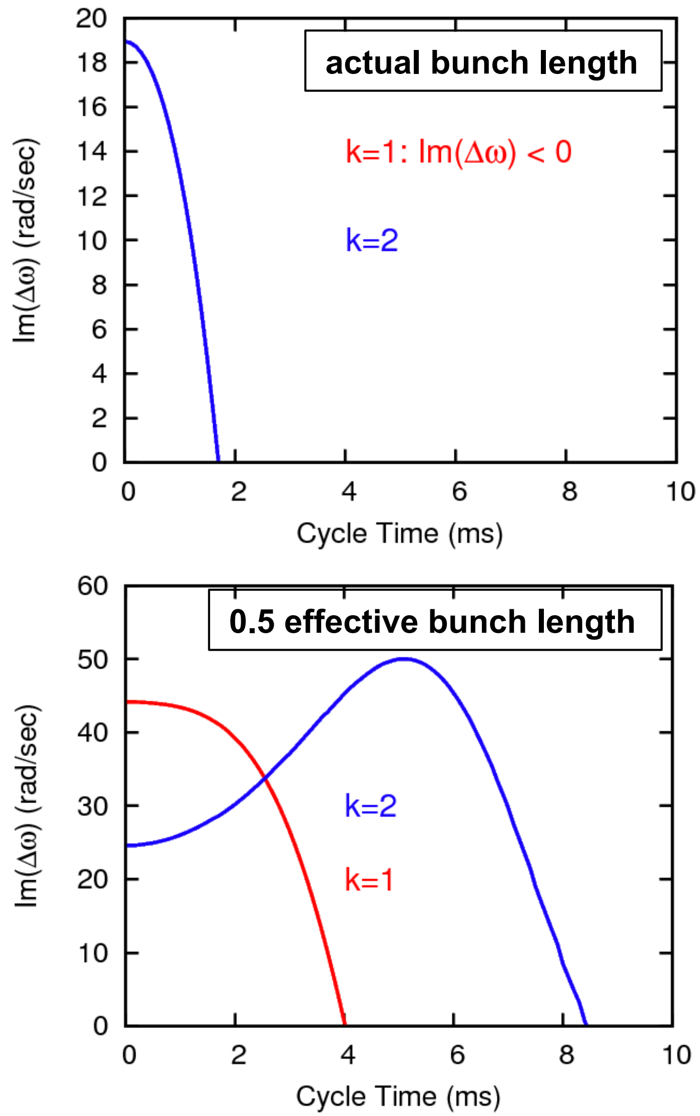
Sacherer theory suggests:

- Reduced bunch length is needed to reproduce the k=1 mode
- Even with the overestimated Thick-Resistive-Wall Impedance the max growth rate corresponds to τ≈20ms (experiment 200×)

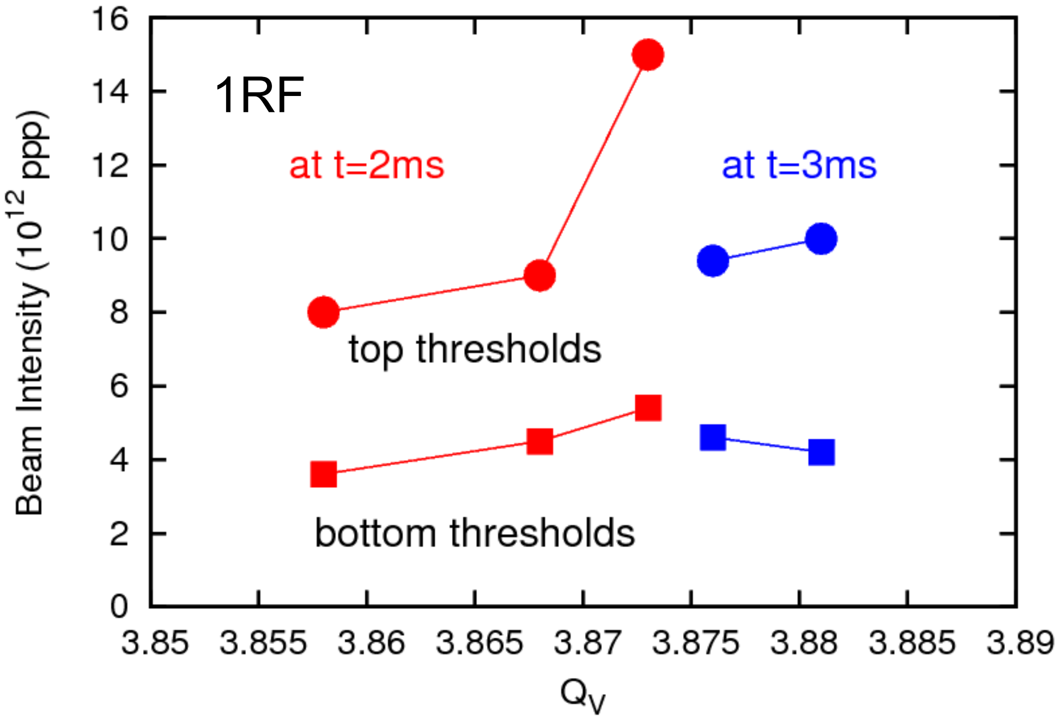
- Another Impedance with a similar Z(f) in ISIS?
- Sacherer theory only partly adequate for these parameters?

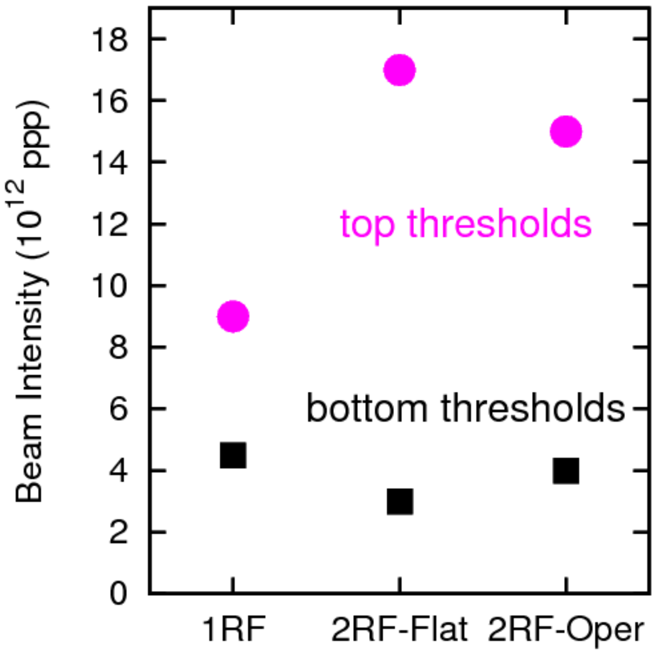
Puzzles to solve in future

(Other approaches: G. Rees, RAL, 1992)



# HEAD-TAIL INSTABILITY IN ISIS




Focus of the present study:
**Intensity Thresholds** of the head-tail instability observed in ISIS

**bottom thresholds**: instability and losses above this intensity
**top thresholds**: no instability and losses above this intensity

Intriguing: repeatable top thresholds for very different bunches

# HEAD-TAIL INSTABILITY: THRESHOLDS

$$\gamma_{MODE} = \gamma_{DRIVE} + \gamma_{DAMPING}$$

$$\gamma_{DRIVE} > 0$$

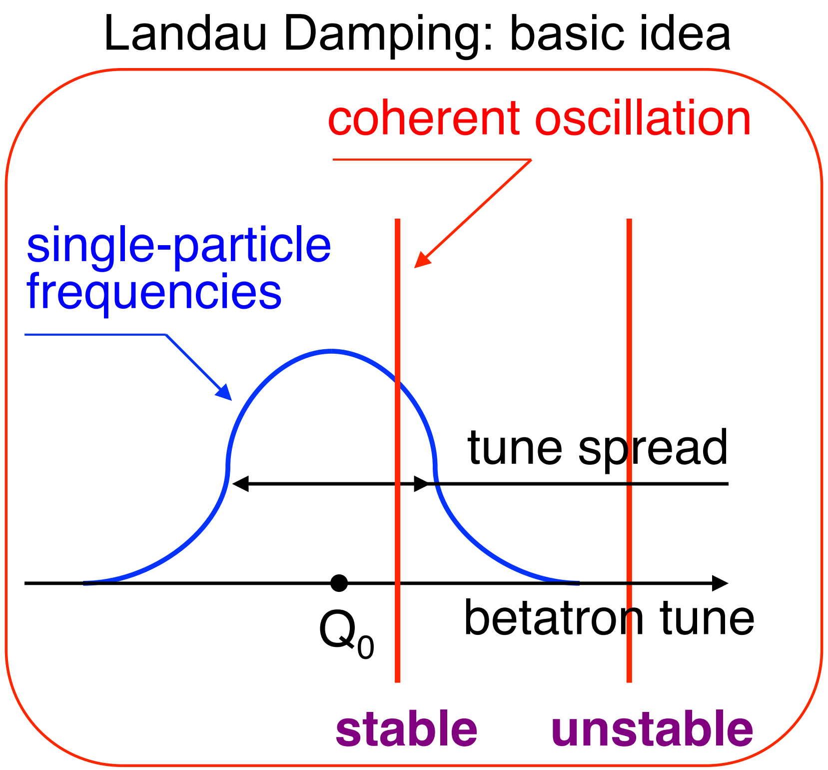
$$\gamma_{DAMPING} < 0$$

Accelerator always has nonlinearities:
produce damping and explains the **bottom thresholds**.
Usually observed for coherent instabilities.

The DRIVE proportional to the intensity: $\gamma_{DRIVE} = c\ Z\ N_{beam}$ .
Bunch parameters do not change much.
The drive can not explain the top threshold.

There must be a non-linear enhancing DAMPING
responsible for the **top thresholds**

# HEAD-TAIL INSTABILITY: DAMPING




Tune Shifts due to chromaticity, space-charge, nonlinearities, impedances.

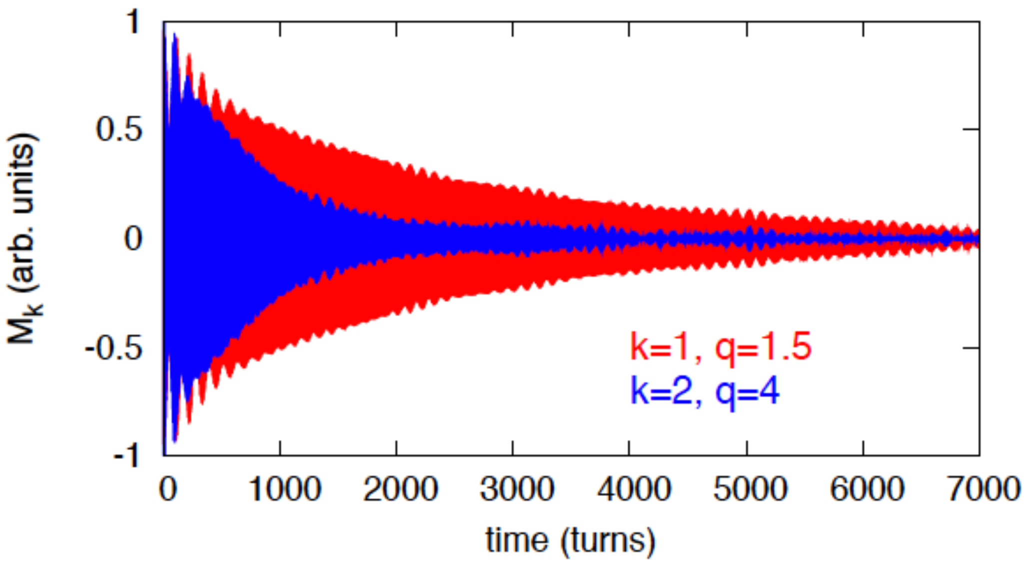
For coasting beam works very well, dispersion relation D.Möhl (1969).
Exception from the simple picture: space charge does not produce Landau damping of its own.

For head-tail modes in bunches more complicated:
Every mode k has it own coherent shift;
Effect of space-charge is different;
Efficient coherent-incoherent interaction is different.

# LANDAU DAMPING DUE TO SPACE-CHARGE

Landau damping in bunches exclusively due to the effect of space charge [Burov PRSTAB 2009], [Balbekov 2009]

Our particle tracking simulations with the PATRIC code




space-charge tune shift

$$\Delta Q_{sc}(\tau) = \frac{\lambda(\tau) r_p R^2}{\gamma^3 \beta^2 Q_0 a^2}$$

space-charge parameter

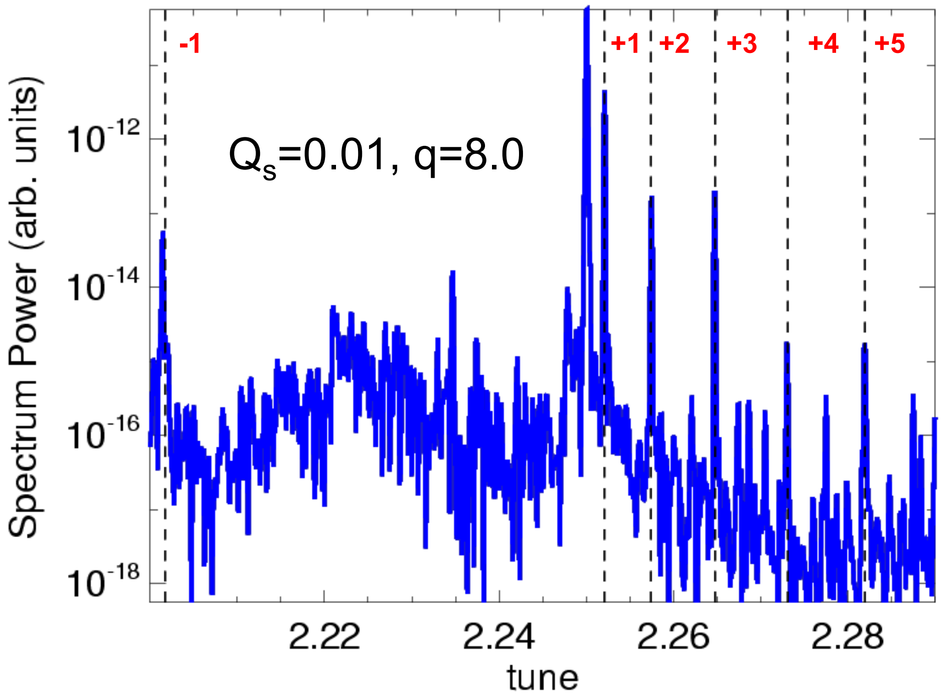
$$q = \frac{\Delta Q_{sc}}{Q_s}$$

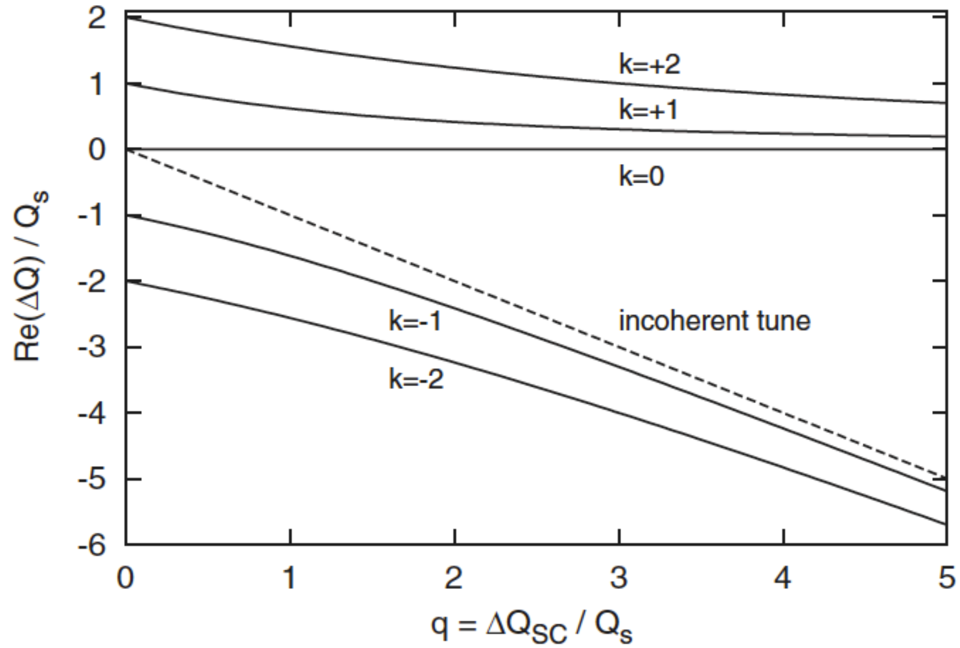
V.Kornilov, O.Boine-Frankenheim, PRSTAB **13**, 114201 (2010)

# HEAD-TAIL MODES WITH SPACE-CHARGE

The theory of an “airbag” bunch:
analytic prediction for arbitrary space charge
M.Blaskiewicz, PRSTAB **1**, 044201 (1998)

$$\Delta Q = -\frac{\Delta Q_{sc}}{2} \pm \sqrt{\frac{\Delta Q_{sc}^2}{4} + k^2 Q_s^2}$$



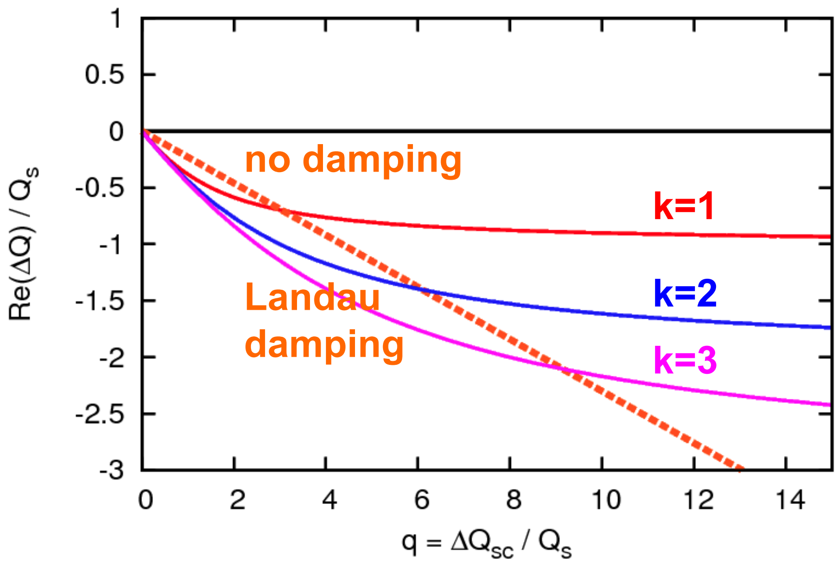

the theory perfectly reproduced in simulations (spectrum from PATRIC, dashed lines: theory):
V.Kornilov, O.Boine-Frankenheim, PRSTAB **13**, 114201 (2010)
The Space-Charge tune shifts also observed and confirmed in experiment:
V.Kornilov, O.Boine-Frankenheim, PRSTAB **15**, 114201 (2012)

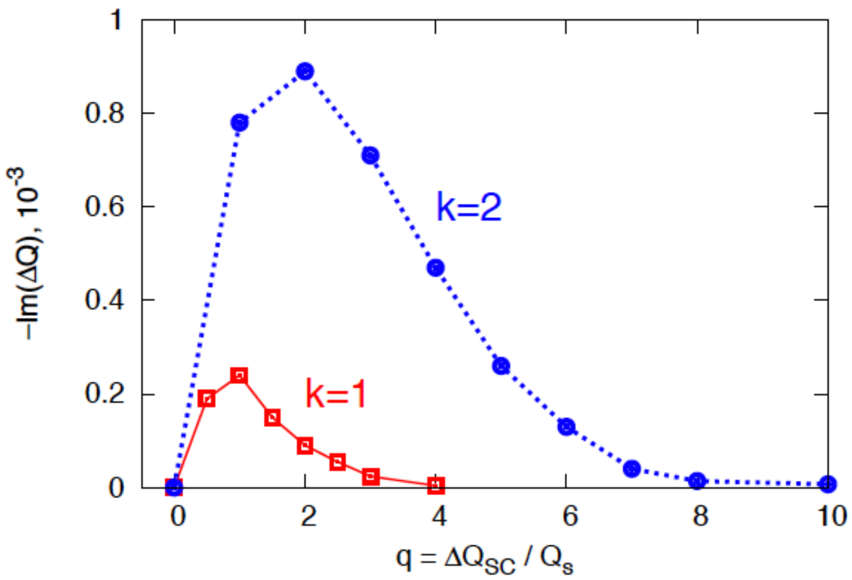
# SIMULATIONS: LANDAU DAMPING

Simulations confirm Landau damping in bunches exclusively due to the effect of space charge

V.Kornilov, O.Boine-Frankenheim, PRSTAB **13**, 114201 (2010)

Landau Damping border $\Delta Q \approx -0.23 \Delta Q_{sc} + kQ_s$

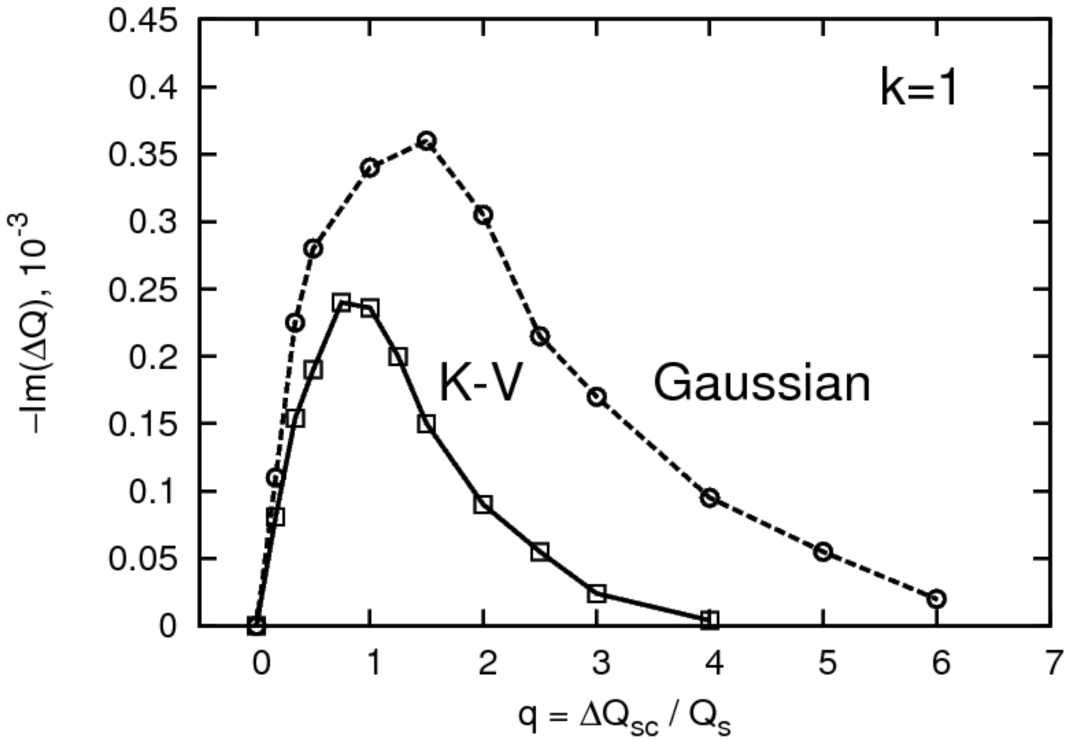



Summary of Landau damping simulations:
region of Landau damping for each mode can be predicted;.
strong space charge $q \gg 2k$ : no Landau damping for the mode.

# SIMULATIONS: LANDAU DAMPING

Simulations so far: with the transverse K-V distribution.
Transverse nonlinear Space-Charge increases tune spread
and should enhance Landau damping.




Damping decrement from PATRIC simulations

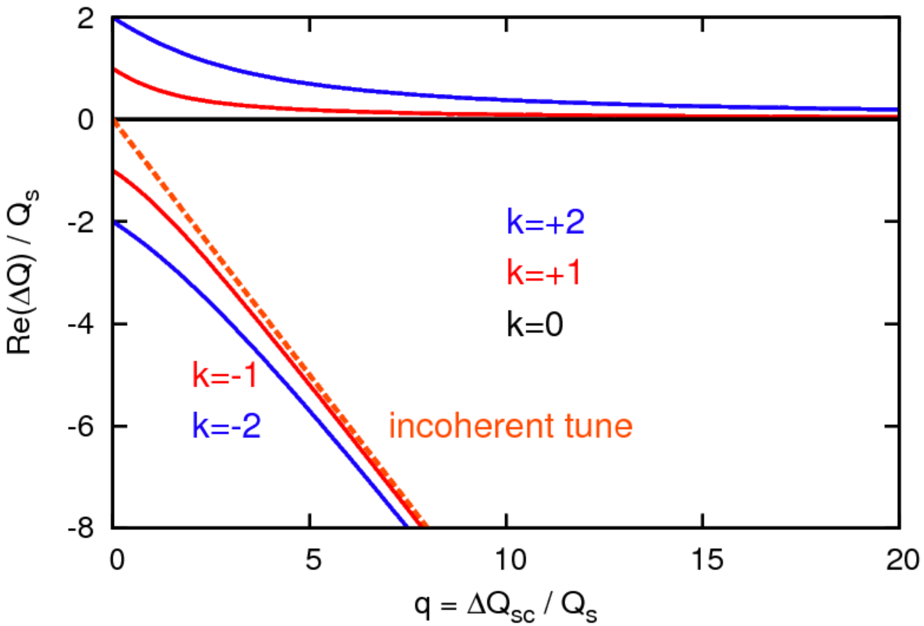
# HEAD-TAIL MODES WITH IMAGE CHARGES

include the effect of the image charges into the airbag theory:
O.Boine-Frankenheim, V.Kornilov, PRSTAB **12**, 114201 (2009)

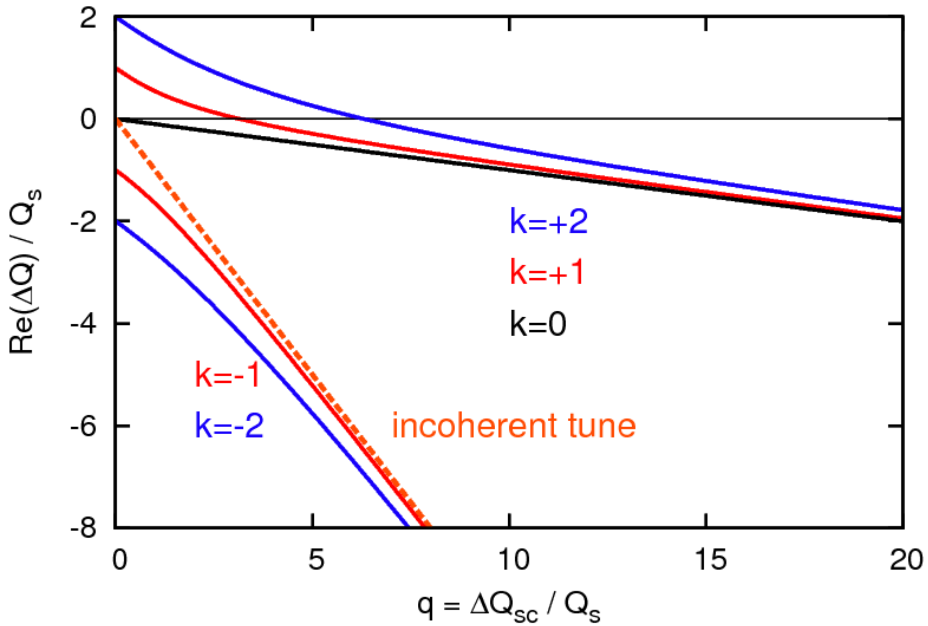
$$\Delta Q_k = -\frac{\Delta Q_{sc} + \Delta Q_{coh}}{2} \pm \sqrt{\left(\frac{\Delta Q_{sc} - \Delta Q_{coh}}{2}\right)^2 + k^2 Q_s^2}$$

reproduced in simulations

$\Delta Q_{coh} = 0$




$\Delta Q_{coh} = 0.1\ \Delta Q_{sc}$

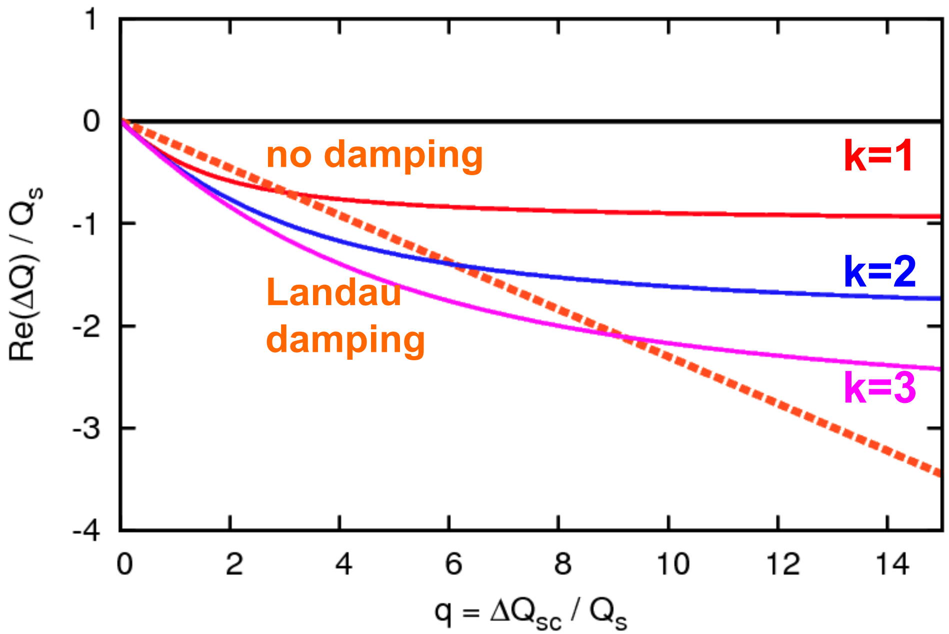


# LANDAU DAMPING DUE TO IMAGE CHARGES

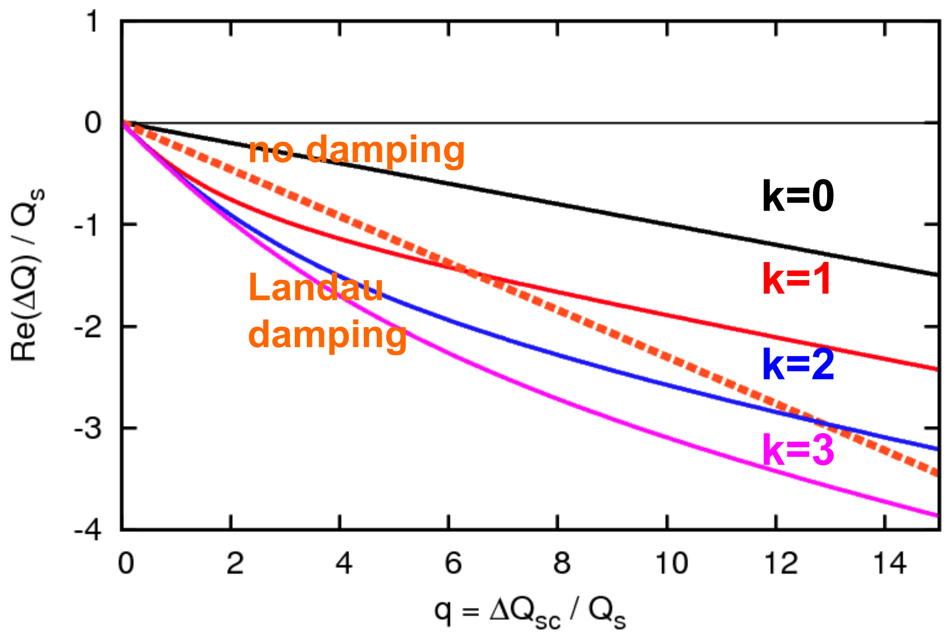
$$\Delta Q_k = -\frac{\Delta Q_{sc} + \Delta Q_{coh}}{2} \pm \sqrt{\left(\frac{\Delta Q_{sc} - \Delta Q_{coh}}{2}\right)^2 + k^2 Q_s^2}$$

this should have an effect on Landau damping:

$\Delta Q_{coh} = 0$



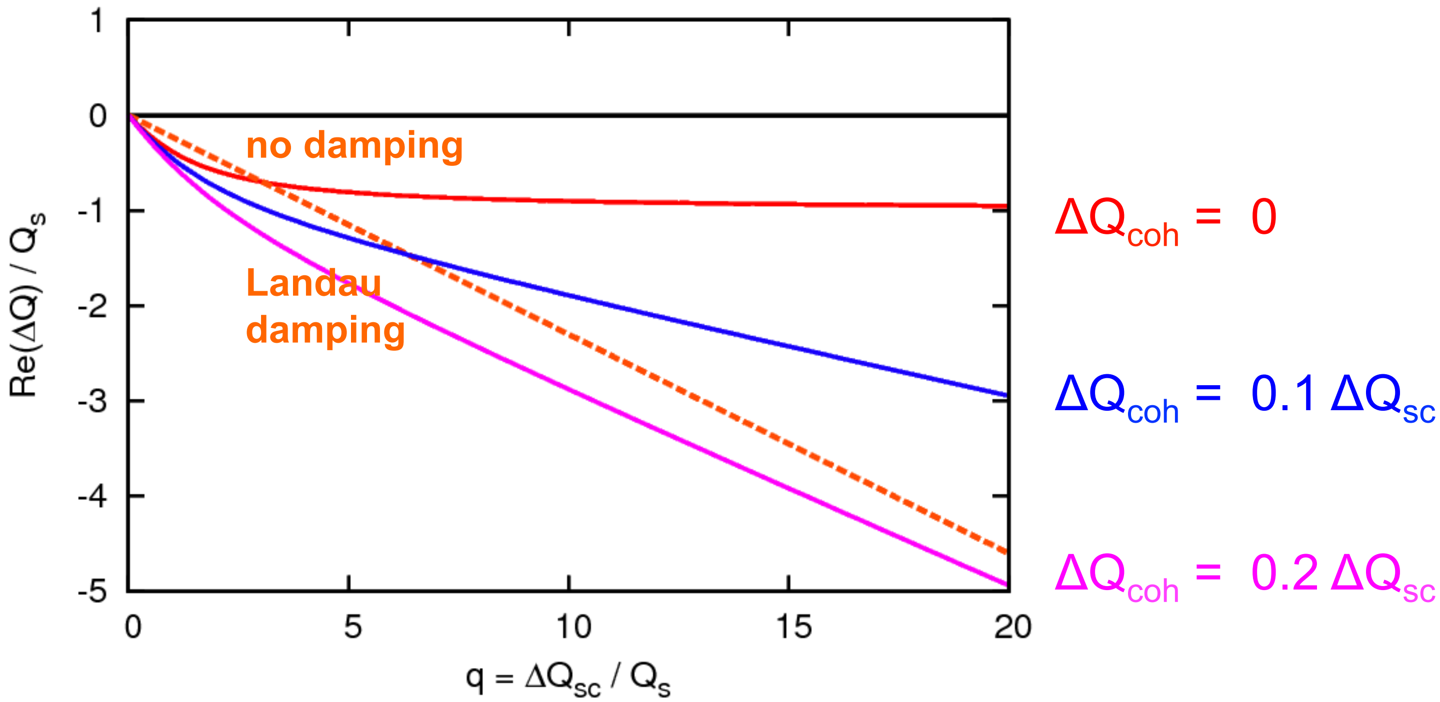

$\Delta Q_{coh} = 0.1\ \Delta Q_{sc}$



# LANDAU DAMPING DUE TO IMAGE CHARGES

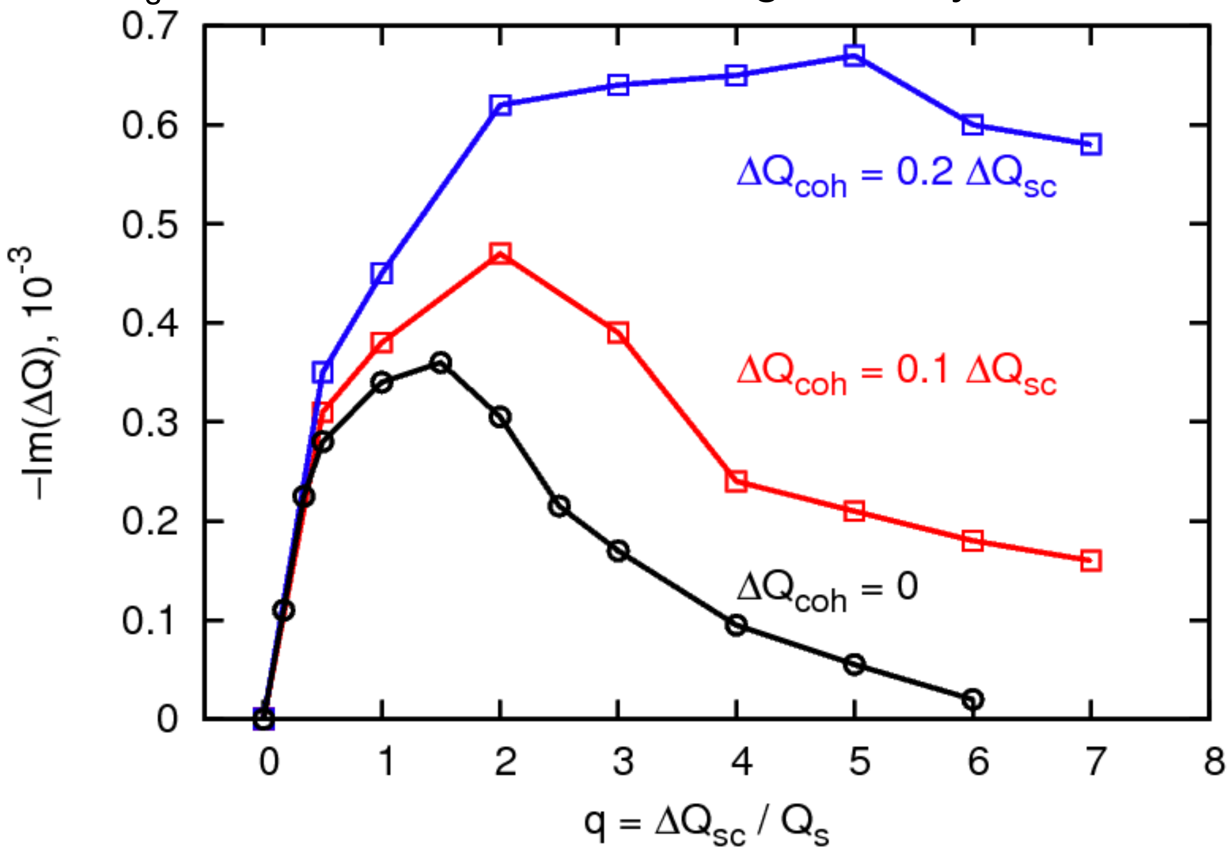
$$\Delta Q_k = -\frac{\Delta Q_{sc} + \Delta Q_{coh}}{2} \pm \sqrt{\left(\frac{\Delta Q_{sc} - \Delta Q_{coh}}{2}\right)^2 + k^2 Q_s^2}$$

if we consider different image charge for the mode k=1:



# SIMULATIONS: LANDAU DAMPING

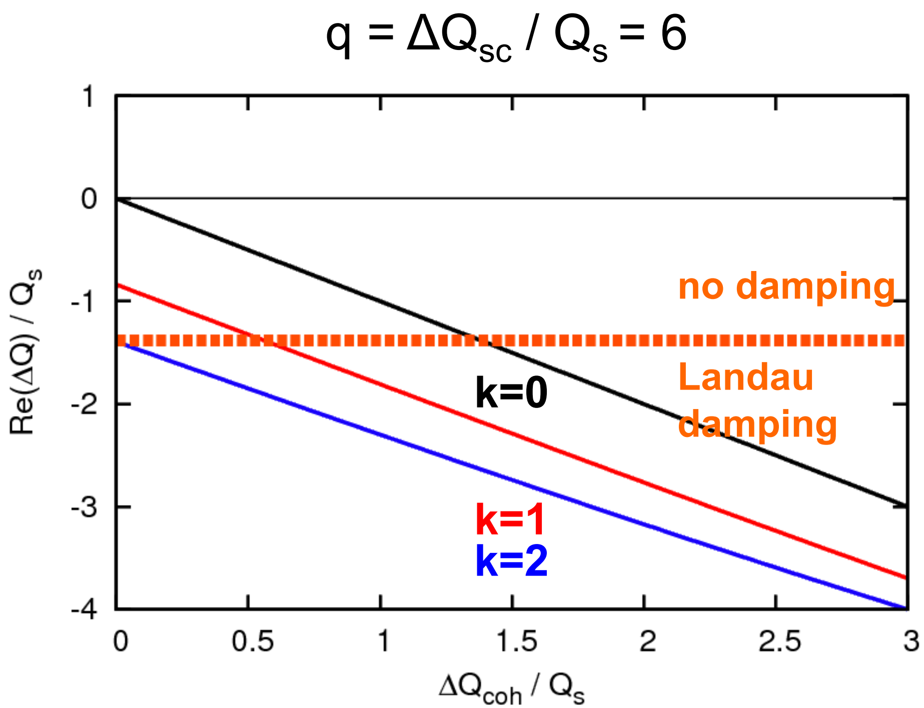
Particle tracking simulations with PATRIC
$Q_s$=0.01, Gaussian bunch longitudinally, transversally

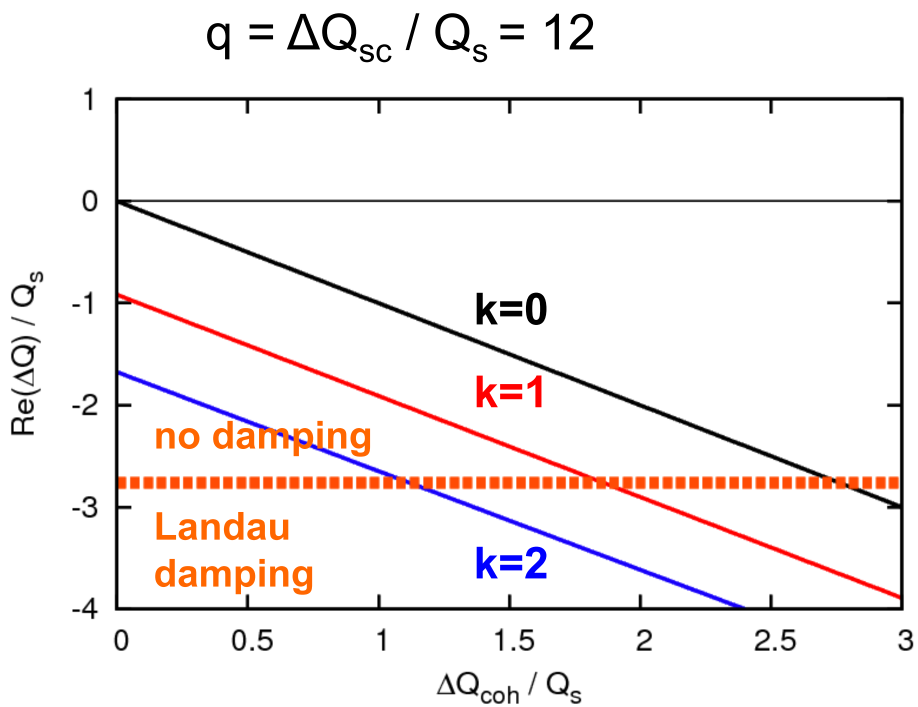



basic agreement with the theory;
detailed predictions of damping close to distribution tails are difficult.

# SIMULATIONS: LANDAU DAMPING

Consider different strength of Space-Charge



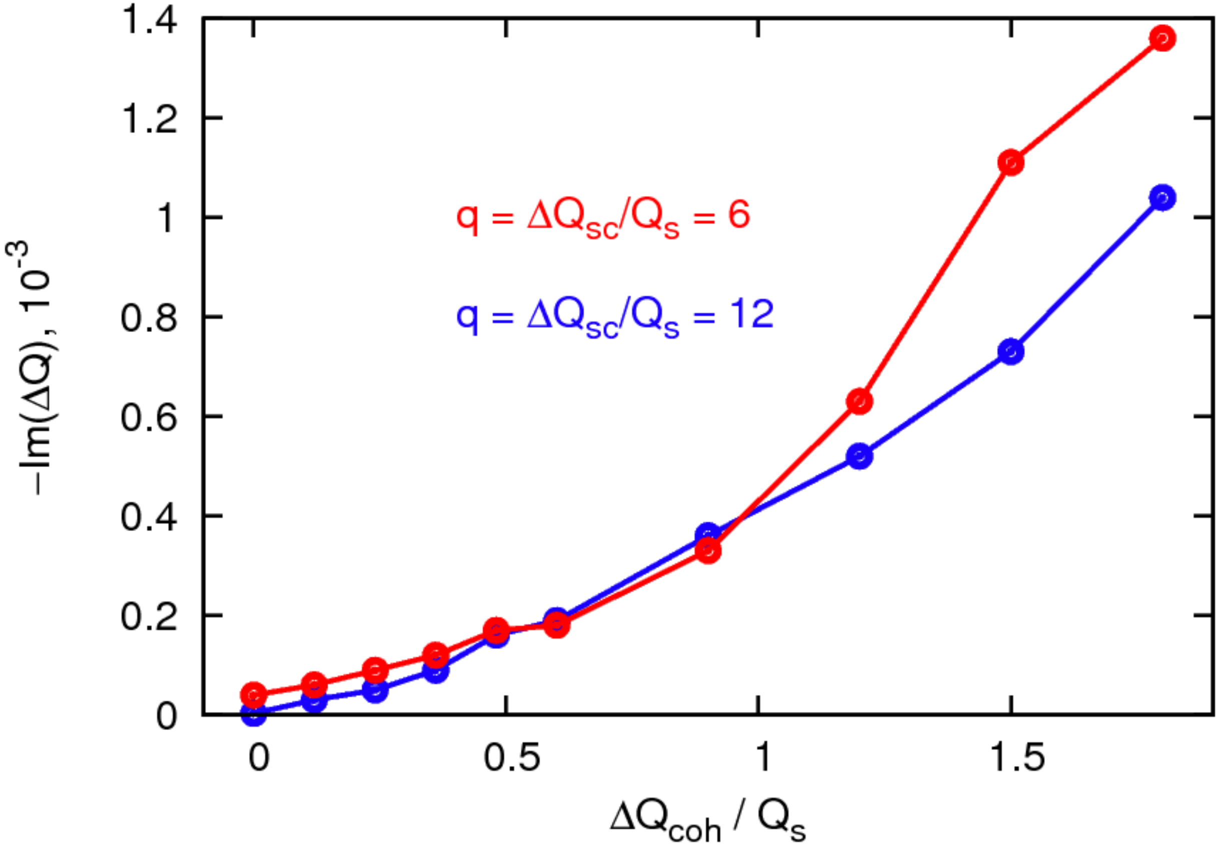




Theory prediction:
keep the strength of space-charge fixed, increase the strength of imaga charges

# SIMULATIONS: LANDAU DAMPING

Particle tracking simulations with PATRIC

$Q_s$=0.01, Gaussian bunch longitudinally, transversally




basic agreement with the theory;
detailed predictions of damping close to distribution tails are difficult.

# SPACE CHARGE IN ISIS BUNCHES

space-charge tune shift

$$\Delta Q_{sc} = \frac{\lambda_0 r_p R}{\gamma^3 \beta^2 \varepsilon_\perp}$$

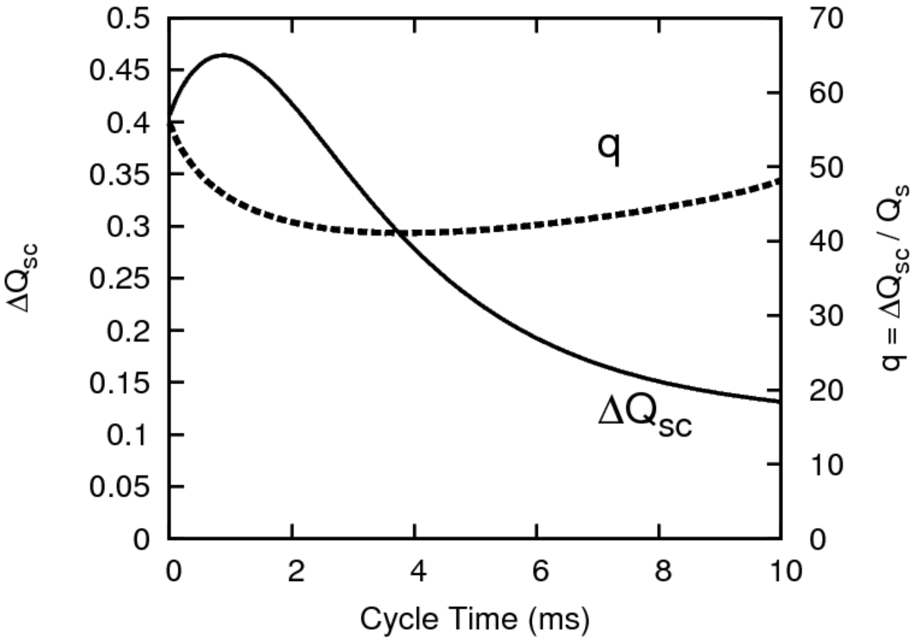
space-charge parameter

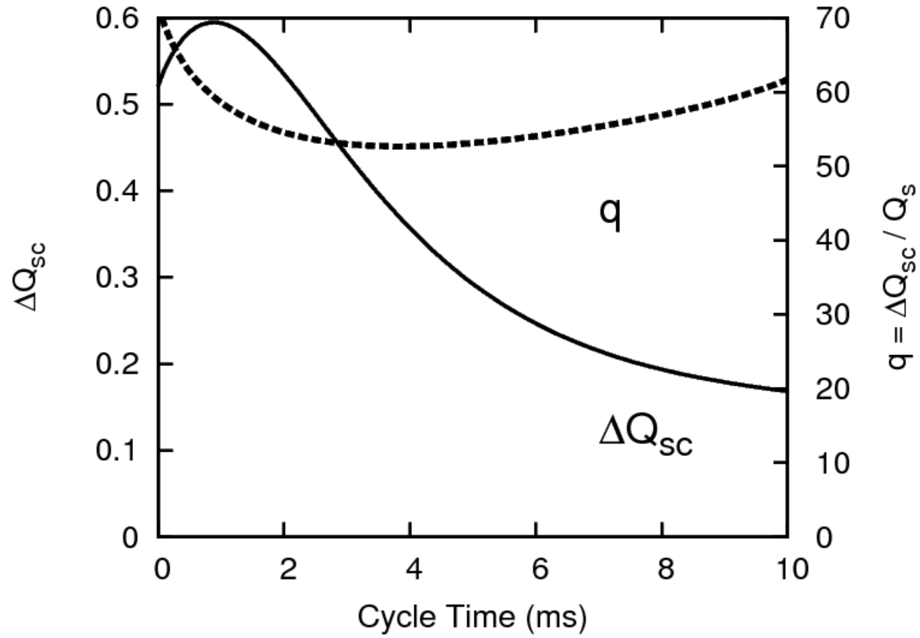
$$q = \frac{\Delta Q_{sc}}{Q_s}$$

parameters of the 1RF bunch

beam 4e12p

beam 8.2e12p




the space-charge parameter q is steady during the cycle

# IMAGE CHARGES IN A BEAM PIPE

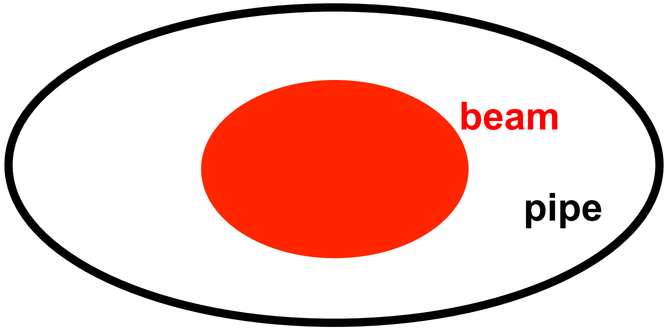
Image Charges are an imaginary impedance
and produce a coherent tune shift, proportional to the own space-charge tune shift:

$$\Delta Q_{\text{coh}} = \frac{\lambda_0 r_p R^2}{\gamma^3 \beta^2 Q_0} \frac{2\xi_{h,v}}{h^2} = \Delta Q_{\text{sc}}\ 2\xi_{h,v} \frac{a^2}{h^2}$$

← **beam size** ($a^2$)

← **pipe size** ($h^2$)

$$\Delta Q_{\text{sc}} = \frac{\lambda_0 r_p R^2}{\gamma^3 \beta^2 Q_0 a^2}$$




large beam/pipe ratio

↓

image charges affect stronger

Round Pipe: $\xi_v = \xi_h = 0.5$

Rectangular Vacuum Chamber,
centered beam (The book K.Y. Ng):

$$\xi_1^V = \frac{K^2(k)}{4}(1-k)^2\ ,$$

$$\xi_1^H = K^2(k)\, k\ ,$$

the modulus *k* calculated from the nome *q*

$$q = e^{-2\pi w/h}$$

half-height *h*, half-width *w*

# IMAGE CHARGES: ISIS PIPE

Rectangular Vacuum Chamber
(The book K.Y. Ng)

$$\xi_1^V = \frac{K^2(k)}{4}(1-k)^2 \,,$$

$$\xi_1^H = K^2(k)\, k \,,$$

Calculation for ISIS:

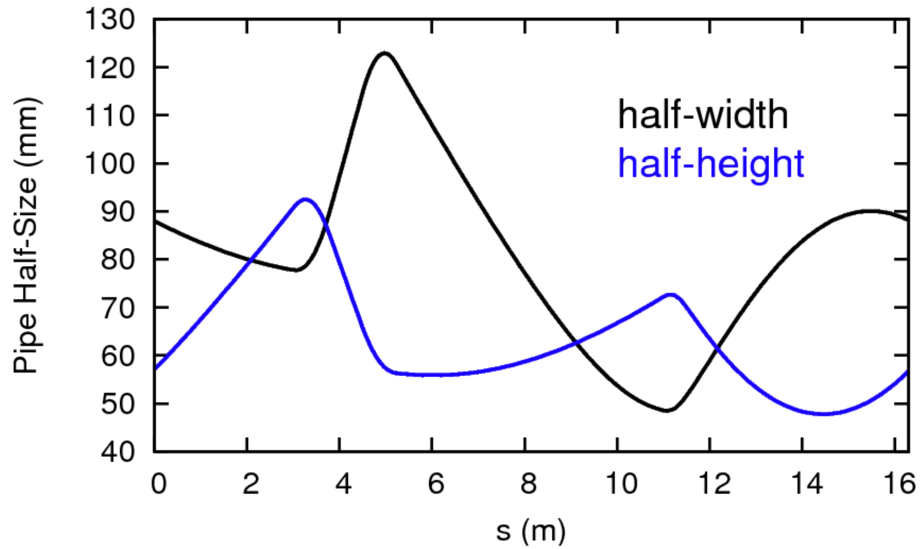
$H$ = $\langle h \rangle$ = 63.42mm

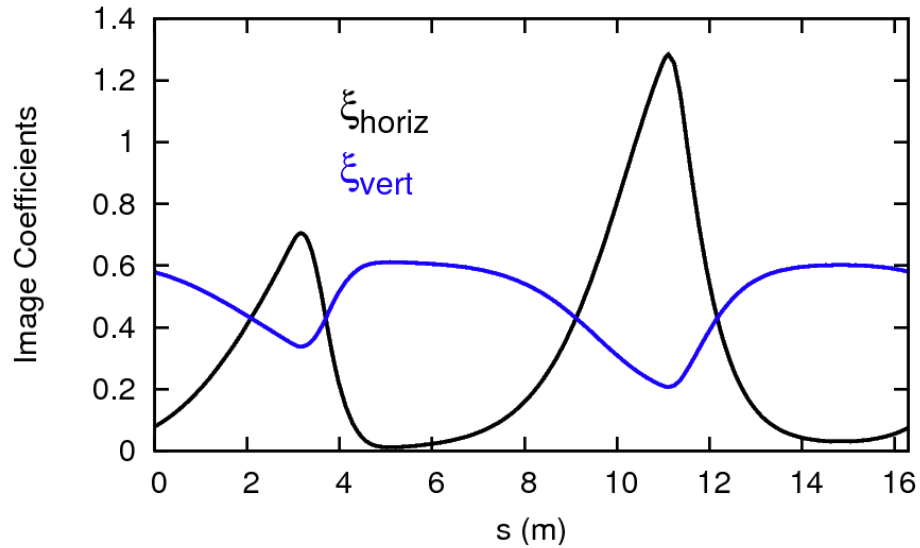
$\langle 2\xi_h(s) / h(s)^2 \rangle H^2 = 0.528$

$\langle 2\xi_v(s) / h(s)^2 \rangle H^2 = 1.13$

results in vertical tune shift:

$$\Delta Q_{\mathrm{coh}} = 0.12 \Delta Q_{\mathrm{sc}}$$



# LANDAU DAMPING FOR ISIS PIPE

ISIS: charge-exchange multi-turn injection, painting in both planes (vertical sweep).

During these experiments, no exact transverse emittance for different intensity was available.

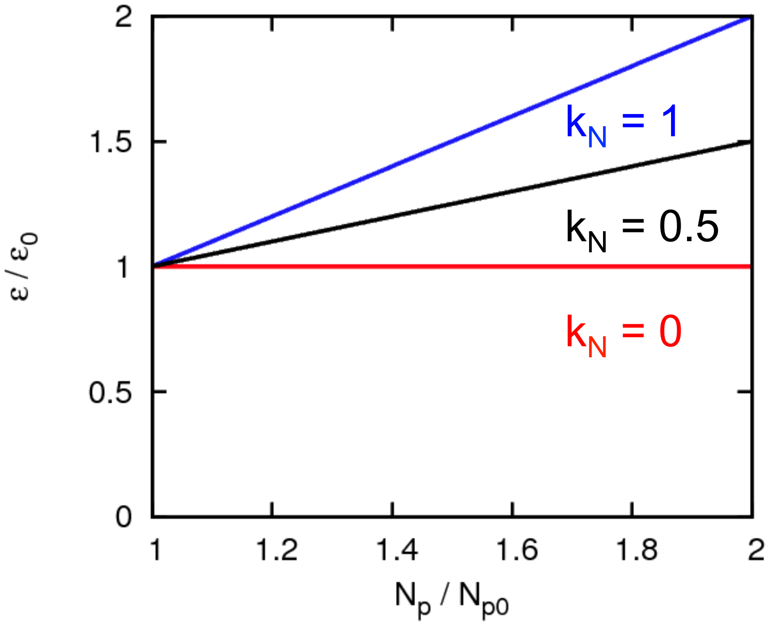
In order to check the model, three emittance(intensity) scenario considered: $k_N$ = 0, 0.5, 1.

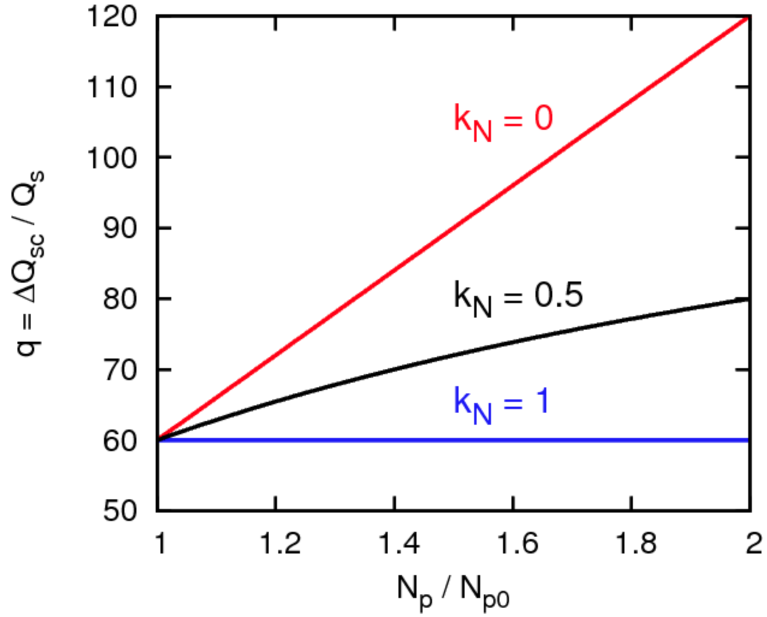
$$\Delta\varepsilon_\perp = k_N \frac{\Delta N_p}{N_{p0}} \varepsilon_{\perp 0}$$

$$\Delta Q_{sc} \propto \frac{N_p}{\varepsilon_\perp}$$

$$\Delta Q_{coh} \propto N_p$$

$$\frac{\Delta Q_{coh}}{\Delta Q_{sc}} \propto \varepsilon_\perp$$



# LANDAU DAMPING FOR ISIS PIPE

$$\Delta\varepsilon_\perp = k_N \frac{\Delta N_p}{N_{p0}} \varepsilon_{\perp 0}$$

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$$\Delta Q_{coh} \propto N_p$$

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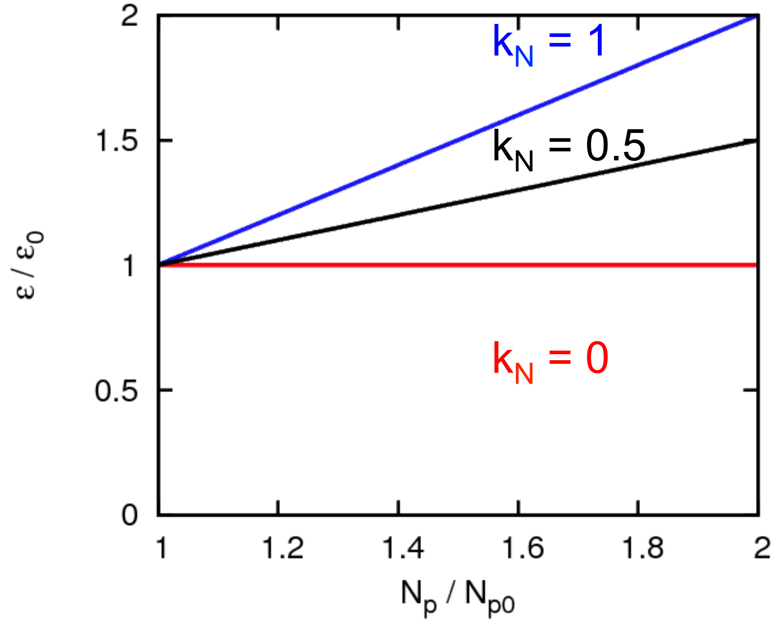
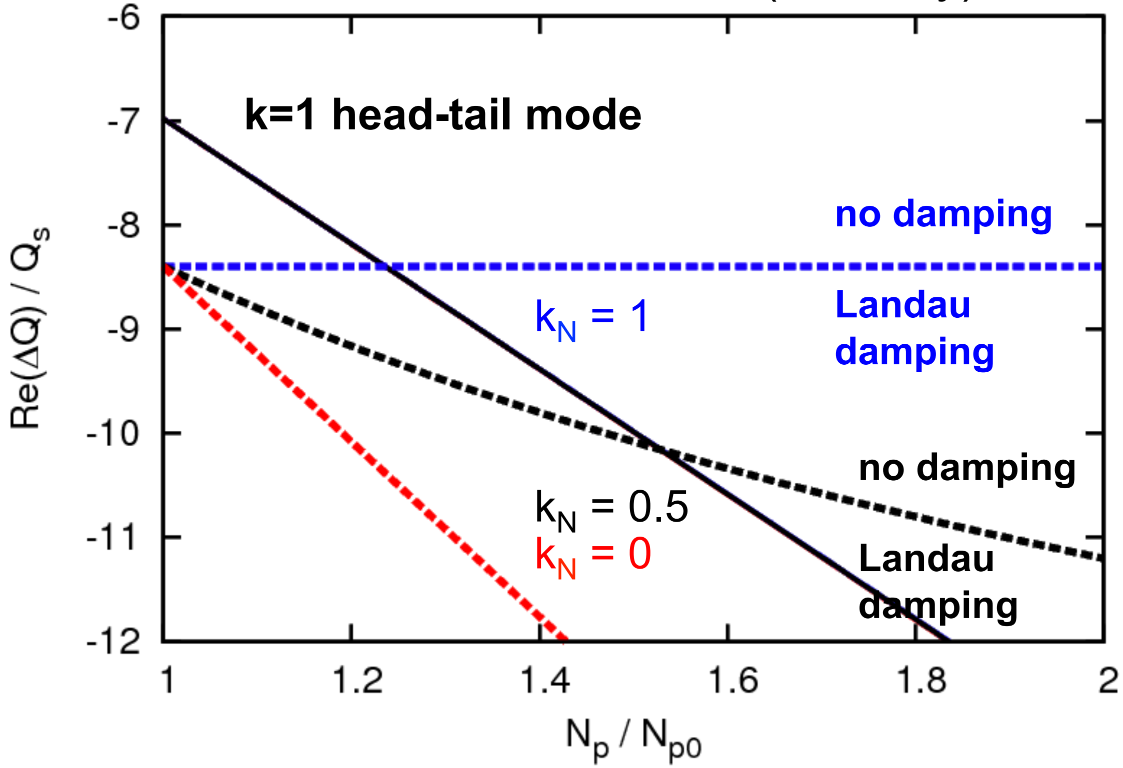
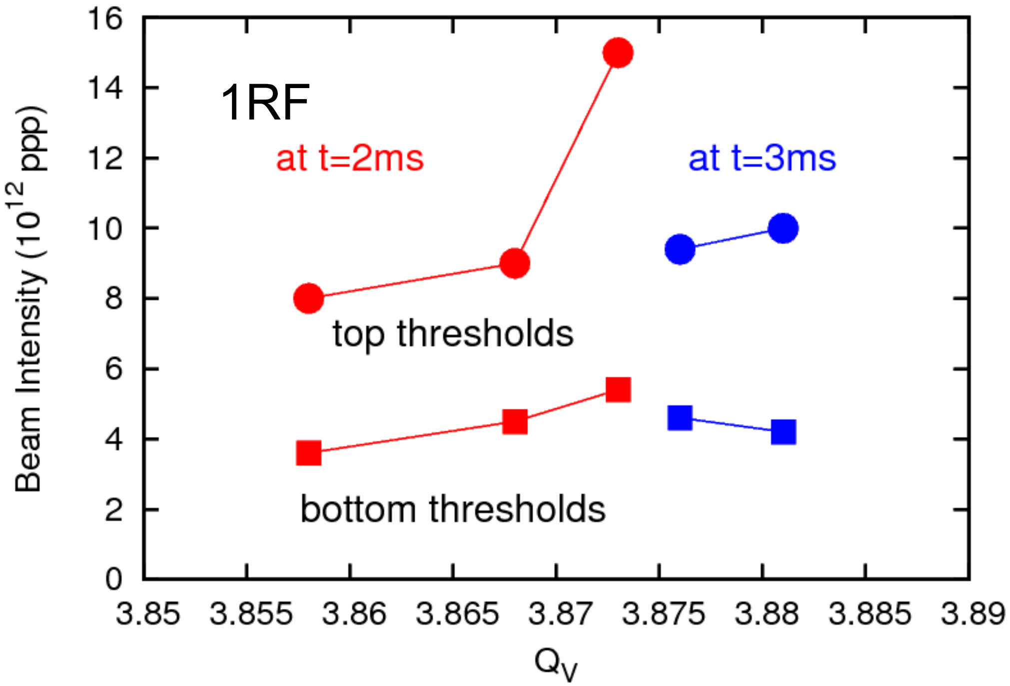


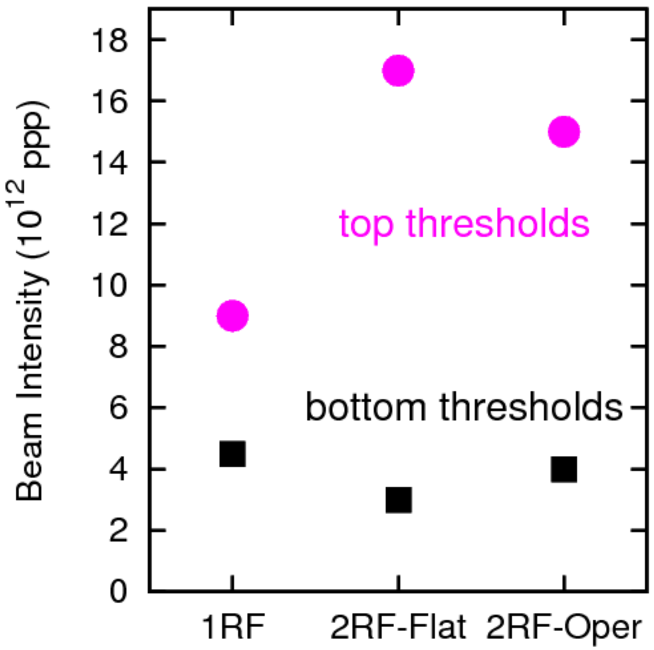

Predictions of the model:
Landau Damping borders of the mode k=1 for three different emittance(intensity)




Intensity threshold for Landau damping due to space-charge and image charges: higher $\Delta Q_{coh} / \Delta Q_{sc}$

# HEAD-TAIL INSTABILITY IN ISIS




$$\Delta Q_{sc} \propto \frac{N_p}{\varepsilon_\perp}$$

$$\Delta Q_{coh} \propto N_p$$

$$\frac{\Delta Q_{coh}}{\Delta Q_{sc}} \propto \varepsilon_\perp$$

As the intensity increases, Landau Damping due to Space-Charge with Image-Charges boosts and can contribute to the **top thresholds**

Additional observation at ISIS during this campaign, not mentioned so far: for the unstable bunch in 1RF, the transverse emittance was increased while keeping all the rest fixed: reproducible stability above ≈90 mm mrad.
**Consistent with this Landau damping idea.**
**An emittance threshold.**

# CONCLUSIONS

Unstable head-tail modes observed in ISIS; beam parameter scans performed; the instability appearance suggests the **resistive-wall-like impedance** as the drive.

**Lower order** and a much **higher growth rate** than expected from the Sacherer theory. Probably bad news for SIS100 U-bunches, where k=4, $\tau \approx 100$ms was expected.

Clear reproducible intensity thresholds found.
Exceptional observation: intensity **top thresholds** (no instability above).

**Landau damping** due to **Space-Charge**, enhanced by the **Image-Charges**, obtained in **simulations**. Qualitative agreement with an airbag-bunch based model.

Calculations show that the unique large $\Delta Q_{coh}/\Delta Q_{sc}$ (**large beam/pipe**) ratio at ISIS should provide Landau damping due to Space-Charge, which can contribute to the top intensity thresholds, and emittance thresholds.
Hence the recommendation: blow-up the beam (trade-off with other losses).

Calculations for SIS100 U-bunches ($\Delta Q_{sc}/Q_s \approx 40$, $\Delta Q_{coh}/\Delta Q_{sc} \approx 0.08$) indicate a possible effective Landau damping (**good news**).