

# RADIO FREQUENCY QUADRUPOLE FOR LANDAU DAMPING IN ACCELERATORS: ANALYTICAL AND NUMERICAL STUDIES

A. Grudiev, K. Li, CERN, Geneva, Switzerland  
 M. Schenk, University of Bern, Bern, Switzerland

## Abstract

It is proposed to use a radio frequency quadrupole (RFQ) to introduce a longitudinal spread of the betatron frequency for Landau damping of transverse beam instabilities in circular accelerators. The existing theory of stability diagrams for Landau damping is applied to the case of an RFQ. As an example, the required quadrupolar strength is calculated for stabilizing the Large Hadron Collider (LHC) beams at 7 TeV. It is shown that this strength can be provided by a superconducting RF device which is only a few meters long. Furthermore, the stabilizing effect of such a device is proven numerically by means of the PyHEADTAIL macroparticle tracking code for the case of a slow head-tail instability observed in the LHC at 3.5 TeV.

## INTRODUCTION

In accelerators, the effect of Landau damping [1] provides a natural stabilizing mechanism against collective instabilities if the particles in the beam have a small spread in their natural (betatron or synchrotron) frequencies, see for example [2] and references therein. If the spread introduced by non-linearities naturally present in an accelerator is too small, a dedicated non-linear element is added to the system. For example, in LHC [3], 84 focusing and 84 defocusing (arranged in 16 families – two per sector), 0.32 m long superconducting magnetic octupoles is used to introduce a betatron frequency spread for Landau damping of the dipole modes. The LHC octupoles have been successfully used to stabilize the beams up to the energy of 4 TeV at which LHC has been operated so far [4]. The effect of the transverse spread however, reduces as the transverse geometrical beam emittance decreases at higher energies due to adiabatic damping.

Recently [5], it has been proposed to use an RFQ to introduce a longitudinal spread of the betatron frequency for Landau damping of the transverse collective oscillations. The basic idea is to use the harmonic dependence of the quadrupolar focusing strength of the RFQ on the longitudinal position of the particles in the bunch. It has been shown that in high energy accelerators, the longitudinal spread is more effective than the transverse one due to the longitudinal emittance of the beam being much larger than the transverse one. The higher efficiency of the longitudinal spread for Landau damping allows for a compact, only a few meters long, RF device based on several 800 MHz superconducting cavities operating in a TM quadrupolar mode to provide the same functionality as the LHC octupoles whose total length is about 56 m. In this paper, the work presented

in [5] is summarized and the first results on the implementation of the RFQ in the PyHEADTAIL macroparticle tracking code and on the simulation of its stabilizing effect on a slow transverse head-tail instability observed in LHC at 3.5 TeV [4] are reported.

## RFQ FOR LANDAU DAMPING

For an ultra-relativistic particle of charge  $q$  and momentum  $p$  traversing an RFQ along the  $z$ -axis at the time moment  $t$ , the transverse kick in the thin-lens approximation is given by

$$\Delta \mathbf{p}_\perp = pk_2(-x\mathbf{u}_x + y\mathbf{u}_y) \cos \omega t, \quad (1)$$

where  $\omega$  is the RFQ frequency,  $\mathbf{u}_\alpha$  is the unit vector along the  $\alpha$  coordinate and  $k_2$  is the amplitude of the normalised integrated quadrupolar strength

$$k_2 = \frac{q}{pc} \frac{1}{\pi r} \int_0^{2\pi} \int_0^L (E_x - cB_y) e^{j\omega z/c} dz \Big| \cos \varphi d\varphi, \quad (2)$$

where  $c$  is the speed of light,  $L$  is the length of the RFQ and  $[r, \varphi, z]$  are cylindrical coordinates. Assuming that the bunch centre ( $z = 0$ ) passes the thin-lens RFQ at  $t = 0$ , the substitution  $t = z/c$  gives the dependence of the quadrupolar strength along the bunch. Substituting this dependence in the expression for the betatron frequency shift due to quadrupolar focusing yields the expression for the variation of the betatron frequency (so-called detuning) along the bunch

$$\Delta \omega_{x,y} = \pm \beta_{x,y} \frac{\omega_0}{4\pi} k_2 \cos(\omega z/c), \quad (3)$$

where  $\omega_0$  is the revolution frequency and  $\beta_{x,y}$  are the horizontal and vertical beta functions, respectively. Taking into account that  $\cos \omega z/c$  can be approximated as  $\sim 1 - (\omega z/c)^2/2$  for small arguments and  $\langle z^2 \rangle = \sigma_z^2 J_z/\varepsilon_z$  after averaging over the synchrotron period  $T_s$ , one can derive the expression for the variation of the betatron frequency in terms of the longitudinal action  $J_z$

$$\langle \Delta \omega_{x,y} \rangle_{T_s} \approx \pm \beta_{x,y} \frac{\omega_0}{4\pi} k_2 \left[ 1 - \frac{1}{2} \left( \frac{\omega \sigma_z}{c} \right)^2 \frac{J_z}{\varepsilon_z} \right], \quad (4)$$

where  $\sigma_z$  and  $\varepsilon_z$  are the RMS bunch length and the longitudinal emittance, respectively. Equation (4) is composed of two terms. The first one is a constant betatron frequency shift which, if necessary, can be compensated by a static magnetic quadrupole. The second term describes the incoherent spread required for Landau damping. It is used to estimate the integrated quadrupolar strength amplitude needed to stabilize a transverse instability with a coherent betatron frequency shift  $\Delta \Omega_{x,y}^{coh}$  according to the following condition

$$\beta_{x,y} \frac{\omega_0}{8\pi} k_2 \left( \frac{\omega \sigma_z}{c} \right)^2 > |\Delta\Omega_{x,y}^{coh}|, \quad (5)$$

which is based on applying the generalized theory of stability diagrams for Landau damping [6] to the case of an RFQ.

According to the Panofsky-Wenzel theorem [7], an RFQ also causes a change of the longitudinal momentum depending on the transverse and longitudinal coordinates of a particle in the bunch. This leads to a transverse spread in the synchrotron frequency. However, for the LHC, this transverse spread is found to be negligible compared to the longitudinal spread due to the large difference between the transverse and longitudinal emittances.

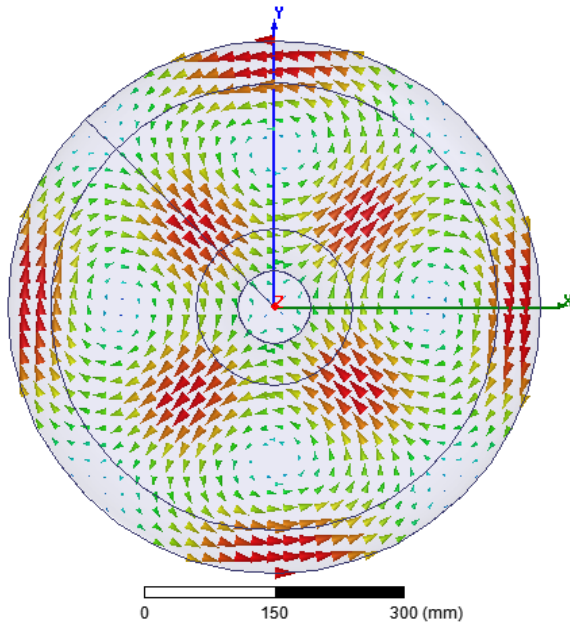


Figure 1: Magnetic field distribution in the transverse plane of the TM quadrupolar mode cavity of the RFQ.

For illustration, a design of an RFQ device with the same functionality as the LHC octupoles has been proposed. From Eq. (5), the required quadrupolar strength is expressed as

$$k_2 = \frac{2}{\pi} \frac{|\Delta\Omega_{x,y}^{coh}|}{\omega_0 \beta_{x,y}} \left( \frac{\lambda}{\sigma_z} \right)^2.$$

Its value  $k_2 = 1.4 \cdot 10^{-5} \text{ m}^{-1}$ , required for Landau damping of a coupled bunch mode with the coherent betatron frequency shift of  $\sim 0.0002\omega_0$ , is calculated for nominal beam parameters of the LHC at 7 TeV [3]:  $\sigma_z = 0.08 \text{ m}$  and beta functions of 200 m at a potential location in IR4 near the main RF system. An RFQ wavelength of  $\lambda = 0.375 \text{ m}$ , i.e. RFQ frequency of 800 MHz is chosen, which is the second harmonic of the main RF frequency and for which the bunch still fits in the RFQ bucket as  $4\sigma_z = 0.32 \text{ m} < \lambda$ . On the other hand, the normalised quadrupolar strength of a cylindrical 800 MHz 0.15 m long pillbox cavity operating in a TM quadrupolar mode is calculated from the complex electromagnetic field map

obtained numerically using the code HFSS [8]. In Fig. 1, the distribution of the magnetic field in the transverse plane of the cavity is shown for illustration. The strength value per cavity is determined to be  $k_2 = 6 \cdot 10^{-6} \text{ m}^{-1}$ , given the maximum values of 46 MV/m and 120 mT for respectively the electric and magnetic fields on the cavity surface. Taking this value as a maximum that can be achieved in one cavity due to limitations on the surface fields coming from an RF superconductivity quench or an electrical discharge in vacuum [9], the total number of cavities needed can be determined to be three. Adding the same factor two margin as for the LHC octupoles, we conclude that six cavities with a total active length of less than a meter can provide the same Landau damping as the 56 m of LHC magnetic octupoles at a nominal current of 500 A. The whole RFQ device, including the RF power couplers and the coupler for lower, same and higher order parasitic modes suppression could be integrated in a single few meters long cryostat.

## NUMERICAL STUDIES AND RESULTS

The performance of an RFQ with the design parameters given in the previous section is evaluated numerically for the LHC by means of PyHEADTAIL, a macroparticle tracking code under development at CERN and successor of the well-established HEADTAIL code [10]. Amongst others, this software allows to study the formation of collective instabilities in circular accelerators and to evaluate appropriate methods for their mitigation.

The PyHEADTAIL model assumes linear periodic transport from one section to another along the accelerator ring in the transverse and longitudinal planes. All the impedances are lumped in a single point where they are adequately weighted by the corresponding beta functions. Macroparticle bunches are initialised and tracked through the ring via the concatenation of linear periodic maps and wakefield kicks. After each linear transport, the effect of chromaticity, i.e. the variation of the betatron tune with the relative momentum deviation  $\Delta p/p$ , is applied by changing the individual phase advance of every single macroparticle in the bunch (incoherent detuning). The amplitude detuning caused by magnetic octupoles [11] possibly installed in the accelerator as well as the detuning introduced by an RFQ (Eq. (3)) are implemented in the same manner.

For the numerical study, an LHC case was identified for which experimental data are available to make direct validation of the simulation results possible. The chosen study case is a single-bunch horizontal slow head-tail instability, which has been observed during LHC commissioning at an energy of 3.5 TeV [4]. It has been shown experimentally that this instability is fully suppressed by means of the magnetic octupoles installed in the LHC. These devices have the sole purpose of introducing an incoherent betatron tune spread for beam stabilization through the process of Landau damping [11]. The octupolar strength, and hence the tune spread, are controlled by the electric currents  $I_f$  and  $I_d$  in the focusing

and defocusing octupole magnets, respectively. Experiments have shown that the threshold value for the mitigation of the abovementioned instability lies between  $I_d = -I_f = 10$  A and  $I_d = -I_f = 20$  A.

Before studying the RFQ, the experimentally observed beam stabilization achieved using the octupole magnets and the threshold values for  $I_f$  and  $I_d$  are reproduced with PyHEADTAIL simulations. The LHC machine and beam parameters used for this study are summarized in Tab. 1. The wakefield kicks are calculated from wake tables computed from the latest version of the LHC impedance model [12]. To simplify the analysis, only the dipolar wake components are considered. It has been verified that including the quadrupolar terms does not significantly change the outcome of the simulations. Due to the comparatively low growth rate of the observed instability with rise times of the order of several seconds, the bunch is tracked over a total number of 300 000 turns which corresponds to a time range of 27 s in the LHC. The bunch is composed of one million macroparticles which was shown from convergence studies to be sufficient to yield accurate results.

Table 1: Main Parameters Used in the PyHEADTAIL Simulations to Reproduce the Experimental Observations Made in LHC.

Parameter	Value
Beam energy	$E$ 3.5 TeV
Beam intensity	$N_b$ $1.05 \cdot 10^{11}$ p/b
RMS bunch length	$\sigma_z$ 0.06 m
Norm. horizontal emittance	$\varepsilon_x$ 3.75 $\mu\text{m}$
Horizontal chromaticity	$Q'_x$ 6
Nb. of macroparticles	$10^6$ p/b
Nb. of turns	$3 \cdot 10^5$ turns

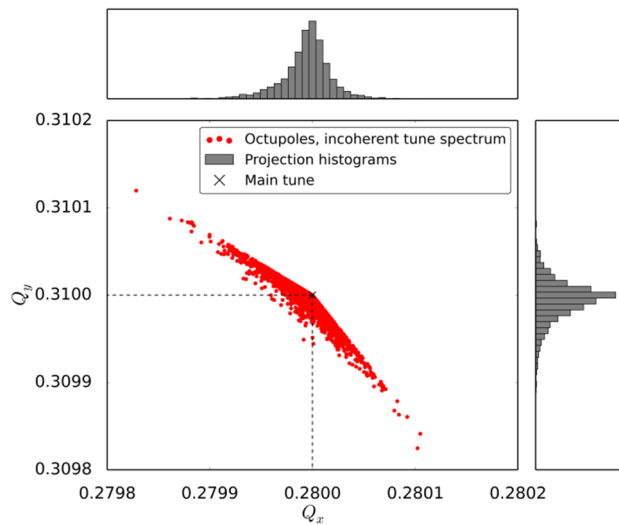


Figure 2: Incoherent betatron tune spectrum  $Q_y$  vs.  $Q_x$  (tune footprint) introduced by the presence of the LHC octupole magnets at  $I_d = -I_f = 20$  A and using the beam parameters given in Tab. 1.

Figure 2 shows the incoherent betatron tune spectrum originating from amplitude detuning caused by the LHC octupole magnets at an electric current of  $I_d = -I_f = 20$  A and using the machine and beam parameters defined in Tab. 1. The spectrum is obtained from PyHEADTAIL simulations incorporating purely linear betatron and synchrotron motion together with the detuning effect of the LHC octupole magnets. The spectral analysis is performed using the SUSSIX code [13]. The shape and distributions of the betatron tune spectrum can be fully understood from the amplitude detuning formula given in [11].

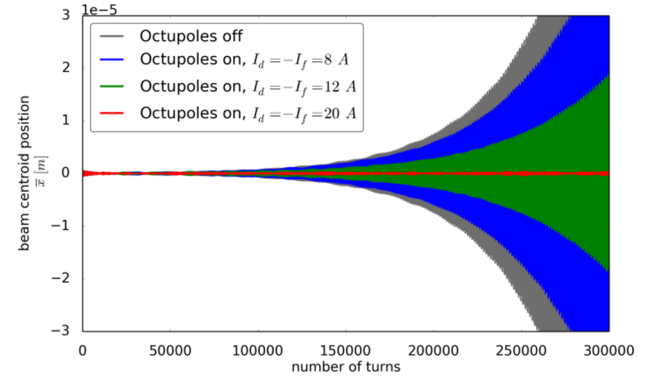


Figure 3: Horizontal beam centroid position vs. number of turns as obtained from PyHEADTAIL using the parameters in Tab. 1. Results are shown for various electric currents in the LHC octupole magnets.

Figure 3 shows the outcome of the PyHEADTAIL simulations using the parameters in Tab. 1. The horizontal bunch centroid position versus the number of turns is plotted for four different values of the electric currents  $I_f$  and  $I_d$  in the octupole magnets. In case the octupoles are switched off (grey curve), a very clean instability is seen with an exponential amplitude growth of the horizontal bunch centroid motion. The frequency analysis of this motion reveals a mode  $m = -1$  instability, in agreement with what has already been determined from experimental data [4]. As can be seen in Fig. 3, starting from electric currents of  $I_d = -I_f = 8$  A (blue) in the octupole magnets, the growth rate of the instability decreases. This is a result of Landau damping setting in, i.e. the incoherent betatron tune spectrum introduced by the octupole magnets becomes wide enough to overlap with the frequency of the coherent (unstable) mode. At values of  $I_d = -I_f = 8$  A and  $I_d = -I_f = 12$  A (green) however, Landau damping is not yet sufficient to fully suppress the instability. This can only be achieved when setting the electric currents to a value of  $I_d = -I_f = 20$  A (red) or higher. The stabilization threshold has been cross-checked with stability diagram theory for magnetic octupoles, yielding  $I_d = -I_f = 18$  A.

The excellent agreement of the results obtained from experimental data, PyHEADTAIL simulations and stability diagram theory proves a solid understanding of the observations made. The fact that the instability is cured unambiguously through the process of Landau damping makes it an ideal case to study the effect of an

RFQ on beam stabilization. In the following, the incoherent betatron tune spectrum originating from the presence of an RFQ is shown and explained first. Afterwards, the octupole magnets used to obtain the results presented in the previous paragraphs are replaced by an RFQ to evaluate its performance in comparison.

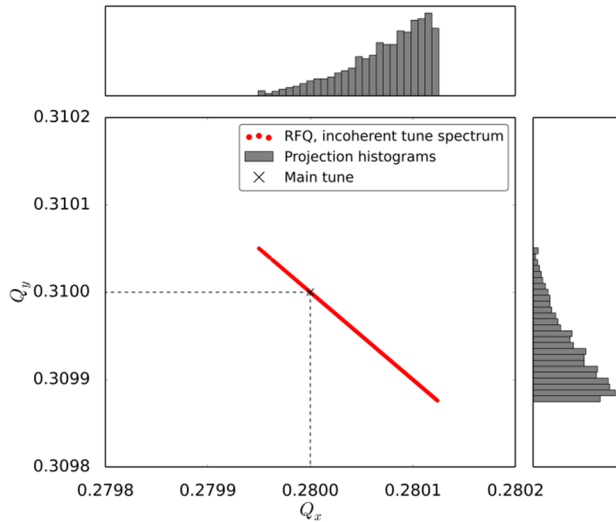


Figure 4: Incoherent betatron tune spectrum  $Q_y$  vs.  $Q_x$  (tune footprint) introduced by the presence of an RFQ at a strength of  $k_2 = 7.8 \cdot 10^{-6} \text{ m}^{-1}$  and using the beam parameters given in Tab. 1.

Similarly to octupole magnets, the detuning caused by an RFQ leads to an incoherent betatron tune spectrum with a certain spread and distribution characteristic for the chosen parameters of the beam and of the device and its location in the accelerator. An example is shown in Fig. 4 for the machine and beam parameters given in Tab. 1. The beta functions at the location of the RFQ are chosen to be 200 m for reasons explained in the previous section. The RFQ frequency is fixed to 800 MHz. The shape and distributions of the betatron tune spectrum can be understood from the detuning formula in Eq. (4). First, as opposed to what is observed when using magnetic octupoles (Fig. 2), the tune shifts  $\Delta Q_x$  and  $\Delta Q_y$  induced by an RFQ are fully correlated as there is only one independent variable  $J_z$ . The tune shift ratio  $\Delta Q_y/\Delta Q_x$  is defined by the negative ratio of the beta functions  $-\beta_y/\beta_x$  at the location of the RFQ. Second, compared to the spectra resulting from magnetic octupoles, the projection histograms in Fig. 4 show much more asymmetric distributions with the betatron tunes of the majority of macroparticles shifted away from the original main tune of the accelerator (black dashed lines). This asymmetry is expected for the case where  $\lambda > \sigma_z$ , i.e. the approximation in Eq. (4) holds. Since the longitudinal phase space distribution is chosen to be Gaussian and centred about zero longitudinal action  $J_z = 0$ , a large fraction of the particles in the beam experience only the constant betatron tune shift, i.e. the first term in Eq. (4). On the other hand, there is only a small number of particles with a large longitudinal action  $J_z$  whose tune shift is

dominated by the second term in Eq. (4). In case the approximation does not hold, i.e. if  $\lambda \leq \sigma_z$ , the incoherent tune spectrum will become more symmetric and uniform.

Figure 5 shows the horizontal bunch centroid position versus the number of turns with and without the presence of the RFQ. Similarly to what has been found for the octupole magnets in Fig. 3, starting from an RFQ strength of  $k_2 = 6.2 \cdot 10^{-6} \text{ m}^{-1}$  (blue), the growth rate of the instability decreases, indicating the presence of Landau damping. Again, at moderate RFQ strengths of  $k_2 = 6.2 \cdot 10^{-6} \text{ m}^{-1}$  and  $k_2 = 6.8 \cdot 10^{-6} \text{ m}^{-1}$  (green), Landau damping is not sufficient to prevent the instability from evolving. Only by increasing the strength to a value of  $k_2 = 7.8 \cdot 10^{-6} \text{ m}^{-1}$  or higher, the instability can finally be fully damped. At the given beam energy of 3.5 TeV, this strength can be provided by a single superconducting cavity of the kind described in the previous section. A comparison of the simulation outcomes presented in Fig. 3 and 5 shows that similarly to octupole magnets, an RFQ can indeed be used for beam stabilization through Landau damping.

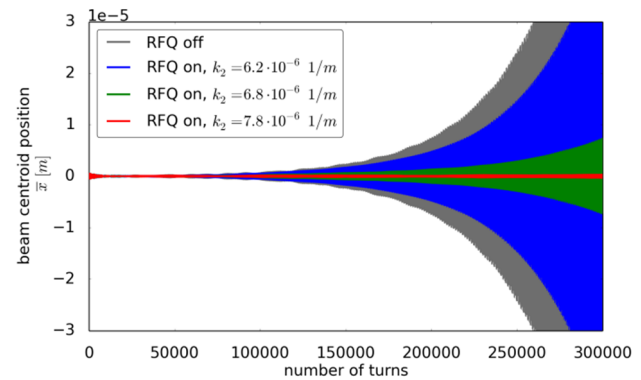


Figure 5: Horizontal beam centroid position vs. number of turns as obtained from PyHEADTAIL using the parameters in Tab. 1. Results are shown for various strengths  $k_2$  of RFQ cavities operating at 800 MHz.

## CONCLUSION

It has been shown that an RF quadrupole introduces a longitudinal spread of the betatron frequencies which can be used for Landau damping of the transverse coupled bunch instabilities. As an example, the required strength of the RFQ providing the same functionality as the LHC magnetic octupoles has been calculated. Furthermore, a possible implementation of the RFQ using a set of superconducting cavities in one few meters long cryostat has been shown. To evaluate numerically the performance of such a device and its effect on the stabilization of transverse beam instabilities, the macroparticle tracking code PyHEADTAIL has been used. A thoroughly analysed and well-understood LHC case at 3.5 TeV served to prove numerically that the RFQ can, similarly to octupole magnets, generate the necessary incoherent betatron tune spread to provide sufficient Landau damping to fully stabilize the beam.

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