MODELING AND FEEDBACK DESIGN TECHNIQUES FOR CONTROLLING INTRA-BUNCH INSTABILITIES AT CERN SPS RING *

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Abstract

The feedback control of intra-bunch instabilities driven by electron-clouds or strong head-tail coupling (transverse mode coupled instabilities MCI) requires bandwidth sufficient to sense the vertical position and apply multiple corrections within a nanosecond-scale bunch. These requirements impose challenges and limits in the design and implementation of the feedback system. This paper presents model-based design techniques for feedback systems to address the stabilization of the transverse bunch dynamics. These techniques include in the design the effect of noise and signals perturbing the bunch motion. They also include realistic limitations such as bandwidth, nonlinearities in the hardware and maximum power deliverable. Robustness of the system is evaluated as a function of parameter variations of the bunch.

INTRODUCTION

The feedback control of intra-bunch instabilities induced by electron-cloud (ECI) or strong head-tail interaction (transverse mode coupled instabilities - TMCI) requires enough bandwidth to sense the vertical position motion and apply correction fields to multiple sections of a nanosecond-scale bunch. Through the US LARP-CERN collaboration a wide-band feedback system is under research and development to control these intra-bunch instabilities. The effort is motivated by the plans to increase the beam current in the Super Proton Synchrotron (SPS) as part of the HL-LHC upgrade.

The feedback controller is implemented based on a digital reprogrammable processing channel, sampling the transverse bunch motion at a rate of 4 GS/s. The approach followed to design the controller is to consider the bunch dynamics as a multi-input multi-output system (MIMO). This conception arise because the multiple samples (multi-input) measuring the transverse motion across the bunch are used input to generate the multiple output samples that defines the control signal driving the kicker device.

During the first part of this development, the feedback control system is using a bank of finite-impulse response (FIR) filters to conduct MDs at CERN SPS ring during January 2013. In this bank, a filters are used to process individually each sample of the input signal. This planning was followed, in part, because of the simplicity of the filter implementation and the definition of its parameters and the limitations imposed by the hardware installed in the machine. The bandwidth of the existent kicker is about 160 MHz, limiting the effective feedback control on a 3.2ns bunch length to the first side-band around the betatron tune. Additionally, the setting of the Q26 lattice in the machine defined the fractional betatron frequency $\omega_\beta = 0.185$ and the fractional synchrotron frequency $\omega_s = 0.0059$ and the phase lag of the FIR filter was not a limitation to damp the transverse bunch dynamics corresponding to the barycentric and head-tail motions [1].

In the second stage of this development, new strip-line kickers with wider bandwidth were installed in the SPS ring and a slotted-coaxial kicker is under development [2,3]. That potentially will define a true wide-band feedback channel able to drive multiple intra-bunch modes. A new challenge in the design of the feedback controller exists due to the re-definition of the SPS lattice from the Q26 to the Q20 optics [4,5]. The new optics in the machine sets a fractional synchrotron frequency $\omega_s = 0.0170$, spreading out the frequency of the satellite bands around the betatron frequency $\omega_\beta = 0.185$. In [6], the design of a controller based on a bank of infinite-impulse response (IIR) filters is analyzed to stabilize the intra-bunch dynamics corresponding to the new Q20 optics. In that pre-design, the phase of the filters is kept almost constant in the frequency range corresponding to the fractional betatron tune and its dominant side-bands ($f_\beta \pm n f_s$). That design uses the bunch dynamics model to define the fundamental parameters of the controller and test the stability and performance robustness of the controller. It does not incorporate specifically the model into the design of the controller.

This paper addresses another methodology for the controller design to stabilize the intra-bunch dynamics of the beam at SPS with Q20 optics. The model of the intra-bunch dynamics is included intrinsically in the controller design providing the maximum information of the bunch modes to be stabilized. This realization gives higher order controllers respect to the FIR/IIR filter banks. In this paper we design a full model-based controller to stabilize the dominant bunch modes, analyze different controller options comparing the stability and performance robustness of the system when the betatron and synchrotron frequency are changed and the initial modal instability (growth rates) are varied. Based on this full controller, simplified versions or reduced order controllers has to be evaluated. The study of these reduced controllers is attractive to simplify the firmware implementation and the setting of the controller parameters in real-time operation.

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FEEDBACK SYSTEM

The first requirement for the controller is to stabilize the intra-bunch dynamics driven by electron cloud and strong head-tail instabilities. Additionally, the feedback system has to be robust to parameter changes in the beam dynamics and different operation conditions of the machine. The controller has to have enough dynamic range to keep the stability-performance of the system for a maximum set of beam transient conditions. Given the conditions that the open loop system is unstable and the feedback channel has delay, it could exist a combination of fast unstable dynamics and long delay in the system that makes the controller unfeasible. The bandwidth of the controller has to be limited to minimize the effect of the receiver noise in the saturation of the power stage. Additionally, the filter has to be able to reject signal perturbations that affects the performance of the feedback system. Feedback control model-based design techniques allows to assess the system stability and address the system performance including in the controller design the rejection to noise and perturbations.

The architecture of the feedback control channel prototype implemented for this application is based on a digital reprogrammable system, sampling the transverse bunch motion at a rate of 4 GS/s. A single bunch controller has been developed to explore new technology and control techniques and it is planned to expand this prototype to allow multi-bunch control. The implementation of this system is based on a reconfigurable FPGA and ADC/DAC operating at 4 GS/s. The system is synchronized with the SPS RF clock and is able to perform diagnostic functions, set feedback parameters and record the bunch motion at selected intervals [7]. A general block diagram of the proposed hardware is depicted in Fig. 1. Analog equalization of the pick-up and cable transfer function is included in the feedback channel. The controller is programmable and has the flexibility to implement FIR / IIR filter banks or more complex control topologies based on the bunch model dynamics.

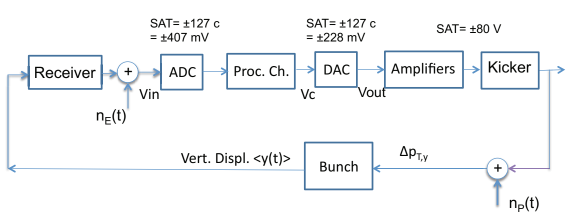

Figure 1: Block diagram control feedback system.

MODEL-BASED CONTROLLERS

During the last decades, research in the feedback control area filled the gap between the classic and modern control theory, including in the controller design the information of the system model, its parameter variations, perturbation and noise to access the stability and performance of the closed loop feedback. [8]. There exists several options to include the bunch model in the design of the controller. We follow in this paper the one based on the observer technique, which is described by the block diagram depicted in Fig. 2.

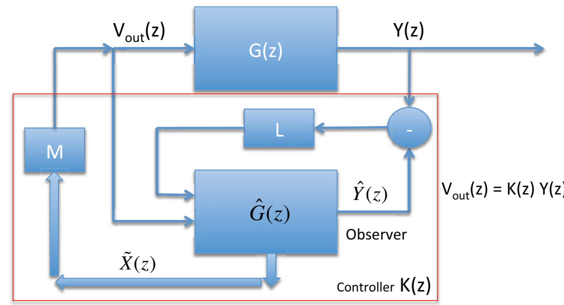

Figure 2: Block diagram model-based controller.

Defining by $G(z)$ the transfer function matrix in $\mathcal{Z}$-domain between the vertical motion of the multiple samples of the bunch $Y(z)$ and the control signal $V_{out}(z)$, the idea is to create an observer based on the bunch dynamical model $\tilde{G}(z)$ and the processed error signal $L(Y(z)-\tilde{Y}(z))$. This observer will allow to estimate in real-time the internal states X of the system that are not included directly in the system output $Y(z)$. The feedback control will generate the correction signal $V_{out}(z) = -M\tilde{X}(z)$ based on the gain matrix M. The controller is defined by the transfer function matrix $K(z)$, such that $V_{out}(z) = K(z)Y(z)$ includes the dynamic model of the bunch and has to gain matrices L and M to adjust the system specifications and robustness.

In this approach is critical the knowledge of the bunch dynamical model. This can be obtained via analytical methods, where the model can include parameter variations based on the different operation conditions of the machine or estimated via identification techniques. The last option is under research and uses the correlation between an injected signal to perturb the bunch and the vertical displacement as response to that excitation [9]. This technique not only is useful to estimate the bunch dynamical model to design the controller but also can be used as diagnostic tool to extract bunch and machine parameters during operation.

Let us assume that the relationship between input-output, $G(z)$, can be represented by a realization in state space $\tilde{G} = \{A, B, C, D\}$. The model of the observer can be expressed as;

$$\tilde{x}(k+1) = A\tilde{x}(k) + Bv_{out}(k) + v(k)$$
$$\tilde{y}(k) = C\tilde{x}(k) + Dv_{out}(k)$$

where $\tilde{y}(k)$ and $\tilde{x}(k)$ are the estimated outputs and internal states of the system, respectively and $v_{out}(k)$ and $v(k)$ are input signals. The matrix D is equal to zero for strictly proper transfer function representations (general physical systems). Then, if $v(k)$ is proportional to the error between the measured and the estimated outputs

$$v(k) = L(y(k) - \tilde{y}(k)) = L(y(k) - C\tilde{x}(k)) \text{ and}$$
$$\tilde{x}(k+1) = (A - LC)\tilde{x}(k) + Bv_{out}(k) + Ly(k) \quad (1)$$

The control signal $u(k)$ is equal to $v_{out}(k) = -M\tilde{x}(k)$, replacing into (1)

$$\tilde{x}(k+1) = (A - LC - BM)\tilde{x}(k) + Ly(k)$$
$$v_{out}(k) = -M\tilde{x}(k)$$

This is the state space representation of the controller with input $y(k)$, output $v_{out}(k)$ and dynamics defined the eigenvalues of the matrix $A - LC - BM$. The control transfer function can expressed in $\mathcal{Z}$-domain by the matrix $K(z)$

$$V_{out}(z) = K(z)Y(z) = -M(z\mathbf{I} - (A - LC - BM))^{-1}LY(z)$$

The order of the controller is defined by the order of the model representing the system (size of the matrix A). The characteristics of the controller and the closed loop system are defined by the gain matrices M and L. There exists several techniques to calculate the gain matrices based upon the system specifications, external perturbations and system uncertainties. In general the number of unknown in the gain matrices is larger than the number of specifications and restrictions imposed to the design. Formulations to calculate the matrix parameters are posed as optimization problems where the system specifications and restrictions are included in the cost function.

In our case, we are designing a stabilizing feedback system or damper. Assuming we do not want to affect the frequency of each bunch mode, the final location of the dominant bunch eigenvalues can be assigned. If the dynamics of the observer is designed such that it is faster than the bunch dynamics, the gain matrices can be evaluated separately where the matrix M adjusts the final position of the dominant bunch eigenvalues and the gain matrix L defines the dynamics of the observer. In the pre-design presented as example in this paper, the dominant bunch modes are damped to the similar rates and two cases are evaluated for the dynamics of the observer. Those cases are compared taking into account the transient response and dynamic range of signals, the robustness of the final system to parameter variations, e.g. variation of the betatron and synchrotron tunes, different unstable modes, etc.

DESIGN OF THE CONTROLLER

Let us assume the bunch dynamical model captures the six dominant modes whose eigenvalues are $\lambda_k = \pm i\,(\omega_\beta + k\omega_s)$ for $k = ..., -6, ..., 0, ..., +6, ...$. The controller is designed such that the final magnitude for those dominant eigenvalues $\lambda_k = -\sigma \pm i(\omega_\beta + k\omega_s)$ are: $\lambda_0 = -0.027 \pm i\,2\pi\,0.185$ and $\lambda_k = -0.019 \pm i\,2\pi\,(0.185 + k\,0.017)$ for $k = -6, ..., 0, ..., +6$. Two controllers are presented and their difference depends of the magnitude of the gain matrix L. In one case, labeled: *Design 1*, the eigenvalues of the controller are complex conjugated while in the *Design 2* the eigenvalues are real or complex conjugated with minimum imaginary components.

Results

Some results of those designs are depicted in Figs. 3 to 8. In Fig. 3 the response of the vertical motion of the multiple slices of the bunch is depicted when the vertical initial offset of the bunch is 1mm and the controller corresponds to *Design 1*. Similarly, Figs. 4 and 5 show the estimated outputs by the observer and the control signals for that case. Figs. 6, 7 and 8 depict the same transient for the controller labeled *Design 2*. It is possible to observe that the initial transient response is more aggressive in the *Design 2* controller requiring more dynamic range in the amplifier driving the kicker. Additionally, because the difference between both controllers is mainly in the dynamics of the observer (definition of gain matrix L) and the dominant dynamics in closed loop of the system is almost the same (definition of gain matrix M, setting of dominant eigenvalues λ_k), the transient response only differs in the first revolutions. During this period, the observer response transitions from the initial state toward the estimated output signal.

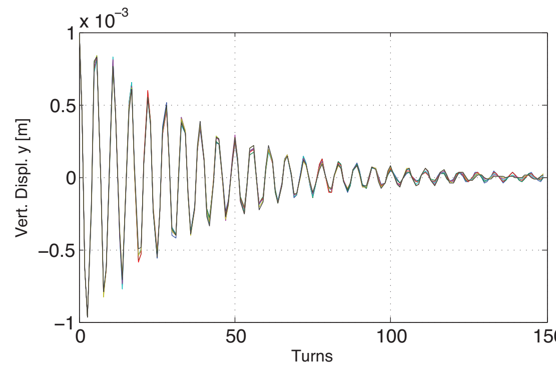

Figure 3: vertical displacement - *Design 1* controller

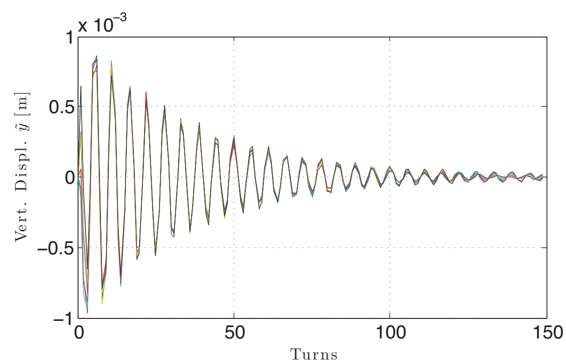

Figure 4: observer output - *Design 1* controller

It is important to evaluate the effect on the final stability and performance of the system if the beam parameters are changed while the controller is kept with its design based on the nominal parameters of the bunch. To analyze that impact, the betatron and synchrotron frequencies are changes as well as the bunch is assumed unstable and quantified by the growth rate per mode. Both designs have similar robustness characteristics and a summary of the results follows. If the betatron frequency is changed keeping the synchrotron frequency constant and assuming that the damping is null for all the modes, the system reaches the stability

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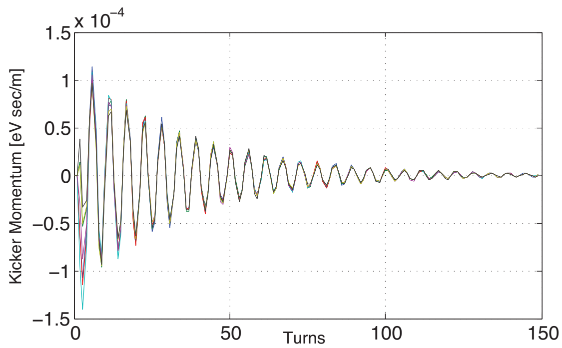

Figure 5: control signal - *Design 1* controller

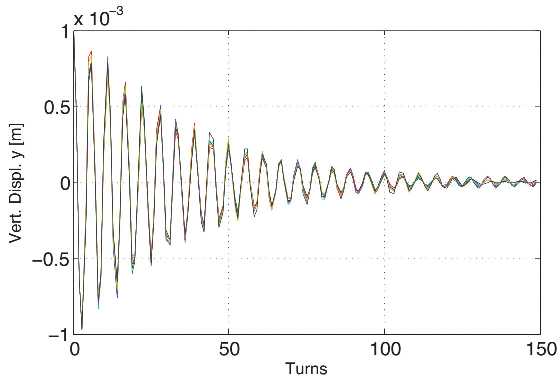

Figure 6: vertical displacement - *Design 2* controller

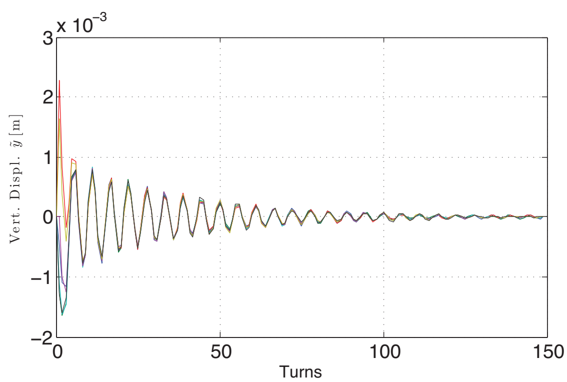

Figure 7: observer output - *Design 2* controller

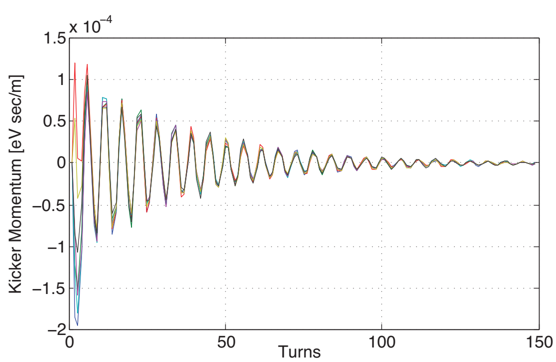

Figure 8: control signal - *Design 2* controller

limits operating in closed loop if either $\omega_\beta < 0.85\,\omega_{\beta 0}$ or $\omega_\beta > 1.2\,\omega_{\beta 0}$, where $\omega_{\beta 0} = 2\pi\, 0.185$ is the nominal value. This limit is mainly defined by the instability of the high order modes. Similarly, it is possible to evaluate the effect of changes in the synchrotron frequency. In this case, the system reaches the stability limits operating in closed loop if either $\omega_s < 0.7\,\omega_{s0}$ or $\omega_s > 1.3\,\omega_{s0}$, where $\omega_{s0} = 2\pi\, 0.017$ is the nominal value.

An important parameter to analyze with this controller is the maximum growth rate or instability that is possible to damp assuming that the kicker amplifier or any other limitation in the feedback channel does not reach its maximum dynamic range. To study this point, two cases of unstable beam were considered. In one case the growth rate for all the modes was assumed the same, while in the other case, only one mode was assumed unstable and the others remained with zero damping. In the case that all the modes are unstable, the designed controller is able to stabilize bunches with growth rates $\sigma \leq 0.03 - 0.035$ 1/turns. For individual modes unstable, the maximum growth rate that the controller can damp per mode are summarized in table **??**.

Table 1: Maximum Growth Rate possible to stabilize

Mode	Growth Rate
0	$\sigma = 0.05$ 1/turns
± 1	$\sigma = 0.05$ 1/turns
± 2	$\sigma = 0.05$ 1/turns
± 3	$\sigma = 0.04$ 1/turns
± 4	$\sigma = 0.04$ 1/turns
± 5	$\sigma = 0.04$ 1/turns
± 6	$\sigma = 0.04$ 1/turns

Remarks about this pre-design

The model-based design technique defines controllers with an order equal to the model used. In general, it is a high order controller. As a MIMO controller, this topology links all the measured variables to calculate each sample of the correction signal $v_{out}(k)$. This issues can limit the implementation and processing in the reconfigurable FPGA due to time involved in the multiple arithmetic operations. It will be important to consider simplified or reduced controllers based on this model-based technique and evaluate the impact on the stability and performance robustness of the system when it is compared with the respect the full-order controller evaluated in this design.

Using this design methodology, where the controller incorporates as much as possible information about system to stabilize, it makes relatively straightforward to incorporate the specification in the design process. As disadvantage, some designs could be sensitive to parameters variations if the model used does not take into account such parameter variations, uncertainties and un-modeled dynamics.

As a final remark, the design requires of the reduced bunch dynamical model. Analytical models of the bunch dynamics can be used incorporating realistic parameters for their

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description. Another option is to evaluate the model in real-time based on measurements in the machine. Identification techniques allows to quantify a reduced model base on the response of the vertical motion of the bunch to signals designed to perform a successful identification. Part of the research in this project is focus on evaluating this techniques to extract the bunch reduced model [9].

CONCLUSIONS

A pre-design of a controller to mitigate the intra-bunch instability has been studied showing good results. This model-based design controller includes in the observer a model of the bunch with multiple modes ($k = -6, ..., 0, ..., +6$). In the design, the location of the dominant eigenvalues of the closed loop system was set to provide satisfactory damping to those dominant bunch modes. This controller topology renders high order systems requiring large processing power.

Future work includes to test the controller performance using more realistic bunch simulators as HeadTail or CMAD. Additionally, the effort will be focused on reducing the order of the controller, balancing the performance, processing power and complexity requirements. These controllers will be compared with the IIR bank filters to define the best option to implement in the FPGA.

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