SIMULATIONS OF THE HEAD-TAIL INSTABILITY ON THE ISIS SYNCHROTRON

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Abstract

ISIS is the pulsed spallation neutron and muon source at the Rutherford Appleton Laboratory in the UK. Operation is centred on a loss limited 50 Hz proton synchrotron which accelerates 3×10^{13} protons per pulse (ppp) from 70 MeV to 800 MeV, delivering a mean beam power of 0.2 MW.

Present studies are focussed on key aspects of high intensity beam dynamics with a view to increasing operational intensity, understanding loss mechanisms and identifying possible upgrade routes. Of particular interest is the head-tail instability observed on ISIS, which is currently a main limitation on beam intensity.

This paper presents initial simulations using HEADTAIL to compare with experimental data taken on the ISIS synchrotron. The details and assumptions of the impedance model and simulations are discussed. Plans for future head-tail measurements, simulations and analysis are outlined.

INTRODUCTION

The transverse head-tail instability represents a possible intensity limit for bunched beams in many synchrotrons including ISIS and its proposed upgrades. The standard theory of Sacherer [1] does not include space charge and associated incoherent tune spreads. However, recent works [2, 3, 4] have proposed theoretical models to treat head-tail motion in the presence of space charge. In parallel with this, numerical simulations are required to analyse beam behaviour with various collective effects included, as noted in [5].

ISIS Synchrotron

ISIS operations centre on a rapid cycling synchrotron (RCS) which accelerates 3×10^{13} protons per pulse (ppp) from 70 MeV to 800 MeV on the 10 ms rising edge of a sinusoidal main magnet field. Injection is via charge exchange of a 70 MeV, 25 mA H^- beam over ~130 turns of the falling main magnet field just prior to field minimum. The unchopped, injected beam is non-adiabatically bunched by the ring dual harmonic RF system (DHRF, h = 2 and 4). Nominal betatron tunes are (Q_x, Q_y) = (4.31, 3.83), with peak incoherent tune shifts exceeding ~0.5. The intensity is loss limited with longitudinal trapping, transverse space charge and the head-tail instability being the main driving mechanisms.

Observations on ISIS have shown that the two proton bunches develop coherent vertical growth approximately 2 ms into the acceleration cycle [6]. The growth is suppressed by ramping the vertical tune away from the integer (Q_y = 4) during that time. However the growth rate scales strongly with intensity and lowering the tune further tends to increase loss associated with the half integer resonance [7]. Work is ongoing to develop a beam feedback system [8] to damp the instability. Recent studies have shown that the instability is present both with just the fundamental (h = 2) RF system [6] as well as with the DHRF system.

This study presents initial simulations of low intensity head-tail dynamics using HEADTAIL [9] to compare with experimental data using single harmonic RF on the ISIS synchrotron. Calculations of chromatic phase shifts and growth rates from Sacherer theory [1] are compared to HEADTAIL results. Plans for future experimental studies alongside simulation and theory work are outlined.

SACHERER THEORY

A non-zero value for the chromaticity, ξ, results in a momentum dependent betatron tune and an accumulated phase shift along the bunch $\chi = \xi Q\omega_0\tau/\eta$, where Q is the tune, ω_0 is the angular revolution frequency, τ is the bunch length in time and $\eta = 1/\gamma_t^2 - 1/\gamma^2$. This chromatic phase shift determines the head-tail mode structure observed through the form factor shown below.

An impedance acting on a beam can introduce a real frequency shift, through its reactive component, as well as instability from its resistive part. The instability growth rate may be calculated for a coasting beam from the equation of motion for a single particle acted on by an impedance. For a bunched beam, the coasting beam growth rate is modified by a sum over the bunch mode spectra. The instability frequency shift [1] is given by

$$\Delta\omega_m = \frac{1}{1+m}\frac{i}{2Q\omega_0}\frac{e\beta}{\gamma m_0}\frac{I_b}{L_b}\frac{\sum Z_\perp(\omega)h_m(\omega-\omega_\xi)}{\sum h_m(\omega-\omega_\xi)}, \quad (1)$$

where m is the oscillation mode number, m_0 is the rest mass, I_b is the bunch current, L_b is the bunch length in metres, $Z_\perp(\omega)$ is the transverse impedance as a function of frequency, $h_m(\omega-\omega_\xi)$ is the envelope of the bunch line spectrum

$$h_m(\omega) = (m+1)^2\frac{\tau^2}{2\pi^4}\frac{1\pm\cos\omega\tau}{[(\omega\tau/\pi)^2-(m+1)^2]^2} \quad (2)$$

where τ is the bunch length in seconds. The associated growth rate from equation 1 is $\tau_m^{-1} = -\mathrm{Im}(\Delta\omega_m)$.

The resistive wall impedance is thought to be the main driving impedance of the head-tail instability on ISIS. The impedance becomes large for small ω predicting large growth rates for the lowest betatron sideband when Q is just below an integer. It can therefore be approximated by

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a narrowband impedance where only one frequency contributes to the summation in the numerator of equation 1. The summation can then be replaced by the impedance at the lowest sideband frequency (Z_{RW}) multiplied by a form factor F_m,

$$\Delta\omega_m = \frac{1}{1+m}\frac{i}{2Q\omega_0}\frac{e\beta}{\gamma m_0}\frac{I_b}{L_b} Z_{RW} F_m(\omega)\,, \tag{3}$$

where F_m is shown in igure 1 for the first few modes and is given by

$$F_m(\omega) = \frac{h_m(\omega)}{\sum h_m(\omega)}\ . \tag{4}$$

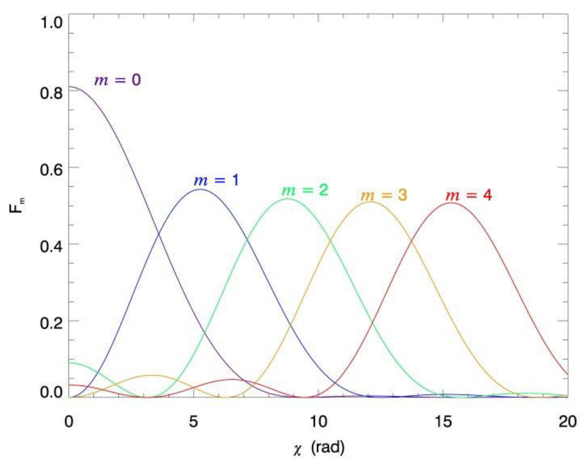

Figure 1: Form factor F_m (defined in equation 4) as a function of the chromatic phase shift χ for modes 0 to 4.

EXPERIMENTAL OBSERVATIONS

ISIS currently operates at the natural value of the machine chromaticity ($\xi_x = \xi_y$ = -1.4, [10]). The impedance acting on the beam leads to a coherent vertical instability early in the 10 ms acceleration cycle. Measurements were taken at a vertical position monitor over 0 - 5 ms during acceleration. A low intensity beam (5×10^{12} ppp) was injected as a simple case, minimising complications due to space charge, to compare with the theoretical model described above. As there is currently no theory to describe head-tail motion for the case of dual harmonic RF, measurements were made in the more straightforward single harmonic (h = 2) case. The vertical tune was increased at 2 and 2.5 ms to 3.87 and 3.75 respectively (from 3.84 and 3.73 for normal single harmonic RF operation) to induce stronger head-tail behaviour.

Figure 2 shows a typical position monitor sum and difference signal over several turns showing clear head-tail motion with mode m = 1, one displacement node along the bunch. Using the tune and natural chromaticity we obtain χ = 9.11 rad. However, putting this into the form factor, equation 3, this corresponds to a larger growth rate for m = 2 mode with a smaller contribution from m = 1 mode, see Figure 1. A modified theory that could explain this observation may be found in [6].

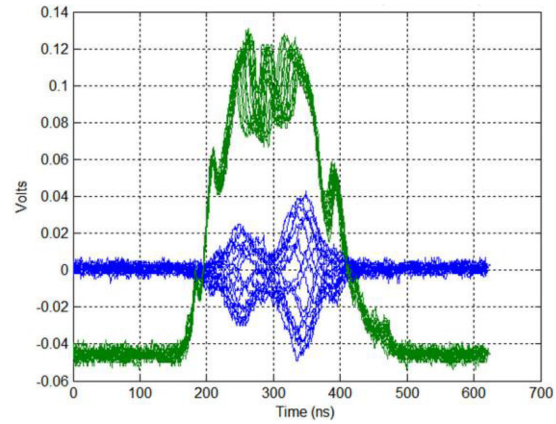

Figure 2: Sum (green) and difference (blue) vertical position monitor signals over several turns at ~1 ms through the ISIS acceleration cycle.

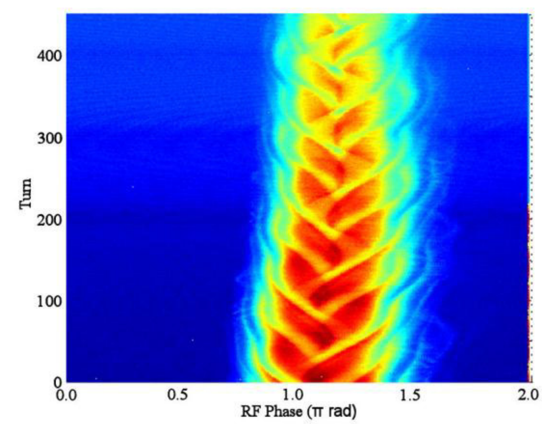

Figure 3: Vertical position monitor sum signal, longitudinal pulse shape as a function of turn from ~1 ms. Red denotes high beam density and blue, no beam.

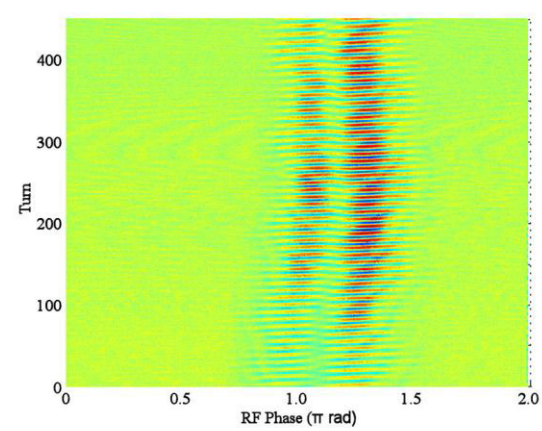

Figure 4: Vertical position monitor difference signal as a function of turn from ~1 ms. Red denotes large positive beam offset, blue negative offset and green zero offset.

Figures 3 and 4 show waterfall plots of the vertical position monitor sum and difference signals respectively for both bunches from ~1–1.5 ms through the ISIS acceleration cycle. During this time the RF frequency

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sweeps between 1.43 – 1.53 MHz. The frequency sweep has been removed from Figures 3 and 4.

Figure 3 clearly shows that there is sustained non-uniform longitudinal structure. This indicates non-optimal longitudinal capture and bunching and could influence the head-tail mechanism. However, the instability growth rate is much faster than the longitudinal motion and, as shown in Figure 4, the $m = 1$ mode is persistent over many hundreds of turns despite the non-uniform longitudinal behaviour and ramping RF frequency. Therefore it is reasonable to assume that these do not play a significant role in the head-tail motion. Figure 4 also shows that there is strong coupling between the two bunches.

SIMULATIONS

The macro-particle simulation code HEADTAIL [9] was used to perform comparative simulations of the low-intensity head-tail behaviour seen on ISIS with single harmonic RF. The code utilises a simple smooth focussing model for transverse motion and applies wakefield kicks to simulate the interaction of the beam with its environment.

It is noted that the HEADTAIL code, and most theoretical models, assume $\beta \sim 1$. However, head-tail behaviour on ISIS is observed at much lower β values ($\beta \sim 0.4$). The importance of this is currently being investigated.

For initial simulations, parameters at 1 ms were assumed with intensities and tunes as per the experiment. Longitudinal motion was simulated with single harmonic RF and acceleration. The thick resistive wall wakefield was assumed with the wake function [5],

$$W_{RW}(z) = -\frac{cL_{RW}}{b^3}\left(\frac{\beta}{\pi}\right)^{3/2}\sqrt{\frac{Z_0}{z\sigma_{RW}}}, \qquad (5)$$

where L_{RW} is the length of the resistive wall, b is the beampipe radius, Z_0 is the impedance of free space and σ_{RW} is the pipe conductivity.

Measurements at ISIS have shown the impedance to be ~200 kΩ/m at 110 kHz [11]. The thick-wall approximation underestimates this impedance as, at low frequencies, the skin depth becomes comparable to or larger than the pipe thickness and the thick-wall approximation is no longer valid. However, a wakefield interpretation of thin resistive wall has not yet been implemented in HEADTAIL. Instead, for these initial, exploratory simulations, the conductivity has been modified to artificially match the impedance at the dominant frequency harmonic.

Figures 5 and 6 show the simulated vertical difference signal over several turns and maximum vertical amplitude over 800 turns. The pattern in Figure 5 shows a $m = 2$ mode oscillation which is consistent with theory given the chromatic phase shift along the bunch. However, it is noted that the growth rate in Figure 6 is much faster than predicted using equation 1 (e-folding time constant of 160 µs as compared to 2.24 ms respectively). In comparison, the experimental growth rate is much closer to the simulated value of order 100 µs.

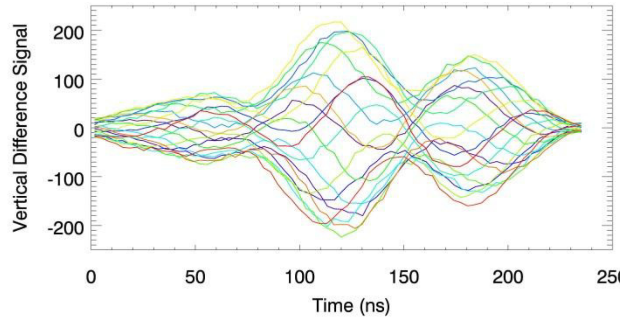

Figure 5: Simulated vertical difference signal over the last 20 turns of the simulation with nominal tune at 1 ms.

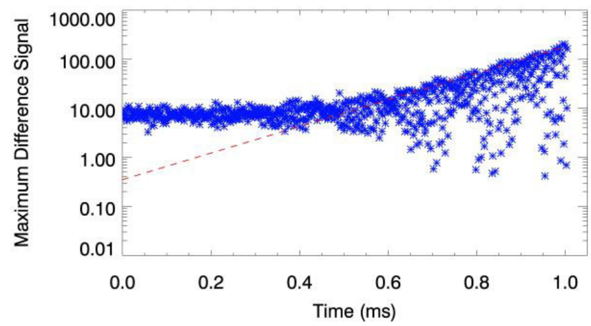

Figure 6: Simulated maximum vertical offset over 800 turns (blue) with exponential fit (red).

Chromaticity Scan

In order to compare better the simulation results to theory, the chromaticity in HEADTAIL was scanned from zero to the operating, natural chromaticity whilst holding all other parameters constant. ISIS does not currently have the capability to control chromaticity so no experimental comparisons were possible. Results are summarised in Figures 7 – 12.

Figures 7 and 8 show head-tail behaviour with zero chromaticity. This leads to strong $m = 0$ mode vertical motion as is predicted from Figure 1 with $\chi = 0$. The growth rate fit in Figure 7 produces an e-folding time of 26 µs. Using the narrowband assumption, theory predicts a much slower e-folding time of 473 µs.

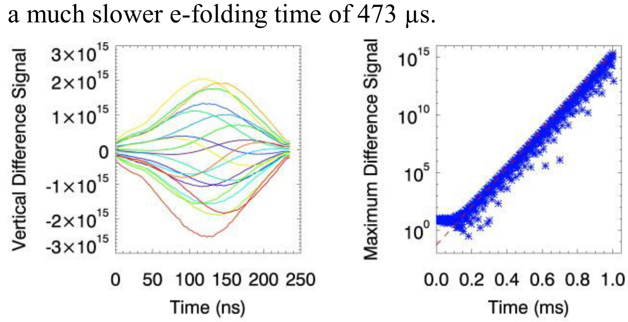

Figure 7: Left - Simulated vertical difference signal over the last 20 turns. Right - Maximum vertical offset over 800 turns (blue) with exponential fit (red) for $\xi = 0$.

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The vertical offset from bunch centre for the zero chromaticity case is shown as a function of longitudinal phase space in Figure 8 for 2000 sample particles. A banded structure is clearly visible across the phase space. Further analysis may allow the ideas of Rees [6] to be explored in more detail.

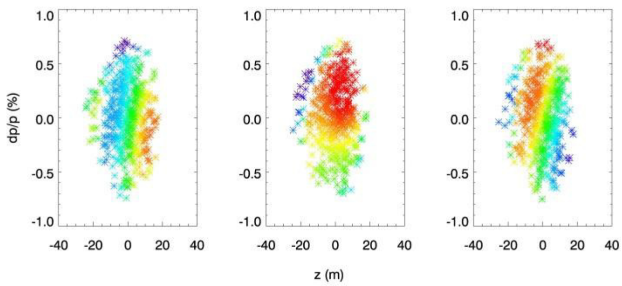

Figure 8: Longitudinal phase space on turn 780 (left), 790 (centre) and 800 (right) for $\xi = 0$. Colour denotes vertical offset for 2000 sample particles from negative (blue) to positive (red) with respect to centre.

Figures 9 and 10 show results of HEADTAIL simulations for an ISIS chromaticity of -0.82. According to theory this results in a maxima for $m = 1$ head-tail motion. This is borne out in Figure 9 with one node, although $m = 0$ and 2 are also present with similar yet smaller growth rates to the dominant $m = 1$ mode. As previously, the simulated growth rate is much faster than in theory (an e-folding time of 62 µs as compared to 1.41 ms).

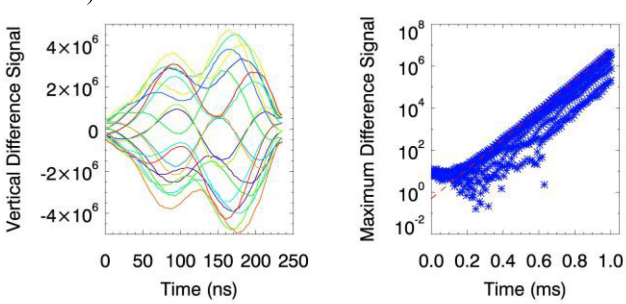

Figure 9: Left - Simulated vertical difference signal over the last 20 turns. Right - Maximum vertical offset over 800 turns (blue) with exponential fit (red) for $\xi = -0.82$ ($\chi = 5.31$ rad).

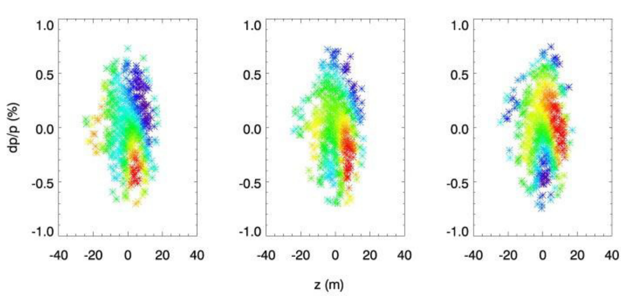

Figure 10: Longitudinal phase space on turn 780 (left), 790 (centre) and 800 (right) for $\xi = -0.82$. Colour denotes vertical offset for 2000 sample particles from negative (blue) to positive (red) with respect to centre.

Figure 10 shows the vertical offset as a function of longitudinal phase space. The displacement structure across the phase space suggests more complicated motion of the higher mode.

Figures 11 and 12 show results from HEADTAIL simulations for ISIS with a chromaticity of -1.2. Sacherer theory suggests a strong m = 2 mode which is shown clearly in Figure 11. From Figure 11 the fitted e-folding time is 140 µs, much faster than that calculated from theory, 2.37 ms.

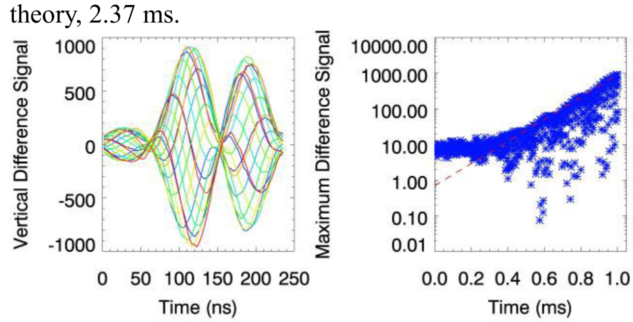

Figure 11: Left - Simulated vertical difference signal over the last 20 turns. Right - Maximum vertical offset over 800 turns (blue) with exponential fit (red) for $\xi = -1.2$ ($\chi = 7.83$ rad).

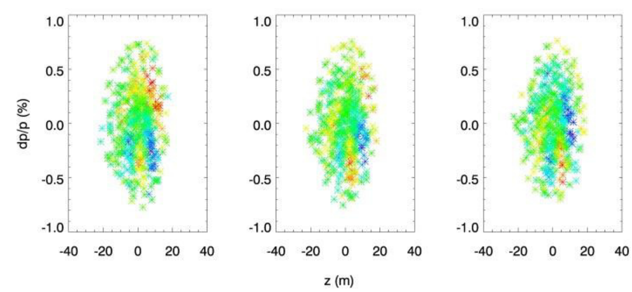

Figure 12: Longitudinal phase space on turn 780 (left), 790 (centre) and 800 (right) for $\xi = -1.2$. Colour denotes vertical offset for 2000 sample particles from negative (blue) to positive (red) with respect to centre.

Figure 12 shows the vertical offset as a function of position in longitudinal phase space for $\xi = -0.82$. There is clear non-uniform structure present with different displacement patterns from lower values of chromaticity.

The difference in the predicted growth rate from theory and that calculated from simulation may be due to the large changes in the thick resistive wall impedance at low frequencies. A small shift in frequency can cause a much larger change in impedance. This will be the subject of further study.

SUMMARY

Head-tail instability has been identified on the ISIS synchrotron and is a key intensity limit for operations. A sample of a recent experimental study into the simplified case of low intensity, single harmonic RF driven head-tail at ISIS has been presented together with current theory based on work by Sacherer [1]. As previously noted [6], the head-tail mode observed on ISIS does not match predictions from theory.

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Complementary to these studies work has begun on developing a beam feedback system to damp instabilities [8]. The proposal sees the installation of two identical strip-line monitors, one to be used as a pick-up and the other as a kicker.

Initial HEADTAIL [9] simulations have been compared to the experimental results and theoretical calculations. The mode structure observed in simulation results has been shown to conform to the theory of Sacherer as chromaticity is varied. However, the growth rates calculated from simulated data are much faster than predicted from theory but are of the same order of magnitude as that observed experimentally. It is thought this may be due to the limitations of the impedance model used and approximations to a narrowband impedance as other, higher harmonics may be important.

FUTURE WORK

A key first step is to make measurements of the resistive wall impedance on ISIS. These are planned using coasting beams in storage ring mode, with the main magnet field set at a DC level appropriate for the 70 MeV injected beam and RF systems switched off. As outlined above, the precise impedance model can cause large changes in beam dynamics and instability. These valuable additions to our existing models will inform further experiments, simulations and influence the design of the planned damping system.

Simulation work is planned to understand better both the HEADTAIL implementation and its relation to the theory of Sacherer. An existing in-house longitudinal code [12] may be modified to implement smooth focussing transverse motion and include a model of the resistive wall impedance. Results will then be compared to theory, experimental results and those of HEADTAIL. It is also planned to analyse the experimental and simulation data with reference to the theory of Rees [6].

Once the case of low intensity, single harmonic RF driven head-tail is understood, studies will move toward high intensity single harmonic behaviour. This will require the addition of a model of transverse space charge in simulations. Finally, it is planned to investigate dual harmonic RF head-tail where there is currently no theory to describe the observations.

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