IDENTIFICATION OF INTRA-BUNCH TRANSVERSE DYNAMICS FOR FEEDBACK CONTROL PURPOSES AT CERN SUPER PROTON SYNCHROTRON*

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Abstract

A promising new approach for designing controllers to stabilize intra-bunch transverse instabilities is to use multi-input multi-output (MIMO) feedback design techniques. However, these techniques require a reduced model and estimation of model parameters based on measurements. We present a method to identify a linear reduced order MIMO model for the vertical intra-bunch dynamics. The effort is motivated by the plans to increase currents in the Super Proton Synchrotron as part of the HL-LHC upgrade where feedback control techniques could be applied to stabilize the bunch dynamics, allowing greater freedom in the machine lattice parameters. Identification algorithms use subspace methods to compute a discrete linear MIMO representation of the nonlinear bunch dynamics. Data from macro particle simulation codes (CMAD and HEADTAIL) and SPS machine measurements are used to identify the reduced model for the bunch dynamics. These models capture the essential dynamics of the bunch motion or instability at a particular operating point, and can then be used analytically to design model-based feedback controllers. The robustness of the model parameters against noise and external excitation signals is studied, as is the effect of the MIMO model order on the accuracy of the identification algorithms.

INTRODUCTION

Electron clouds and machine impedance can cause intra-bunch instabilities at the CERN Super Proton Synchrotron (SPS). The high current operation of the SPS for LHC injection requires mitigation of these problems. Modern control techniques can be used to stabilize the bunch. These techniques are powerful tools allowing us to evaluate and understand the performance and the limits of the system beforehand. Yet, they require reduced order models of intra-bunch dynamics to design optimal or robust controllers for wide-band feedback systems. System identification techniques can be used to get these required reduced order models.

Nanosecond-scale bunch stabilization is more challenging compared to the case of rigid body dipole coupled bunch oscillations. It requires sufficient bandwidth to sense transverse motion at multiple locations along the bunch and apply correction signals to the corresponding parts of the bunch. Apart from these technological constraints, modeling the intra-bunch dynamics is also more challenging compared to the case of modeling the beam dynamics including bunch to bunch interactions.

The feedback system senses the vertical positions at multiple locations within the nanosecond-scale bunch. Control filters use these measurements to calculate correction signals and apply them back onto the bunch using the kicker as actuator. A 4 Gs/Sec. digital feedback system has been developed to process the motion signals and generate the correction actions [1]. Due to very the fast intrinsic time characteristics of the system, a parallel computation control filter architecture has been developed. A very similar method had been used for bunch by bunch feedback control systems [2].

In this paper, we show the use of system identification techniques to estimate parameters of linear models representing single bunch dynamics. We define the form of the reduced order model. We pose the identification problem in a least squares form [3] for given input-output data set. After a brief discussion of identification constraints, we show results of identification applied to data from SPS measurements and nonlinear macro particle simulation codes.

MODEL AND IDENTIFICATION

Reduced Order Model and Identification

Any linear dynamical system can be represented in state space matrix form. A discrete time system sampled at every revolution period k with p inputs and q outputs is represented by

$$\begin{aligned} X_{k+1} &= AX_k + BU_k \\ Y_k &= CX_k \end{aligned} \tag{1}$$

where $U \in R^p$ is the control variable (external excitation), $Y \in R^q$ is the vertical displacement measurement, $A \in R^{n\times n}$ is the system matrix, $B \in R^{n\times p}$ is the input matrix, and $C \in R^{q\times n}$ is the output matrix. For a MIMO system, the model order determines the complexity. In this study, we assumed time invariant dynamics which means having constant A, B and C matrices in the state space model. When it comes to the interactions between the bunch with electron clouds or during energy ramping operations, time variant dynamics has to be accounted for tune shifts, changing beam parameters, etc.

System identification techniques require exciting the system with appropriate signals and observing the response.

* Work supported by the U.S. Department of Energy under contract DE-AC02-76SF00515 and the US LHC Accelerator Research program (LARP).
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ISBN 978-3-95450-173-1

A dynamical model is estimated based on the measured input-output data. In particular for the bunch dynamics identification, the bunch is driven by a wide-band kicker using a persistent excitation and measuring the vertical displacements along the 3.2ns bunch length. The signals used in the identification process are input $U(k)$, where the vector represents the multiple momentum kicks applied to the different locations along the bunch and the output $Y(k)$ are the vertical displacements measured at those locations.

$$Y(z) = \left[D^{-1}(z)N(z)\right]U(z) \tag{2}$$

where [] represents the transfer function matrix ($\in R^{q\times p}$) for a system with p inputs and q outputs. $D(z)$ and $N(z)$ represent denominator and numerator of each discrete time transfer function between input-output couples.

$$N(z)U(z) - D(z)Y(z) = 0 \tag{3}$$

$$U(z) = \sum_{i=0}^{T} U_i z^i, \quad Y(z) = \sum_{i=0}^{T} Y_i z^i \tag{4}$$

$$D(z) = \sum_{i=0}^{m} D_i z^i, \quad N(z) = \sum_{i=0}^{n} N_i z^i \tag{5}$$

$$\left[N_r \mid -D_r\right]\begin{bmatrix}U(k)\\ Y(k)\end{bmatrix} = 0 \tag{6}$$

Given the input and output signals, the estimation of the parameter matrices N_r and D_r is obtained by solving the last linear equation. There are many different subspace based methods to solve this linear MIMO problem. These methods use projections or singular value decomposition (SVD) to cast the problem into linear least squares [4]. We follow [3] where the construction of the data matrix from input and output signals, the solution algorithm for Equation 6 and the relationship between the transfer function coefficients and the observable canonical state space form are shown. Assuming full observability of the system, we can represent our state space in discrete time observable canonical form. This will enable us to estimate the minimum number of parameters [3]. However, even for linear systems there are two well known limitations for identification. These are the effect of noise and lack of persistent excitation.

Persistent Input and Noise Sensitivity

Input signal design and persistent excitation are critical aspects of system identification. Given a quasi-stationary input of u with a dimension nu and with a spectrum $\phi_u(\omega)$, $\phi_u(\omega) > 0$ should hold for at least n distinct frequencies for u to be a persistent excitation [5]. Random noise would be ideal to excite all the modes in the system but requires high excitation power and bandwidth. The hardware used in these measurements puts constraints on both power and bandwidth. The design of an input signal for identification under given constraints becomes an important question for the future studies.

Noise affects the performance of the identification, and in certain cases can make identification impossible. We can quantify its effect by adding noise at different power levels into a known system until the identification can no longer clearly estimate the known dynamics. We drive a synthetic 2×2 coupled MIMO system using a band limited frequency chirp signal. Random noise at different power levels was added to the output signals. The effect of noise is tested by running the identification algorithm on input - output data as we increase the noise level. Parameters of the model and the corresponding modes of the identified model are estimated for different noise level cases. We quantify the effect of the signal to noise ratio (SNR) by comparing the estimated parameters of the system with the original 2×2 MIMO system. Figure 1 shows the impact of noise on the estimation of system parameters. For identification algorithm to perform well, we need a SNR $>\sim 8$.

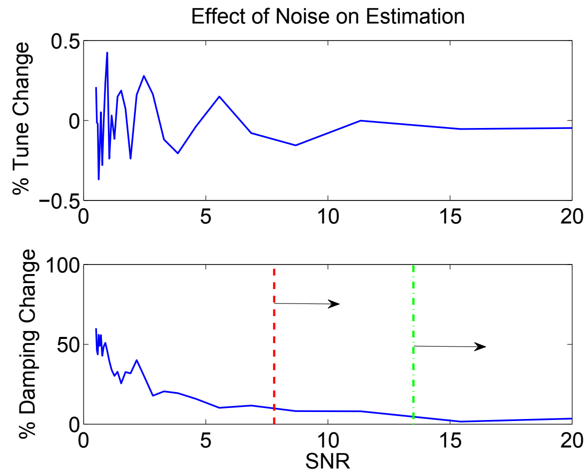

Figure 1: Deviation of estimated natural tune and damping of the 1^{st} mode from the true value for different SNR values. Red line shows min SNR to get errors less than 10%, green line is for errors less than 5%.

APPLICATIONS

Multiples MDs have been conducted at the CERN SPS ring to drive the bunch with different excitations (open loop) and also test feedback controllers to stabilize the bunch (closed loop). Those measurements were conducted using a single bunch in the machine with intensities of about $1-1.5\times10^{11}$ protons at the injection energy, 26 GeV and Q26 lattice configuration [6, 7]. The driven tests with different excitation signals have been designed such that the kicking signal is a persistent input for the system, and the collected data can be use to study the identification algorithms and quantify a reduced parametric model of the beam dynamics. Similarly, data obtained from macro-particle simulation codes (CMAD-HEADTAIL) has been used to test the identification algorithms and compare the dynamic model results with those obtained from machine measurements.

SPS Measurements

The hardware installed in the CERN SPS allows us to drive the bunch with limited bandwidth. The kicker used for

ISBN 978-3-95450-173-1

these tests has a maximum bandwidth of 160 MHz, limiting the spectrum of the momentum applied to the bunch. This limit conditions the number of modes that can be identified in the dynamic model because the final momentum is not a persistent input to detect high order transverse modes in the bunch. New kicker designs, fabrication and installation in the CERN SPS ring are in progress to be able to drive the bunch at higher frequencies [8].

The existing limited bandwidth kicker forces us to set our reduced model to detect low order modes corresponding to frequencies up to the second sideband ($2f_s$) around the betatron frequency (f_β). We use both mode 0 (barycentric shape along longitudinal axis) and mode 1 (a 200 MHz single cycle sine wave shape) excitation signals for which the amplitude is modulated with a frequency chirp [1]. The chirp covers $f_\beta \pm 2f_s$ in $\sim$ 15000 turns. Time alignment between excitation signals and bunch gives us the flexibility to excite a specific mode more dominantly. To improve the SNR used in the identification, the vertical motion signal is processed using a time-varying band-pass filter whose central frequency follows the frequency variation of the excitation signal applied.

Tailoring the reduced model to the low-order modes, it is possible to use 4 coupled 2^{nd} order differential equations to capture the low-order dynamics of the bunch (mode 0 - barycentric motion and mode ±1 - head-tail motion). The input-output relationship (momentum kick to vertical displacement) of the bunch is defined by a 4 $\times$4 MIMO system with $p = 4$, $q = 4$ and $n = 8$. This MIMO model sets the input and output vectors dimension (equation 1) to 4 for each sampling instance k. The measurement set-up acquires either 16 or 32 samples across the bunch at each sampling time k for the momentum kick and the vertical displacement signals. To do identification with the corresponding MIMO model, each sample in vectors U_k and Y_k ($\in R^{4\times1}$) is calculated averaging either 4 or 8 consecutive non overlapping samples of the 16 or 32 samples long original data (e.g $U(1,k), Y(1,k)$ is the average of samples 1-4, $U(2,k), Y(2,k)$, the average of samples 5-8 ...etc).

Using these input-output signals, the identification algorithm is evaluated and some results are analyzed. Figure 2 shows the time evolution of the 4 components of the vertical displacement vector Y_k for about 10000 turns. Measured data is represented by the blue trace and the response of the identified model is the red trace. It is important to notice that the reduced order model is linear time invariant and cannot capture external perturbations or parameter variations in the bunch. Still, the envelope of the amplitude of the intra-bunch vertical motion is captured in the time domain. The plots on the right show measurements and the response of the model in frequency domain for the same samples.

Figure 3 shows another data with strong excitation of both mode 0 and 1^{st} sideband (mode 1). We also see some motion around 2^{nd} and 3^{rd} sideband. On the left, we see the RMS spectrogram of the driven measurement with clear mode 0, mode 1, mode 2 and mode 3 excitation around turns $\sim$ 7000, $\sim$ 11500, $\sim$ 15500 and and $\sim$ 15500. On the right side, we show the RMS spectrogram of bunch's vertical motion predicted by reduced model. It is important to notice from the measured data (Fig. 3 - left) the effect of nonlinearities either in the driving system or in the bunch. The spectrogram analysis of the measured signal shows that the 2^{nd} ($f_\beta + 2f_s = 0.189$) and 3^{rd} sidebands are excited before the chirp excitation drives the bunch motion at that particular frequency. As expected, our linear model is able to capture dominant characteristics and linear dynamics such as motions at mode 0, mode 1 and mode 2 tunes, but not the effect attributed to the non-linearity in the system.

Nonlinear Simulations

Similar techniques were applied to data obtained by nonlinear macro particle simulation codes such as HEADTAIL and CMAD. These tools are especially very useful and helpful because they are accessible to study beam dynamics without beam time in the machine. As soon as simulations are benchmarked with machine measurements, the intra-bunch dynamics can also be identified using data from these simulations. As opposed to machine conditions and experiments, simulations have control over noise, disturbances, etc. This gives more flexibility and control to check the performance of the identification algorithm.

In the simulation, the bunch is represented by 64 slices. The bunch is excited by a chirp signal similar to the one used in the experimental tests at CERN SPS. The shape (along longitudinal axis) of the excitation signal is a 200 MHz sine wave (single cycle in a 5ns RF bucket) for a given turn and the amplitude of this head-tail shaped signal is modulated by a frequency sweep. The sweep covers from the 2^{nd} lower sideband to 2^{nd} upper sideband where betatron tune is 0.18 and synchrotron separation is 0.017 as given in Q-20 optics.

Figure 4 shows on the left side the vertical displacement data across the bunch for 1000 turns. Due to the shape of excitation signal across the bunch in this specific case, most of the excitation energy is coupled to ±1 lateral bands and we see very small amount of barycentric oscillations due to residual kick. With appropriate excitation signals we can study many multi-modal dynamics with the help of these simulations.

Figure 4 shows on the right side the response of the reduced order model to the same excitation in time domain. The relative error, which is calculated based on the maximum deviation of the model's response from the original simulation data, is less than $\sim$ 10%. The reduced order model can capture the dominant dynamics of the bunch successfully. Similarly, results can also be seen in frequency domain as show in Fig. 5 where on the left we see the spectrogram of the vertical displacement data from the simulation and on the right we have the spectrogram of the vertical displacement obtained by the reduced order model.

One of the important identification parameters that we can study is the estimation of the MIMO model order. Two different approaches could be used. In the first approach, as briefly explained in the "SPS Measurement" section we proposed the order of the system based on the persistent

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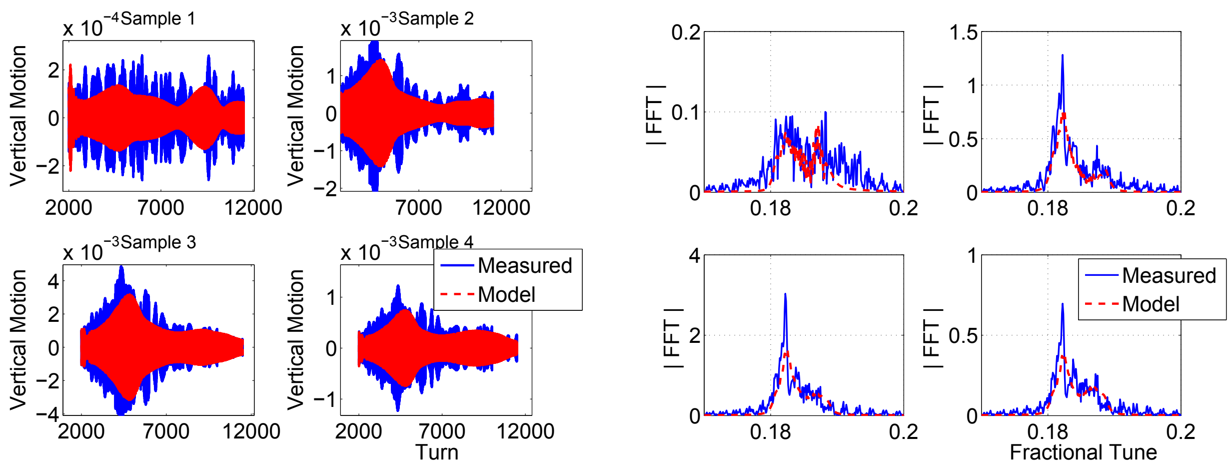

Figure 2: Comparison of the reduced order model responses with machine measurements in time (amplitude of vertical motion vs turns) and frequency domain. Dominant modes are mode 0 and mode 1 (1^{st} upper sideband).

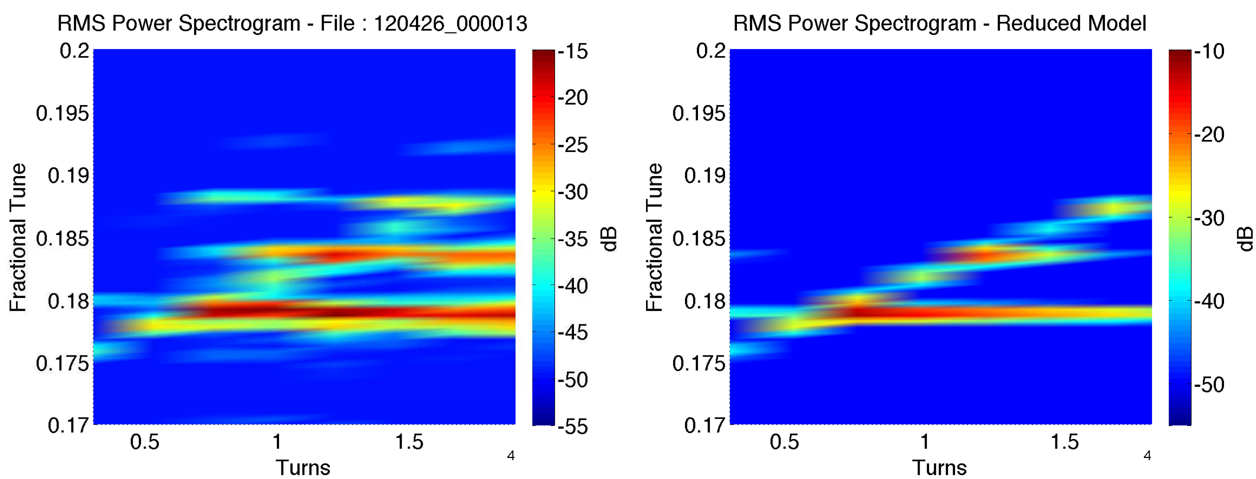

Figure 3: On the left we see the spectrogram of physical measurement showing chirp excitation where we excite mode 0, mode 1, mode 2, and mode 3 excitation around turns ~ 7000, ~ 11500, ~ 15500, and ~ 17500 respectively. On the right, we see the same excitation and analysis applied to the reduced order model capturing linear dynamics.

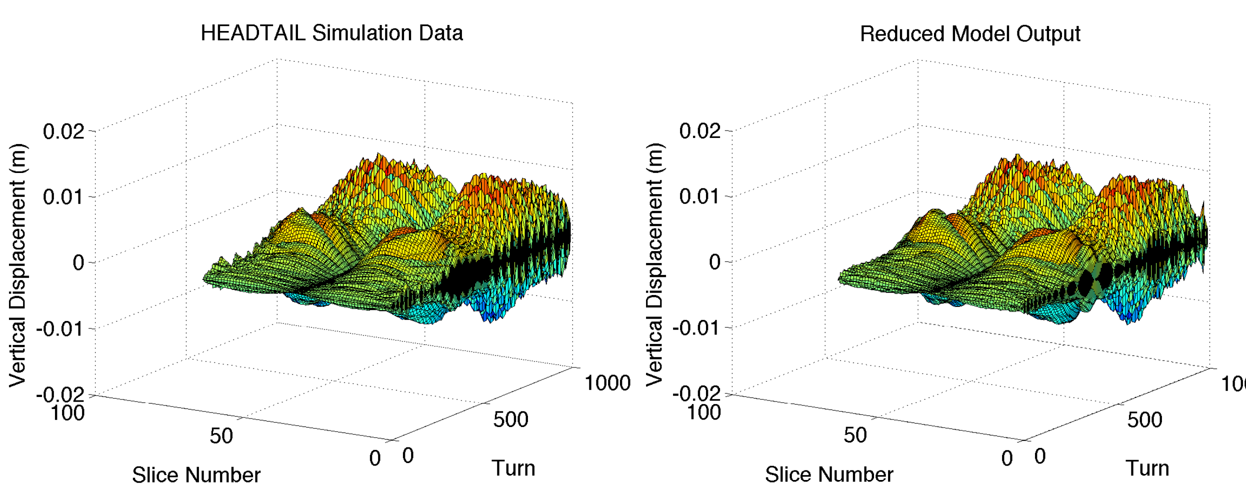

Figure 4: Comparison of the reduced order model response with HEADTAIL simulation data in time domain. Relative error between these two results are less than ~ 10%. The reduced model and HEADTAIL simulation are driven by same excitation signal.

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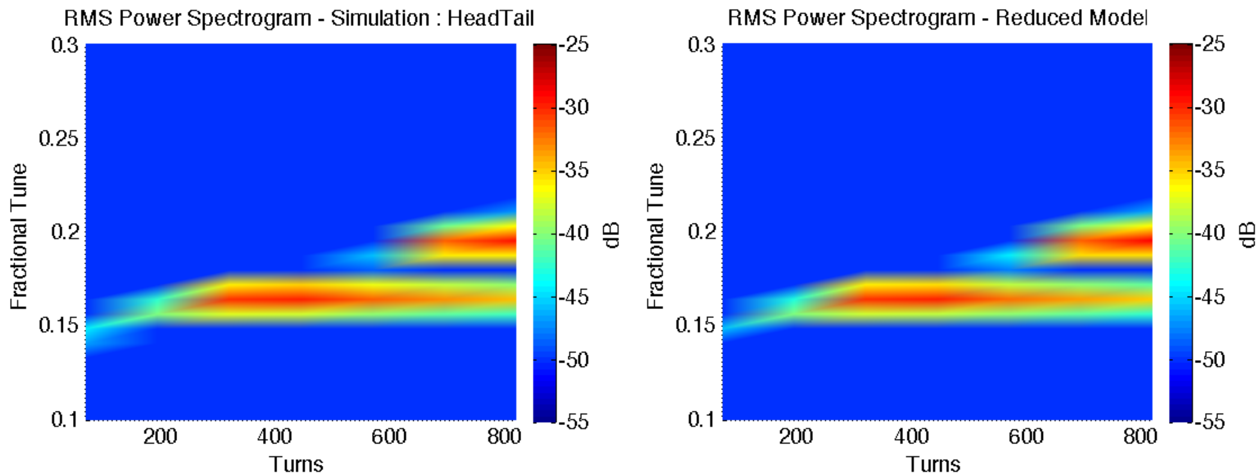

Figure 5: Comparison of the reduced order model response with HEADTAIL simulation data in frequency domain.

input signal. The model order was estimated based on the modes that the excitation signal is able to drive in the bunch and we constrained the input-output signals to identify that reduced model (e.g. we averaged the measured samples in order to reduce the number of inputs and outputs to identify a 4×4 MIMO system with $p = 4$, $q = 4$ and $n = 8$.). With the simulation data, we used the second approach in the analysis. All the individual samples across the bunch were taken into account to set $\mathrm{N} \times \mathrm{N}$ MIMO system with N inputs, N outputs and 2N states. Identification is performed based on an $\mathrm{N} \times \mathrm{N}$ MIMO model and then a model reduction technique is applied to the result based on Henkel Singular Value (HSVD) analysis to get a minimum order balanced realization of the model [4]. HSVD analysis indicates that relative contributions of the first 6 states of 128 states ($N = 64$ case) are noticeable higher than the contributions of the remaining states. Therefore, we can conclude that there are 6 states - 3 modes as dominant dynamics in the system and a reduced order model with 6 states (order of 6) is enough to capture dominant dynamics driven by this excitation signal.

CONCLUSION AND FUTURE WORK

Model-based control design techniques for intra-bunch instabilities require a reduced model of the intra-bunch dynamics. We propose reduced order models and show initial results of the identification of those models. We identify parameters of a reduced order model that captures mode 0, mode 1 and mode 2 dynamics from the CERN SPS machine measurements. The natural tunes, damping values and the separation of modes associated with the motion seen in measurements are estimated correctly using a linear model. We also show similar results using macro particle simulation codes data. Dominant dynamics are captured with a reduced order model and simulation data is regenerated successfully in time domain. Future work is aimed at estimating more internal modes as the wideband kicker will be available early 2015. Availability of the new wideband kicker also requires careful analysis of persistency and optimality of the new excitation signals for the estimation of higher order internal modes. Optimal and robust controllers will be designed using identified reduced order models. These new model based control architectures will be compared with the existing parallelized control filter architecture in terms of performance, processing power and complexity requirements. We plan to evaluate new controllers using macro particle simulations and test in the SPS with single bunch mid 2015.

ACKNOWLEDGEMENTS

We thank J. Dusatko, H. Bartosik, K. Li, A. Drago, G. Kotzian, U. Wehrle, the CERN AB RF group, the CERN operations team and the US-Japan Cooperative Program for their vital help.

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ISBN 978-3-95450-173-1