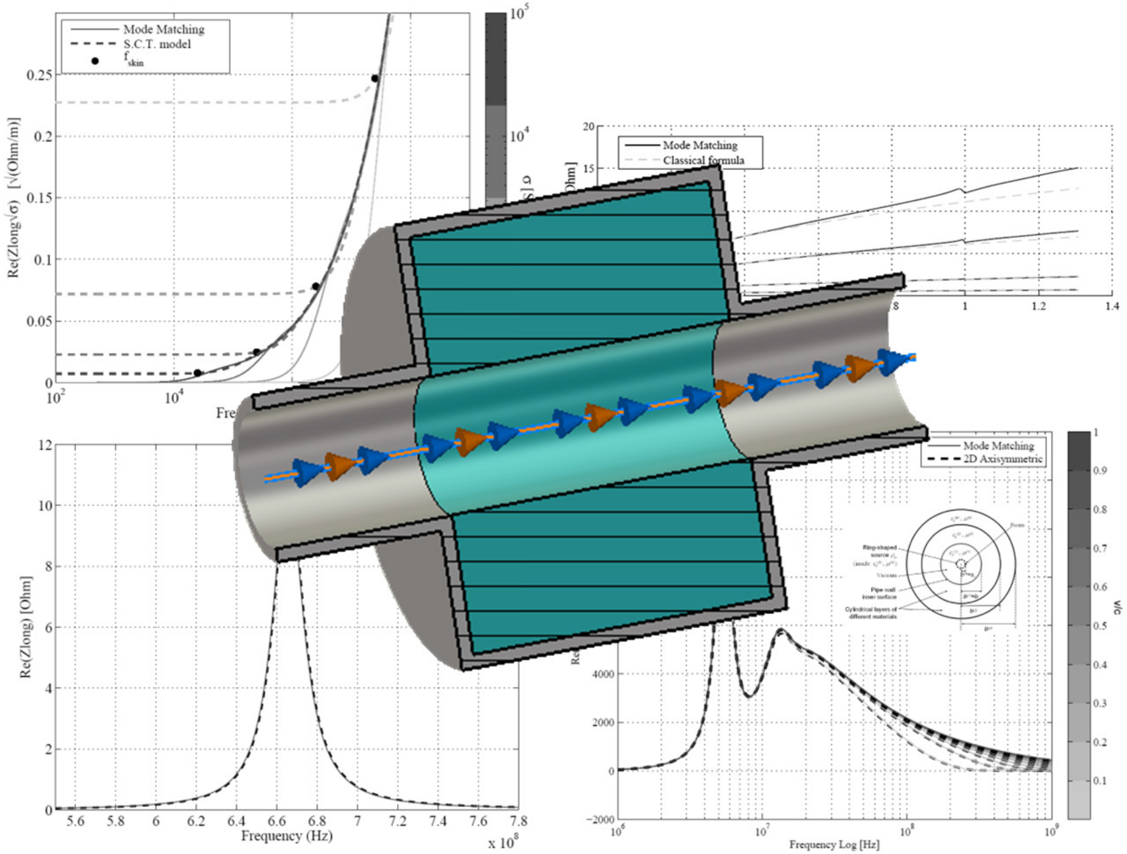
IMPEDANCE STUDIES OF 2D AZIMUTHALLY SYMMETRIC DEVICES OF FINITE LENGTH

N.Biancacci, V.G.Vaccaro

E.Métral, M.Migliorati, L.Palumbo, B.Salvant



- Motivation
- The Mode Matching Method
- Field expressions
- Field matching Vs Mode matching
- General tests
- Convergence
- General tests
- Applications

Acknowldgement: H.Day, A.Mostacci, C.Zannini, CERN ICE/Impedance colleagues.

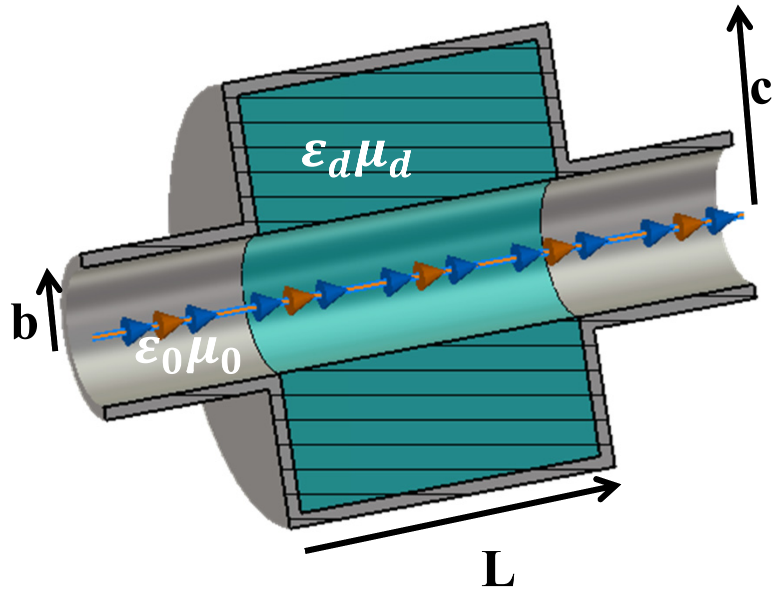
Motivation

Beam coupling impedance is one of the main issues for high intensity beams in accelerators

Having a reliable model for calculating impedance is useful to make projections of beam stability in different conditions.

In this direction, we developed a simple model: a cylindrical cavity loaded with a toroidal material connected to circular infinite beam pipes. This enables us to:

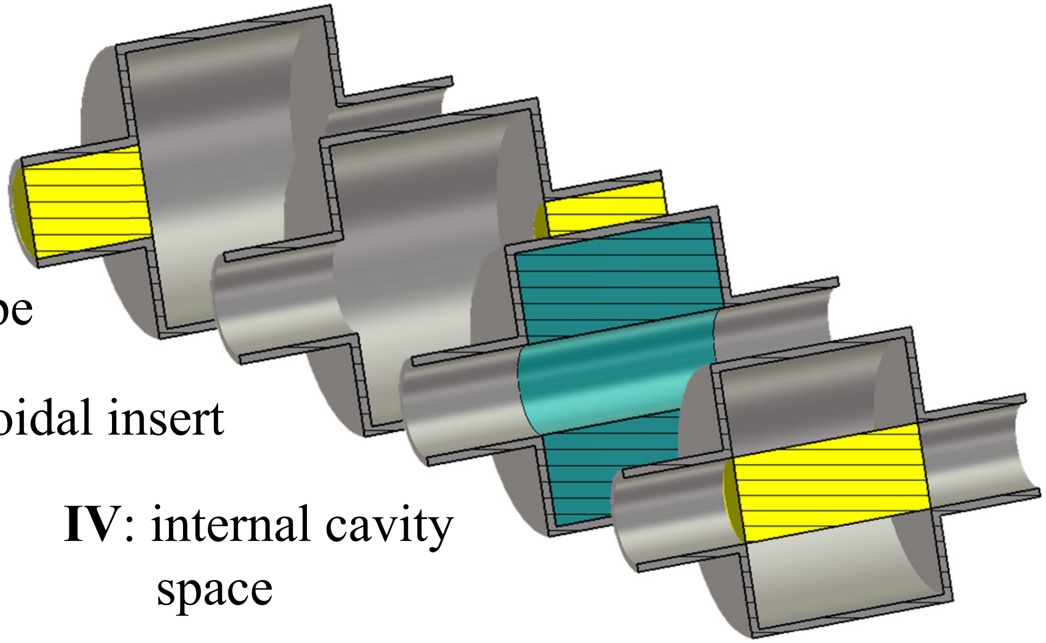
- Evaluate the impact of finite length on longitudinal impedance with respect to the usual 2D approximation.
- Evaluate the impact of non relativistic β (low energy machines)
- Evaluate the power losses induced by the beam for different materials



Fields and longitudinal impedance are calculated by means of **Mode Matching Method**.

Mode Matching Method

The **structure** is divided in 4 regions:

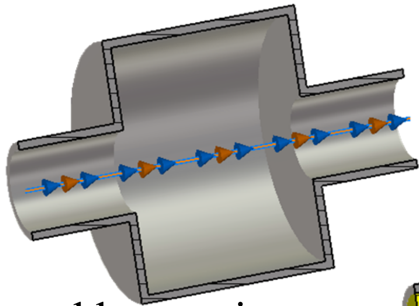


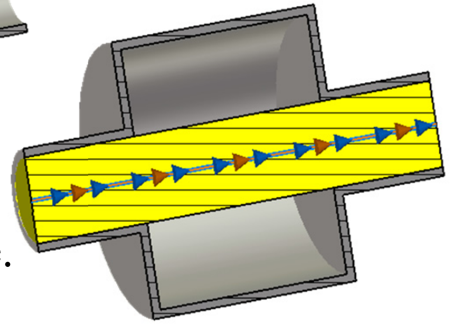
I: left beam pipe

II: right beam pipe

III: toroidal insert

IV: internal cavity space





The **source current** is a thin ring of charge travelling at the center of the structure with velocity βc .

The **source field** $\bar{E}^{(source)}$ is the induced e.m. field in the prolonged beam pipe.

The **scattered field** $\bar{E}^{(scattered)}$ is the field scattered in the structure from the source.

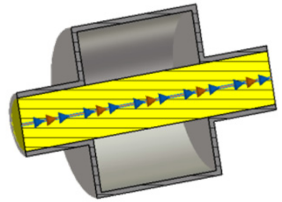
$$\bar{E}^{(tot)} = \bar{E}^{(source)} + \bar{E}^{(scattered)}$$

Fields expressions

In each region we can derive the fields starting from the electric longitudinal component [1,2].
Only TM waves are considered due to symmetry driven by the source field.

Source field:

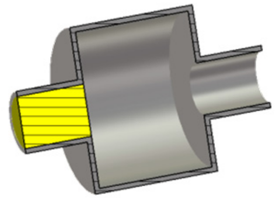
$$E_z^{(source)} = \frac{j\alpha_b Q\, Z_0}{2\pi\gamma^2\beta b}\left[K_0(u) - \frac{K_0(x)}{I_0(x)} I_0(u)\right] e^{-jk_0 z}$$



Scattered field:

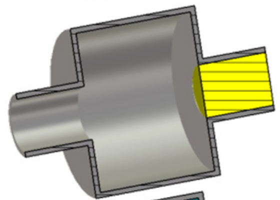
I

$$E_z^{(left)} = \sum_p^{\infty} \boldsymbol{C_p} \frac{J_0(\alpha_p r/b)}{b\sqrt{\pi} J_1(\alpha_p)} e^{j\tilde{\alpha}_p z/b}.$$



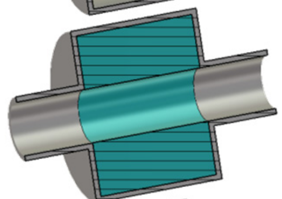
II

$$E_z^{(right)} = \sum_p^{\infty} \boldsymbol{D_p} \frac{J_0(\alpha_p r/b)}{b\sqrt{\pi} J_1(\alpha_p)} e^{-j\tilde{\alpha}_p (z-L)/b}$$



III

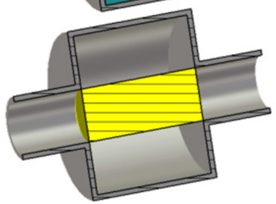
$$E_z^{(rad)} = \sum_s^{\infty} \boldsymbol{A_s} W_s(\hat{\alpha}_s r/b) cos(\alpha_s z/b)$$



IV

$$\bar{E}^{(cav)} = \sum_{p,s} \boldsymbol{V_{ps}}\ \bar{e}_{ps}.$$

4 vector unknowns



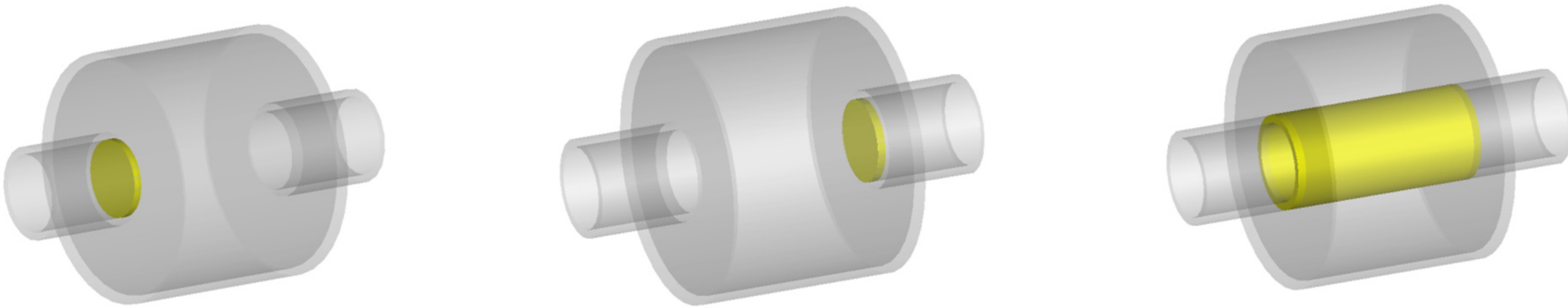
Field Matching Vs Mode Matching

With 4 unknowns we need some procedure to find the 4 equations necessary to solve the problem:

Field matching: impose continuity of tangential E, H field components on the separating surfaces (usual approach in infinite long structures, e.g. 2D multilayer pipes, steps in or out, kickers, etc..)

Mode matching: Decompose the field in summation of modes and try to "*match*" each mode coefficient by proper field projection on the correspondent mode (enclosed volumes as in finite length structures) [3,4].

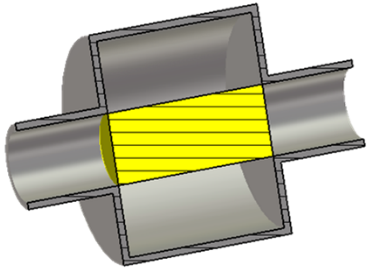
- **3 equations** come using **field matching** for magnetic field on internal boundary surfaces.



- Using the field matching also for the electric field is not possible...

Field Matching Vs Mode Matching

- In region IV we wrote an expansion of internal fields as:



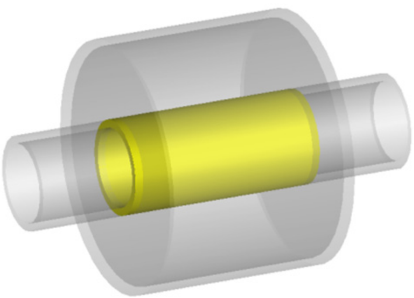
$$\bar{E}^{(cav)} = \sum_{p,s} \boldsymbol{V_{ps}}\ \bar{e}_{ps}$$

$$\bar{H}^{(cav)} = \sum_{p,s} I_{ps}\bar{h}_{ps}$$

- A general e.m. field can always be decomposed in sum of irrotational and solenoidal modes (Helmholtz Theorem).

- In this structure, irrotational modes do not couple, the $\bar{e}_{ps}$ and $\bar{h}_{ps}$ constitutes a complete set of solenoidal modes.

- To be complete, this set has to satisfy these boundaries on the surface *S*:



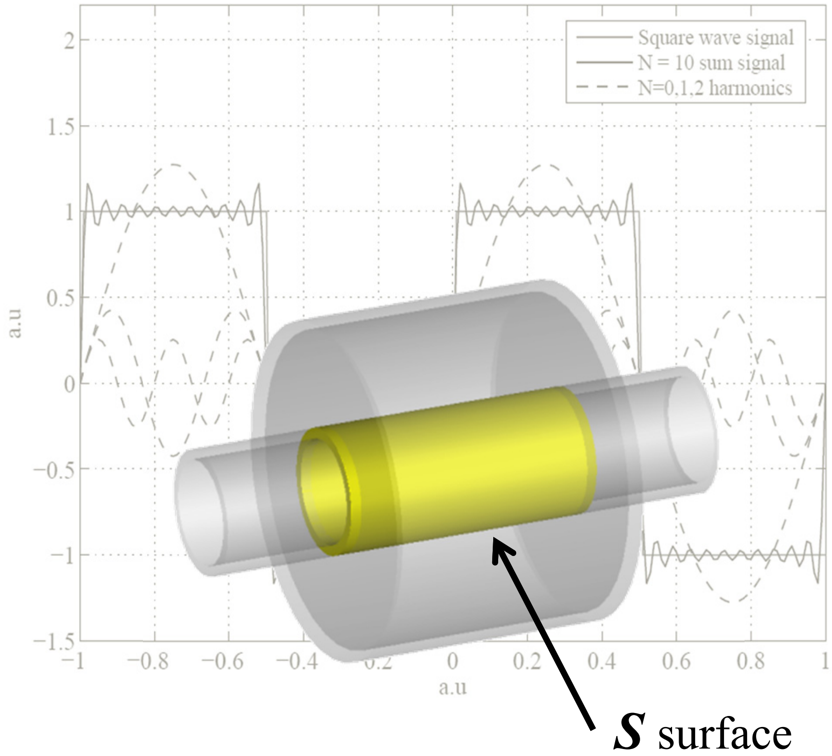
$$\left.\begin{aligned} \bar{n}_o \times \bar{e}_n &= 0 \\ \bar{n}_o \cdot \bar{h}_n &= 0 \end{aligned}\right\}$$

<u>This means that the tangential electric field is null at the boundaries and we cannot use field matching</u>.

Field Matching Vs Mode Matching

- The **4th equation** comes using the **mode matching** for the electric field on internal boundary surfaces.

  Parallelism: Following the same idea that a square wave signal can be constructed by means of **non-uniformly converging** series of sinusoidal waves, at the same way, the eigenvectors will be non-uniformly converging to the electric field at the boundary.



$$f(x) = \sum_{n} \boldsymbol{b_n} \, sin(nx)$$

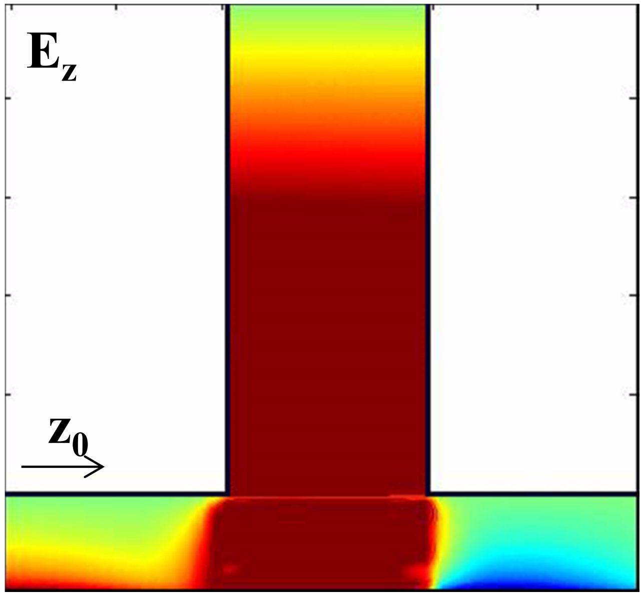
$$\boldsymbol{b_n} = \frac{1}{L} \int_{0}^{2L} f(x) \sin\left(\frac{n\,\pi x}{L}\right) d\,x$$

$$\bar{E} = \sum_{p,s} \boldsymbol{V_{ps}} \, \bar{e}_{ps}$$

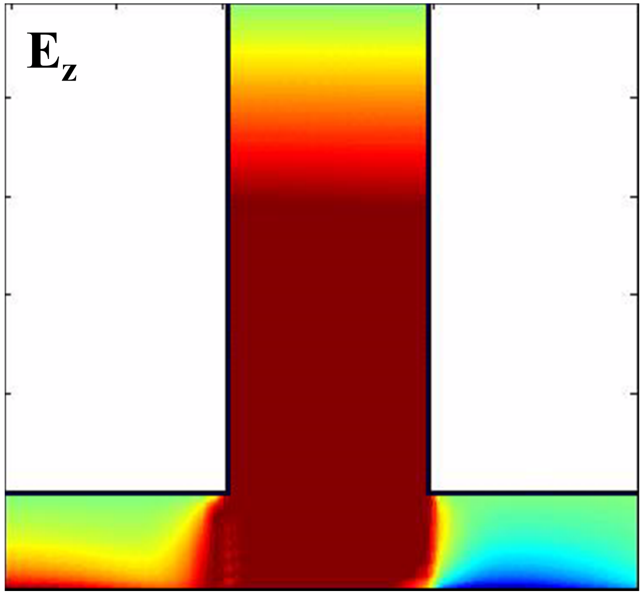
$$\boldsymbol{V_{ps}} = \frac{\alpha_0 b}{(\alpha_0^2 - \alpha_{ps}^2)} \oint_S \left(\bar{E} \times \bar{h}_{ps}^*\right) \cdot \hat{r}_0 \, dS.$$

Convergence

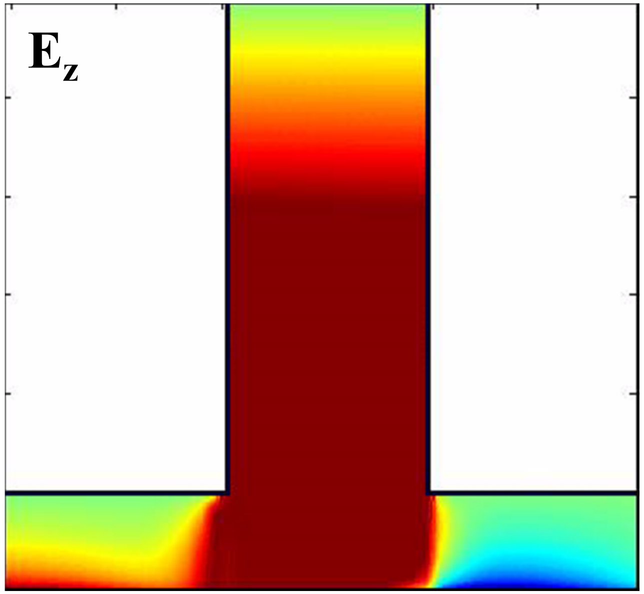
Being a non-uniformly convergent series, the field in the cavity is very slowly converging.
Here an example of longitudinal electric field below cut off :
P= # radial modes, S= # longitudinal modes



P=5, S=5



P=15, S=15



P=30, S=30

NB: What matters to us is **impedance**. Being an integrated parameters over the symmetry axis, it is possible to apply **Kneser-Sommerfeld formula** in order to close the slow converging series.

General tests

A series of tests were done to verify the code:

1. **Comparison with the classical thick wall formula**

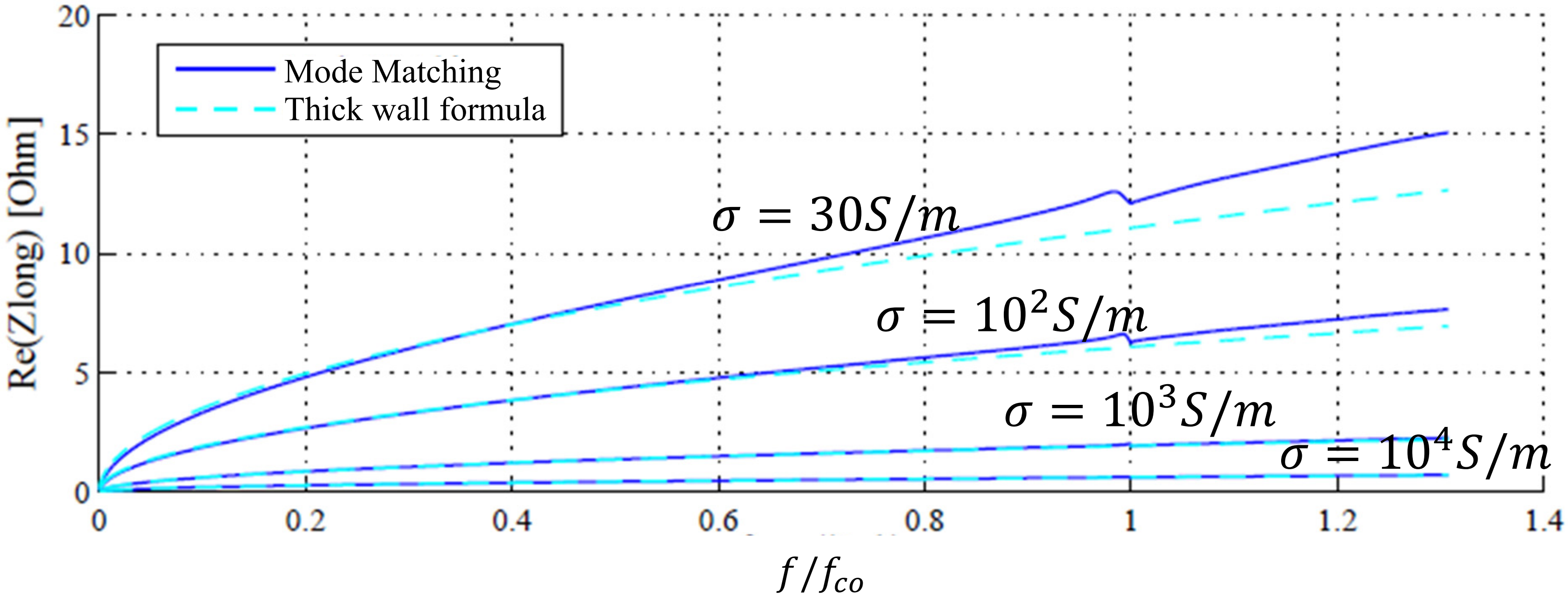
2. **Comparison with CST particle studio**

3. **Comparison Shobuda-Chin-Takata's model (SCT)**

General tests

1. Comparison with the classical thick wall formula:

$$Z_{long} = \frac{1+j}{\sqrt{2}} \frac{L}{2\pi b} \sqrt{\frac{Z_0 \omega}{\sigma c}}$$

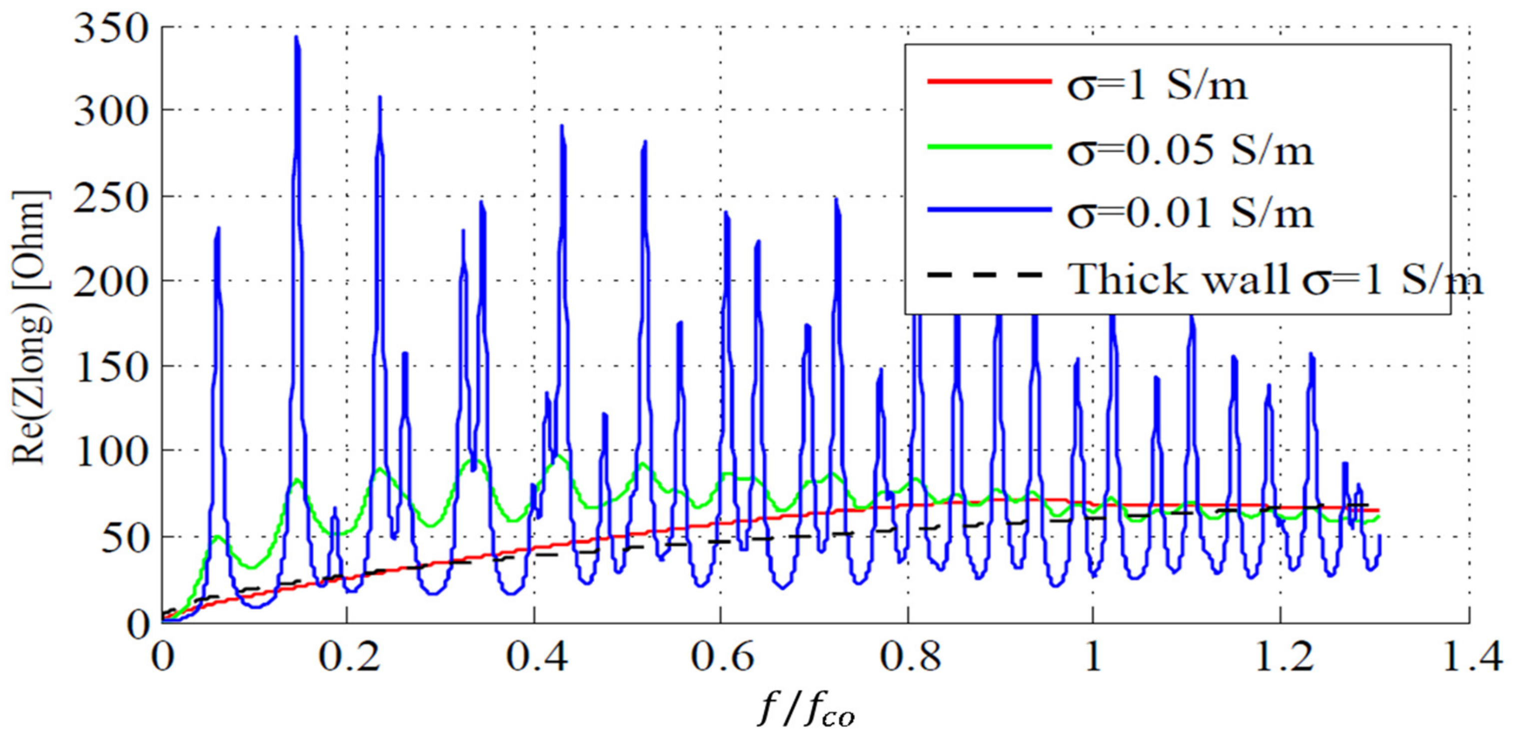


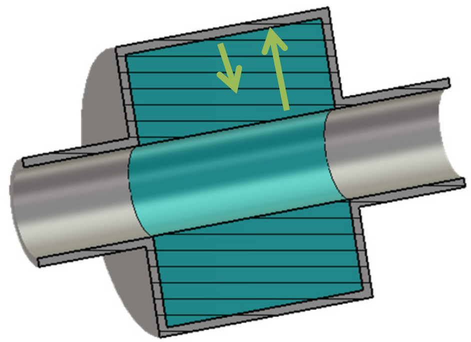
For low conductivities, the real part of the impedance is a lower estimation in classical formula. Opposite behavior is found to be for the imaginary part.

Good agreement for high conductivities.

Dimensions: b=5cm, c=30cm, L=20cm; **Material:** $\varepsilon_d = 8\varepsilon_0 - j\,\sigma/\omega$; $\mu_d = \mu_0$; $f_{co} = \alpha_1 c\,/2\pi b$

General tests



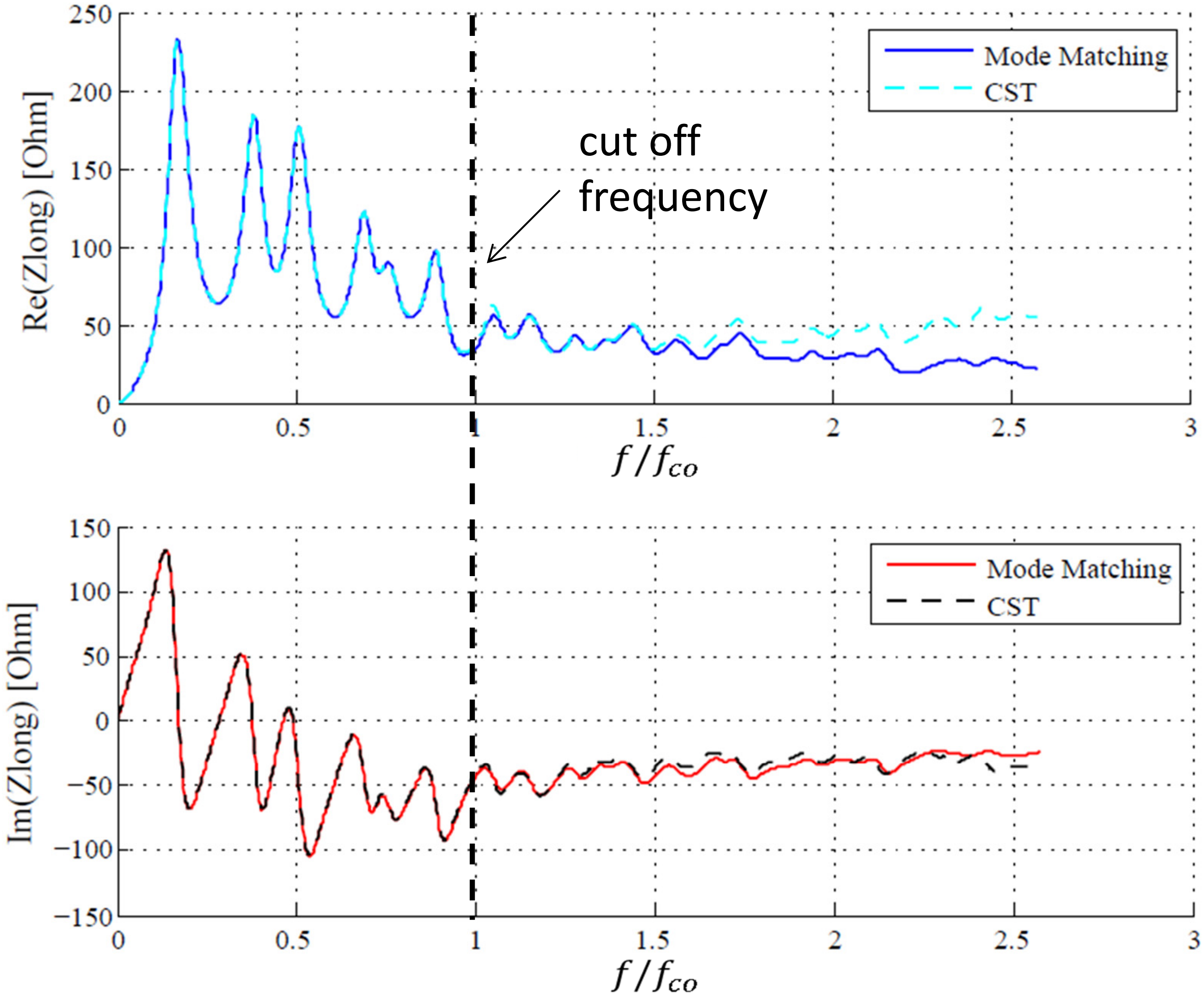


For low conductivities, the field start penetrating: resonances from transverse and longitudinal plane start to be visible.
Thick wall formula is no more applicable.

Dimensions: b=5cm, c=30cm, L=20cm; **Material:** $\varepsilon_d = 8\varepsilon_0 - j\,\sigma/\omega$; $\mu_d = \mu_0$; $f_{co} = \alpha_1 c\,/2\pi b$

General tests

2. Comparison with CST particle studio:

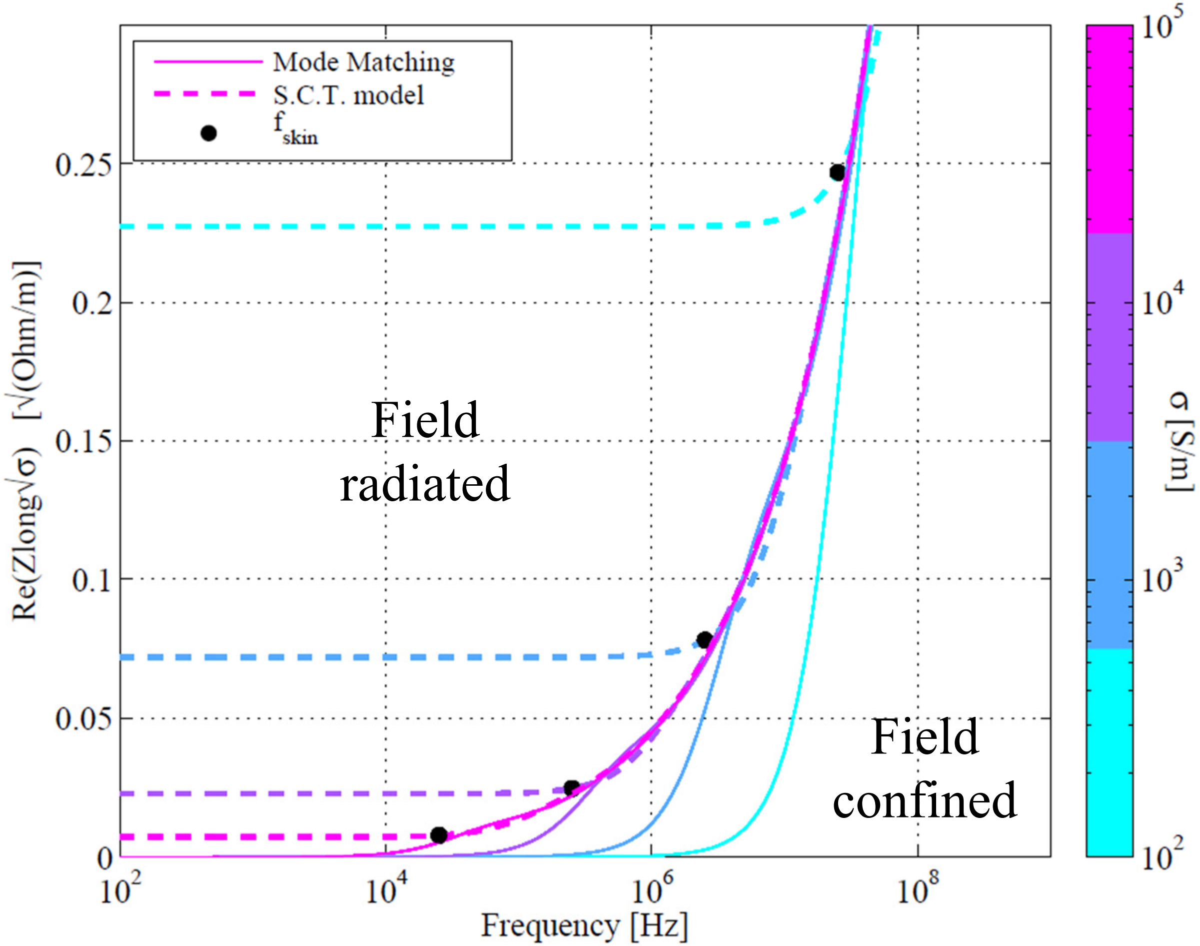


Very good agreement.
The discrepancy at half the simulated bunch spectrum is thought to be due to the CST division by very small bunch spectrum.

Dimensions:
b=5cm,
c=30cm,
L=20cm.
Material:
ε_{dr}=1,
μ_{dr}=1,
σ=1e-2 S/m

General tests

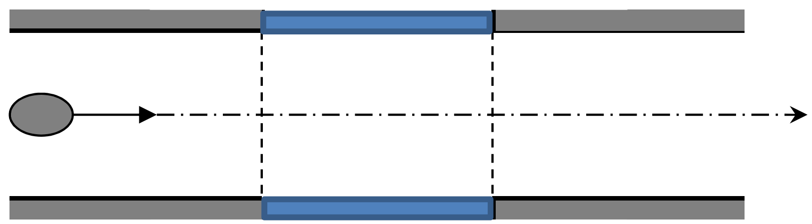
3. Comparison Shobuda-Chin-Takata's model (SCT):



Dimensions: b=7cm, c=8cm, L=1cm.
Material: ε_{dr}=1, μ_{dr}=1, σ=1e6 S/m

Main differences between the two models [5]:

- In SCT no longitudinal modes are considered (short insert respect to beam pipe radius);
- In SCT there is no PEC layer bounding the insert.



The two model can be anyway compared for frequencies above the frequency in which the skin depth starts to be comparable with the cavity thickness.

Applications

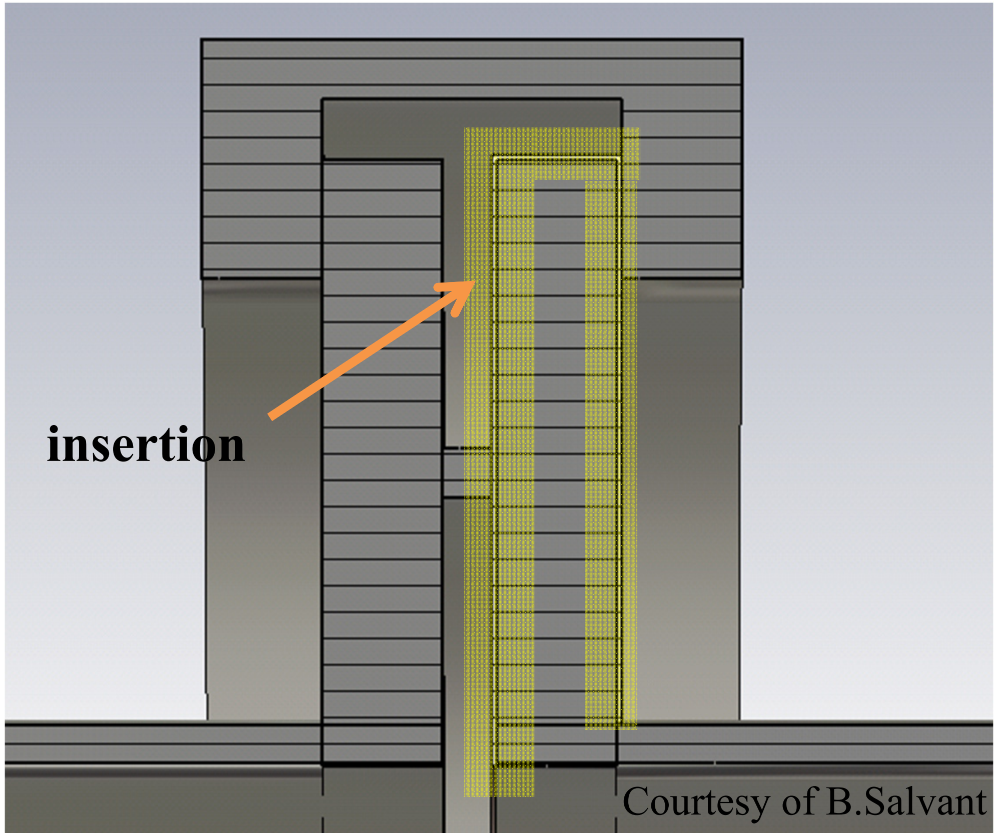
1. **Impedance of an SPS enamel flange (thin insert)**

2. **Non-ultrarelativistic cases: impedance dependence on β**

3. **Non-ultrarelativistic cases: power loss dependence on β**

Applications

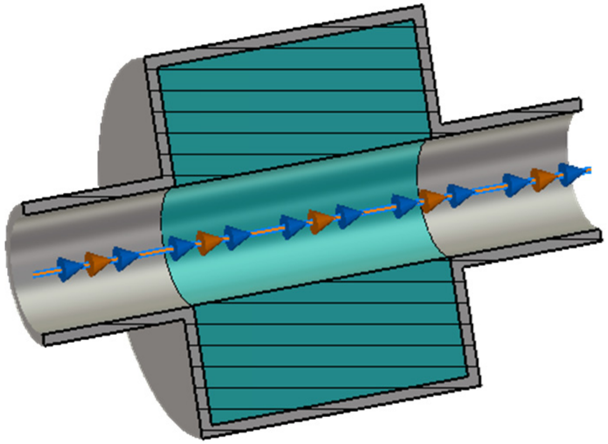
1. Impedance of an SPS enamel flange (thin insert):



- Thin isolating insertions between beam pipe flanges in the CERN SPS could present impedance peaks at enough low frequency to overlap with the bunch spectrum.
- The <u>thickness</u> of these inserts is very short, order of <u>200um</u>, maybe difficult for simulators.
- A crosscheck of the Mode Matching was therefore done to stress its capabilities.

Applications

1. Impedance of an SPS enamel flange (thin insert):

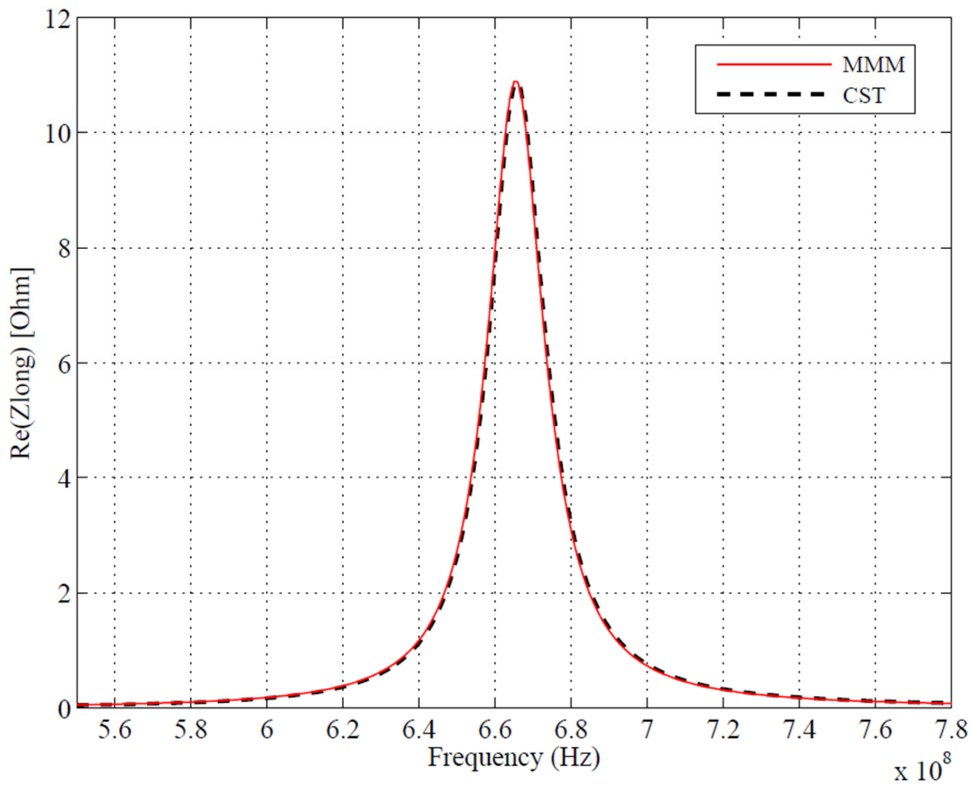


We considered a device length of 800 um to be able to do comparisons with CST.

Dimensions: b=5cm, c=9cm, L=800um.
Material: ε_{dr}=9.9, μ_{dr}=1, σ=1e-2 S/m

Applications

1. Impedance of an SPS enamel flange (thin insert):



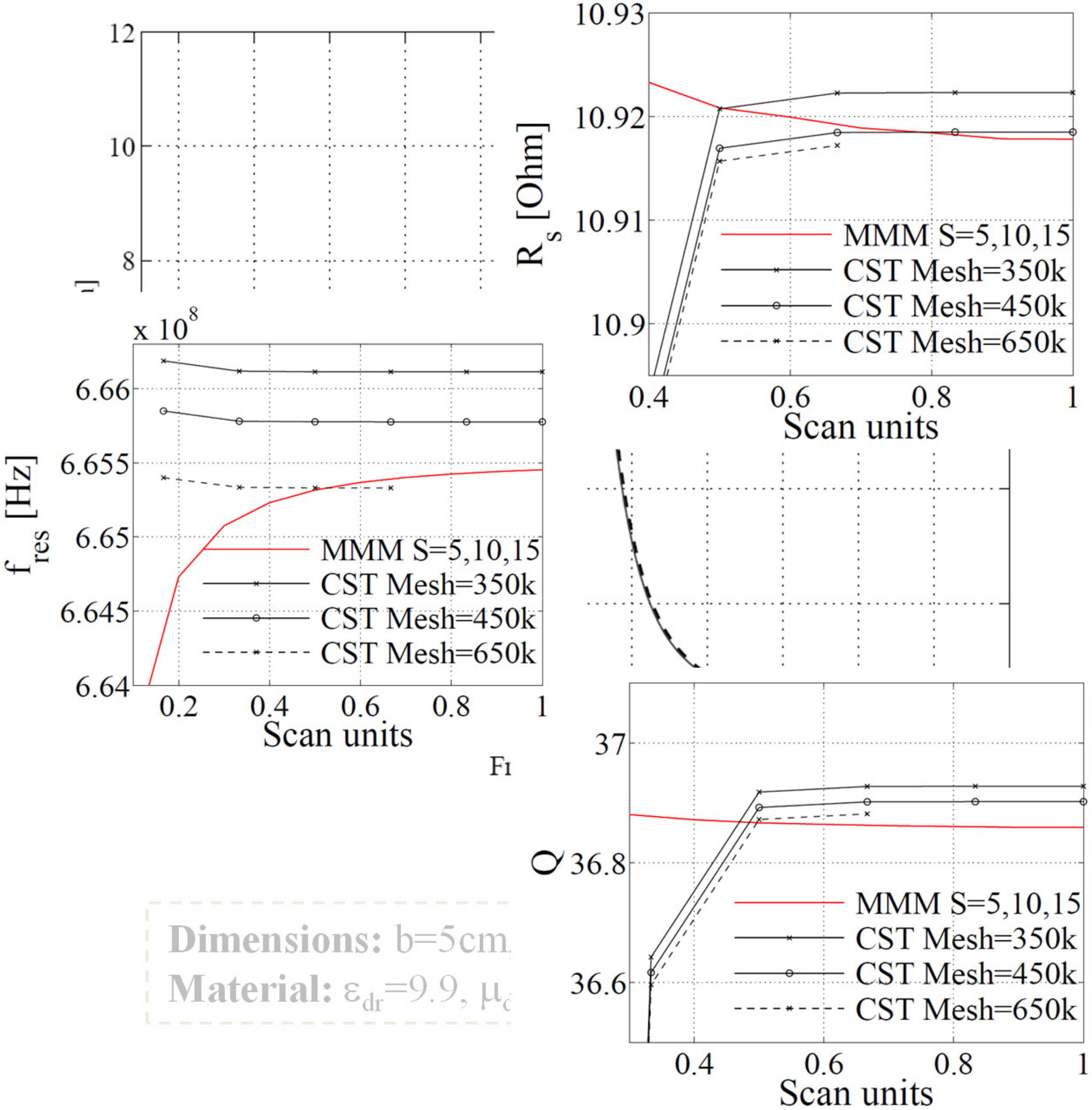
We considered a device length of 800 um to be able to do comparisons with CST.

For the geometry used, a first resonance is seen at 660 MHz.

Dimensions: b=5cm, c=9cm, L=800um.
Material: ε_{dr}=9.9, μ_{dr}=1, σ=1e-2 S/m

Applications

1. Impedance of an SPS enamel flange (thin insert):



We considered a device length of 800 um to be able to do comparisons with CST.

For the geometry used, a first resonance is seen at 660 MHz.

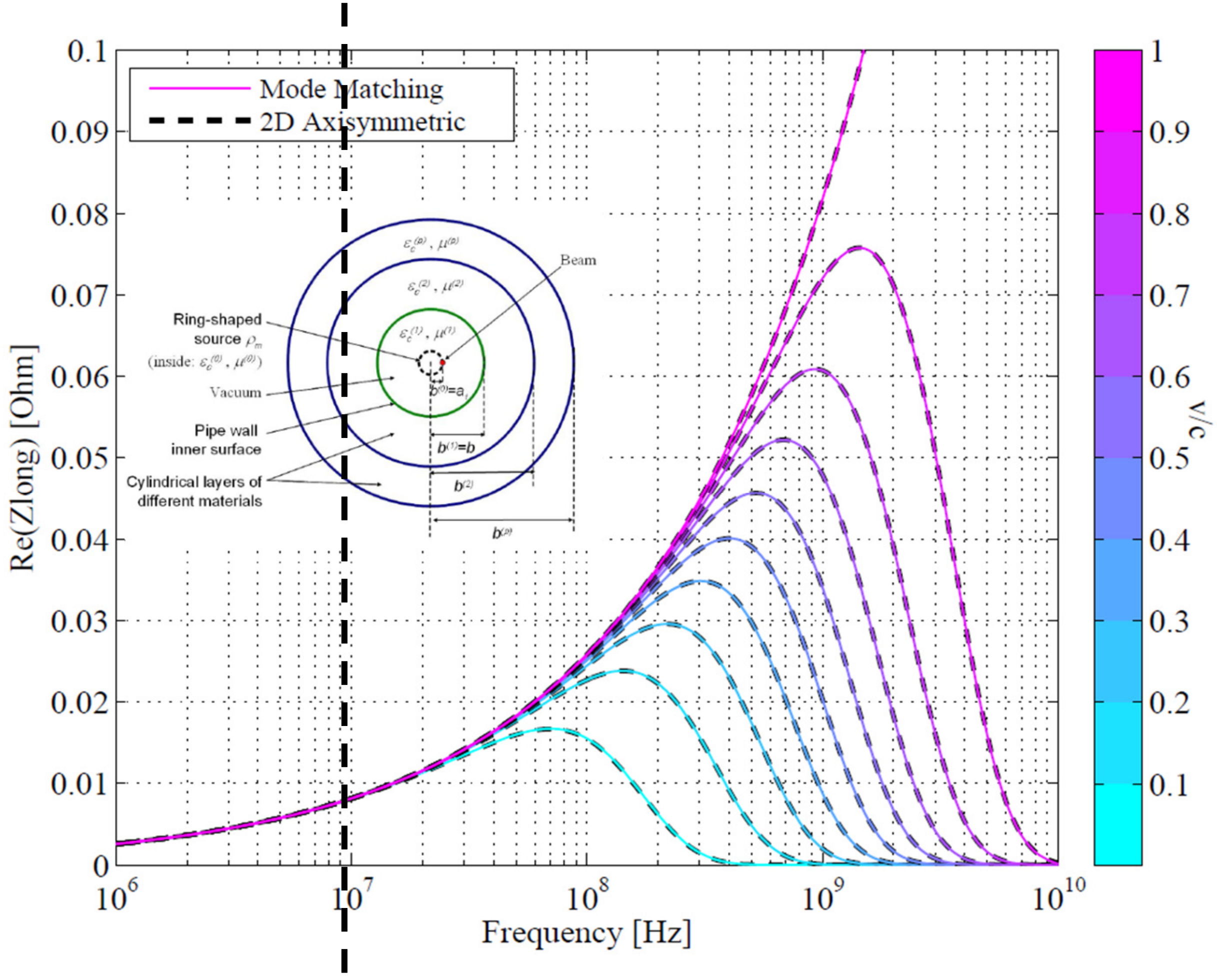
Convergence test:

	Scan unit	Min	Max
CST	Mesh	350k	650k
	L_{wake}	10m	60m
MMM	# P modes	5	50
	# S modes	5	15

Agreement is below a percent!

Applications

2. Non-ultrarelativistic cases: impedance dependence on β



Dimensions: b=5cm, c=30cm, L=20cm.
Material: $\varepsilon_{dr}=1$, $\mu_{dr}=1$, $\sigma=5.98e6$ S/m

The method do not have any restriction on beam speed.

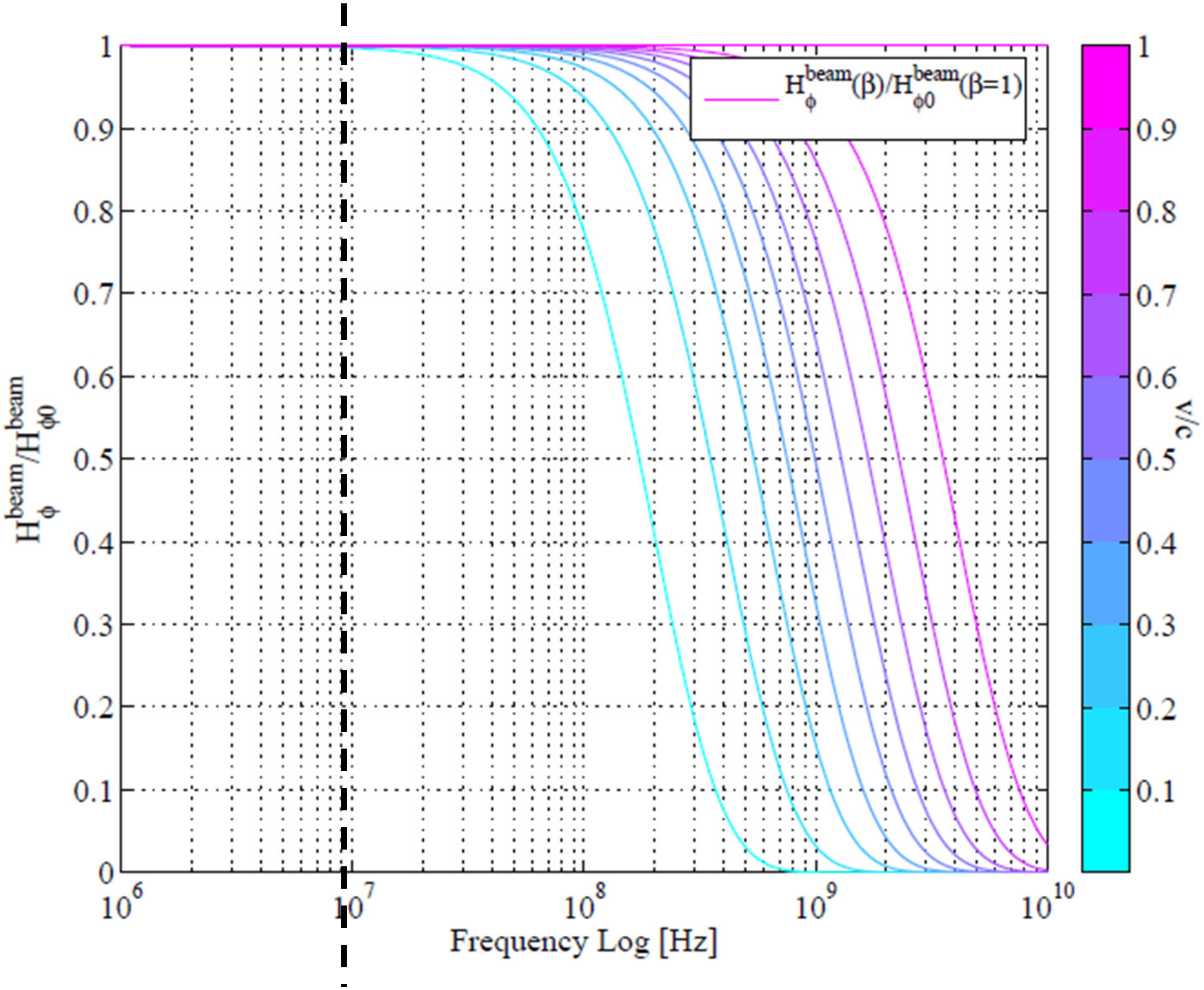
The method was compared with the 2D axisymmetric infinite length model developed by N.Mounet, E. Métral [6].

Since the finite length is not playing a significant rule for the geometry under study and the conductivity used, **the two methods are in this case equivalent**.

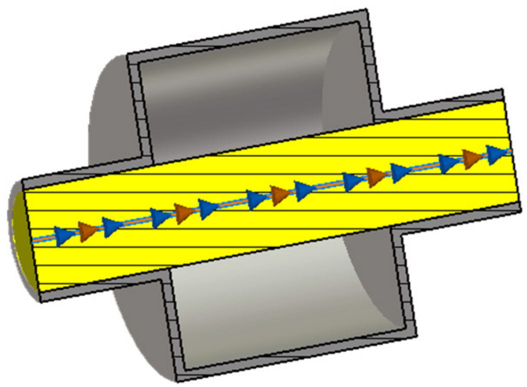
NB: The impedance is changing significantly only above 10 MHz

Applications

2. **Non-ultrarelativistic cases: impedance dependence on β**



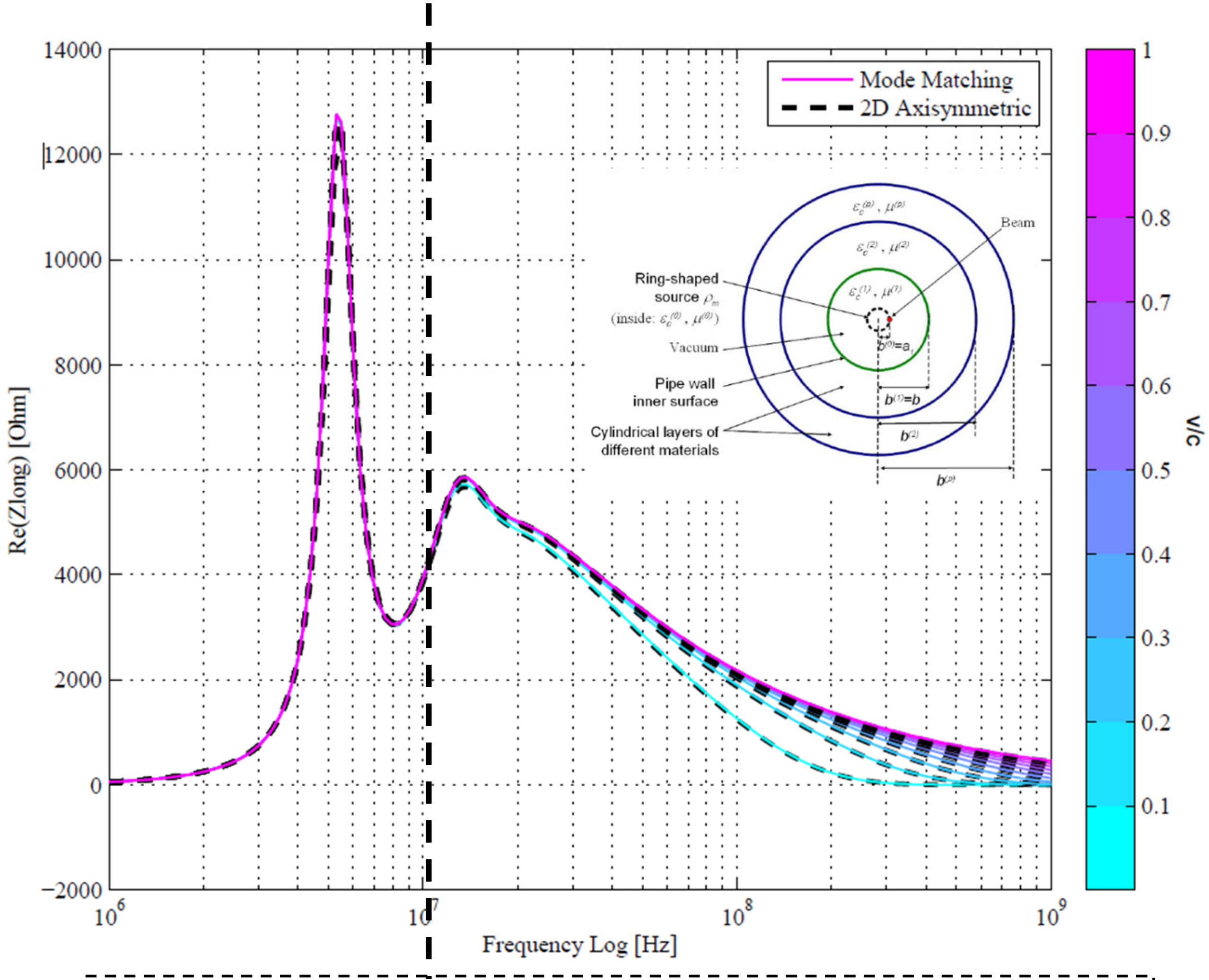
It can be shown that, in the structure under study the azimuthal magnetic field **H_φ is the only component of the source field that couples**



Dimensions: b=5cm, c=30cm, L=20cm.

Applications

2. Non-ultrarelativistic cases: impedance dependence on β



Similar behavior we had in case of a Ferrite 4A4 insert:

Below 10MHz impedance is **not significantly changing** with β.

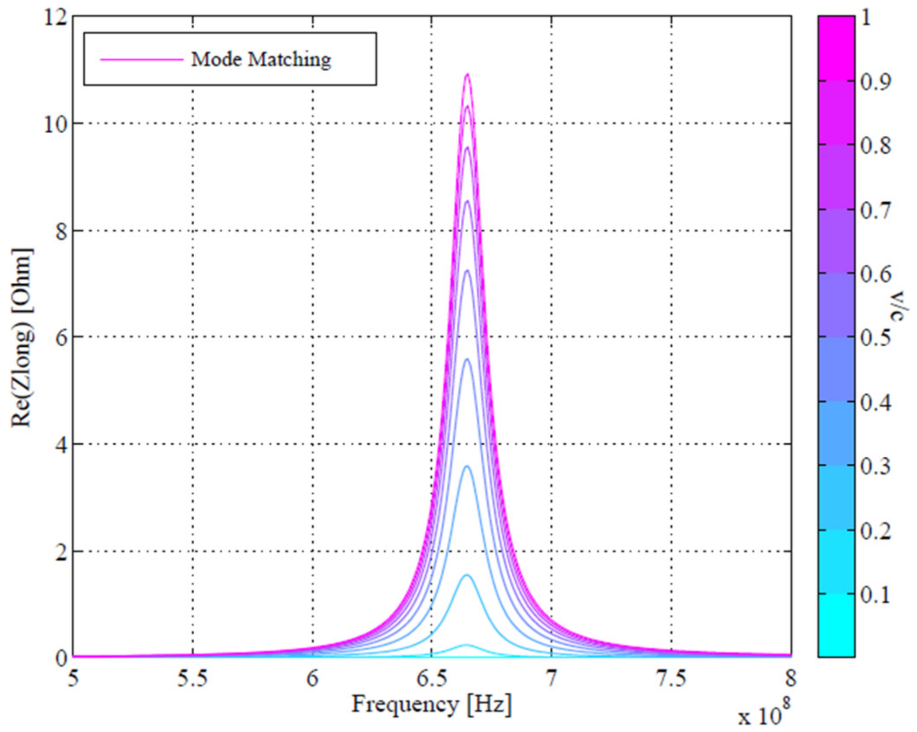
Above 10MHz the driving term **becomes weaker** as well as the impedance.

Dimensions: b=5cm, c=30cm, L=20cm.
Material: ε_{dr}=12, μ_{dr}=1 + 460 / (1 + j f / 20e6), σ=1e-6 S/m.

Applications

3. Non-ultrarelativistic cases: power loss dependence on β

Applying poynting theorem it is possible to estimate power losses behavior with β in the volume insert from conductivity, magnetic and electric relaxation.



Dimensions: b=5cm, c=9cm, L=800um.
Material: ε_{dr}=9.9, μ_{dr}=1, σ=1e-2 S/m

In case of the thin insert we have:

- Q constant (depends on material properties)
- f_{res} constant (depends on geometry)
- R_s decreases with β (depends on driving source)
- W_m decreases with β (depends on driving source)
- P_l decreases with β (depends on driving source)

β	f_{res} [Hz]	R_s [Ω]	Q	W_m [J]	P_l [W]
1	6.65E+08	10.94	36.90	1.27E-08	1.45
0.8	6.65E+08	9.56	36.90	1.11E-08	1.27
0.6	6.65E+08	7.26	36.88	8.45E-09	0.97
0.4	6.64E+08	3.59	36.83	4.18E-09	0.48
0.2	6.64E+08	0.24	36.51	2.73E-10	0.03

Conclusions and outlook

Conclusions:

- The mode matching method was presented from both theoretical and applicative point of view.
- A series of successful benchmarks were presented with:
  - ✓Thick wall formula
  - ✓CST simulations
  - ✓Shobuda-Chin-Takata’s model
  - ✓N.Mounet-E.Métral’s model
- Low beta studies: the impedance and power loss are dependent on the coupling with the driving source field (magnetic in the model case)

Outlook:

- Implementation of MMM for transverse impedance: ready…. On refinement.
- More tests with CST for accuracy dependence on conductivity.
- Tests on impedance dependence on finite length for non ultrarelativistic cases
- ….

謝 謝 !!

References

[1] R.L Gluckstern, B.Zotter, "Longitudinal Impedance of a Resistive Tube of Finite Length", CERN unpublished.

[2] L. Palumbo, V.G. Vaccaro, M. Zobov, "Wake Fields and Impedance", CAS school 1994.

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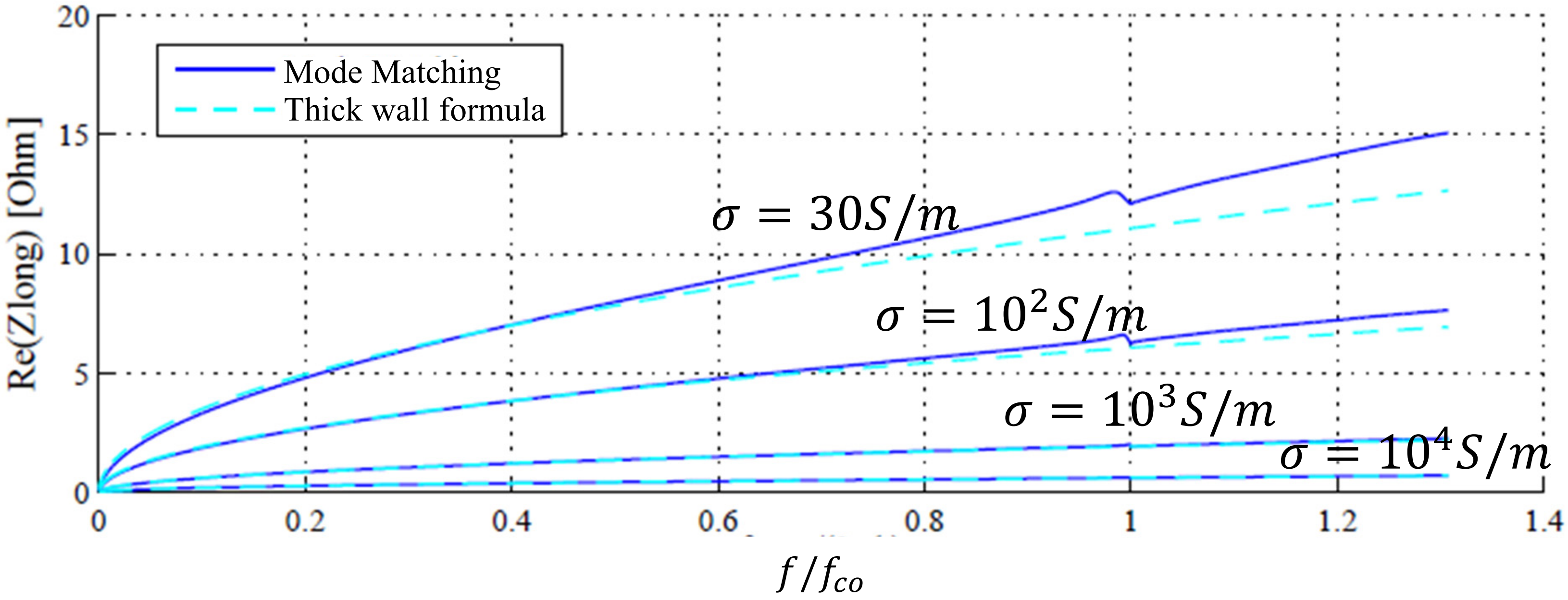
[4] J. G. Van Bladel, "Electromagnetic Fields",(IEEE Press Series on Electromagnetic Wave Theory, 2007.

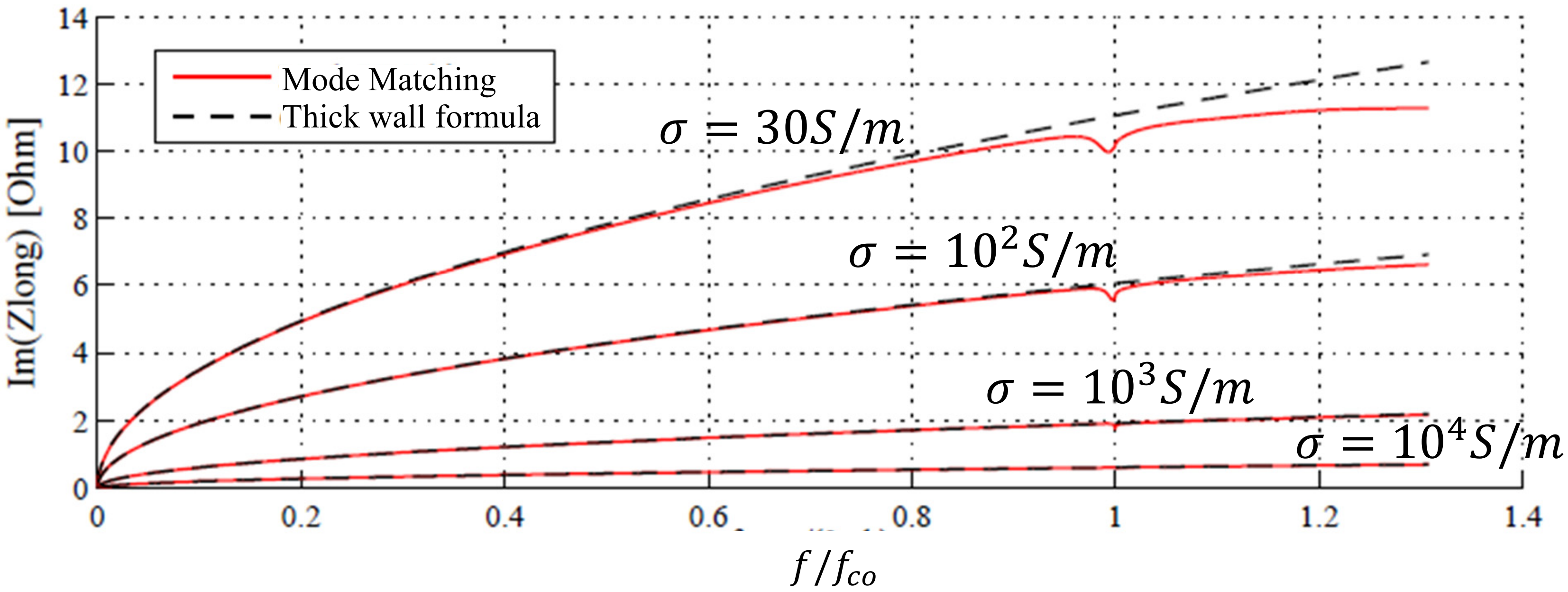
[5] Y. Shobuda, Y.H. Chin, K. Takata "Coupling impedances of a resistive insert in a vacuum chamber", PhysRevSTAB, 2009.

[6] N.Mounet, E. Métral, "Impedances of an Infinitely Long and Axisymmetric Multilayer Beam Pipe: Matrix Formalism and Multimode Analysis", IPAC 2010.

Additional material



Mode Matching
Thick wall formula
Re(Zlong) [Ohm]
$\sigma = 30S/m$
$\sigma = 10^2 S/m$
$\sigma = 10^3 S/m$
$\sigma = 10^4 S/m$
f/f_{co}



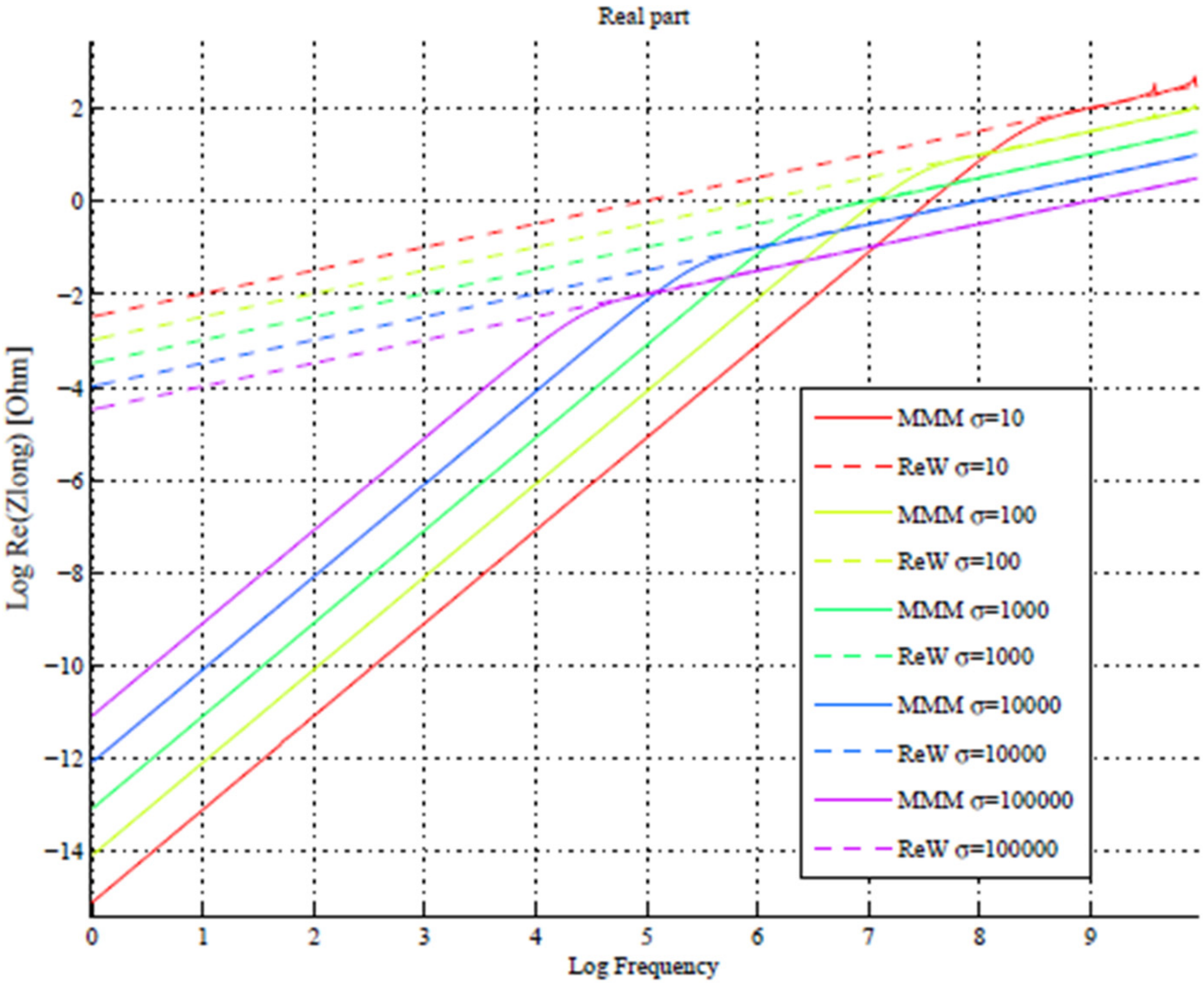
Mode Matching
Thick wall formula
Im(Zlong) [Ohm]
$\sigma = 30S/m$
$\sigma = 10^2 S/m$
$\sigma = 10^3 S/m$
$\sigma = 10^4 S/m$
f/f_{co}

1-Thick Wall Formula: Test for high conductivity σ (1/1)

Varying conductivity

Thick wall formula:

$$Z_{long} = \frac{1+j}{\sqrt{2}} \frac{L}{2\pi b} \sqrt{\frac{Z_0 \omega}{\sigma c}}$$



Parameters:

Inner radius=5cm

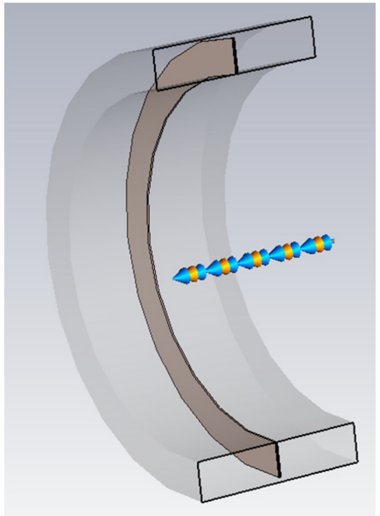
Outer radius=30cm

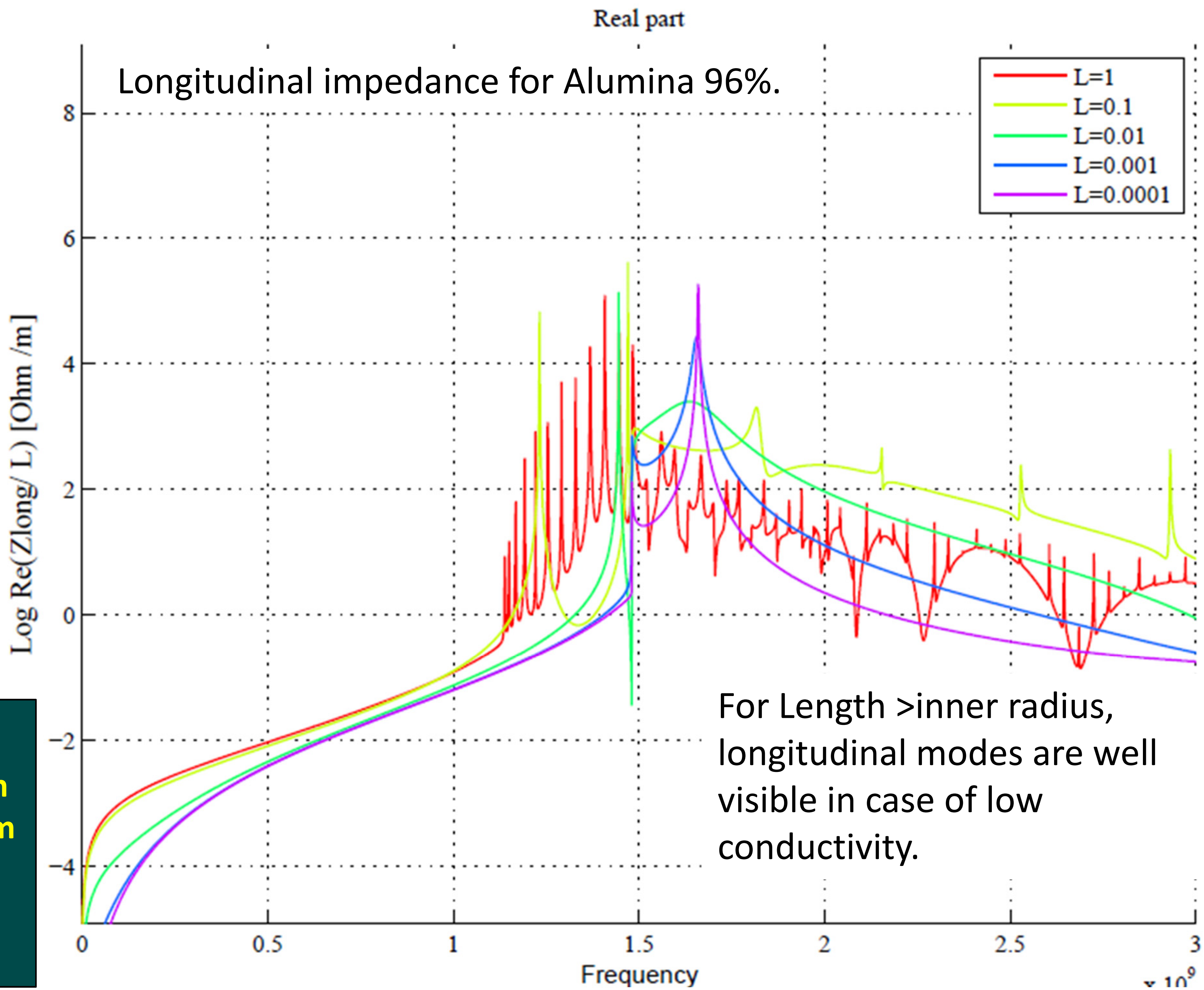
Length=20cm

εr=1

σ variable

3- Length dependence of impedance (1/2)





Parameters:
Inner radius=7.7cm
Outer radius=9.2cm
Length=variable
εr=9.4
σ = 10^{-12} S/m