Space charge effects in cyclotrons

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Outline

◇ Over 40 years of work on this topic:

- Injection current limitation: Vertical space charge.
- Extraction current limitation: Longitudinal space charge.

◇ Space charge in the TRIUMF cyclotron – revisited:

- Development of a 3D simulation tool including effect of neighboring turns using periodic boundary conditions in the radial direction.

Vertical space charge

Incoherent tune shift is a well-known effect in synchrotron theory:

$$\Delta(\nu_z^2)_{SC} = -\frac{2}{\pi}\frac{NRr_p}{\beta^2}\left[\frac{1}{b(a+b)} + \frac{\epsilon_1}{h^2}\right],$$

with N: number of particles,
R: orbit radius,
r_p: classical proton radius (1.54$\times 10^{-18}$ m),
β: ratio of the particle velocity to the speed of light,
a and b: horizontal and vertical beam size, resp.
h: metal chamber half-height,
ϵ_1: the Laslett image coefficient, $\simeq$ 0.2 for parallel plates.

In cyclotrons, since the vertical tune is generally much smaller than the horizontal tune, the current limit is reached when the vertical focusing nearly vanishes.

Vertical space charge

Ways to push the space charge limit further away:

◇ $\propto \frac{1}{\beta^2}$: increase the injection energy.

This is why high power cyclotrons use external ion sources.

Drawback: increases beam power lost at injection.

◇ Increase the vertical focusing.

Vertical focusing in cyclotrons comes manly from the azimuthal field variations (*i.e.* edge focusing).

Drawback: machine less compact. Necessarily the injection orbit radius must be large compare to the magnetic gap.

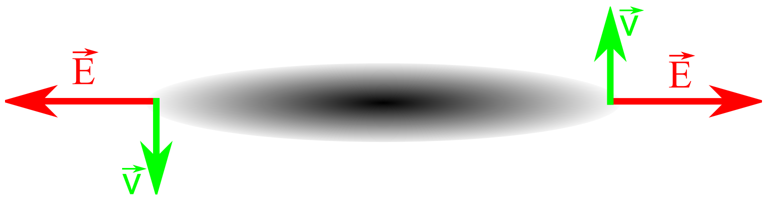
Current limitation from transverse space charge

No hard limit

but a price to pay

Longitudinal-radial effects

• Intuitive effect of longitudinal space charge forces: particles at the head of the bunch gain energy, particles at the tail lose energy.



• The absence of longitudinal focusing leads to the accumulation the energy spread (M.M. Gordon, Int. Cyclotron Conf. 1969).

• Similar to the case of a synchrotron at transition.

• Because of the **non-zero dispersion**: longitudinally dependent radial motion, **reducing turn separation.**

If the energy gain accumulates

From Gauss's law, the space charge potential of a cylindrical beam in a cylindrical pipe is:

$$V = \frac{\lambda}{4\pi\epsilon_0}(1 + 2\log(b/a)),$$

where a is the beam radius, b is the beam pipe, and λ lambda is charge per unit length.

Since the electric field is the gradient of the potential, the longitudinal space charge force is proportional to the derivative of λ.

If the energy gain accumulates

Generalized for the non-cylindrical geometry, the accumulated energy spread is given by "Joho formula" (see W. Joho, Int. Cyclotron Conf. 1981):

$$\Delta U_{\mathrm{SC}} = (2800\,\Omega)\hat{I}\,\frac{n^2}{\beta_{\max}}.$$

Since the energy gain per turn $V = \Delta E/n$ must exceed this, we find that the current capability is proportional to the cube of the energy gain per turn $\boldsymbol{V^3}$.

If the energy gain accumulates

History of PSI show a good agreement with this scaling law, **even though the details of the space charge interaction do not match the original Joho analysis.**

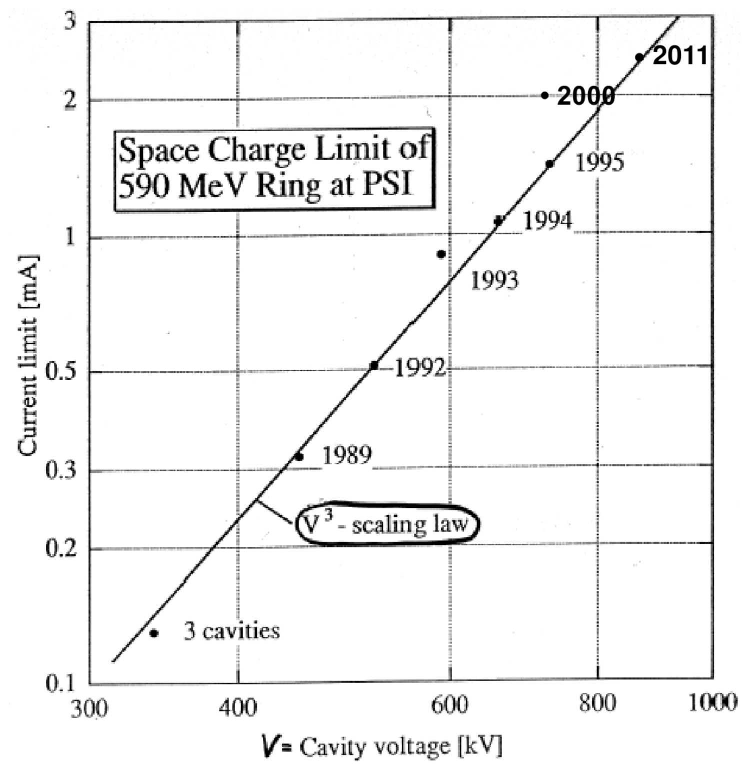


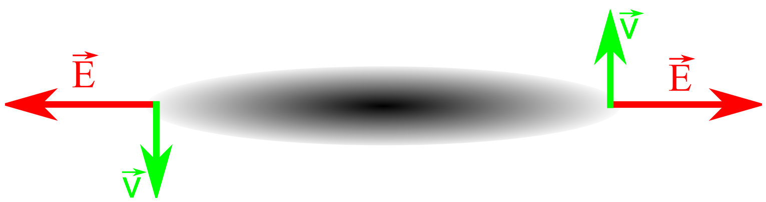
Fig. 1: From: W. Joho, PSI. (link to the talk it is taken from)

Longitudinal-radial effects

(1) **No phase stability**: energy gain accumulates.

(2) Because of the **non-zero dispersion**: longitudinally dependent radial motion.

But this is an incomplete picture!

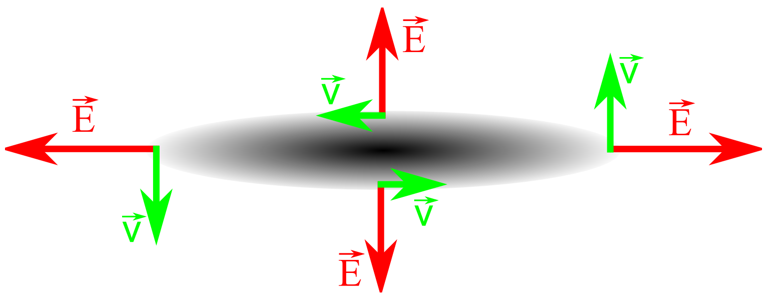


Longitudinal-radial effects

(1) **No phase stability**: energy gain accumulates.

(2) Because of the **non-zero dispersion**: longitudinally dependent radial motion.

(3) **Simplecticity**: it was realized during the late 1980's that the longitudinally dependent radial motion comes with and azimuthally dependent radial motion.



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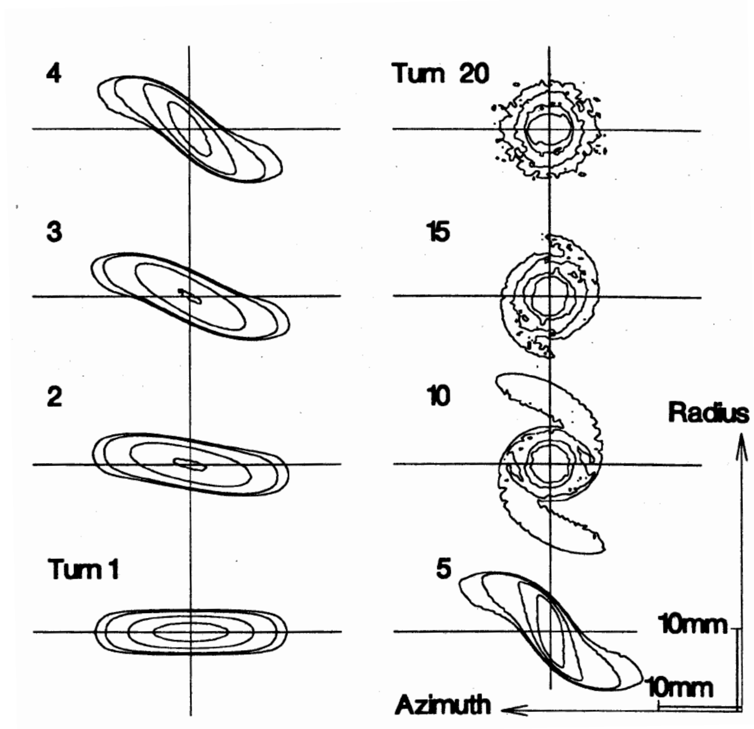


Fig. 2: `PICN` Simulation results (PSI injector II). From: S. Adam Int. Cyclotron Conf. 1995.

Stationary distribution

Important landmark: Kleeven Part. Acc, 1989, vol 24.: distribution which are cylindricaly symmetric are stationary.

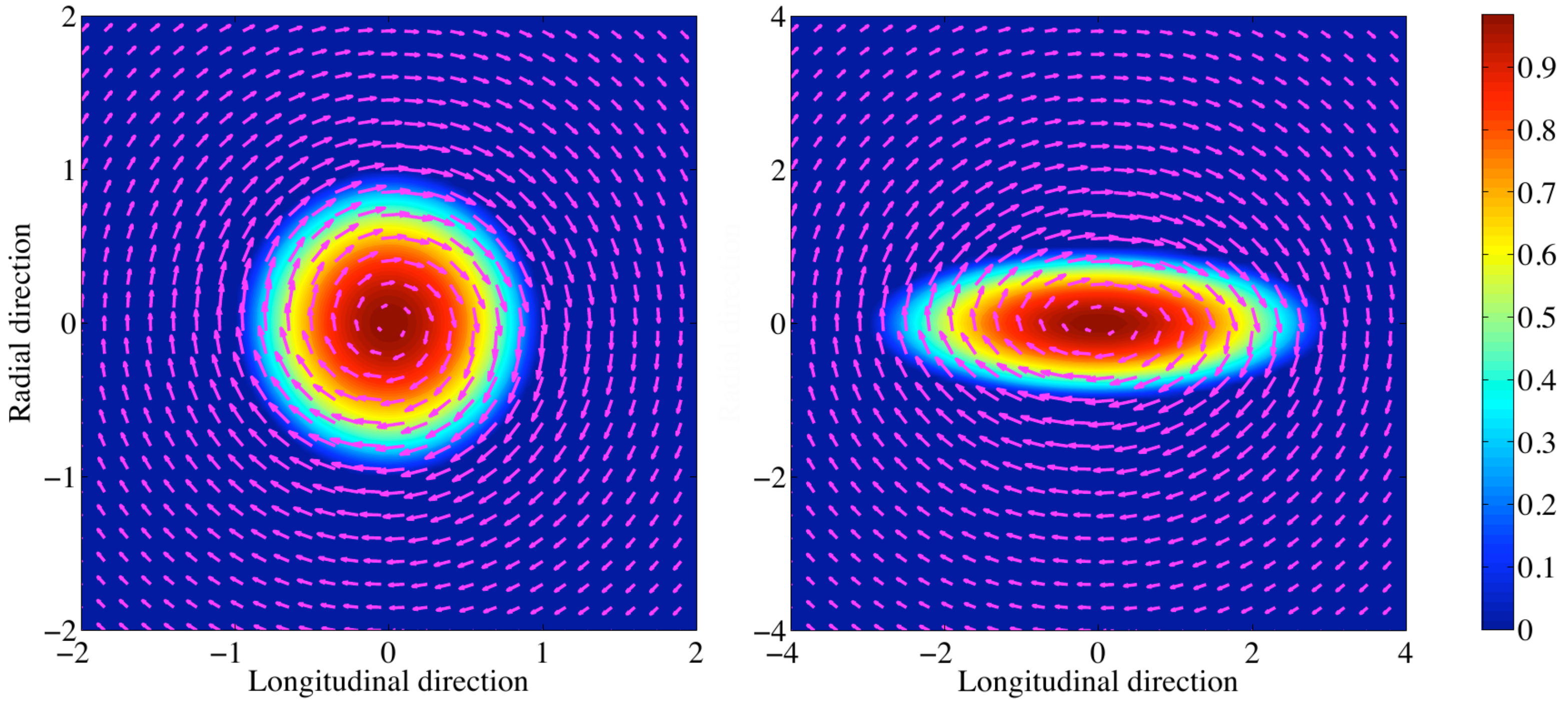


Fig. 3: Arrows show the particle velocity, in the beam frame. From A.J. Cerfon (NYU), J.P. Freidberg, F.I. Parra (MIT), after numerical resolution of the **Vlasov equation** (2012).

Fragmentation of long bunches

Long bunches tend to break up into several disks:

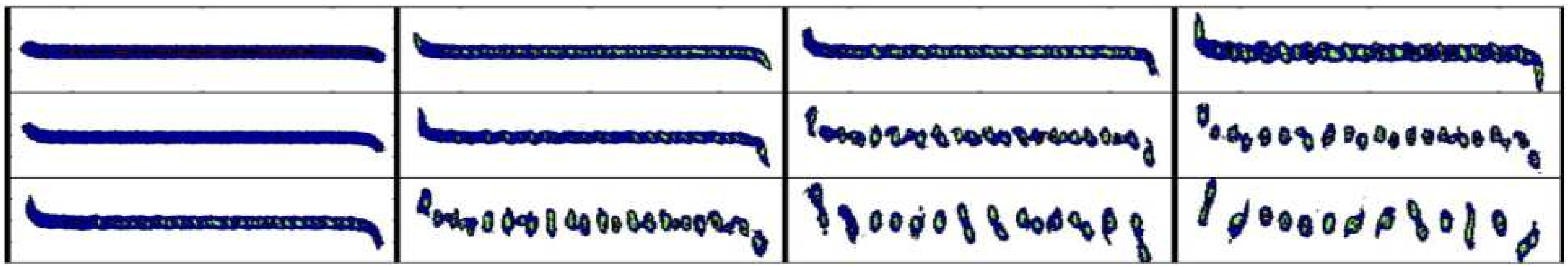


Fig. 4: Simulation results obtained with `CYCO` – a 3D PIC tracking code. From Pozdeyev, Rodriguez, Marti, Phys. Rev. ST AB 12 (2009).

Fragmentation of long bunches

Experimental evidence obtained with SIR (MSU):

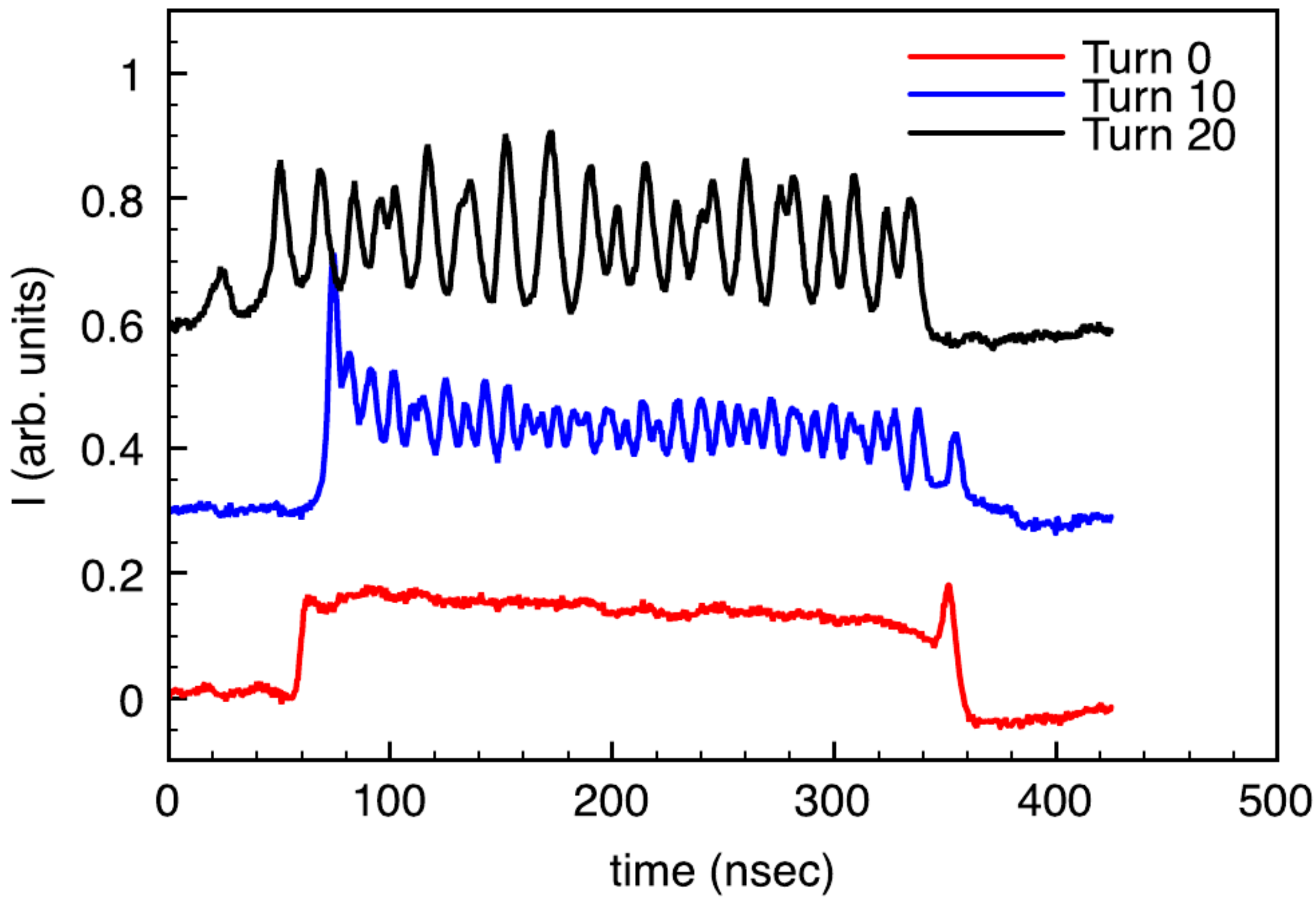


Fig. 5: From E. Pozdeyev, Phys. Rev. ST AB 12 (2009).

Effect of neighboring turns

Simulation of neighboring turns (multibunch tracking) carried out at PSI using the 3D particle in cell (PIC) parallel code `OPAL`.

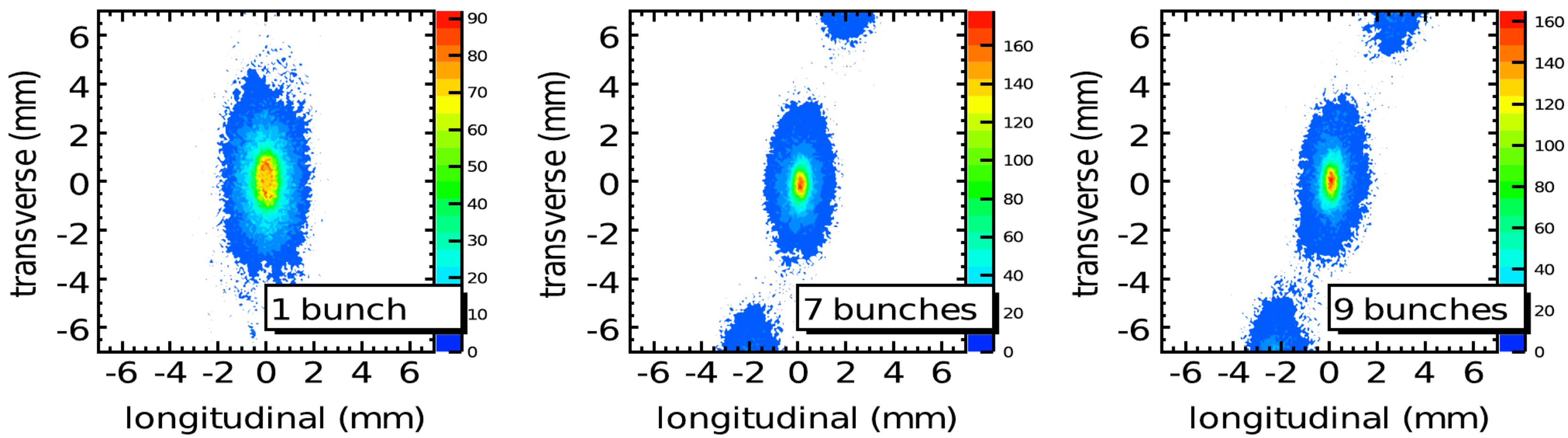


Fig. 6: Simulation results from `OPAL-CYCL` showing a bunch after 130 turns in the PSI ring cyclotron. Figure from: J.J. Yang, A. Adelmann et al. Phys. Rev. ST AB 13 (2010).

Longitudinal space charge

- For short bunches: we have got a stationary charge distribution. **How much current you can extract depends only on how well you can match the charge distribution to the stationary one at injection!**

- Is this distribution still stationary in the presence of **neighboring turns**, or for **long bunches**? Entering here a "gray zone" where things are not yet fully understood.

- The ultimate current limit of cyclotrons has not yet been found!

Space charge in the TRIUMF cyclotron – revisited.

◇ TRIUMF 500 MeV H^- cyclotron : no turn separation is required for extraction.

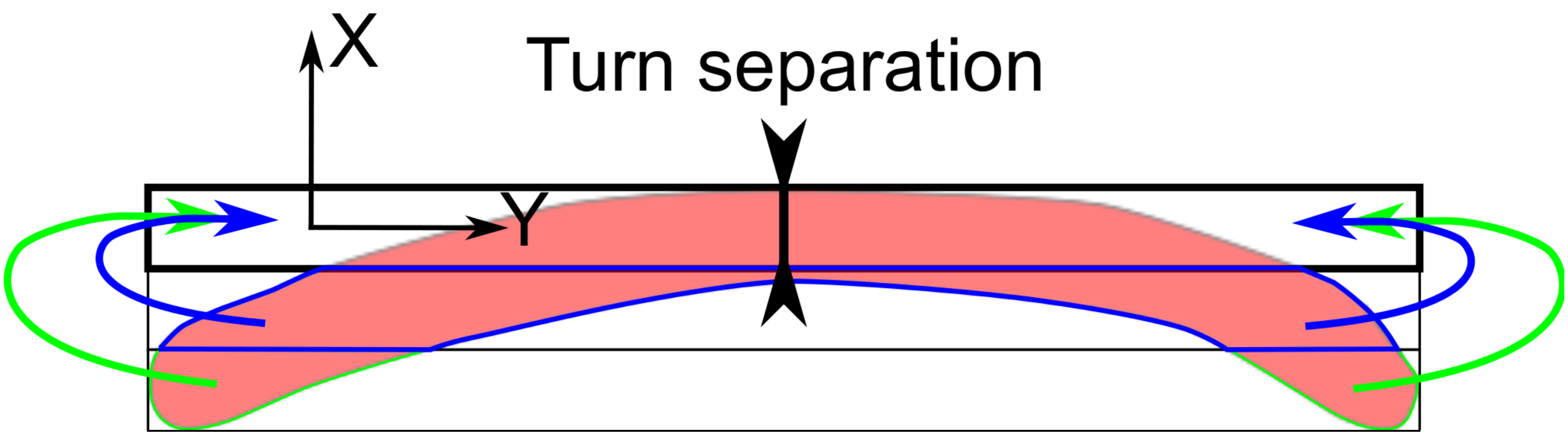
◇ Phase acceptance of about 60°.

◇ Bunches are very long, and have a very large energy spread between the head and the tail of a bunch.

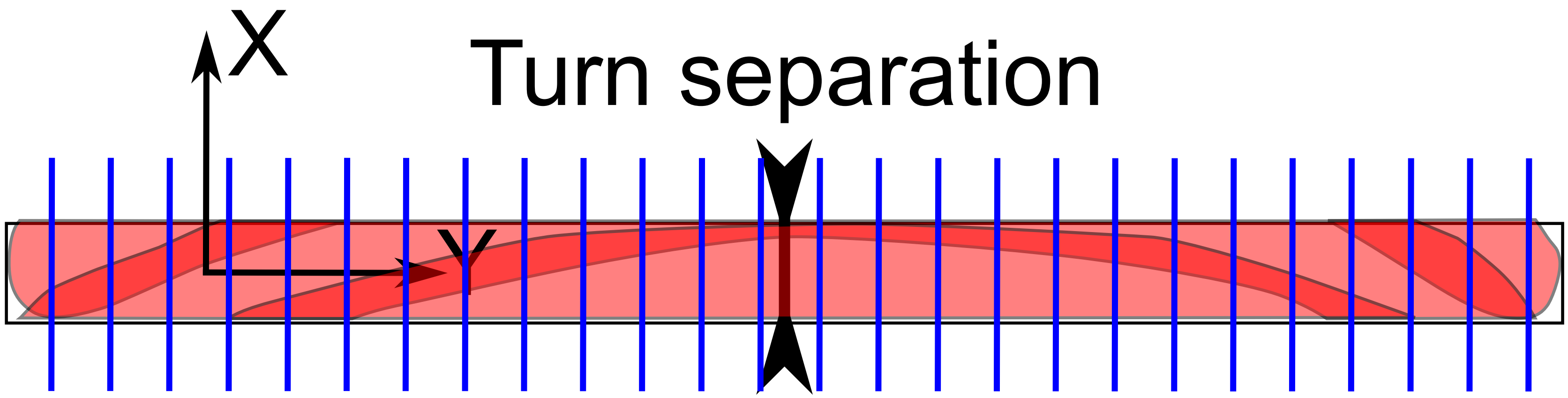
◇ Solving Poisson equation in a PIC code over such a large volume would require a significant computation time.

Radial periodicity

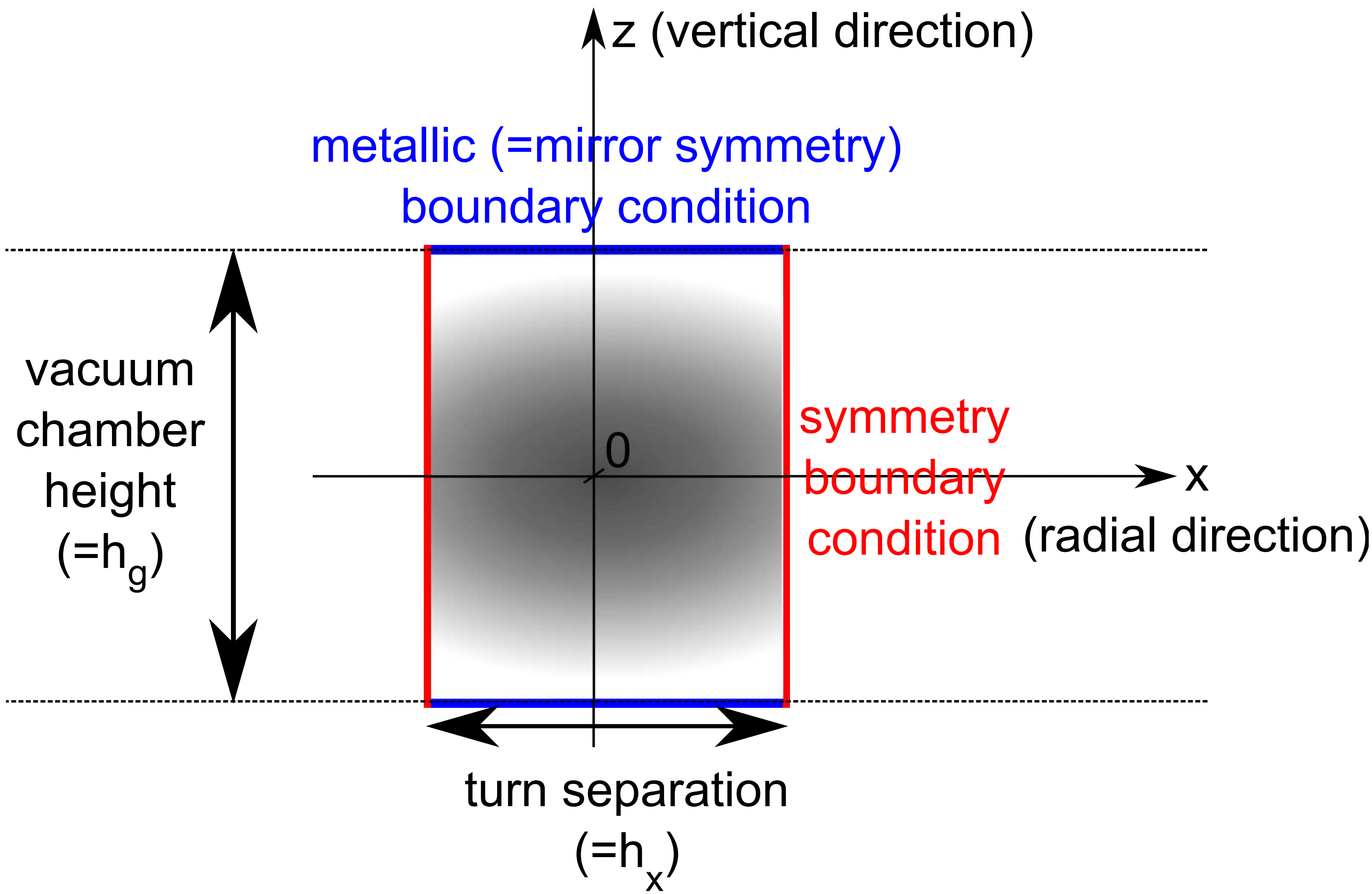
If one makes the assumption that the beam shape evolves slowly compared to the turn to turn time scale, one can significantly reduce the computation time by using periodic boundary conditions in the radial direction. This idea was originally proposed by Pozdeyev as a possible way to improve his code `CYCO`.



Radial periodicity



Algorithm


z (vertical direction)
metallic (=mirror symmetry)
boundary condition
vacuum
chamber
height
(=h_g)
0
symmetry
boundary
condition
x
(radial direction)
turn separation
(=h_x)

Algorithm

For each of these slices, the charge distribution can now be expressed as a sum of Fourier harmonics:

$$\sigma(x,z) = \sum_{lm} \sigma_{lm} \exp(i\omega_l x) \cos(\omega_m z),$$

with $\omega_l = 2\pi \frac{l}{h_x}$, and $\omega_m = 2\pi \frac{m}{2h_g}$.

Let's assume that the potential produced by the (l,m)–th harmonic can be written in the form:

$$\phi_{lm}(x,z,y) = \exp(i\omega_l x) \cos(\omega_m z) \cdot f_{lm}(y) + const.$$

Algorithm

Injecting this potential into Laplace equation leads for the (l, m)–th harmonic to:

$$\frac{\mathrm{d}^2 f_{lm}}{\mathrm{d}y^2} - \left(\omega_l^2 + \omega_m^2\right) f_{lm} = 0.$$

The general solution of this equation can be written as:

$$f_{lm} = C_1 \exp(\omega_{lm} y) + C_2 \exp(-\omega_{lm} y),$$

with C_1 and C_2 two real numbers, and $\omega_{lm} = \sqrt{\omega_l^2 + \omega_m^2}$.

For the potential to be bounded:

$$f_{lm} = C_{lm} \exp(-\omega_{lm} |y|).$$

Algorithm

To find the value of the constant C_{lm}, let's calculate the electric potential using the Coulomb law at $x = y = z = 0$ ($\omega_m \neq 0$):

$$\phi_{lm}(0,0,0) = C_{lm}$$

$$= \iint_{-\infty}^{+\infty} \frac{\sigma_{lm} \exp(i\omega_l x) \cos(\omega_m z)}{\sqrt{x^2 + z^2}} \mathrm{d}x \mathrm{d}z = 2\pi \frac{\sigma_{lm}}{\omega_{lm}}.$$

Algorithm

To sum up:

$$\sigma(x,z) \xrightarrow{FFT} \sigma_{lm} \xrightarrow{\quad \times \dfrac{2\pi}{\omega_{lm}} \mathrm{e}^{-\omega_{lm}|y|} \quad} \phi_{lm} \xrightarrow{FFT^{-1}} \phi(x,z)$$

Thanks!

Thank you for your attention!