Test of Optical Stochastic Cooling in Fermilab

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Contents

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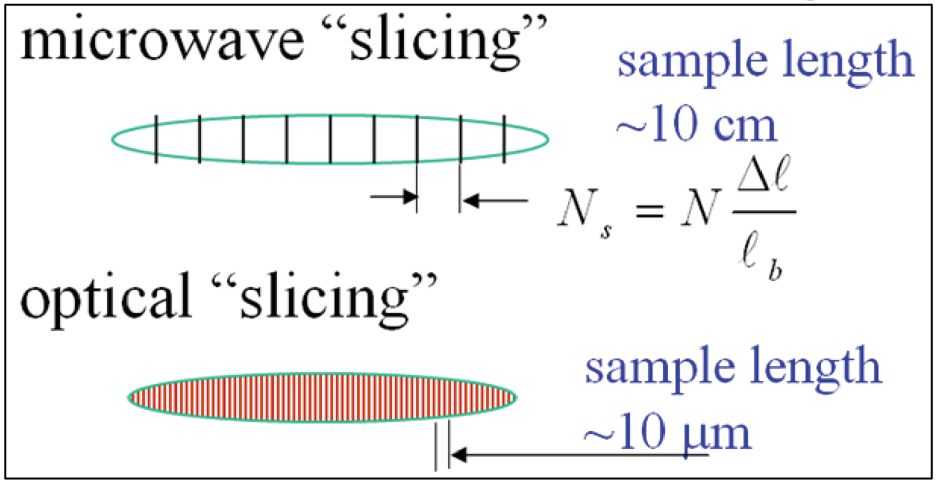
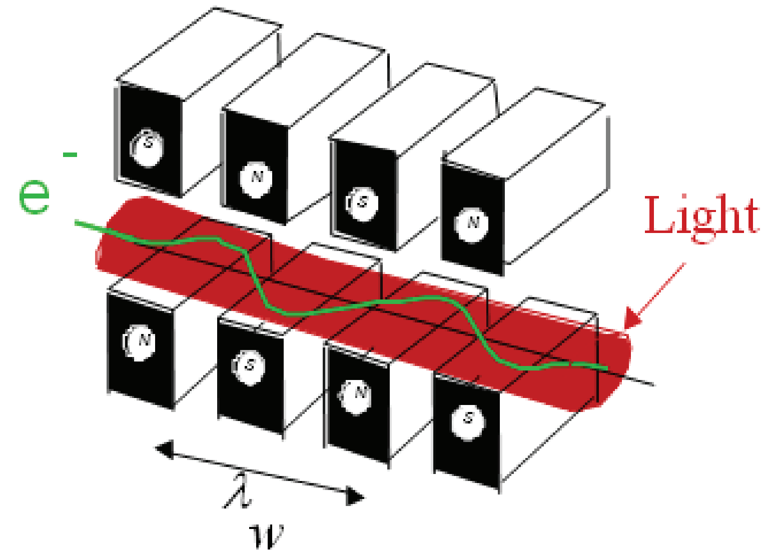
Principles of Optical Stochastic Cooling

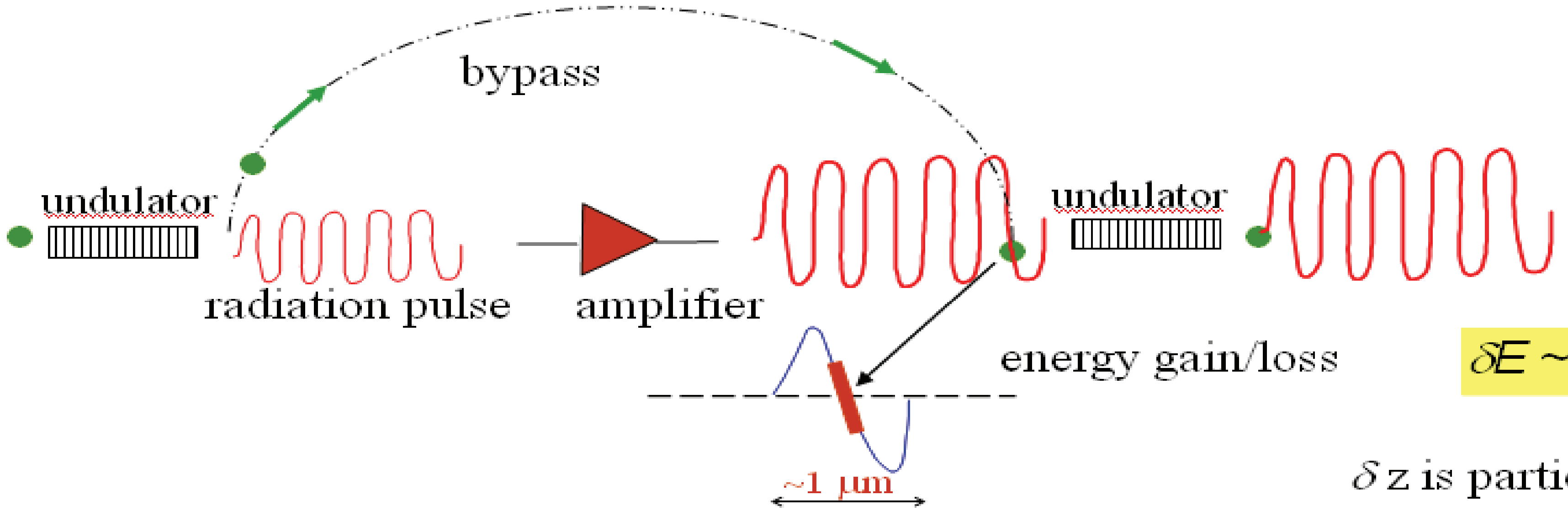
- Suggested by Zolotorev, Zholents and Mikhailichenko (1994)
- OSC obeys the same principles as the microwave stochastic cooling, but exploits the superior bandwidth of optical amplifiers ~ 10^{14} Hz
  - can deliver damping rates 4 orders of magnitude larger than usual (microwave) stochastic cooling



- Pickup and kicker must work in the optical range and support the same bandwidth as the amplifier
  - Undulators were suggested for both pickups and kickers
  - Undulator are effective for longitudinal kicks
  - Transverse kick is suppressed for ultra-relativistic beam $\left(\mathrm{F} = e\left(\mathbf{E} - [\boldsymbol{\beta}\mathbf{B}]\right) \xrightarrow{\beta\to c} 0\right)$



Principles of Optical Stochastic Cooling (continue)

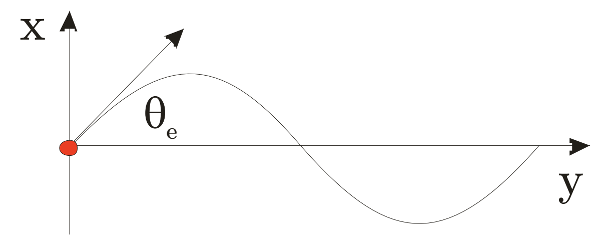


$\delta E \sim \sin(k\,\delta z)$

δz is particle delay

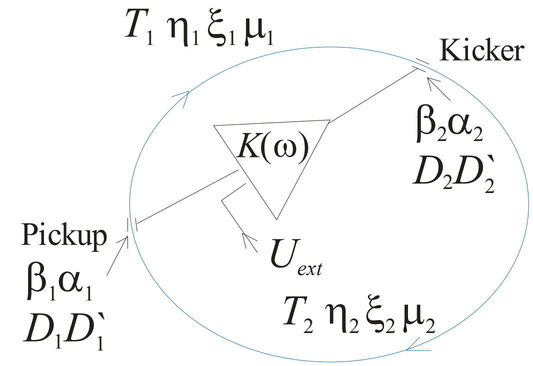
- Radiation wave length

$$\lambda = \frac{\lambda_{wgl}}{2\gamma^2}\begin{cases}\left(1+\gamma^2\left(\theta_e^{\,2}+\theta^2\right)\right) & -\ \textit{helical undulator}\\ \left(1+\gamma^2\left(\frac{1}{2}\theta_e^{\,2}+\theta^2\right)\right) & -\ \textit{flat undulator}\end{cases}$$



- To obtain transverse cooling one needs coupling between transverse and longitudinal degrees of freedom
  - It can be achieved by locating pick-up and kicker in positions with nonzero dispersion function

Basics of OSC - Damping Rates



- Pickup-to-Kicker Transfer Matrix
  - Vertical plane is uncoupled and we omit it

$$\mathbf{M}^{pk}=\begin{bmatrix} M_{11} & M_{12} & 0 & M_{16} \\ M_{21} & M_{22} & 0 & M_{26} \\ M_{51} & M_{52} & 1 & M_{56} \\ 0 & 0 & 0 & 1 \end{bmatrix},\quad \mathbf{x}=\begin{bmatrix} x \\ \theta_x \\ s \\ \Delta p/p \end{bmatrix}$$

M^{pk} - pickup-to-kicker matrix
M^{kp} - kicker-to-pickup matrix
$M = M^{pk}M^{kp}$ – ring matrix

  - Symplecticity ($\mathbf{M}^T\mathbf{U}\,\mathbf{M}=\mathbf{U}$) binds up M_{51},M_{52} and M_{16},M_{26}
  - All matrix elements can be expressed through $\beta,\alpha,D,D',\eta\,(\text{or}\,\eta_1)$
- Partial momentum compaction (slip factor) is related to M_{56} ($\mathrm{v}=c$)

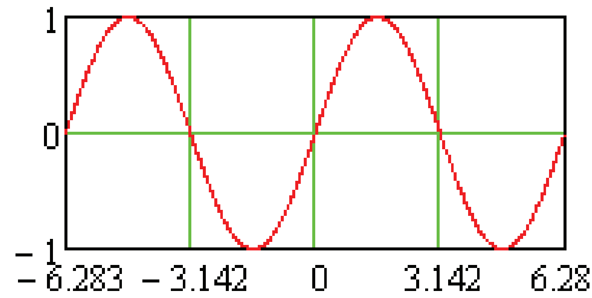
$$-\eta_1=-\alpha_{1\to 2}=\frac{M_{51}^{pk}D_1+M_{52}^{pk}D_1'+M_{56}^{pk}}{2\pi R}$$

  - M_{56} is positive if a particle with $+\Delta p$ moves faster than the ref. particle
- Linearized longitudinal kick: $\displaystyle\frac{\delta p}{p}=\kappa\,\Delta s=\kappa\left(M_{51}^{pk}x_1+M_{52}^{pk}\theta_{x1}+M_{56}^{pk}\frac{\Delta p}{p}\right)$
- Perturbation theory for symplectic undamped motion yields cooling rates

$$\boxed{\begin{aligned}\lambda_1\equiv\lambda_x&=-\frac{\kappa}{2}\left(2\pi R\eta_1+M_{56}^{pk}\right)\\ \lambda_2\equiv\lambda_s&=\pi\kappa R\eta_1\end{aligned}}\quad\Leftrightarrow\quad\boxed{\lambda_x+\lambda_s=-\frac{\kappa}{2}M_{56}^{pk}}$$

Basics of OSC - Cooling Range

- The cooling force depends on Δs nonlinearly



$$\frac{\delta p}{p}=\frac{\Delta E_{max}}{E}\sin(k\delta s)=\frac{\Delta E_{max}}{E}\sin\left(a_x\sin(\psi_x)+a_p\sin(\psi_p)\right)$$

where a_x & a_p are the lengthening amplitudes due to $\perp$ and L motions measured in units of laser phase ($a = k\,\delta s$)

- The form-factors for damping rate

$$\boxed{\lambda_1(a_x,a_p)=F_1(a_x,a_p)\lambda_1 \quad , \qquad \lambda_2(a_x,a_p)=F_2(a_x,a_p)\lambda_2}$$

- For L cooling for particle with amplitudes a_x & a_p

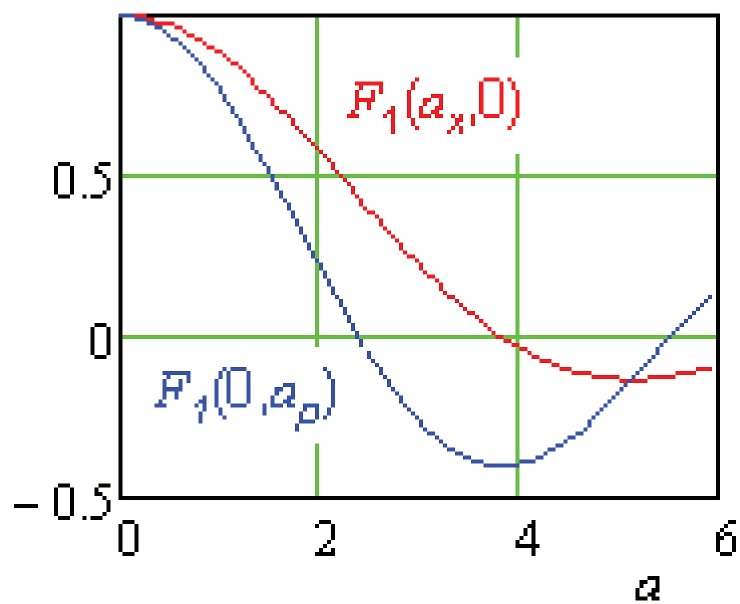
$$F_2(a_x,a_p)=\frac{2}{a_p}\oint\sin\left(a_x\sin\psi_x+a_p\sin\psi_p\right)\sin\psi_p\frac{d\psi_x}{2\pi}\frac{d\psi_p}{2\pi}$$

$\Rightarrow$ $$\boxed{F_2(a_x,a_p)=\frac{2}{a_p}J_0(a_x)J_1(a_p)}$$

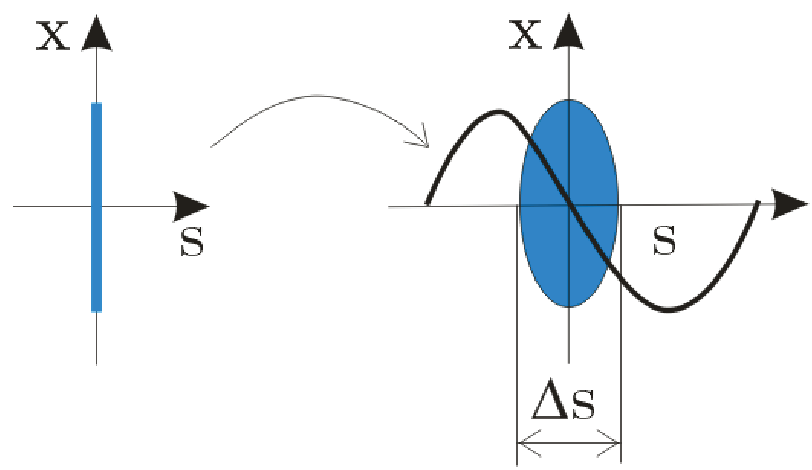
- Similar for transverse motion

$\Rightarrow$ $$\boxed{F_1(a_x,a_p)=\frac{2}{a_x}J_0(a_p)J_1(a_x)}$$

- Damping requires both lengthening amplitudes be smaller than $\mu_0 \approx 2.405$



Basics of OSC - Sample Lengthening on Pickup-to-Kicker Travel



- Zero length sample lengthens on its way from pickup-to-kicker

  $$\sigma_{\Delta s}{}^2 = \int \left(M_{1_{51}} x + M_{1_{52}} \theta_x + M_{1_{56}} \tilde{p} \right)^2 f\left(x, \theta_x, \tilde{p}\right) dx d\theta_x d\tilde{p}$$

  where $\tilde{p} = \Delta p / p$

  - Performing integration one obtains for Gaussian distribution

$$\sigma_{\Delta s}{}^2 = \sigma_{\Delta s\varepsilon}{}^2 + \sigma_{\Delta sp}{}^2$$

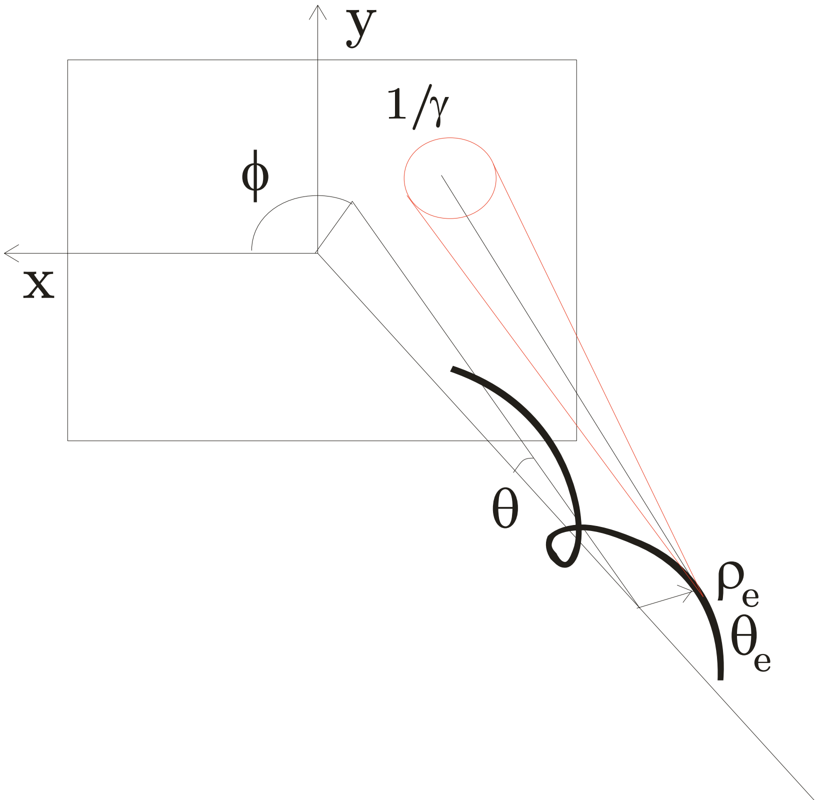
$$\sigma_{\Delta s\varepsilon}{}^2 = \varepsilon \left(\beta_p M_{51}{}^2 - 2\alpha_p M_{51} M_{52} + \gamma_p M_{52}{}^2 \right)$$

$$\sigma_{\Delta sp}{}^2 = \sigma_p{}^2 \left(M_{51} D_p + M_{52} D'_p + M_{56} \right)^2$$

  - Both Δp/p and ε contribute to the lengthening
- While in linear approximation β_p and α_p do not affect damping rates they affect sample lengthening and, consequently the cooling range

$$\sigma_{\Delta s\varepsilon} k \le \mu_0 \qquad \mu_0 \approx 2.405$$
$$\sigma_{\Delta sp} k \le \mu_0$$

Basics of OSC – Radiation from Undulator



- Liénard-Wiechert potentials and E-field of moving charge in wave zone

$$\begin{cases} \varphi(\mathbf{r},t) = \left.\dfrac{e}{\left(R - \boldsymbol{\beta}\cdot\mathbf{R}\right)}\right|_{t-R/c} \\ \mathbf{A}(\mathbf{r},t) = \left.\dfrac{e\mathbf{v}}{\left(R - \boldsymbol{\beta}\cdot\mathbf{R}\right)}\right|_{t-R/c} \end{cases} \Rightarrow$$

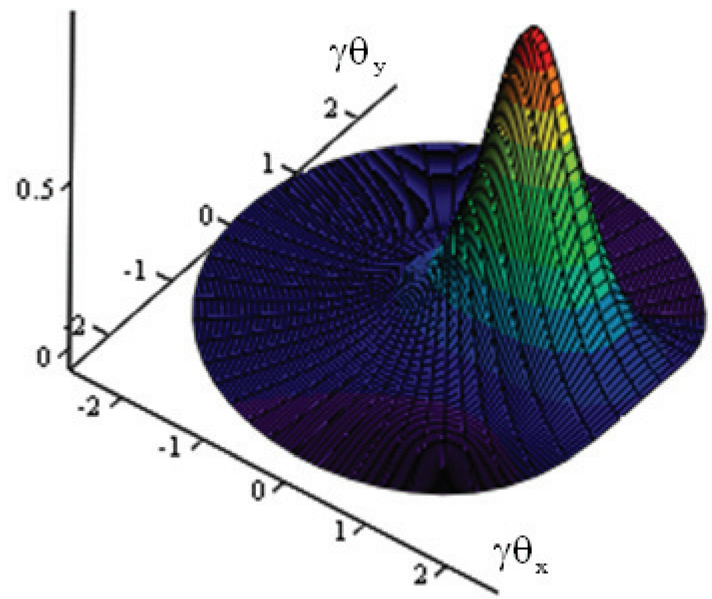
$$\mathbf{E}(\mathbf{r},t) = \frac{e}{c^2}\left.\frac{\left(\mathbf{R}-\boldsymbol{\beta}\cdot R\right)\left(\mathbf{a}\cdot\mathbf{R}\right) - \mathbf{a}R\left(R - \boldsymbol{\beta}\cdot\mathbf{R}\right)}{\left(R-\boldsymbol{\beta}\cdot\mathbf{R}\right)^3}\right|_{t-R/c}$$

where $\mathbf{a} \equiv \dfrac{d\mathbf{v}}{dt}$

- Radiation of ultra-relativistic particle is concentrated in 1/γ angle
- Undulator parameter:

$$K \equiv \gamma\theta_e = \frac{\lambda_{wgl}}{2\pi}\frac{eB_0}{mc^2}$$

- For K ≥ 1 the radiation is mainly radiated into higher harmonics

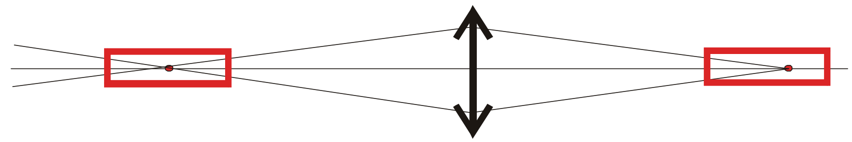


E_x for K=1

Basics of OSC – Radiation Focusing to Kicker Undulator

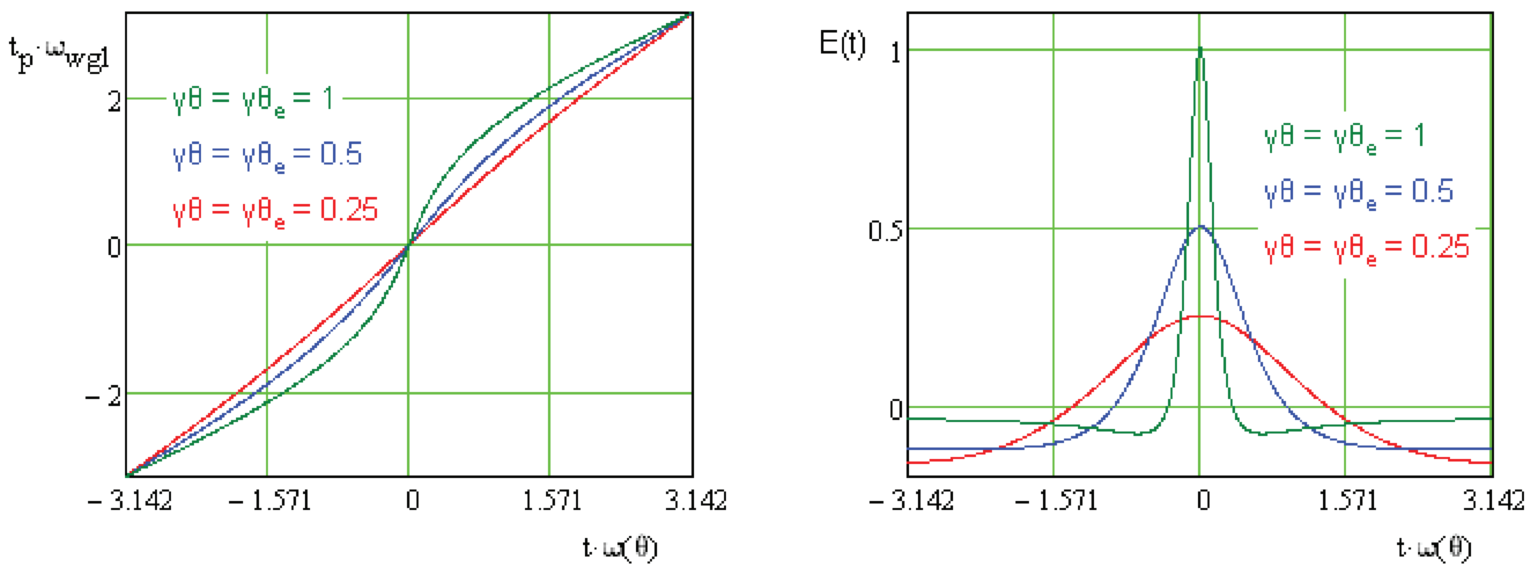
- Modified Kirchhoff formula

$$E(r)=\frac{\omega}{2\pi i c}\int_S \frac{E(r')}{|r-r'|}\mathrm{e}^{i\omega|r-r'|}\,ds'$$



=> $$E(r)=\frac{1}{2\pi i c}\int_S \frac{\omega(r')E(r')}{|r-r'|}\mathrm{e}^{i\omega|r-r'|}\,ds'$$

- Effect of higher harmonics
  - Higher harmonics are normally located outside window of optical lens transparency and are absorbed in the lens material

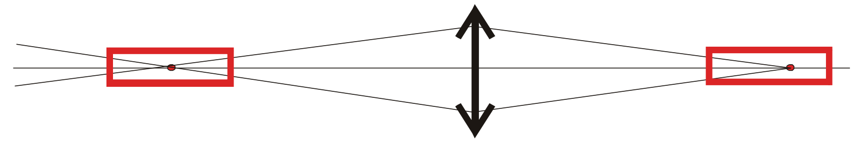


Dependences of retarded time (t_p) and E_x on time for helical undulator

- Only first harmonic is retained in the calculations presented below

Basics of OSC – Longitudinal Kick for K<<1

- For $K << 1$ refocused radiation of pickup undulator has the same structure as radiation from kicker undulator. They are added coherently:

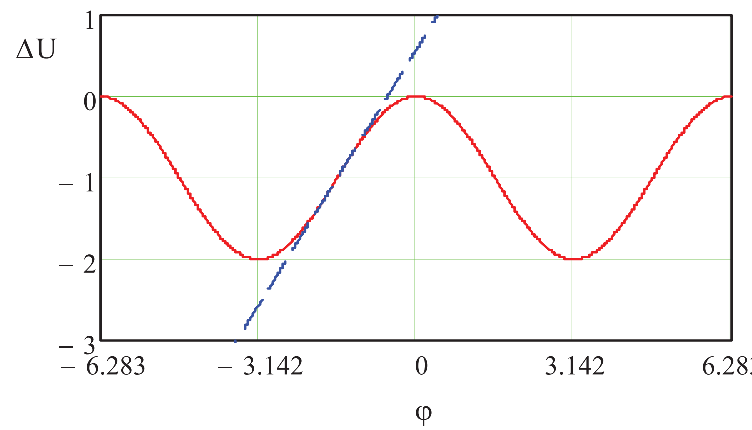


$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 e^{i\phi} \xrightarrow{\mathbf{E}_1 = \mathbf{E}_2} 2\cos\left(\phi/2\right)\mathbf{E}_1 e^{i\phi/2}$$

⇨ Energy loss after passing 2 undulators

$$\Delta U \propto \left|E^2\right| = 4\cos\left(\phi/2\right)^2 \left|\mathbf{E}_1{}^2\right| = 2\left(1+\cos\phi\right)\left|\mathbf{E}_1{}^2\right| = 2\left(1+\cos\left(kM_{56}\frac{\Delta p}{p}\right)\right)\left|\mathbf{E}_1{}^2\right|$$

- Large derivative of energy loss on momentum amplifies damping rates and creates a possibility to achieve damping without optical amplifier



  - SR damping: $\lambda_{\|\_SR} \approx \dfrac{2\Delta U_{SR}}{pc} f_0$
  - OSC: $\lambda_{\|\_OSC} \approx f_0 \dfrac{2\Delta U_{wgl}}{pc}\left(GkM_{56}\right) \xrightarrow{kM_{56}(\Delta p/p)_{\max} = \pi} f_0 \dfrac{2\Delta U_{wgl}}{pc}\left(\dfrac{G}{\left(\Delta p/p\right)_{\max}}\right)$

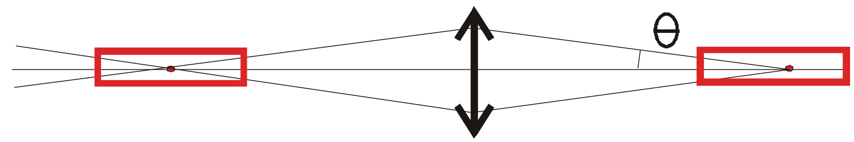
    where G - optical amplifier gain, $(\Delta p/p)_{max}$ - cooling system acceptance

⇨ $\lambda_{\|\_OSC} \propto B^2 L \propto K^2 L$ - but cooling efficiency drops with K increase above ~1

Basics of OSC – Longitudinal Kick for K<<1(continue)

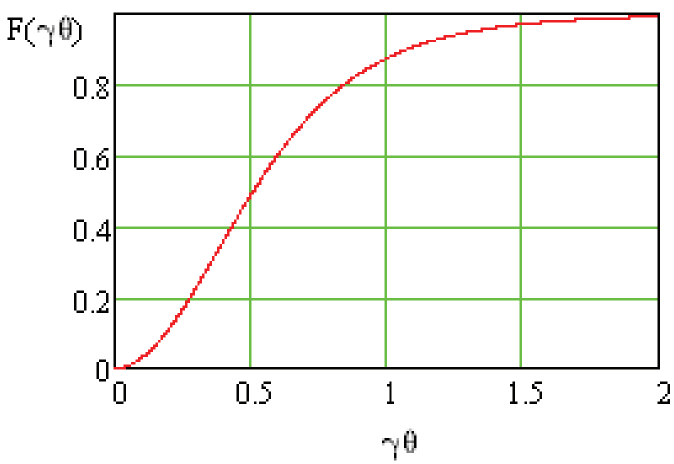
- Radiation wavelength depends on θ as

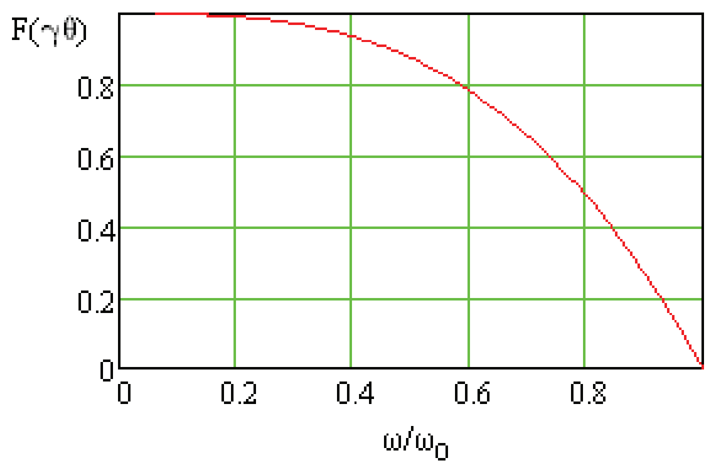
$$\lambda = \frac{\lambda}{2\gamma^2}\left(1+\gamma^2\theta^2\right)$$



Limitation of system bandwidth by (1) optical amplifier band or (2) subtended angle reduce damping rate

$$\lambda_{\|\_SR} = \lambda_{\|\_SR0} F\left(\gamma\theta_{\mathrm{m}}\right), \qquad F\left(x\right) = 1 - \frac{1}{\left(1+x^2\right)^3}$$





- For narrow band: $\Delta U_{wgl} = \Delta U_{wgl0}\left(\frac{3\Delta\omega}{\omega}\right), \quad \frac{3\Delta\omega}{\omega} << 1$

where $\Delta U_{wgl0} = \frac{e^4 B^2 \gamma^2 L}{3m^2c^4}\begin{cases} 1, & \text{Flat wiggler} \\ 2, & \text{Helical wiggler}\end{cases}$ the energy radiated in one undulator

Basics of OSC – Radiation from Flat Undulator

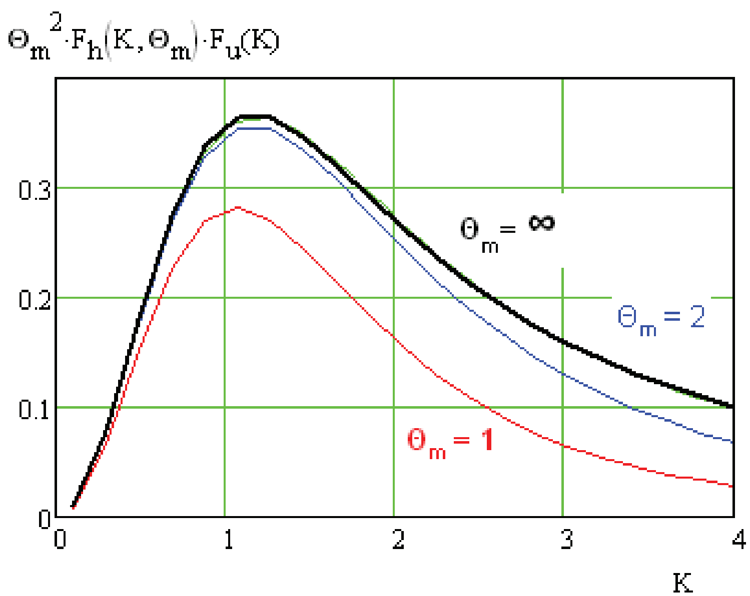
- For arbitrary undulator parameter we have

$$\Delta U_{OSC\_F} = \frac{1}{2}\frac{4e^4 B_0^{\,2}\gamma^2 L}{3m^2c^4} GF_f\left(K,\gamma\theta_{\max}\right)F_u\left(\kappa_u\right)$$

$$F_u\left(\kappa_u\right) = \mathrm{J}_0\left(\kappa_u\right) - \mathrm{J}_1\left(\kappa_u\right), \quad \kappa_u = K^2 / \left(4\left(1+K^2/2\right)\right)$$

Fitting results of numerical integration yields:

$$F_h\left(K,\infty\right) \approx \frac{1}{1+1.07K^2+0.11K^3+0.36K^4}, \quad K \equiv \gamma\theta_e \le 4$$



- Dependence of wave length on θ:

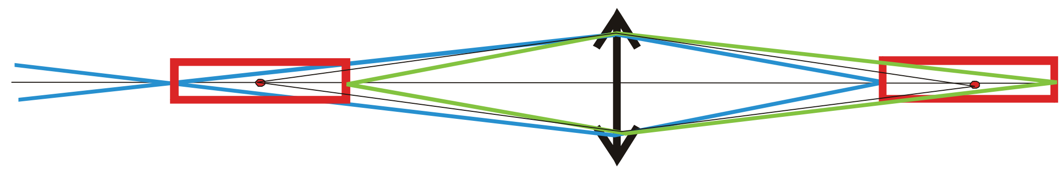
$$\lambda \approx \frac{\lambda_{wgl}}{2\gamma^2}\left(1+\gamma^2\left(\theta^2+\frac{\theta_e^{\,2}}{2}\right)\right)$$

$$K \equiv \gamma\theta_e$$

- Flat undulator is "more effective" than the helical one
- For the same K and λ_{wgl} flat undulator generates shorter wave lengths

- For both cases of the flat and helical undulators and for fixed B a decrease of λ_{wgl} and, consequently, λ yields kick increase
  - but wavelength is limited by both beam optics and light focusing

Basics of OSC – Correction of the Depth of Field

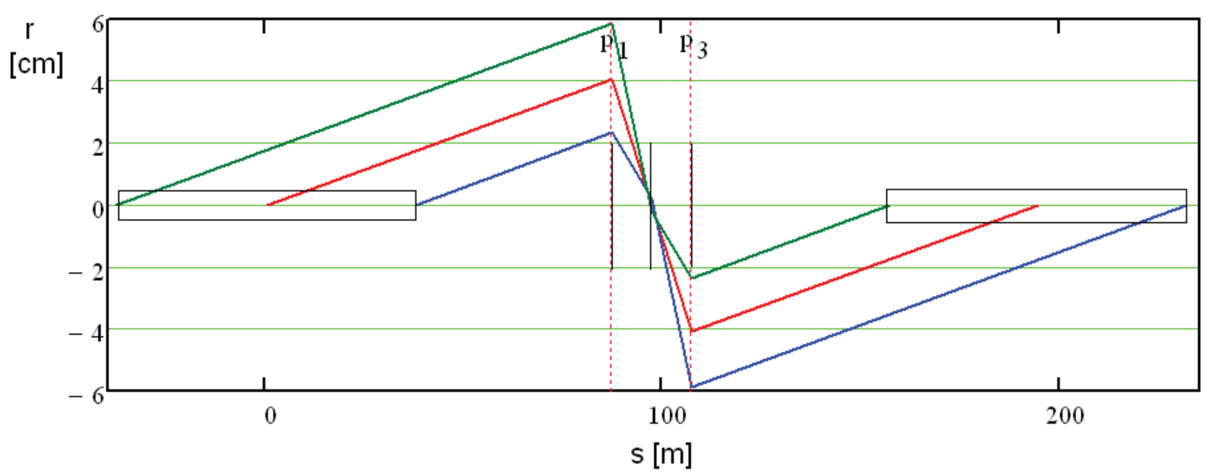


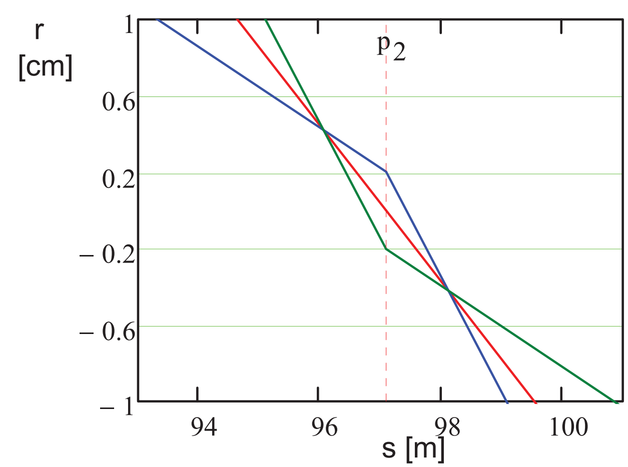
- It was implied above that the radiation coming out of the pickup undulator is focused on the particle during its trip through the kicker undulator
  - It can be achieved with lens located at infinity

$$\frac{1}{2F+\Delta s}+\frac{1}{2F-\Delta s}=\frac{1}{F}\quad\rightarrow\quad\frac{1}{F-\Delta s^2/4F}=\frac{1}{F}\quad\xrightarrow{F\rightarrow\infty}\quad\frac{1}{F}=\frac{1}{F}$$

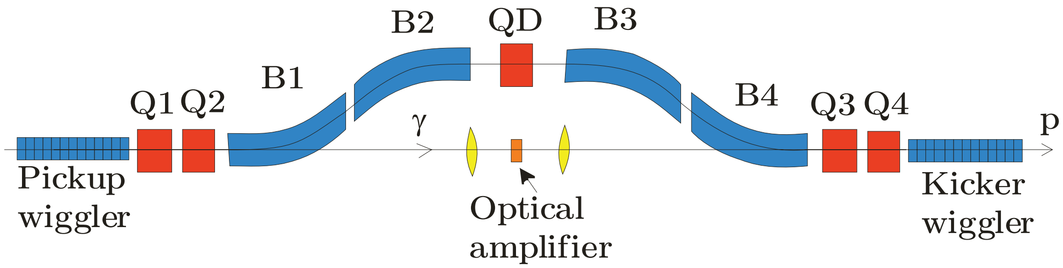
  - but this arrangement cannot be used in practice
- *A* 3-lens telescope can address the problem within limited space

$$\begin{bmatrix}1 & L\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\-F_1^{-1} & 1\end{bmatrix}\begin{bmatrix}1 & L_1\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\-F_2^{-1} & 1\end{bmatrix}\begin{bmatrix}1 & L_1\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\-F_1^{-1} & 1\end{bmatrix}\begin{bmatrix}1 & L\\0 & 1\end{bmatrix}=\begin{bmatrix}-1 & 0\\0 & -1\end{bmatrix}$$

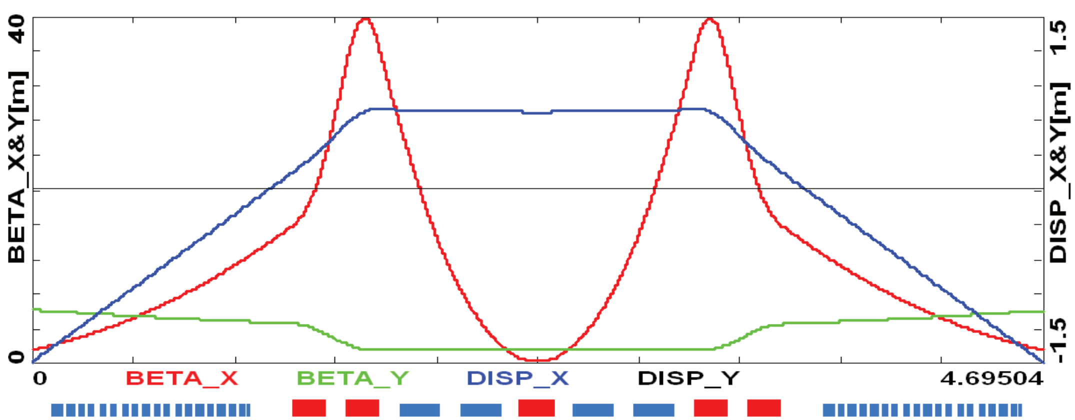




IOTA Beam Optics



- Optics is optimized for 800 nm wavelength
- Large ratio of cooling rates to get reasonably large cooling ranges
- Large sensitivity to optics errors



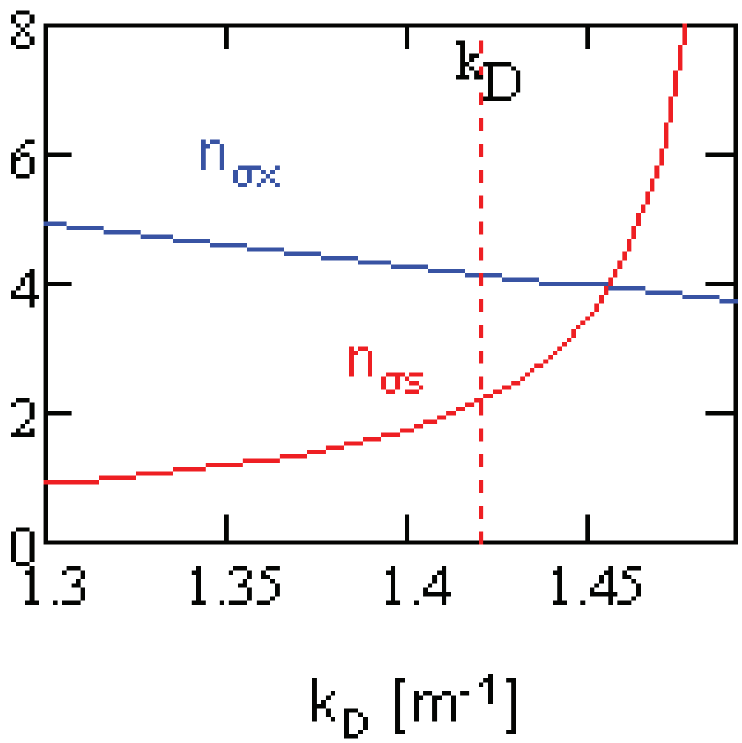
Major parameters of chicane optics

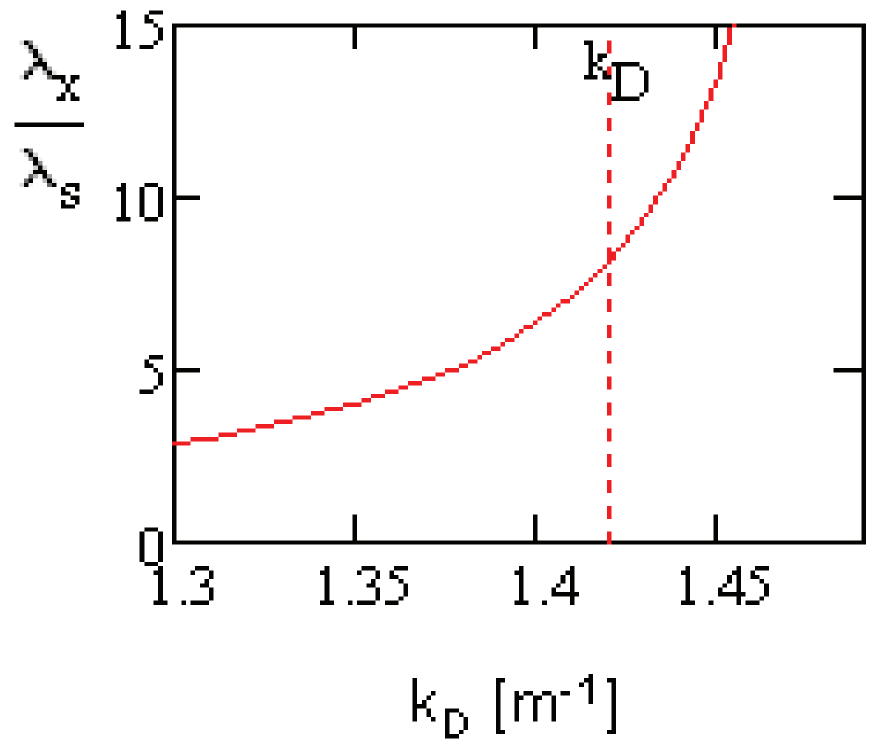
M_{56}	8.7 mm
Cooling rates ratio, λ_x/λ_s	7.5
Hor. beam separation	40 mm
Delay in the chicane	4.5 mm
Cooling ranges (no OSC)	3.5 / 2
Dipole magnetic field	4 kG
Dipole length	18 cm
GdL of central quad	1.52 kG

Main Parameters of IOTA storage ring

Circumference	38.7 m
Beam energy	150 MeV
Bending field	7.2 kG
Betatron tune	3.5 ÷ 7.2
Max. β-function	3 ÷ 9 m
Emittance, rms	3 nm
Rms $\Delta p/p$, σ_p	$1.5\cdot10^{-4}$
SR damp.rate (ampl.) $\lambda_s/\lambda_\perp$	4 / 2 s^{-1}

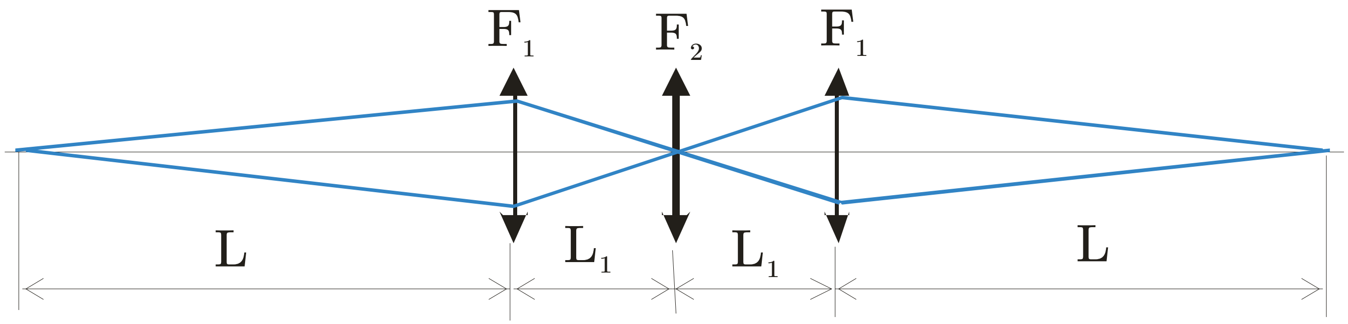
Beam Optics Sensitivity to errors





Dependencies of cooling ranges (left) and ratio of damping rates on focusing strength of central quad.

Main parameters of OSC

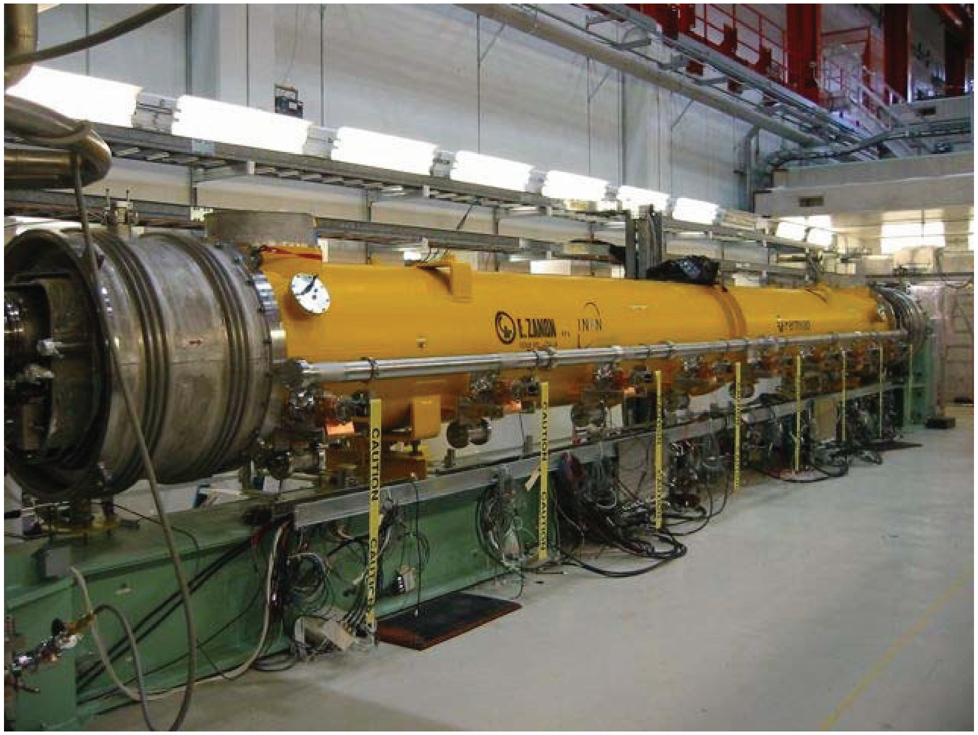


Light optics layout for passive cooling

Undulator parameter, K	1.5
Undulator period, $2\pi c/\omega_u$	6.53 cm
Number of periods, *m*	14
Total undulator length, L_w	0.915 m
Distance between undulators	3.6 m
Telescope length, $2L_1$	0.25 m
Telescope aperture, $2a$	40 mm
Lens focal distances, F_1 / F_2	116 / 4.3 mm
Damping rates of passive OSC(x/y/s)	100/100/25 s^{-1}
Damp. rates 20 Db gain & 10% band	300/300/75 s^{-1}

IOTA – Test ring for Non-Linear Optics and Optical Stochastic Cooling

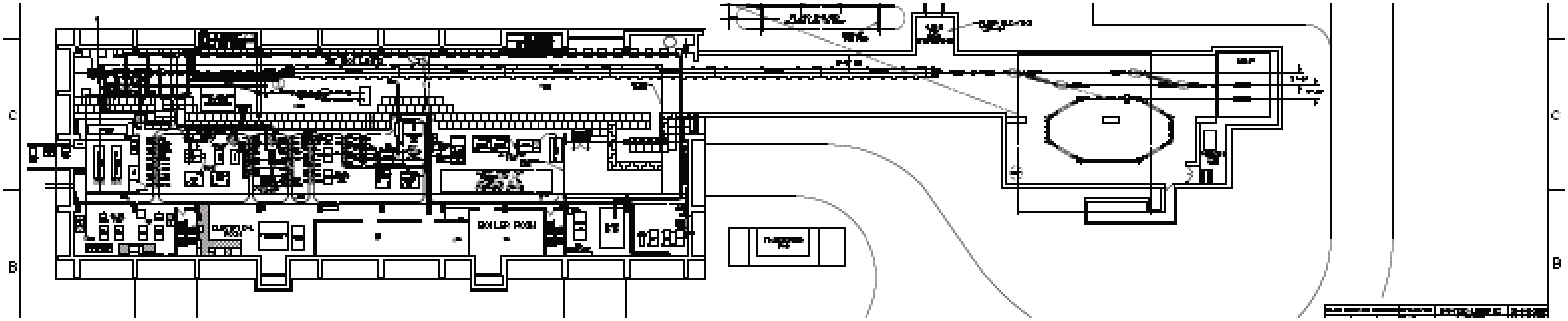
- Small test ring in NML building
- It is planned to test both OSC scenarios: with and without optical amplifier
- ASTA injector (~20 MeV) would be sufficient for filling the ring

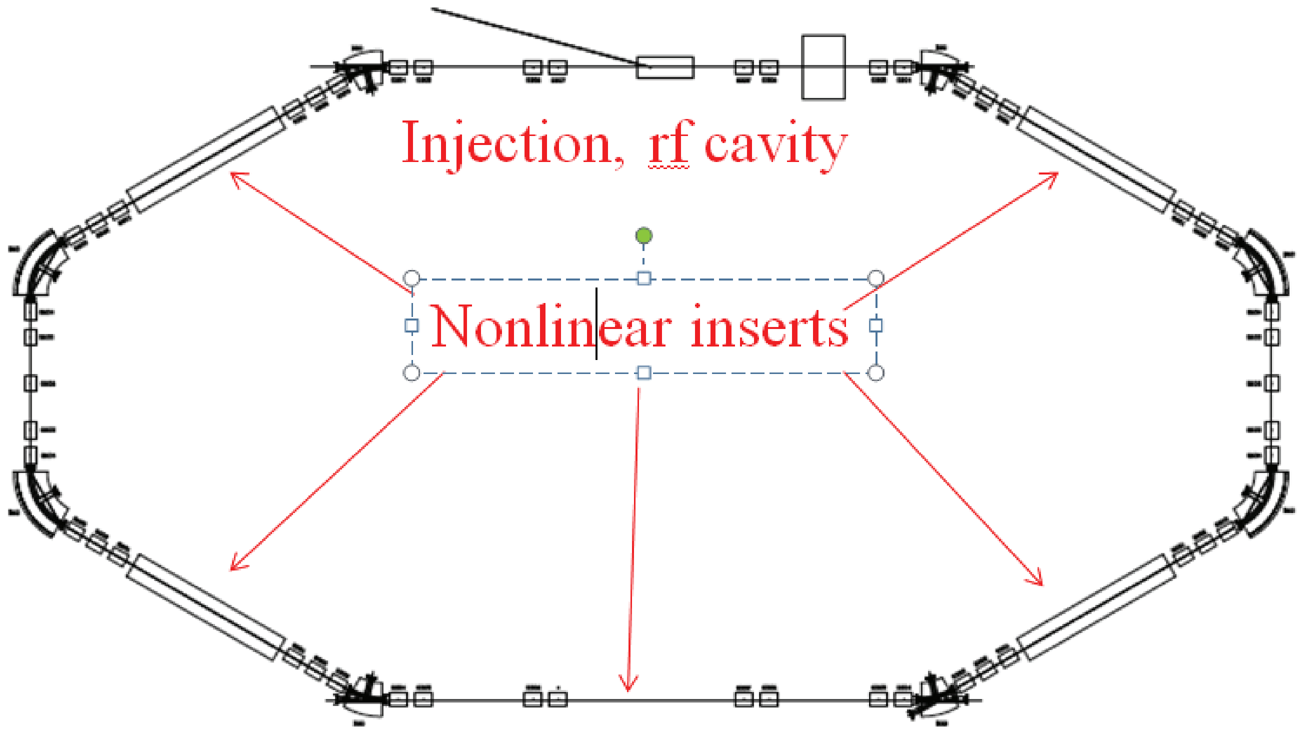


SC 1.3 GHz linac

Empty room for IOTA

IOTA Schematic and Main Parameters

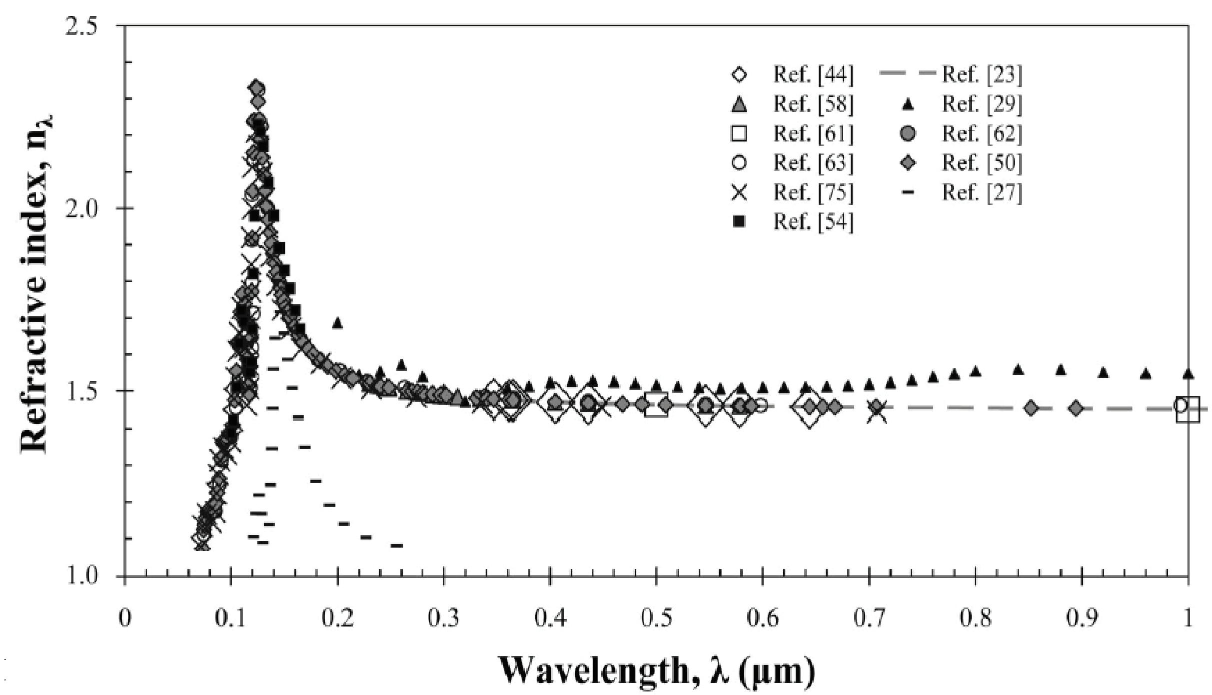




- pc = 150 MeV, electrons (single bunch, $5 \cdot 10^8$)
- ~36 m circumference
- One of non-linear inserts will be replaced with OSC section

Conclusions

- Optical stochastic cooling looks as a promising technique for the LHC
  - It would allow well controlled luminosity leveling and
  - Potentially can double its average luminosity
- Experimental study of OSC is planned in Fermilab
  - It is aimed to validate cooling principles and to demonstrate cooling with and without optical amplifier
  - More work is required to clarify details of the cooling scheme and formulate a technical proposal
  - Chromaticity in the light optics looks as the major problem
    - Combination of glasses with normal and abnormal dispersion looks as a possible way to address the problem



Backup Slides

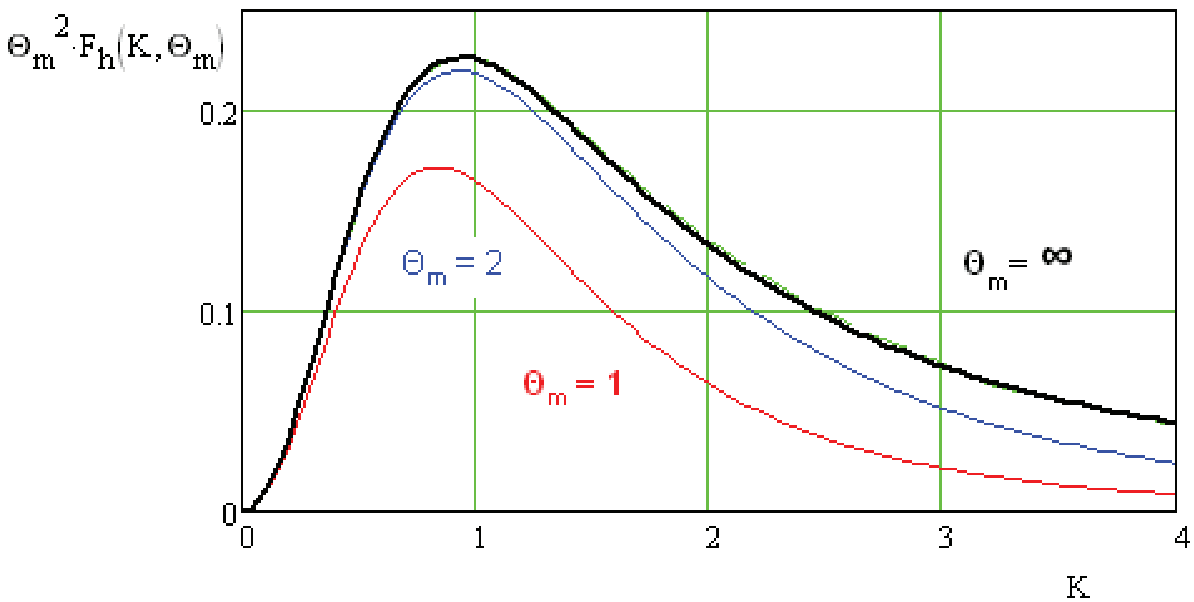
Basics of OSC – Radiation from Helical Undulator

- Assuming that the lens is "located at infinity" and only first harmonic of undulator radiation contributes to the electric field at the focal point one obtains the total kick value:

$$\Delta U_{OSC\_H} = \frac{4e^4 B^2 \gamma^2 L}{3m^2 c^4} G F_h\left(K, \gamma\theta_{\max}\right) \propto K^2 F_h\left(K, \gamma\theta_{\max}\right)$$

Fitting of numerical integration yields:

$$F_h\left(K, \infty\right) \approx \frac{1}{1 + 2.15K^2 + 1.28K^4}, \quad K \equiv \gamma\theta_e \le 4$$



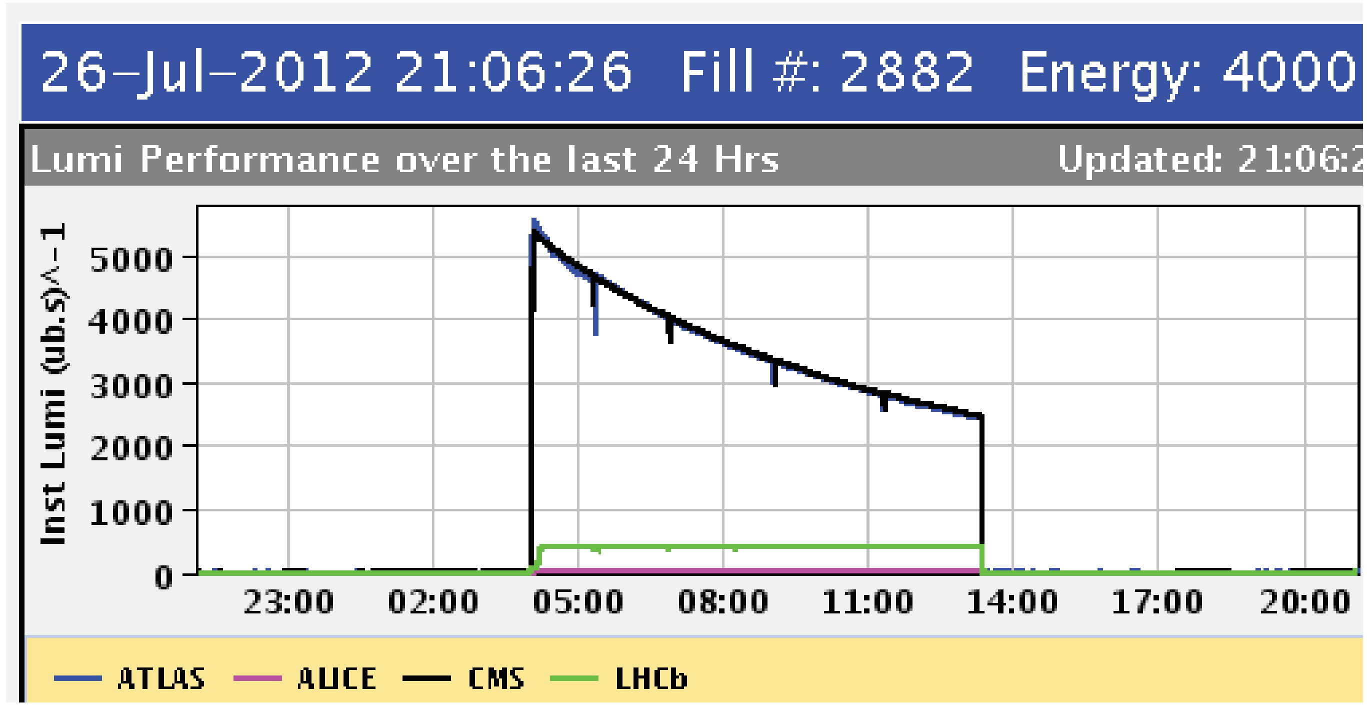
- Dependence of wave length on θ: $\lambda \approx \frac{\lambda_{wgl}}{2\gamma^2}\left(1 + \gamma^2\left(\theta^2 + \theta_e^{\,2}\right)\right), \quad K \equiv \gamma\theta_e$

Optical Stochastic Cooling for the LHC

How fast we need to cool

- Typical luminosity lifetime ~10-15 hour

$$L = \frac{N_1 N_2}{4\pi\varepsilon\beta} f_0 n_b$$



- ⊥ emittance growth is the main source of luminosity loss
- Thus the emittance damping time of about 10 hours is required
  - It corresponds to the amplitude damping rate of 20 hours
- In most of future scenarios 10 hours damping time (in amplitude) should be sufficient

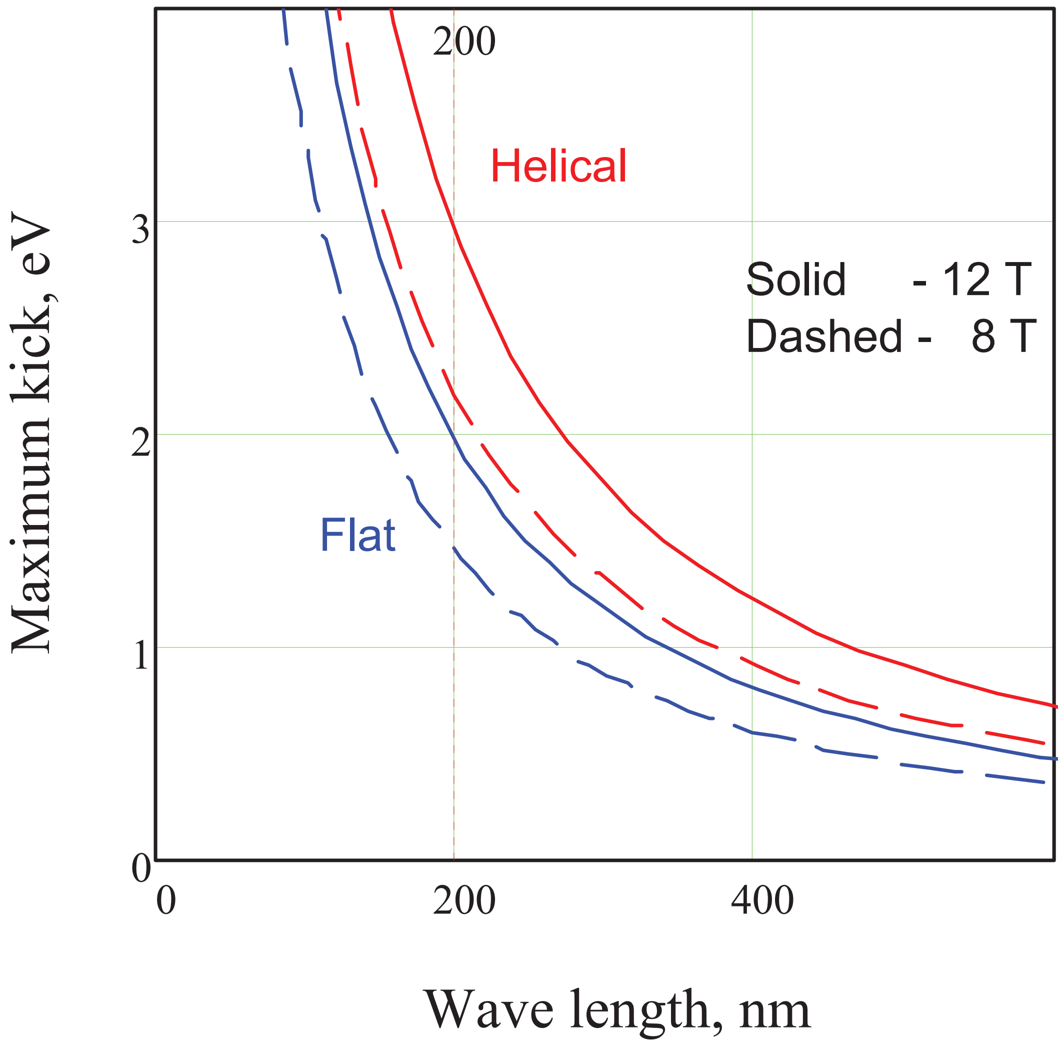
Main parameters for LHC OSC

Beam energy	6 TeV
Bunch population	$1.5 \cdot 10^{11}$
Number of bunches	2808
Initial rms norm. emittance	2 mm mrad
Initial momentum spread	$0.95 \cdot 10^{-4}$
Basic Wave Length of OSC	200 nm
Undulator type	helical
Undulator parameter	2
Undulator magnetic field	12 T
Undulator period	3.3 m
Undulator aperture	2*3.5 cm
Number of periods	23
Undulator length	75 m
Total power of SR from one undulator	33 W
Longitudinal cooling range	3.1σ
Transverse cooling range	5.1σ
Longitudinal amplitude cooling time†	18 h
Transverse amplitude cooling time†‡, $\tau_x = \tau_y$	9.5 h

† Takes into account loss in four lenses and kick reduction due to finite radius of particle motion in undulator

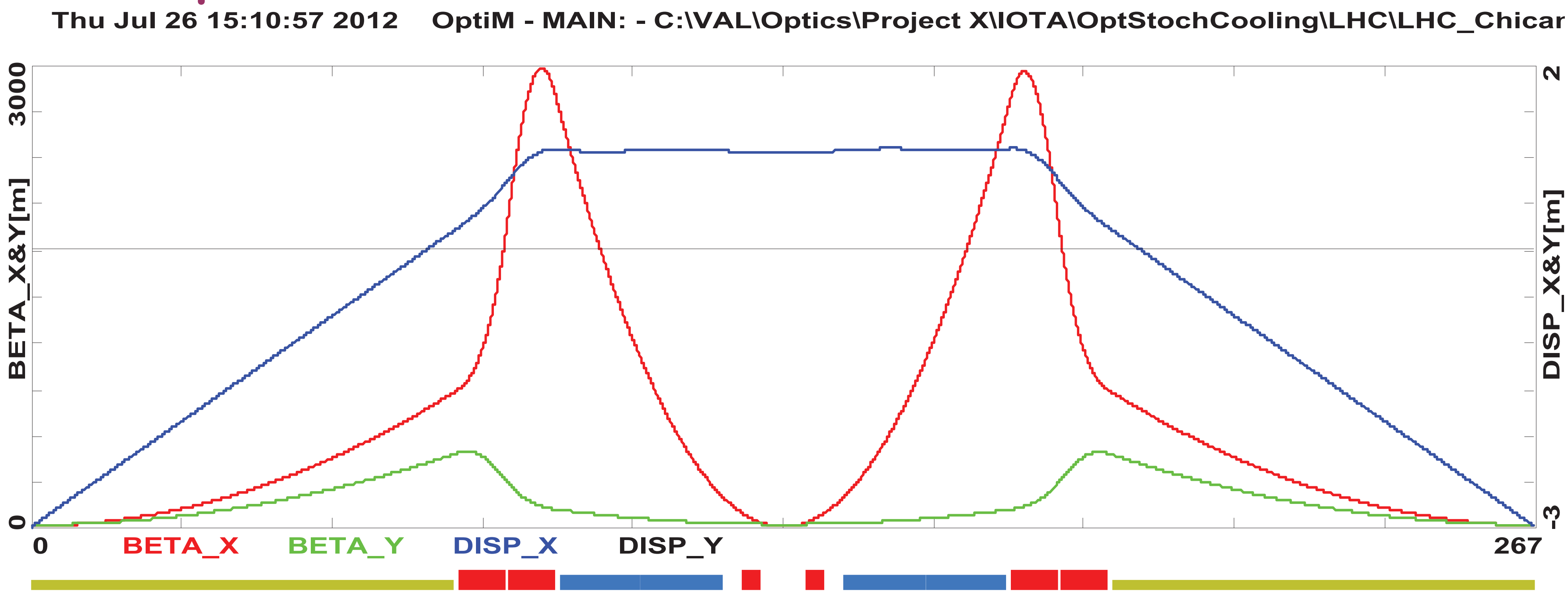
$\ddagger$Takes into account that both $\perp$ planes are damped due to x-y coupling

Comparison of Helical and Flat Undulators



- Helical undulator makes about 1.5 times stronger kick for given light wavelength, magnetic field and undulator length

Beam Optics for LHC OSC



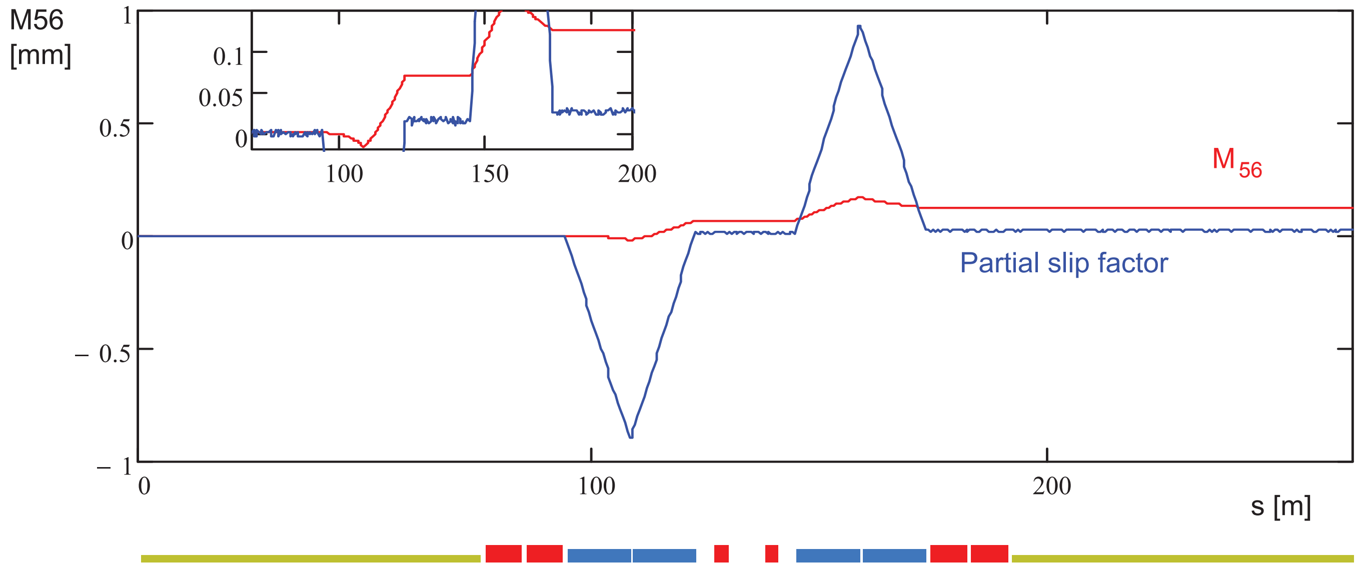
Beta-functions and dispersion for OSC section; undulators are shown by yellow rectangles

Total length of cooling section	270 m
Magnetic field in chicane dipole	10 T
Chicane dipole length	14 m
Chicane dipole aperture	2*60 mm
Horizontal beam offset in chicane	122 mm
Delay in the chicane	0.69 mm
M_{56}	1.25 mm
Partial M_{56}	0.26 mm

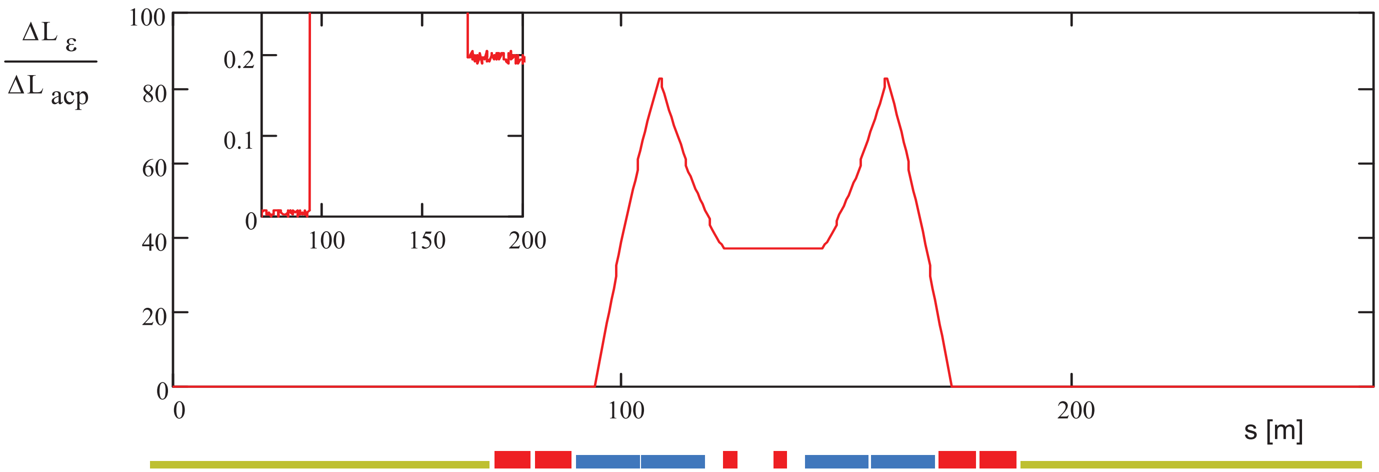
Parameters of Quadrupoles

	L [m]	G [kG/cm]
Qf	8	17.2
Qd	8	-15.7
Qc	3	-2.45

Sample lengthening & Longitudinal Beam Optics



M_{56} and partial slip factor through cooling section

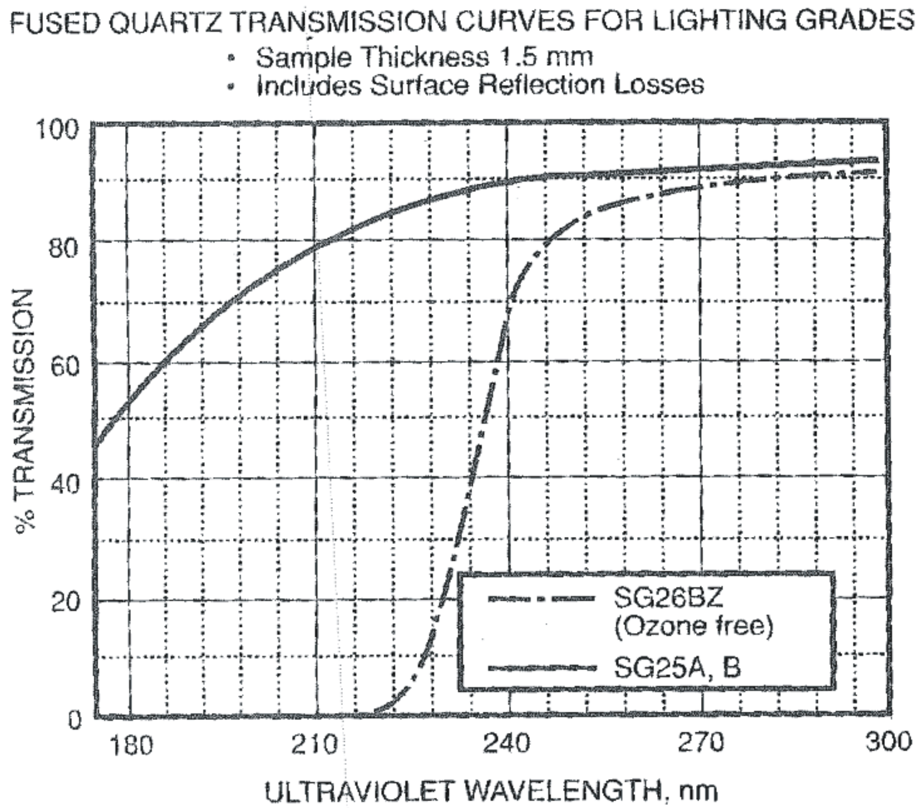


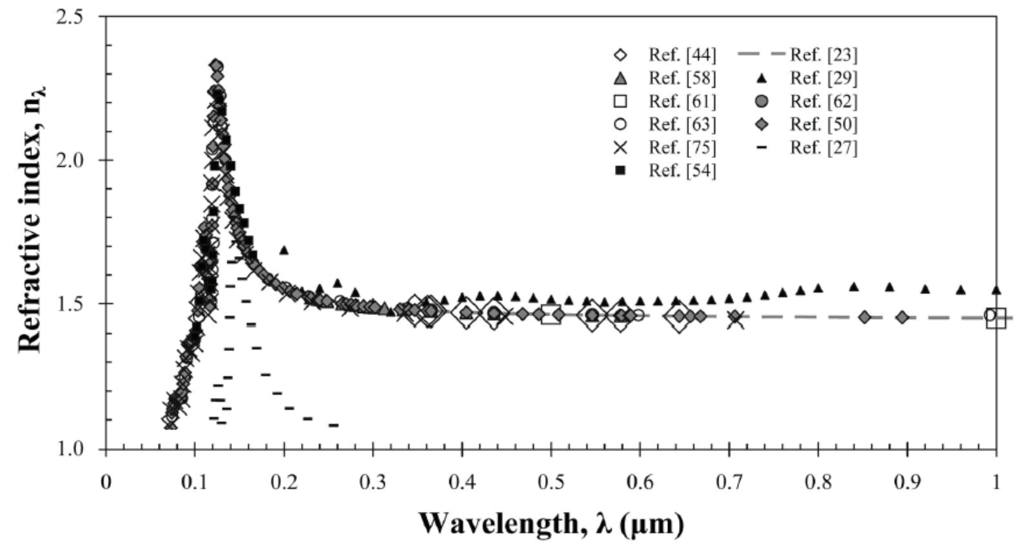
Ration of sample lengthening due to betatron motion to the transverse cooling acceptance through the cooling section

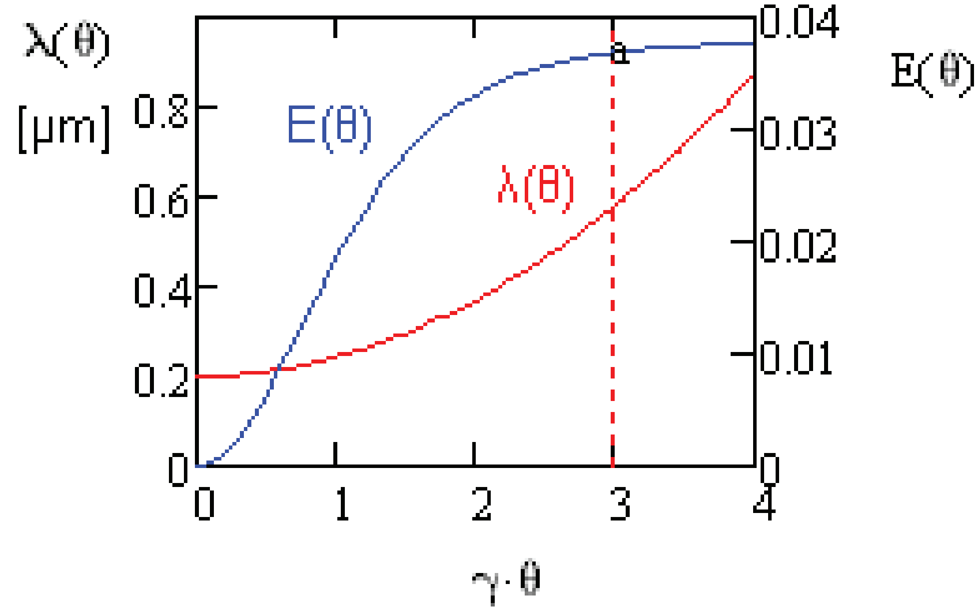
- High accuracy of beam optics control is required to prevent uncontrolled sample lengthening

Light Optics for LHC OSC

- Only first harmonic of undulator radiation is taken into account in the above damping rate estimate
- Higher harmonics are absorbed in the lens





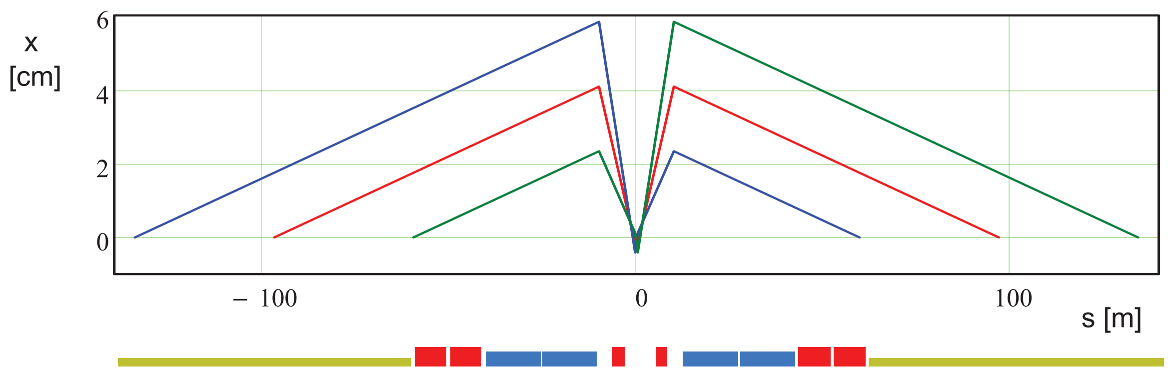


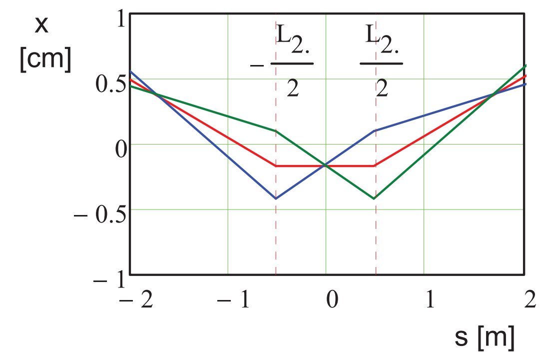
Dependence of the first harmonic wave length on angle (red) and kick strength for a lens with radius determined by subtended angle θ (a = $L\theta$, blue, arbitrary units)

Data are taken from:
http://www.hep.ucl.ac.uk/~jolly/pepperpot/Quartz%20optical%20properties.pdf

Light Optics for LHC OSC (continue)

- 4 lens telescope with total length of 20 m and 1 m space between 2 central lenses
  - Large length increases focusing length and decreases lens thickness



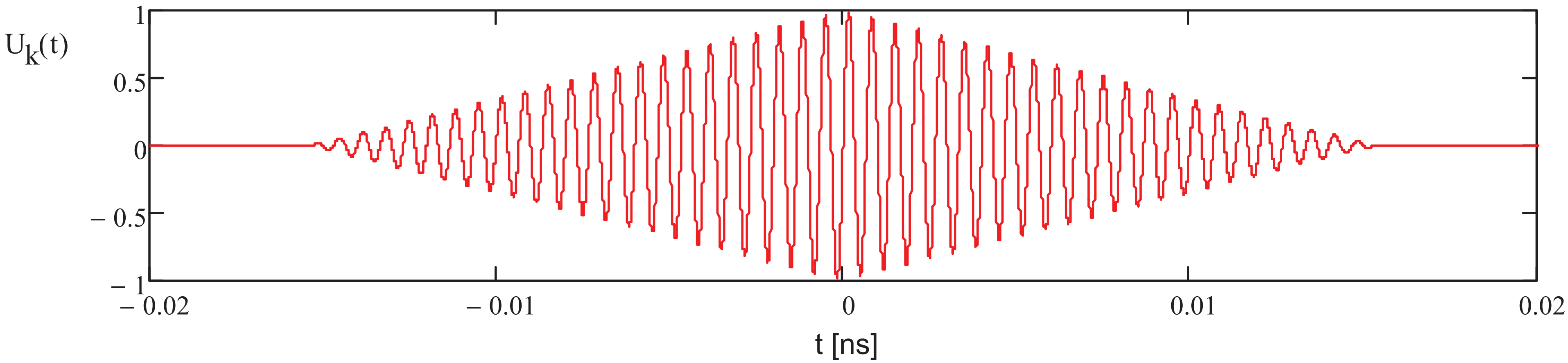


	L1,4	L2,3
Lens focusing distance, cm	825	35.5
Lens radius, mm	59	15
Lens thickness (quartz, n=1.5), mm	0.42+0.11	0.10+0.08
Total delay in 4 lenses, mm	0.69	

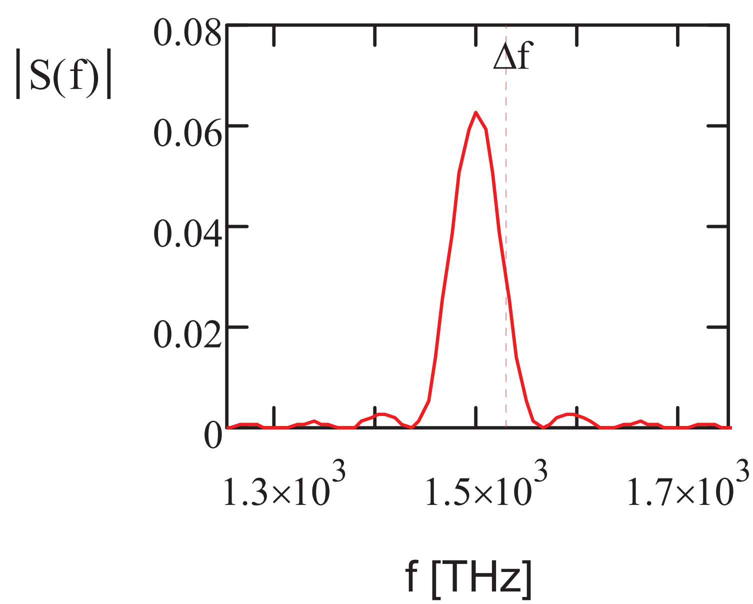
- Chromaticity of lens focusing is corrected by adjustments of lens thickness on the radius

Effective Bandwidth

- Bandwidth is determined by number of undulator periods
  - ~60 THz for 23 periods



- High accuracy for delay control
  - $\Delta L/L \sim 2\cdot10^{-5}$
    - $\sim\lambda/15$=13 nm (25 deg) versus total delay of 0.69 mm



Comments for the LHC OS cooling

- Passive optical stochastic cooling is sufficient to prevent emittance dilution and perform luminosity leveling
  - Operation in UV is required to achieve this goal
- Cooling effectiveness grows with undulator magnetic field
  - Using larger B would increase cooling
- Beam optics manipulations allows one to adjust redistribution of cooling decrements between different degrees of freedom
- Further studies are required for:
  - compensation of quartz chromaticity in the light optics
  - effect of higher harmonics of radiation on cooling
  - effect on beam focusing non-linearities on sample lengthening
    - better cooling for the core is expected
- Experimental proof is highly desirable
  - It can be performed in a small ring with electrons (E~150 MeV)