DYNAMICS OF PARTICLES IN A TILTED SOLENOIDAL FOCUSING CHANNEL *

Hongping Jiang, Shinian Fu
Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, China

Abstract

We use the paraxial ray approximation equations to analysis the dynamics of particles in a tilted solenoidal focusing channel. In this case, the particles' initial canonical angular momentum is nonzero, so we need to add the term of centrifugal potential to the dynamics equation of particles. And in the dynamics equation this centrifugal potential term is nonlinear, which results in the emittance growth. In practice, we also need to consider the spherical aberration's effect on emittance growth and the linear part of the space-charge force of a Kapchinskij-Vladimirskij [1] distribution beam in the dynamics equation of particles.

INTRODUCTION

Building a high quality transport line is recognized as one of critical issues in the low energy section of a high intensity RF linac. For an RF linac front end used in ADS and Project X, which employs superconducting spoke-type cavities, it was found that solenoid-based lenses can provide the needed focusing[2],[3][4]. The solenoid-based lenses are assembled and aligned in the cryomodules. The level of acceptable misalignment established for HINS linac is 0.3mm lateral and 5 mrad tilt [3], the alignment and assembly in the cryomodules are difficulty. Beam emittance can grow, if the beam passes a solenoid-based focusing lens off-centre. So we analyse the dynamics of particles in a tilted solenoid-based focusing channel.

DYNAMICS OF PARTICLES

In the paraxial approximation the radial equation for a particle in a solenoidal focusing magnetic field reads [5]

$$\frac{d^2r}{ds^2}+k_0^2r-\frac{p_\theta^2}{m^2c^2\gamma^2\beta^2r^3}=0. \qquad (1)$$

s is the coordinate along the beam axis, r is the particle radius, and $k_0^2=\frac{q^2B_0^2}{4m^2c^2\beta^2\gamma^2}$ is the focusing strength parameter, $p_\theta=\gamma m r^2\dot{\theta}+qrA_\theta$ is the canonical angular momentum of the particle, where B_0 is the magnetic field on the beam axis, βc is the axial velocity of particles, c is the speed of light in vacuum, and q, m, and $\gamma=(1-\beta^2)^{-1/2}$ are, respectively, the charge, mass, relativistic factor of beam particles, $\dot{\theta}$ is the azimuthal velocity of particle, A_θ is the azimuthal part of vector potential of the field in cylindrical coordinates.

For the sake of simplicity we usually consider that the canonical angular momentum is zero, ($p_\theta=0$), and the last term in the paraxial radial equation vanishes ($\frac{d^2r}{ds^2}+k_0^2r=0$), this equation is quite familiar to us. But if we take the errors of alignment into account, the canonical angular momentum is not zero, in this case, we must turn to the Equation (1), not the equation we usually used.

Considered a solenoidal focusing channel is tilted, scaled ofφ_0, we can get the expression of the canonical angular momentum

$$p_\theta=\gamma\beta c m r_0\varphi_0 cos\left(\theta_0\right). \qquad (2)$$

r_0 and θ_0 are the particles' initial value of radius and azimuth. The value of canonical angular momentum is relative not only with the tilted angleφ_0 also with the particles' initial conditions (r_0,θ_0). Substituting Eq. (2) into the Eq. (1) gives the radial equation

$$\frac{d^2r}{ds^2}+k_0^2r-\frac{\left(\varphi_0 r_0 cos\left(\theta_0\right)\right)^2}{r^3}=0. \qquad (3)$$

When the canonical angular momentum is different from zero, the last term adds an effective repulsive core or a centrifugal potential, and is a defocusing force. In this case, the particle never crosses the axis ($r\neq 0$).

We rewrite the radial equation as follow

$$\frac{d^2r}{ds^2}+k^2r=0. \qquad (4)$$

$k^2=k_0^2-\frac{(\varphi_0 r_0 cos(\theta_0))^2}{r^4}$ is the focusing strength parameter including the nonlinear defocusing part from the initial canonical angular momentum. In order to ensure the beam is good focused, we should have perfect alignment, in the ideal case, the tilted angle should be zero. From the formula we can evaluate the tilted angle φ_0 by seeing the $-\frac{(\varphi_0 r_0 cos(\theta_0))^2}{r^4}$ as a perturbation of the focusing strength, when the particles radius is larger than 1mm ($r\geq 1$mm, we consider the repulsive core radius is about1mm). For simplicity, we select the particles' initial conditions ($r_0=1$mm$,\theta_0=0$) as representative. k_0^2 is about 10^{-4} mm^{-2}, we need $-\frac{(\varphi_0 r_0 cos(\theta_0))^2}{r^4}$ is smaller than 10^{-6}mm^{-2}, that means the tilted angle φ_0 is about 1mrad.

*E-mail: jianghp@ihep.ac.cn

ISBN 978-3-95450-118-2

In practice, the focusing lenses have spherical aberration, which means the solenoidal focusing magnetic field$B$ is function of radius r.

$$B(r) = B_0\left(1+\frac{r^2}{R_0^2}\right). \tag{5}$$

B_0 is the magnetic field on the beam axis, R_0 is a parameter that defines the scale of the aberration.

Considering the spherical aberration, the radial equation becomes:

$$\frac{d^2r}{ds^2}+k_0^2\left(1+\frac{r^2}{R_0^2}\right)r-\frac{\left(\varphi_0 r_0 cos\left(\theta_0\right)\right)^2}{r^3}=0 . \tag{6}$$

The solenoidal focusing lenses are used to transport the low energy proton, so the space-charge force is must considered. For a K-V distribution beam, the space-charge is linear [6]. In the radial equation, the space-charge term is $-K\frac{r}{r_m^2}$, where $K=\frac{I}{I_0}\frac{2}{\beta^3\gamma^3}$ is the generalized perveance and r_m is the beam envelope radius, I is the beam current, $I_0=\frac{4\pi\varepsilon_0 mc^3}{q}$ is characteristic current of proton.

Then the radial equation gives

$$\frac{d^2r}{ds^2}+k_0^2\left(1+\frac{r^2}{R_0^2}\right)r-K\frac{r}{r_m^2}-\frac{\left(\varphi_0 r_0 cos\left(\theta_0\right)\right)^2}{r^3}=0 . \tag{7}$$

The equations have included the linear solenoidal focusing force, the nonlinear part of spherical aberration's effect, the linear defocusing space-charge force, and the nonlinear defocusing force due to the misalignment (tilted angle φ_0).

CONCLUSION

In summary, we have obtained the dynamics equation of particles in a tilted solenoidal focusing channel, based on paraxial approximation. From the equation we have analysed the acceptable tilted angle $\varphi_0$is about1 mrad, when we see the $-\frac{(\varphi_0 r_0 cos(\theta_0))^2}{r^4}$ as a perturbation of the focusing strength. I.Terechkine has analysed the beam emittance growth due to aberrations, and don't take the effect of beam rotation. The beam particles' initial conditions are $r(0)=r_0, \dot{r}(0)\approx\beta c\varphi_0 sin(\theta_0)$. Because the particles' initial conditions is relative to the $sin(\theta_0)$, the beam is anisotropy [7]. In the future, we will do self-consistent simulations study for more details about the emittance growth due to the tilted focusing channel.

REFERENCES

[1] I.M. Kapchinskij and V.V. Vladimirskij, in Proceedings of the International Conference on High Energy Accelerators (CERN, Geneva, 1959), p.274.

[2] I.Terechkine, Beam Emittance Growth due to Aberrations in Focusing Lenses, TD-10-005, FNAL, March 2010.

[3] I.Terechkine, Analysis of a Concept Focusing Lens for SSR0 Section of PX Linac,TD-10-013, FNAL, June 2010.

[4] G. Gavis, et al, Pre-Production Solenoid-Based Focusing Lenses for Superconducting Sections of HINS Linac,TD-09-029,FNAL, 2009.

[5] M. Reiser, Theory and Design of Charged-Particle beams (Wiley and Sons, New York, 1994).

[6] F. J. Sacherer, IEEE Trans. Nucl. Sci. 18, 1105 (1971).

[7] I. Hofmann, L. J. Laslett, L. Smith, and I. Haber, Part. Accel.13, 145 (1983).

ISBN 978-3-95450-118-2