Space Charge
in Isochronous Regime (IR)

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*work partly done at Michigan State University
in 2001-2003 (experiments and simulations)
together with J.A. Rodriguez

Isochronous regime

- Several types of machines operate / run into IR:
  - rings for precise nuclear mass spectrometry
  - some isochronous-optics light sources.
  - hadron synchrotrons during transition crossing
  - cyclotrons (FFAG?)

- Studies of beam dynamics of intense beams around transition have been conducted and documented (including text books, K. Ng)

- Effect of space charge (SC) on transverse motion and coupling of radial and longitudinal motion must be included in consideration in IR (usually omitted)

Longitudinal impedance at short λ

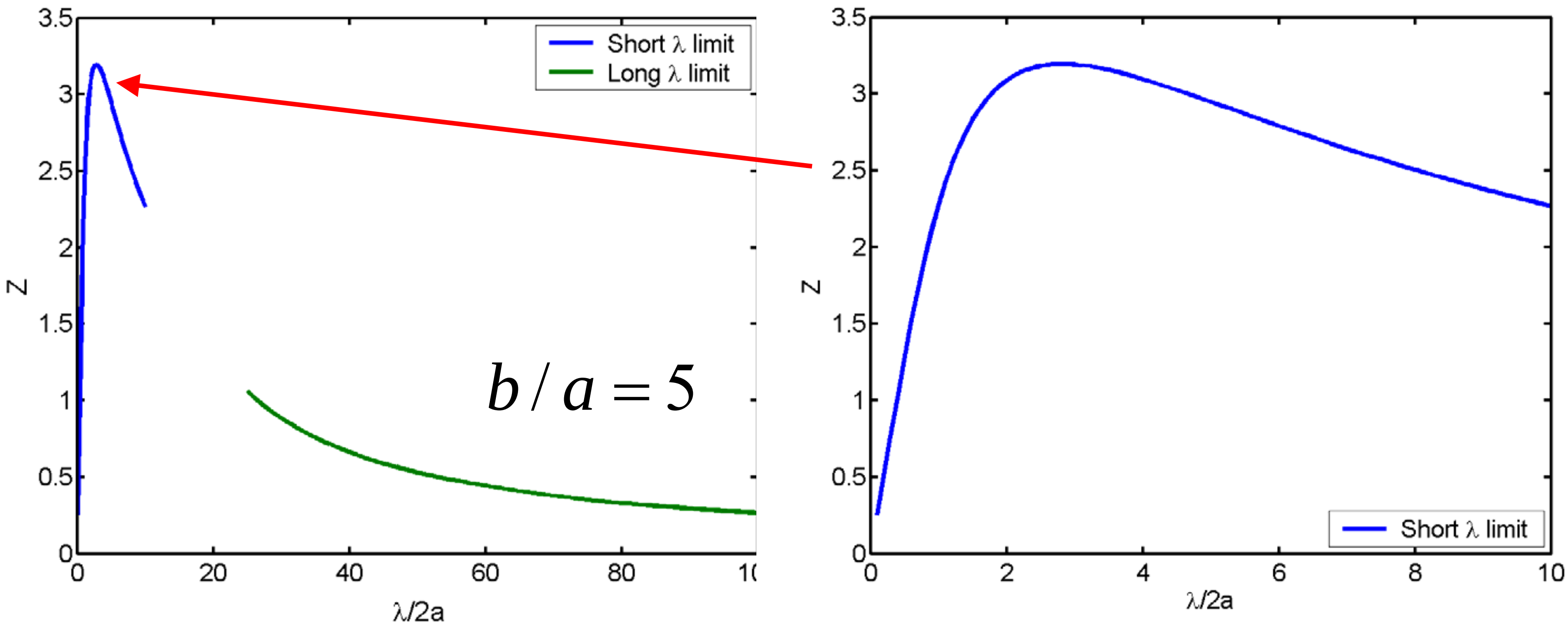
$$Z_{\|}(k) = ik\frac{Z_0 R_0}{\gamma^2 \beta}\left(\frac{1}{2} - \ln\left(\frac{a}{b}\right)\right), \quad k = \frac{2\pi}{\lambda}$$

-Long wavelength approximation (includes image charges)

$$Z_{\|}(k) = i\frac{2Z_0 R_0}{ka^2 \beta}\left(1 - \frac{ka}{\gamma}\cdot K_1\left(\frac{ka}{\gamma}\right)\right)$$

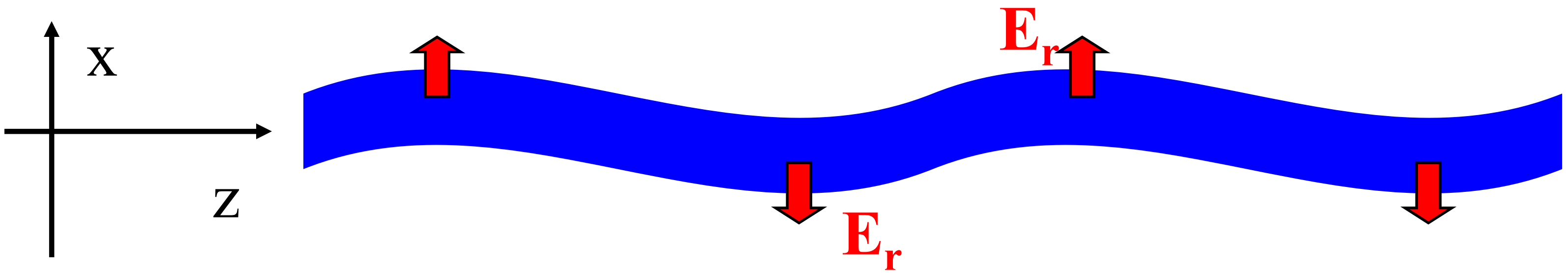
-Short wavelength approximation (no image charges)

SC impedance peaks at short wavelength: λ_m~2.5 ∅



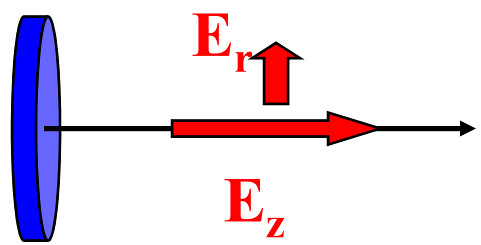
Transverse SC field

Linear charge density modulation => Energy modulation => Radius modulation => Radial electric field



The radial field comes from the snaky shape and can come from image charges. (We neglect images assuming flat vacuum chamber (like in a cyclotron).)

Field from a slice



$$E_r \approx -\frac{E_z'}{2} x$$

The radial field due to snaky shape

$$E_r = 2\pi\rho \cdot x \cdot (1 - ka \cdot K_1(ka))$$

Dispersion function and slip factor

$$x'' + \frac{\nu_0^2}{R_0^2} x = \frac{1}{\rho}\frac{\delta p}{p} + \frac{eE_r}{m\beta^2 c^2}$$

Steady state solution $x_{ss} \approx \frac{R_0}{\nu_0^2}\left(1 + 2\left(\frac{-\delta\nu}{\nu}\right)_{SC}(1 - ka\cdot K_1(ka))\right)\frac{\delta p}{p}$

Exactly at the transition $\eta_s = \alpha_p - \frac{1}{\gamma_{tr}^2} = 0$

If there is dispersion function error, the slip factor is $\eta_s \approx \frac{\delta\eta}{R_0}$

$$\eta_s \approx 2\alpha_p\left(\frac{-\delta\nu}{\nu}\right)_{SC}(1 - ka\cdot K_1(ka))$$

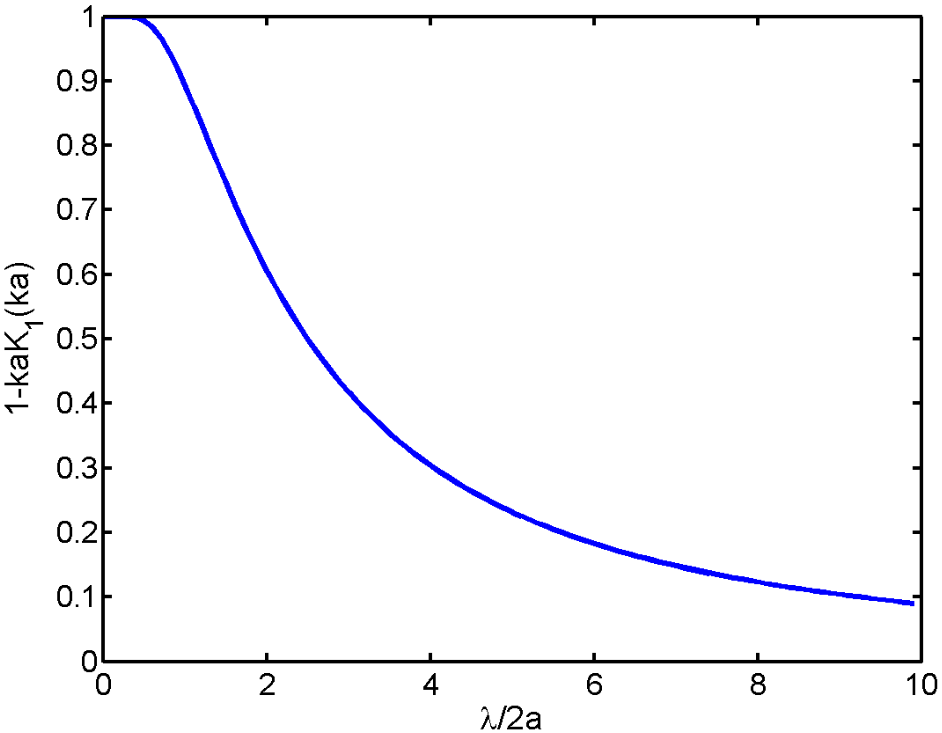
Negative Mass instability below γ_{tr} ?! Sure…

Growth rate with SC in Isoch. Reg.

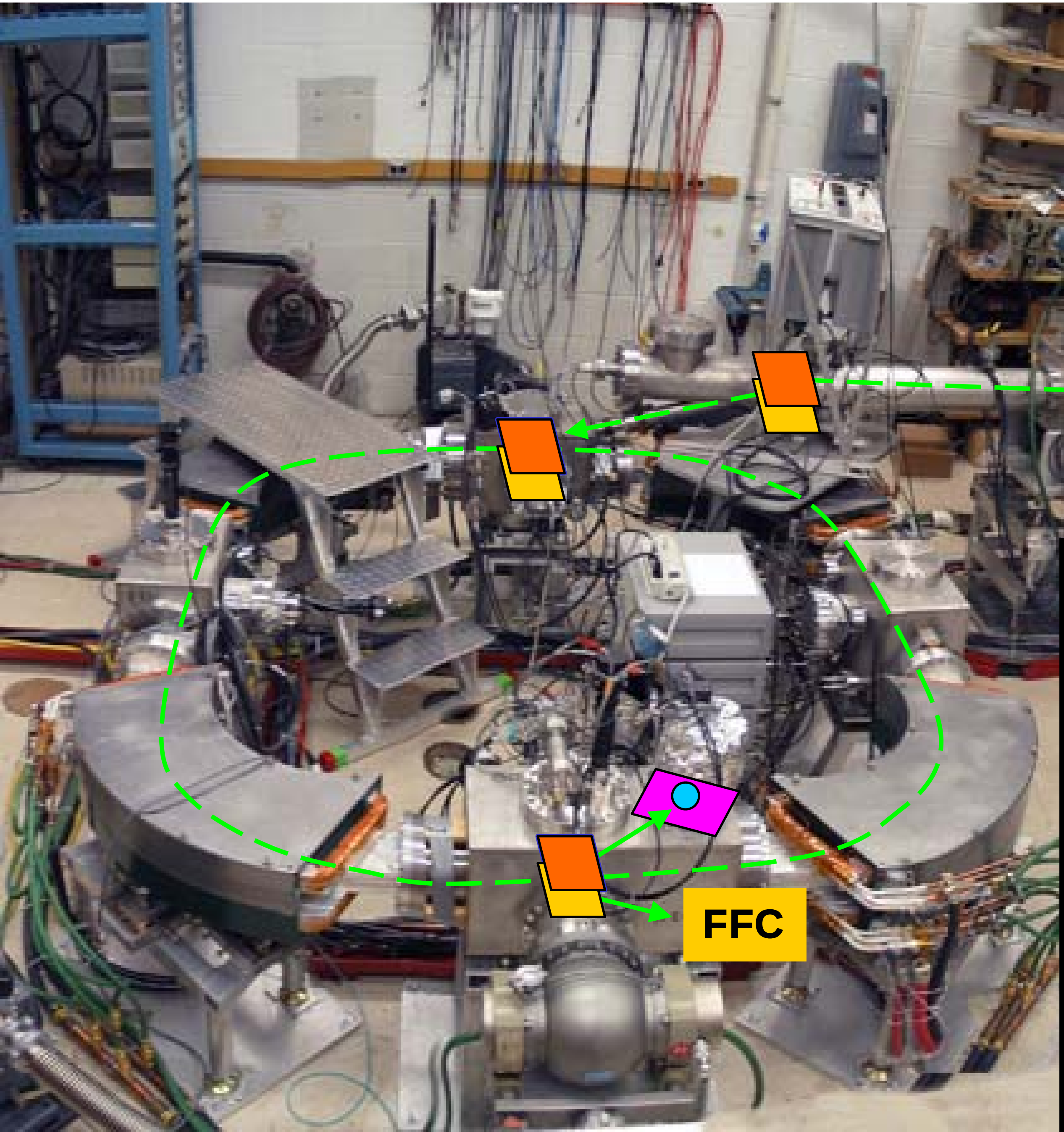
The growth rate for the microwave instability: $\tau^{-1}(k) = \omega_0 \sqrt{-i \dfrac{\eta_s e I_0 k R_0 Z_{\|}}{2\pi\beta^2 E}}$

$$\tau^{-1}(k) \approx \frac{4\sqrt{2}\pi \left(\dfrac{-\delta\nu}{\nu}\right)_{SC} (1 - ka \cdot K_1(ka))}{T_0}$$

$4\sqrt{2}\pi \approx 18$



Small Isochronous Ring (SIR), Circa end of 2003

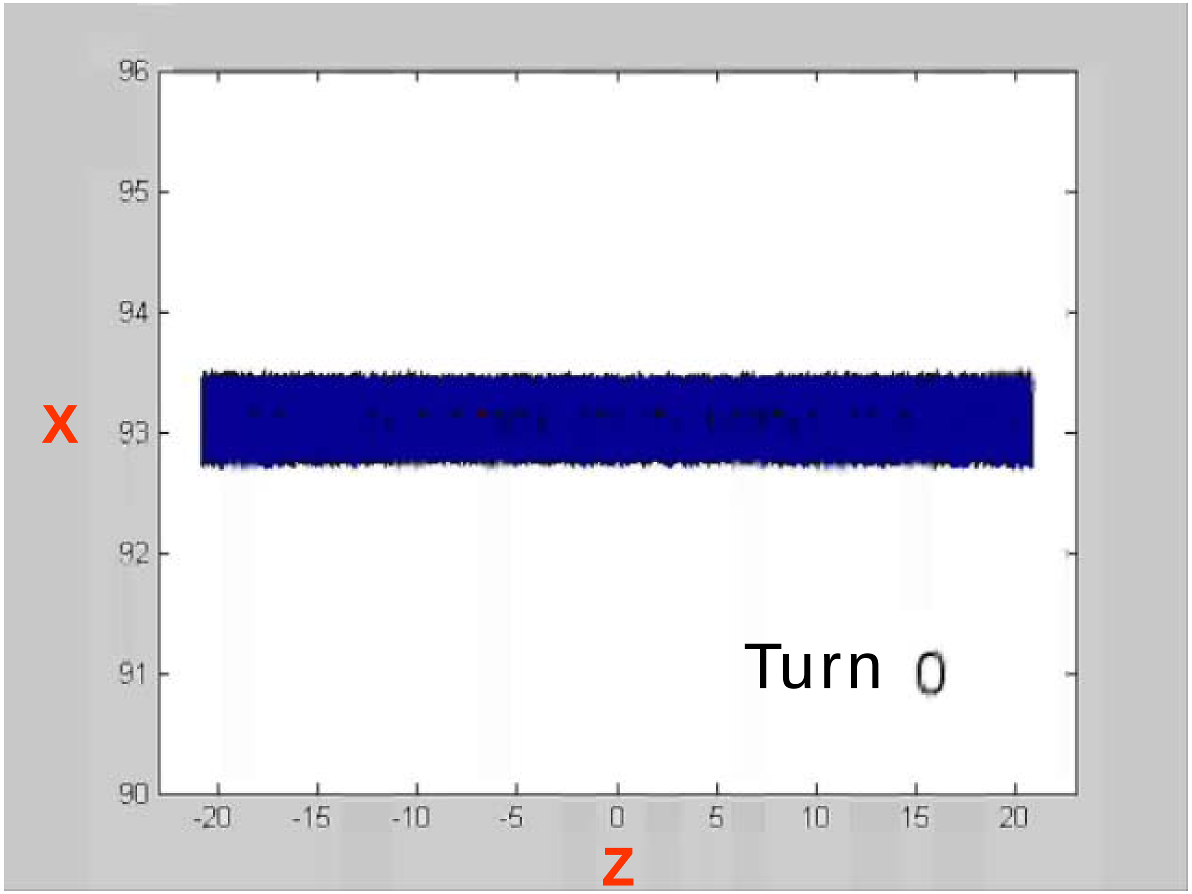


Beam	H_2^+
Energy	10-30 keV
ν_x, ν_y	1.14, 1.11
$\alpha_p - 1/\gamma^2$	1.7e-4
T	5 μsec
C	6.6 m
N_{turns}	100
I_{peak}	0-25 μA

Beam dynamics simulations in SIR

CYCO simulations ($N_p = 3 \cdot 10^5$)

Bunch breaks up within a few turns throughout the bunch



Contour plot of bunch charge density in median plane for turns 0 to 75

$I_{peak} = 10\ \mu A$

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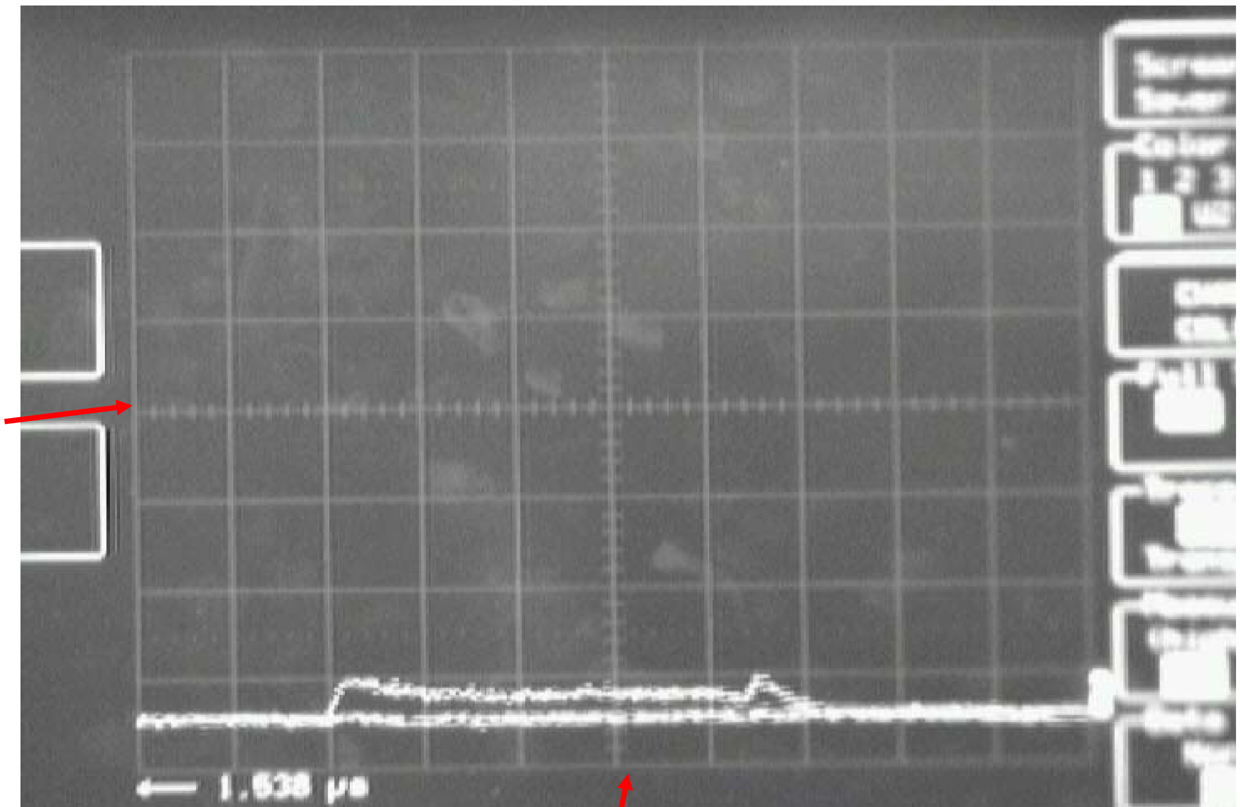
Experimental results: Longitudinal beam dynamics

Measured longitudinal bunch profile
Turn# 10 (fixed), Current increases

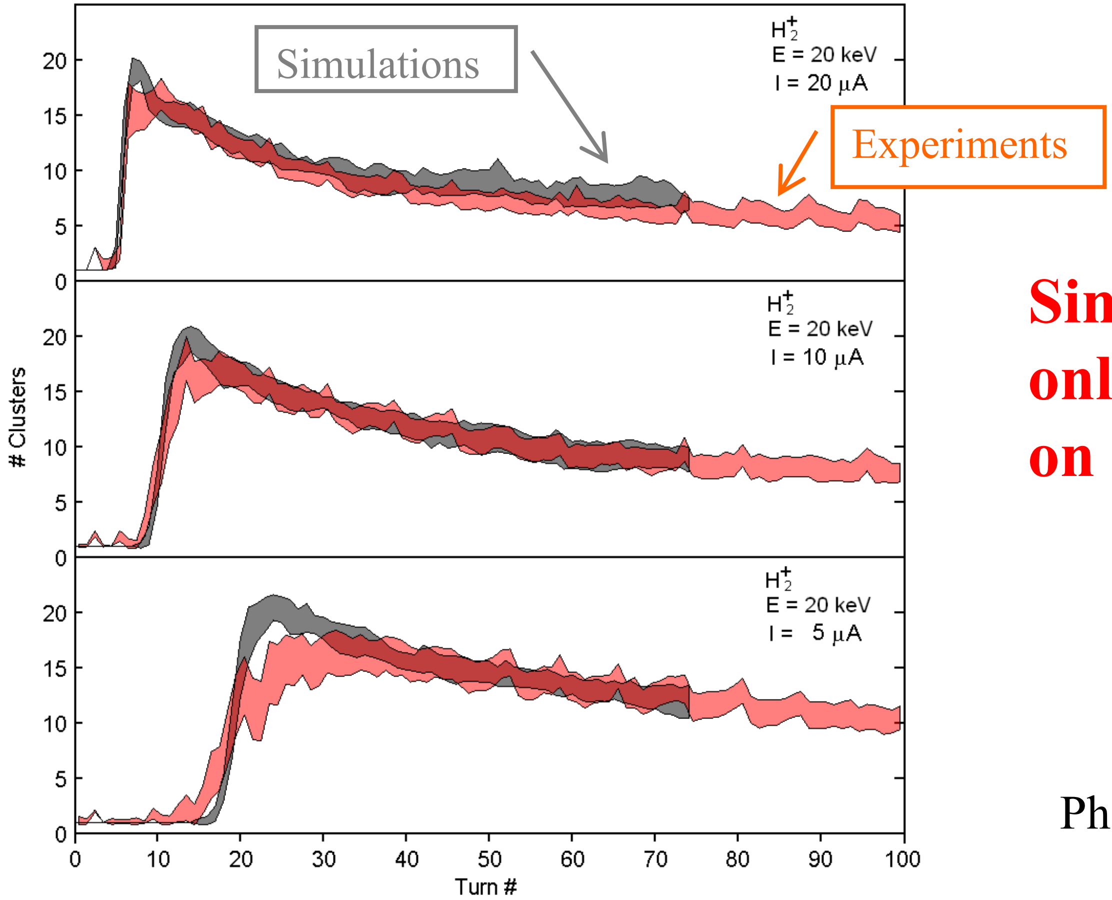
Vertical axis: peak current measured by Faraday Cup



Horizontal axis: arrival time to the Faraday Cup (equivalent to Z)

Comparison
Experiments to Simulations

of clusters as a function of # of turns
for 5, 10 and 20 μA



Simulations included only SC and image charges on the vacuum chamber

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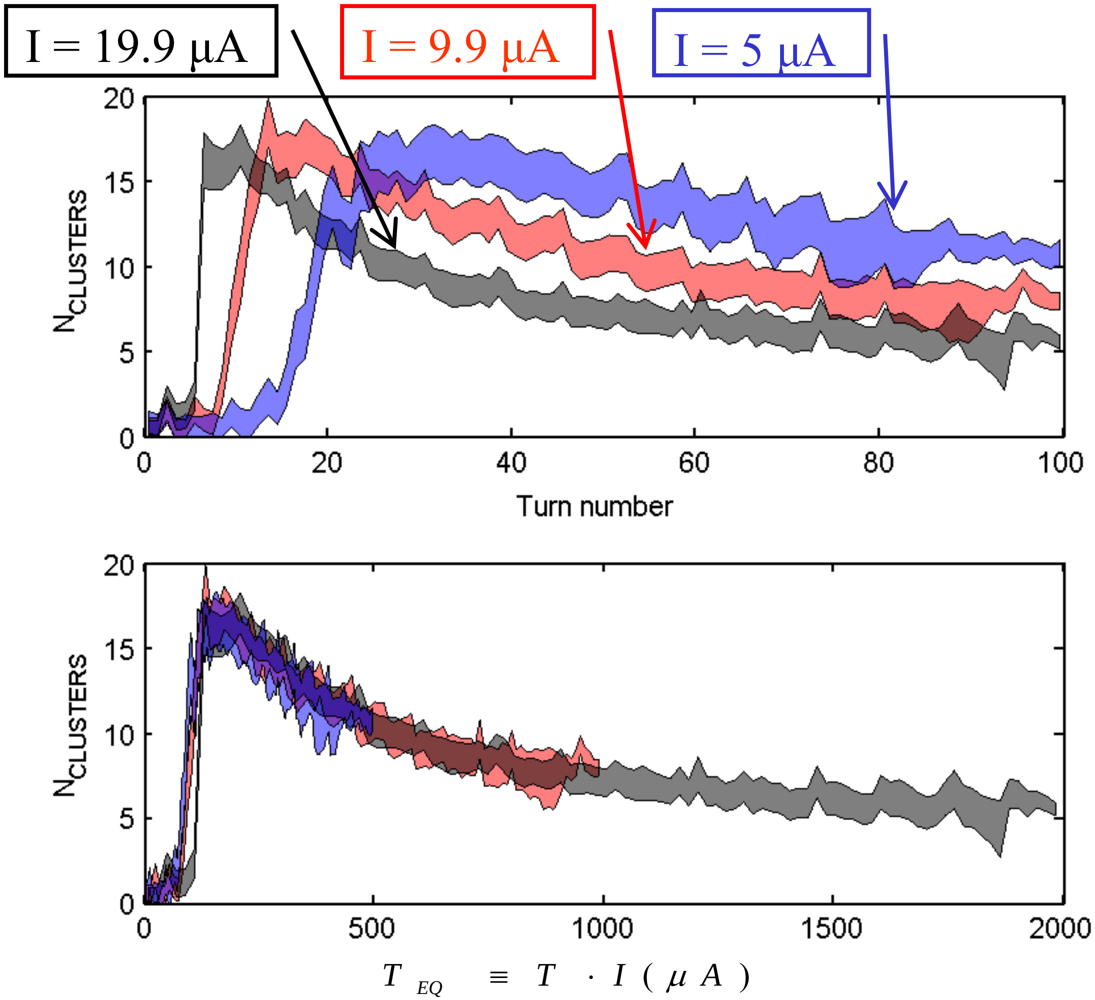
Experimental Results: Scaling with Beam Current



Growth rate depends linearly on the beam current!!! Not as sqrt(I)!!!

I_0(μA)	δν/ν	τ (turn) + SC	τ (turn) NO SC
5	-0.011	4.9	51.3
9.9	-0.023	2.5	36.2
19.9	-0.045	1.3	26.5

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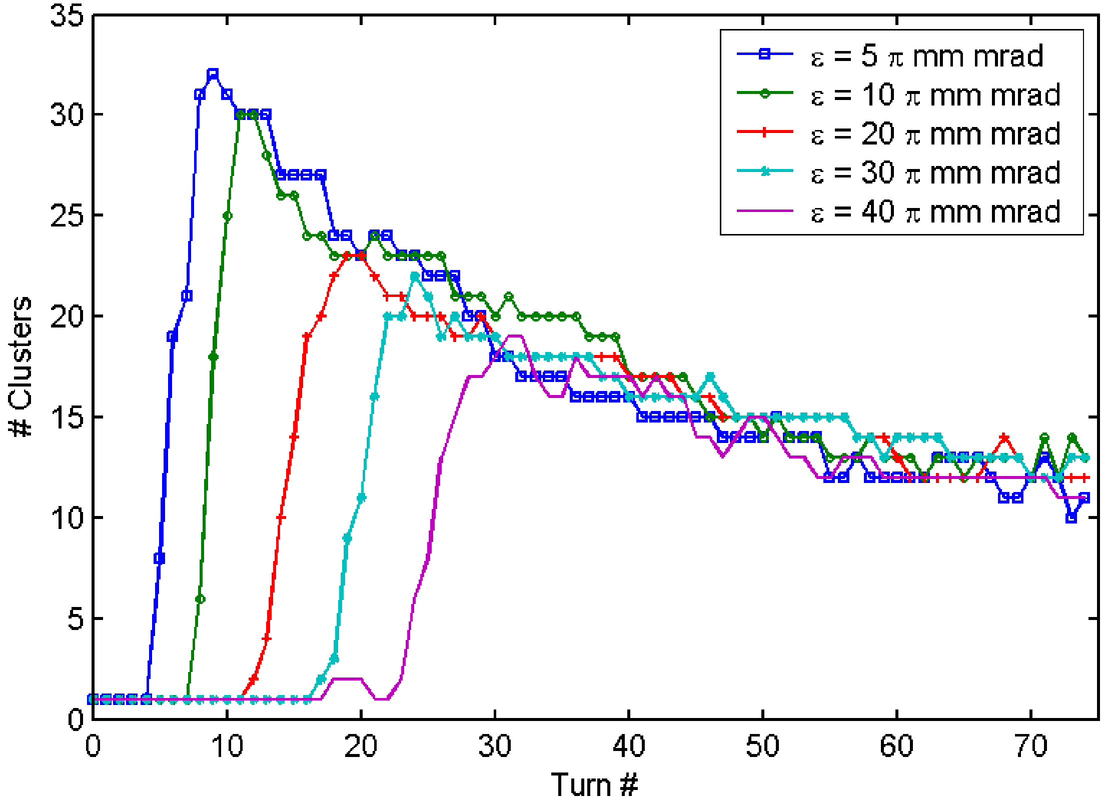
Simulation results: Dependence on Emittance

Simulations: number of clusters vs. turn # for different beam emittance

$$\tau^{-1}(k) \sim \left(\frac{-\delta\nu}{\nu}\right)_{SC} \sim \frac{1}{\varepsilon}$$



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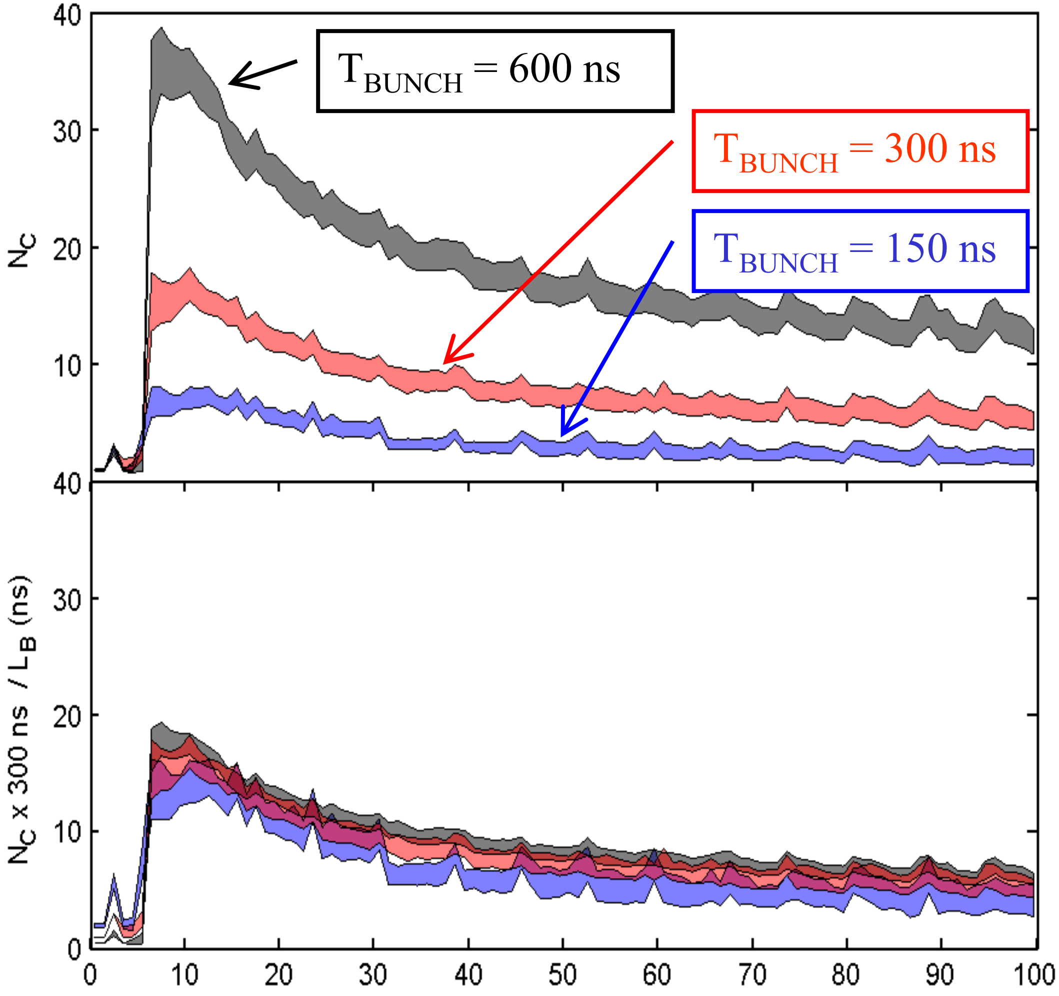
Experimental Results: Scaling with Bunch Length



Breakup happens throughout the bunch (no roll-up from the ends as thought by some).

Size of clusters and number per unit length does not depend on current.

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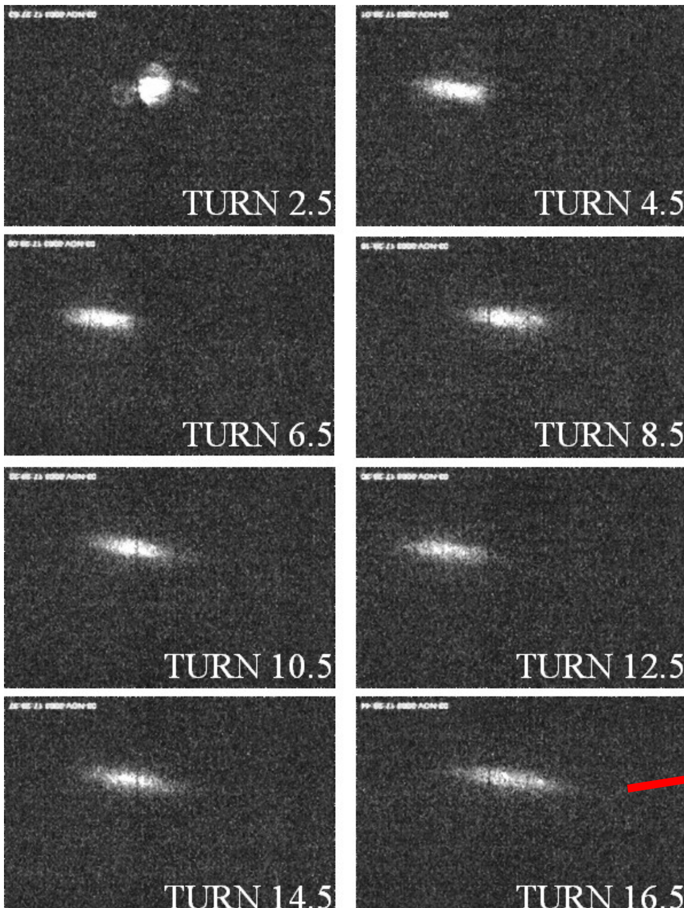
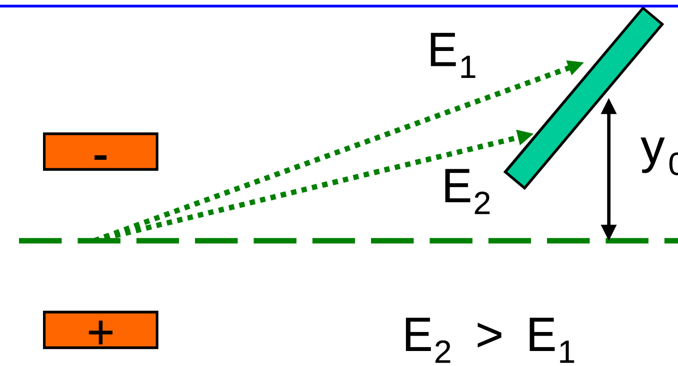
Experimental results: Transverse beam dynamics

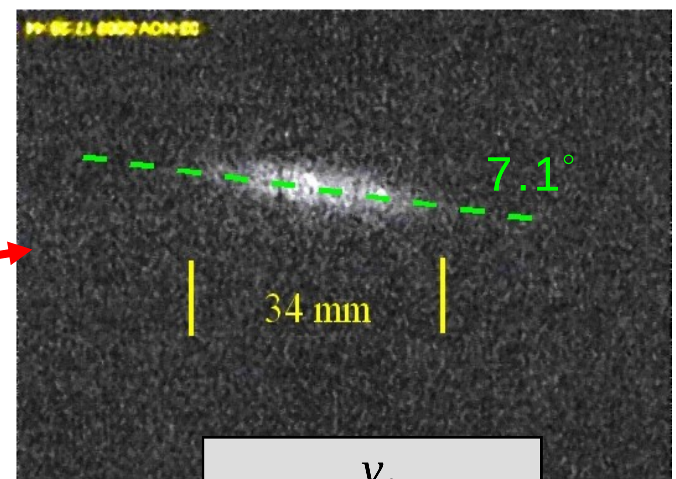
Energy spread grows from 0 to 5% in 10-20 turns



$$\frac{\delta E}{E} = 2\frac{\sigma_{16} - \sigma_2}{\eta} = 5\%$$

I=20 μA, E=17 keV, T=1 μsec





$$\theta \approx -2\frac{y_0}{\eta} = 7.1^\circ$$

Conclusions

- Effect of space charge (SC) on transverse motion and coupling of radial and longitudinal motion can play a crucial role at IR (usually omitted)
- This can drive Negative Mass Instability at and below γ_{tr}
- Simulation results (CYCO) and experimental data (SIR) agree remarkably well
- They show that
  - the instability causes very fast beam fragmentation and energy spread growth
  - the growth rate is proportional to the beam current and inversely proportional to the beam emittance
- Landau damping most likely exist through modification of the dispersion function non-coherently

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