Space Charge and Resonances in High Intensity Beams

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- Motivation
- The "Montague resonance" case
- Searching for scaling laws
- Fourth and sixth order structure resonances
- Conclusion

Acknowledgment:

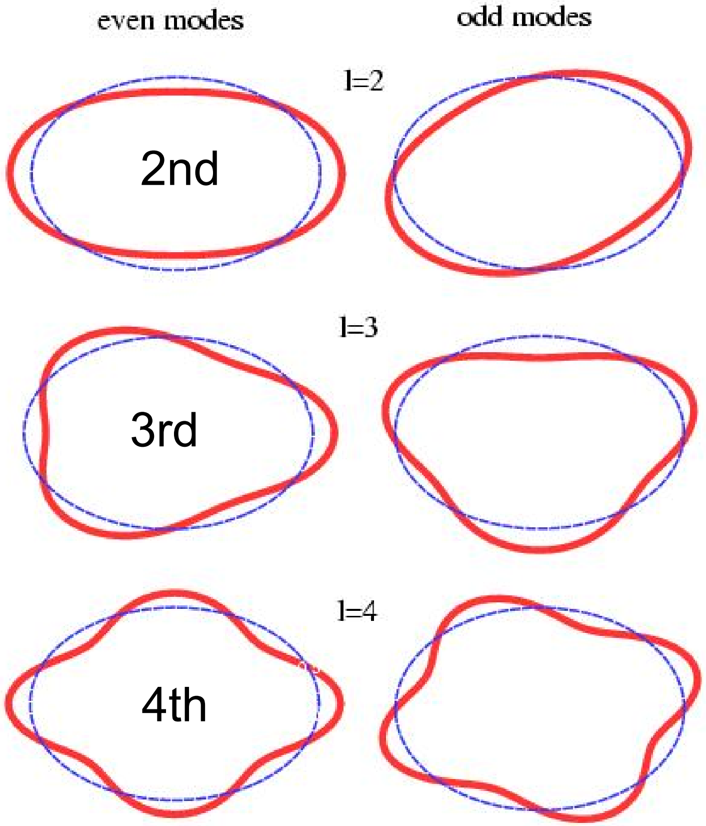
G. Franchetti (MICROMAP), D. Jeon (PARMILA)

Motivation

- space charge driven resonant effects
  - **space charge nonlinearity can itself be source of resonance (linacs and rings)**
  - more common: space charge is modifying other sources of resonance (rings mainly) - *not the subject here*
- usually studied by simulation
  - gives reliable answers for specific cases
  - but: systematics often missing
  - underlying mechanisms not always clear
- efforts to explore if common principles exist to reduce emittance growth phenomenon to fewer basic parameters → **scaling laws**

Here: nonlinear space charge force

well-known envelope mismatch + 2:1 halo is second order (gradient) resonance

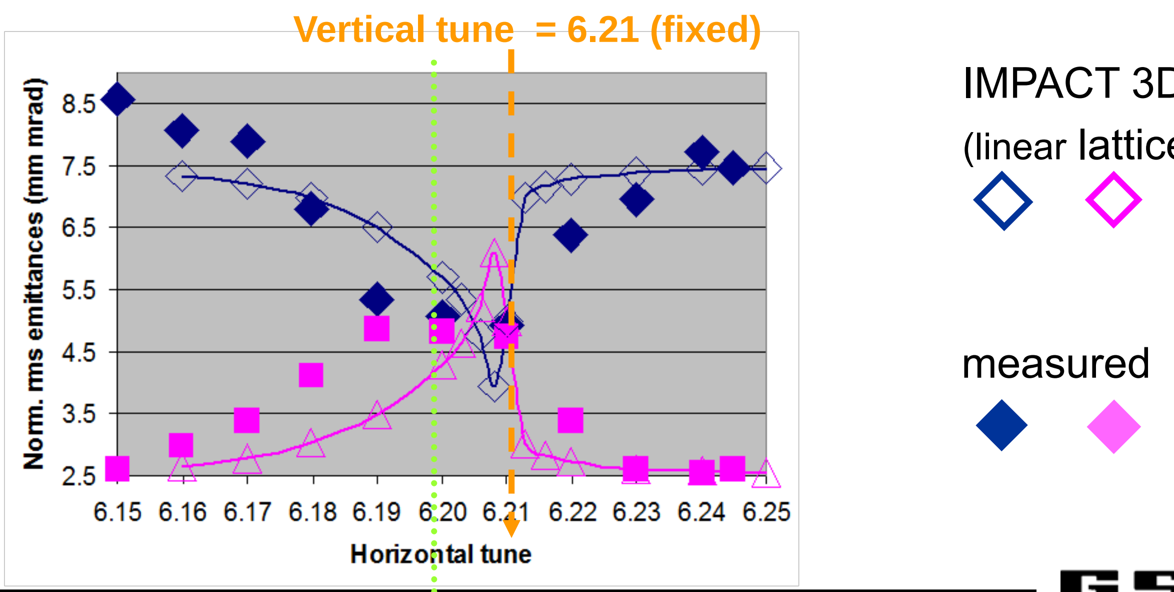


here we are concerned about **nonlinear case** using:

- space charge pseudo-octupole (4th order resonance)
- space charge pseudo-dodecapole (6th order resonance)

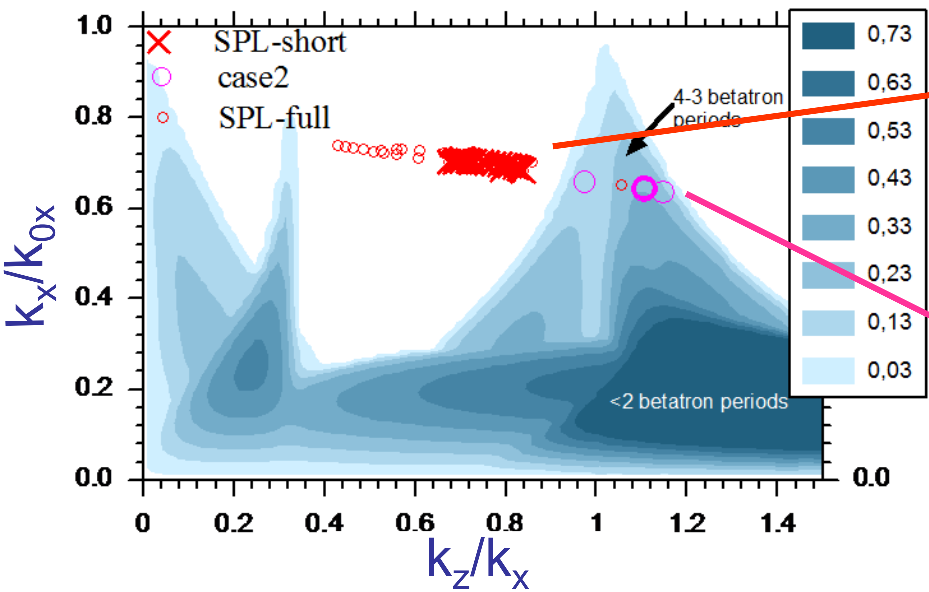
Starting point:
Montague emittance coupling resonance

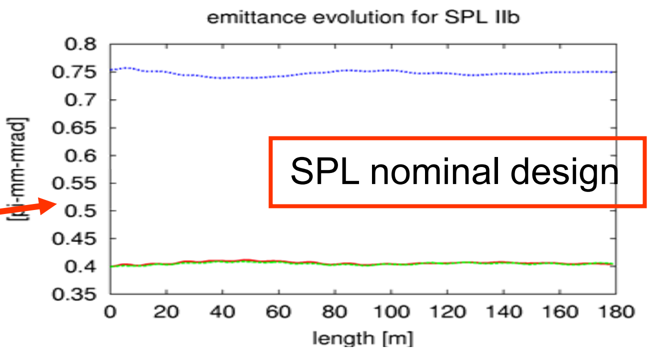
- originally studied for CERN PS by Montague (CERN Report No. 68-38, 1968) on a single particle basis
- later comparison of fully self-consistent Vlasov study & simulation (I.H. G. Franchetti, O. Boine-Frankenheim, J. Qiang and R. Ryne, PRSTAB 2003) → applies to rings and linacs
- experiment at CERN PS in 2003 → successful comparison with theory (E. Metral et al., HB2004)

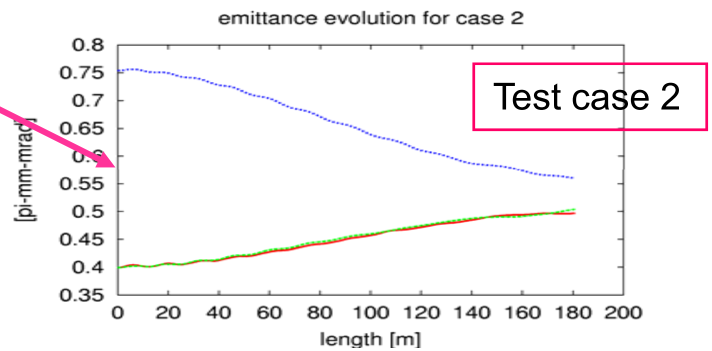


Vlasov equation perturbation theory →
"Stability charts" for different $\varepsilon_z/\varepsilon_{x,y}$=2,3, ...
(applied to SNS, CERN-SPL etc.)

CERN SPL study (F. Gerigk/CERN, 2002):







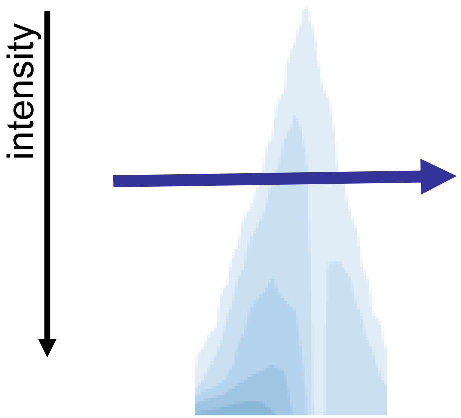
„Undesirable" longitudinal → transverse emittance transfer due to

$2k_z - 2k_x \sim 0$ "internal" resonance (k_z/k_x~1 or σ_z/σ_x~1)

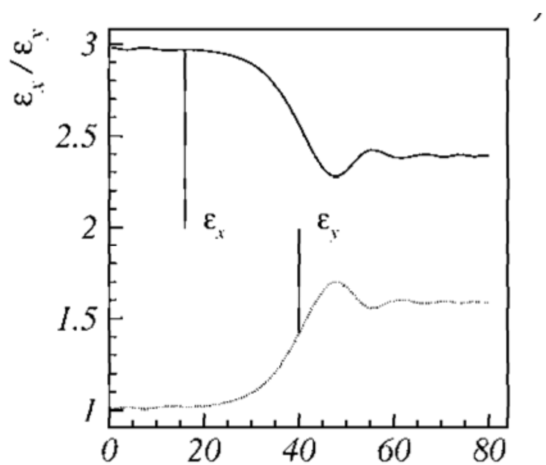
or: $2Q_x - 2Q_y \sim 0$ between x and y in circular machine (Montague condition)

Can we make "simpler" predictions on this resonant emittance transfer by finding a proper scaling law?

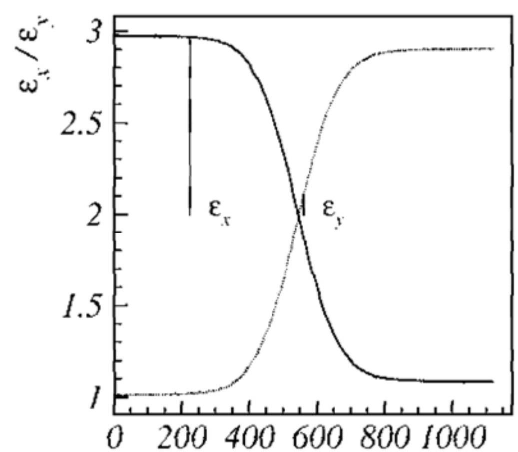
Simple lattice: crossing of emittance exchange resonance in 2D simulation:



stop-band width increasing with intensity



faster crossing



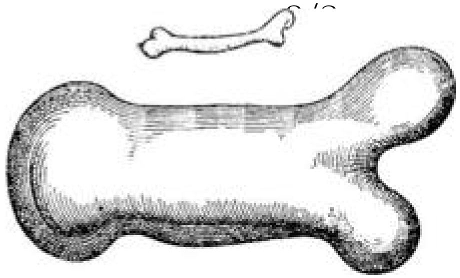
slower crossing

how to relate emittance exchange to stop-band width and crossing speed?

Search for scaling laws

A famous scaling law found by Galilei (1638) for "stability" of bone structure:

The thickness must scale like the linear size to the three-halves power .



"I believe that a little dog might carry on his back two or three dogs of the same size, whereas I doubt if a horse could carry even one horse of his own size."

Child-Langmuir: I_{max} = 4/9 ε_0 $(2e/m)^{1/2}$ $V^{3/2}$ is exact in 1D, but in 2D or 3D still verified by simulation with fitted coefficients

Scaling laws relate one measurable quantity to another one

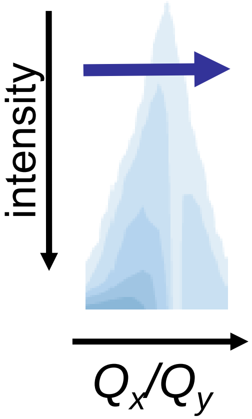
- although underlying phenomena **may be complex**
- "simple" **power laws ^1/2, ^1, ^3/2, ^2, ^3** if **proper variables** are used → reflects underlying physics
- may require **simulation to fit coefficients** to complex situations

For Montague crossing we found a scaling law starting from Vlasov based analytical expression for stop-band:

stop-band width (analytically, 2003):

$$\Theta = \frac{3}{2}\left(\sqrt{\frac{\varepsilon_x}{\varepsilon_y}} - 1\right)\Delta Q_x$$

~ ΔQ_x = space charge tune spread



confirmed that emittance exchange rate obeys similar expression (coefficient yet undetermined):

$$\frac{\delta\varepsilon_y}{\delta t} \sim \left(\sqrt{\frac{\varepsilon_x}{\varepsilon_y}} - 1\right)\frac{\Delta Q_x}{Q_{ox}}$$

hypothesis: assume that effect of crossing depends on product of both!

(if broader, longer interaction with resonant force!)

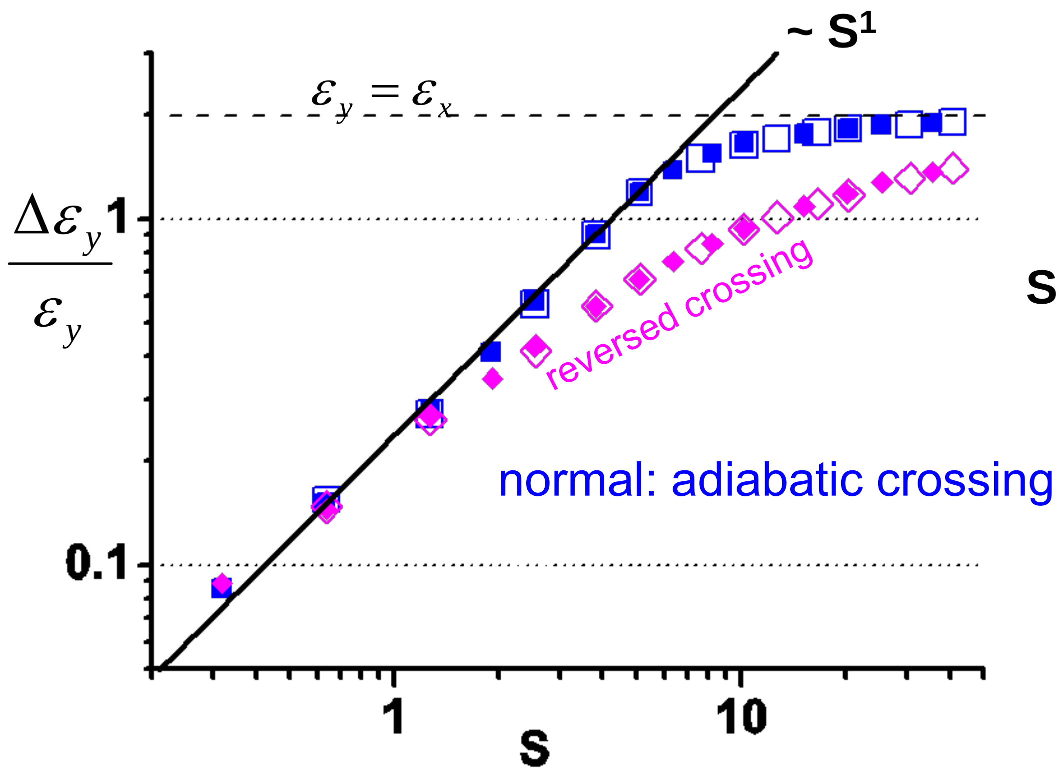
This ansatz for a scaling law was clearly confirmed by MICROMAP simulations → determine coefficient!

"ring notation"

$$\frac{\Delta\varepsilon_y}{\varepsilon_y} = \alpha_{2,2}\left(\sqrt{\frac{\varepsilon_x}{\varepsilon_y}} - 1\right)^2 \frac{(\Delta Q_x)^2}{d\left(Q_{0,x}\right)/dn}$$

"linac notation → 3D problem"

$$\frac{\Delta\varepsilon_\perp}{\varepsilon_\perp} = \alpha_{linac}\left(\sqrt{\frac{\varepsilon_z}{\varepsilon_\perp}} - 1\right)^2 \frac{(\Delta\sigma_z/360^o)^2}{d\left(\sigma_{0,z}/360^o\right)/dn}$$



$$S \equiv \frac{(\Delta Q_x)^2}{d\left(Q_{0,x}\right)/dn}$$

S: scaling or "similarity" parameter

$$\frac{\Delta\varepsilon}{\varepsilon} \propto S^1$$

$$\alpha_{2,2} = \frac{0.18}{\sqrt{Q/R}} \qquad \alpha_{linac} = \frac{1.35}{\sqrt{\sigma_{z0}/L}}$$

L: focusing cell length

Validity of scaling law was tested for UNILAC by PARMILA simulations (3 D!!!) by D. Jeon (SNS), 2007

- allows quick check, if crossing is "harmful"
- design with "fast crossing" avoiding exchange can be easily estimated
- experimental verification seems possible. if "perfect" matching can be achieved → *talk by Lars Groening in Linac working group*

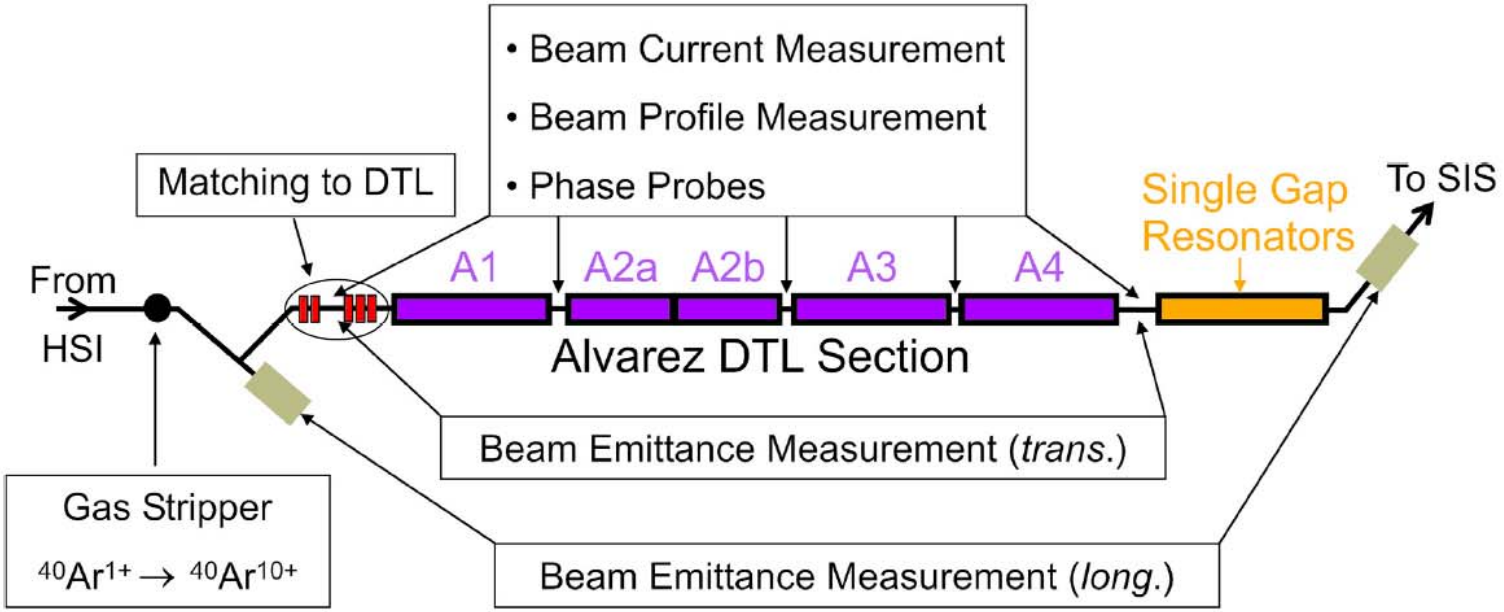
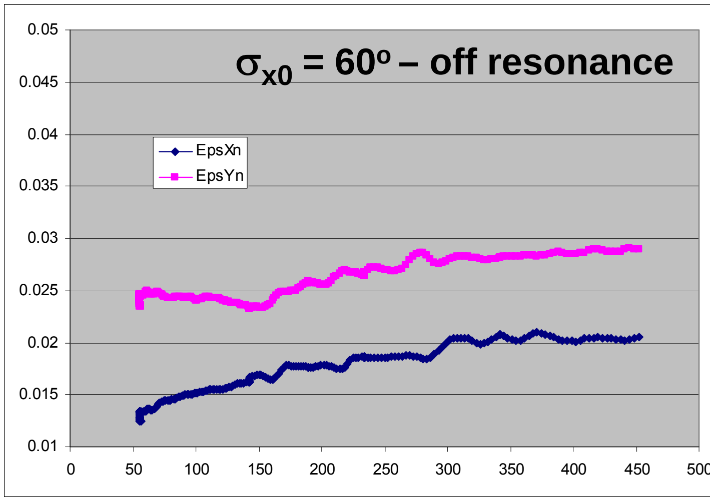
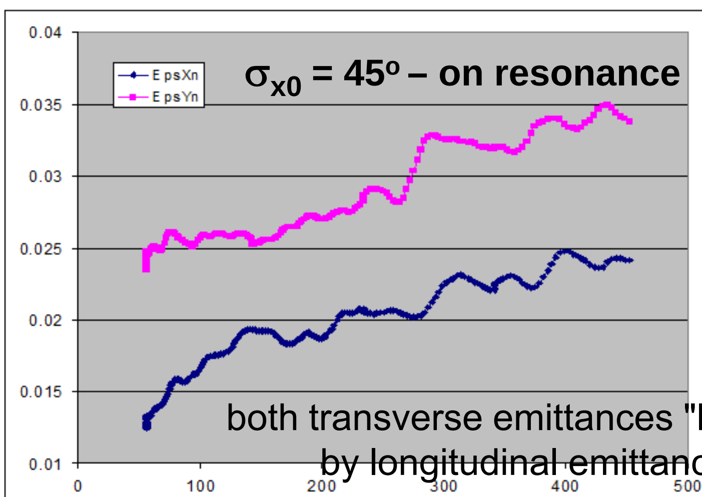


Figure 2.1: Schematic set-up of the experiments.

PARMILA results for well-matched beam in UNILAC

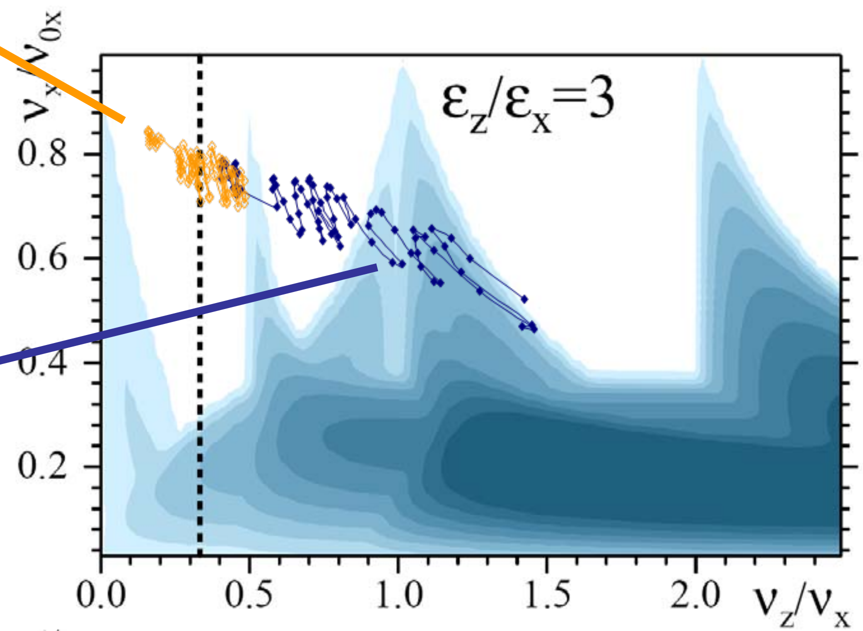
σ_{z0} = 45° – fixed





both transverse emittances "heated" by longitudinal emittance

- used several combinations of intensities and crossing rates
- checked factor α_{linac} for constancy: ~ 0.5
- work in progress: → IMPACT-study in 3D to determine "exact" 3D value of α_{linac} (indication that in 3D larger than in 2D)



Can we extend the ansatz $\Delta\varepsilon/\varepsilon \sim S^n$ to other space charge driven nonlinear resonances?

- fourth order space charge structure resonance driven by space charge "pseudo-octupole" (non-uniform initial beam) if 4Q = nxN (N number of super-periods/cells); or weaker, if purely error driven
- linacs: with phase advance per cell $\sigma_0 > 90^0$
- rings: non-scaling FFAG with Q crossing over many units
- KEK PS at 500 MeV stripping injection and Q → 7 (4Q = 7x4) (Igarashi et. al, PAC03)

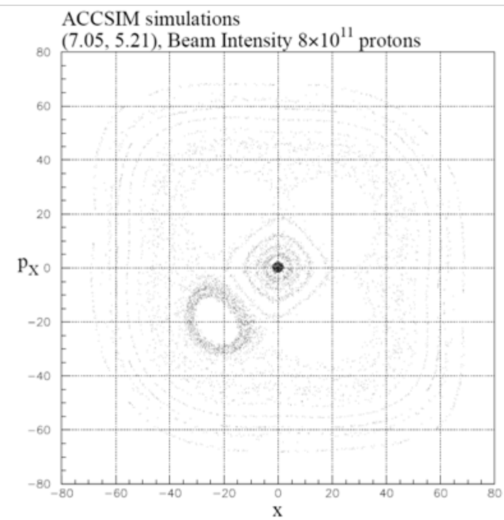


Figure 4: ACCSIM simulation of the $x - p_x$ phase space plots of 20 test particles when the horizontal tune was 7.05 and the injection beam intensity was 8×10^{11} protons.

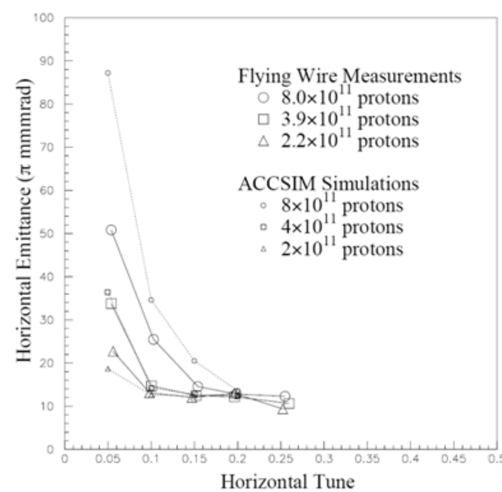
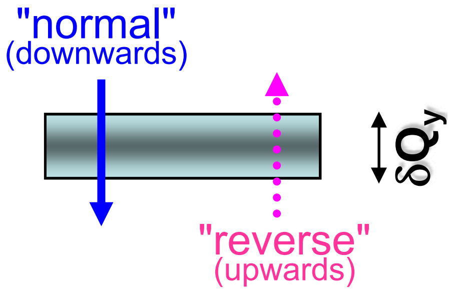


Figure 6: ACCSIM results of the horizontal 87% emittance as a function of the horizontal tune are shown in small symbols and dotted lines. Flyingwire measurement results are shown in large symbols and solid lines.

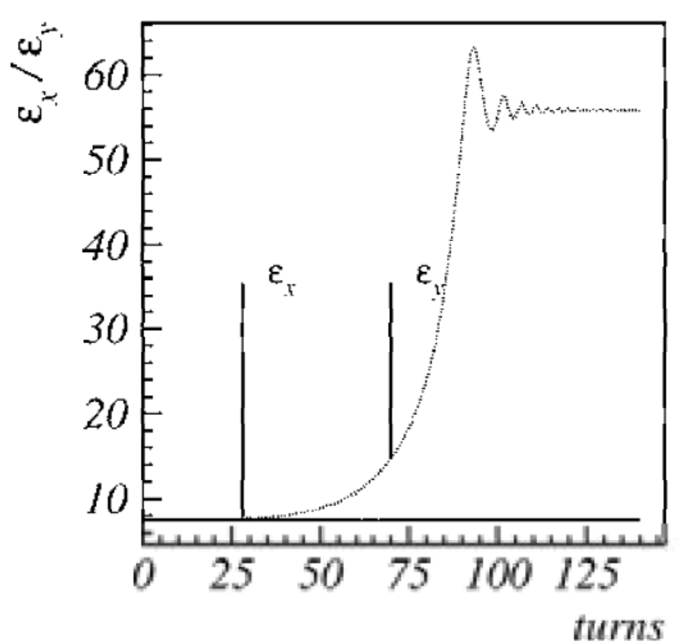
Studied 4Q=12 resonance in doublet lattice with Q→3 (90^0 phase advance per cell)

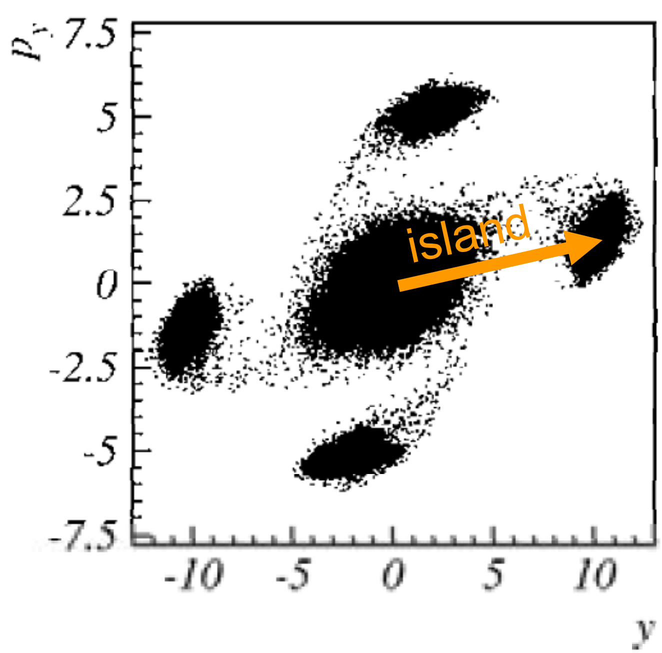


- normal: downwards crossing (same direction as space charge tune shift)
- resonance islands emerge from beam center
- particles trapped in islands if **slow crossing process**

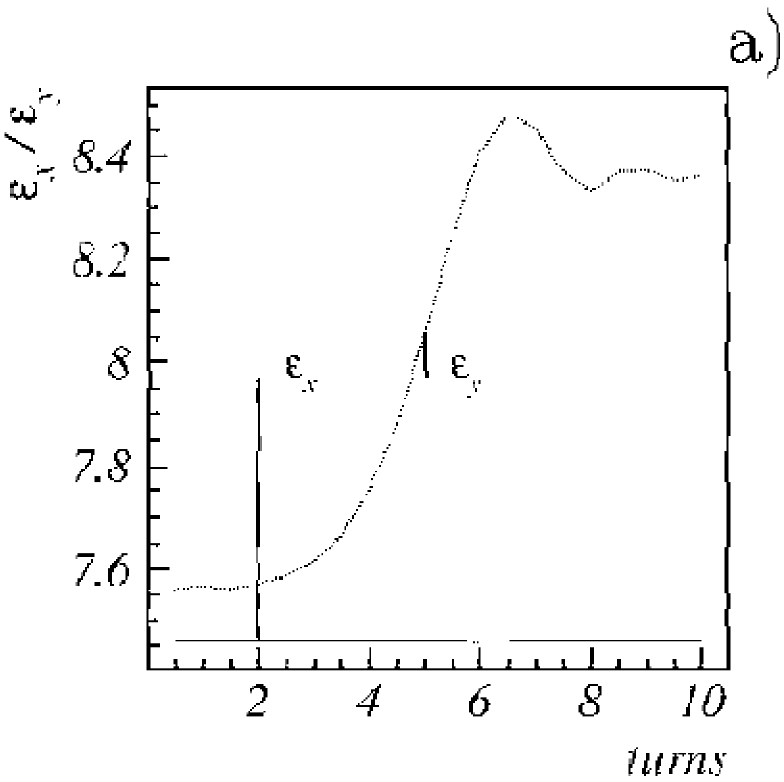
example: tune ramped downwards across stop-band width in 140 turns with ΔQ_{Gauss}= -0.3

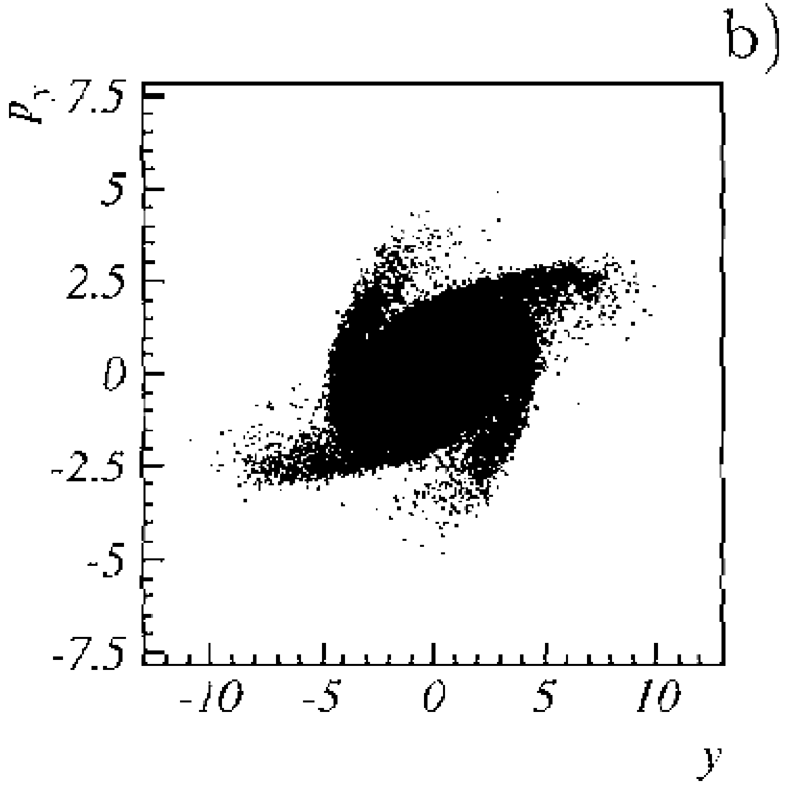
warning: 3D: modulation of space charge (→ island distance) by synchrotron motion **may suppress trapping**



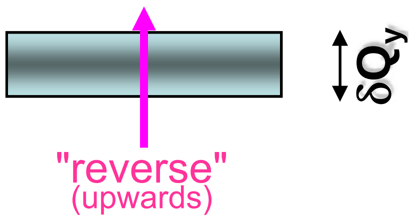


fast crossing (10 turns): only scattering on nonlinear potential – no trapping is possible



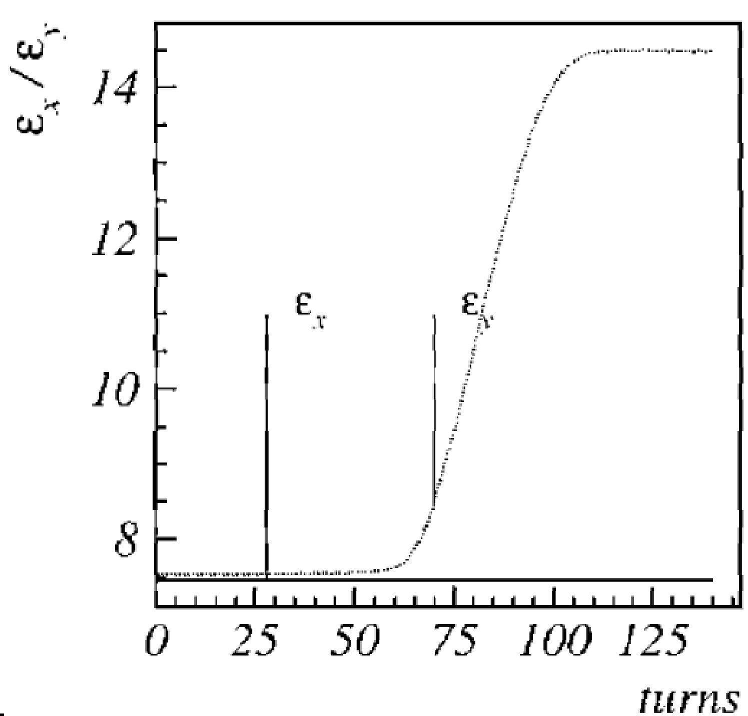


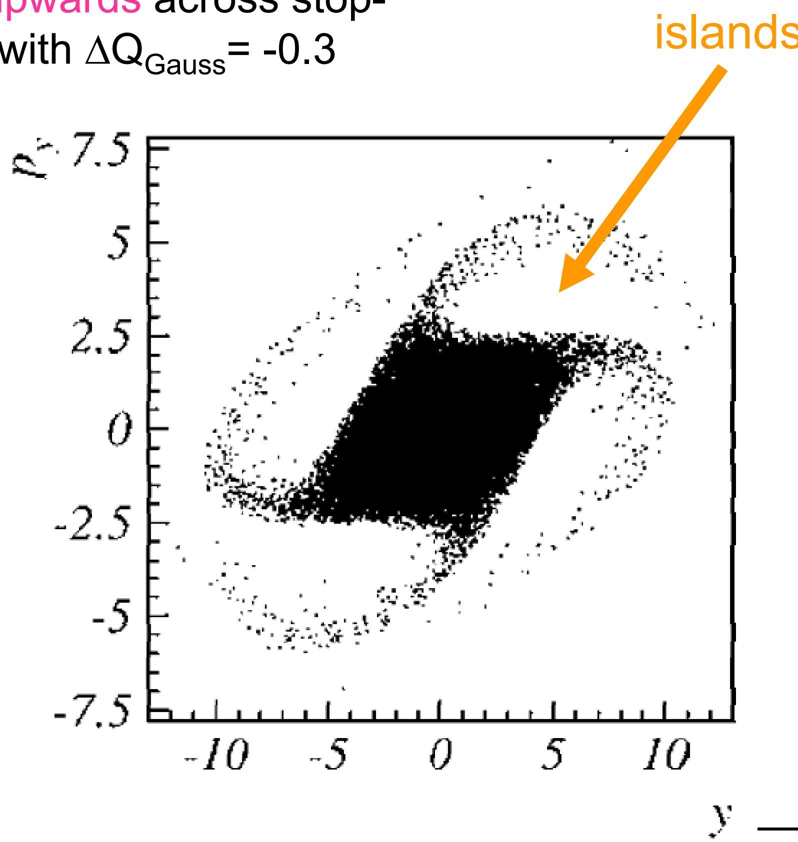
Reverse crossing



reverse: upwards crossing (against direction of space charge tune shift) – *resonance islands come in from infinity* ***void and remain void***

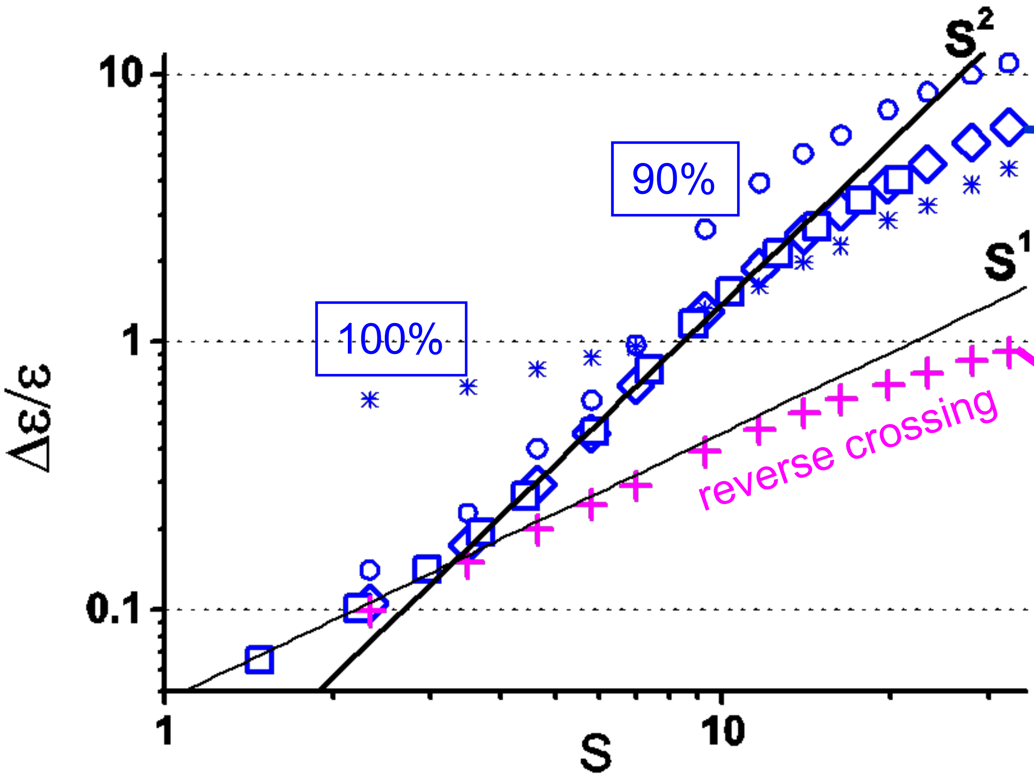
example: tune ramped upwards across stop-band width in 140 turns with ΔQ_{Gauss}= -0.3

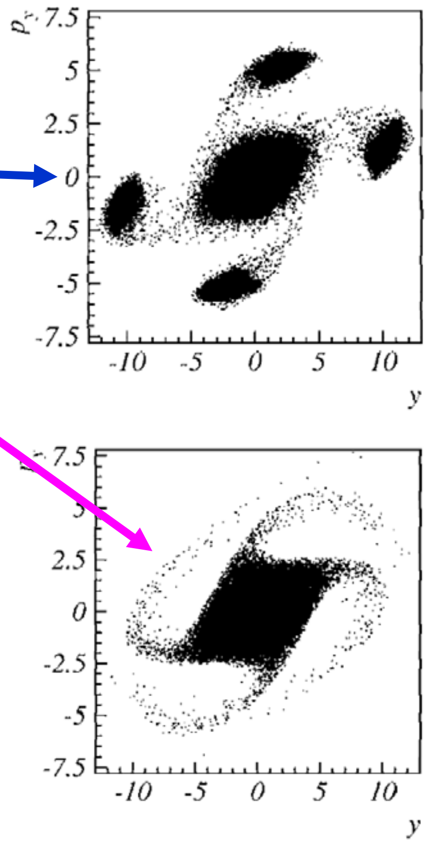




Searched for expression containing again S ≡ $(\Delta Q)^2/dQ/dn$ and found **two regimes:** scattering - trapping

RMS emittance growth due to substantial fraction of total number of particles growing in amplitude





"trapping" regime

$$\frac{\Delta\varepsilon}{\varepsilon} \propto S^2$$

"scattering" regime

$$\frac{\Delta\varepsilon}{\varepsilon} \propto S^1$$

fast or reverse crossing

Scaling coefficient depends on cell length

"trapping" regime:

$$\frac{\Delta\varepsilon}{\varepsilon} = \alpha_4 S^2$$

$S \equiv (\Delta Q)^2/dQ/dn$

α_4 ~

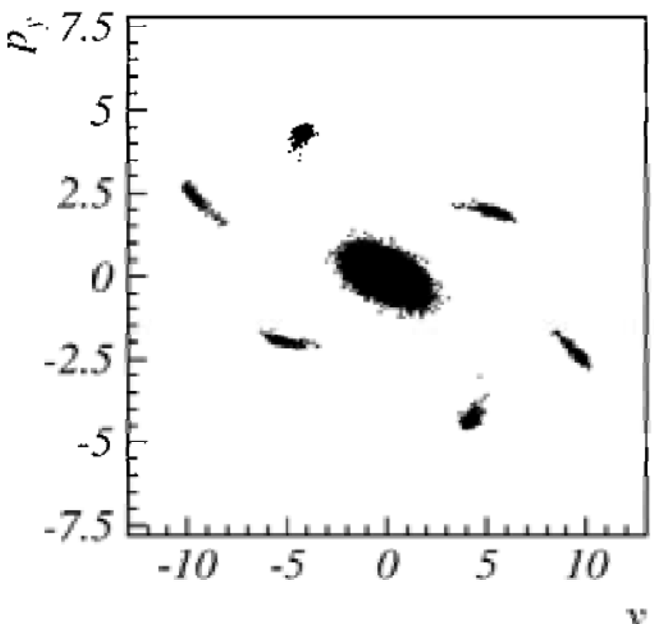
- $0.22/L^2$, L: length of 90^0 focusing cell in m
- less for triplet focusing or non-structure (error)
- =0 for constant focusing

example: non-scaling FFAG with L=3 m doublet cells
requires dQ/dn > 0.35 ΔQ^2 for $\Delta\varepsilon/\varepsilon$<20%
→ dQ/dn > 0.015 for ΔQ~-0.2

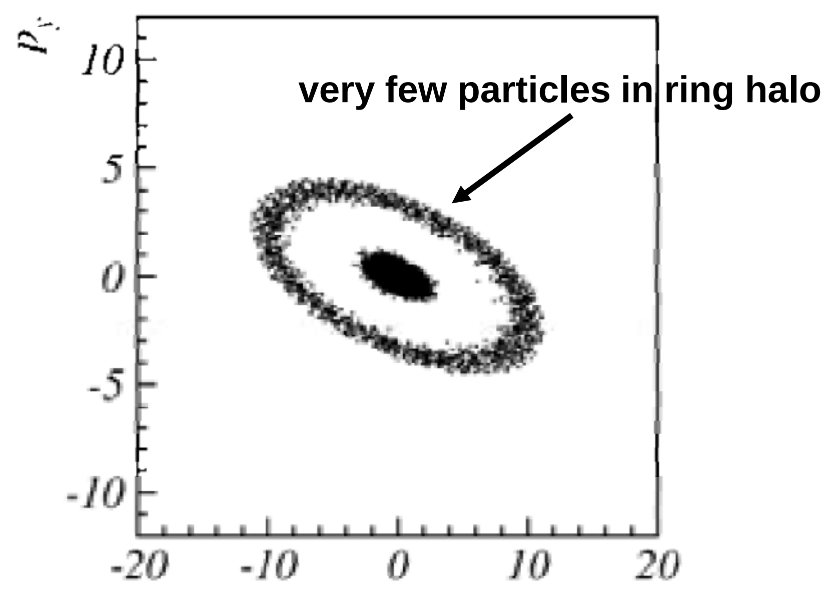
Sixth order resonance

- sixth order space charge structure resonance driven by space charge "pseudo-dodecapole" if 6Q = nxN (N number of super-periods/cells)
- linacs: with phase advance per cell $\sigma_0 \sim 60^0$
- rings: **non-scaling FFAG ...**

downwards crossing over 900 turns:



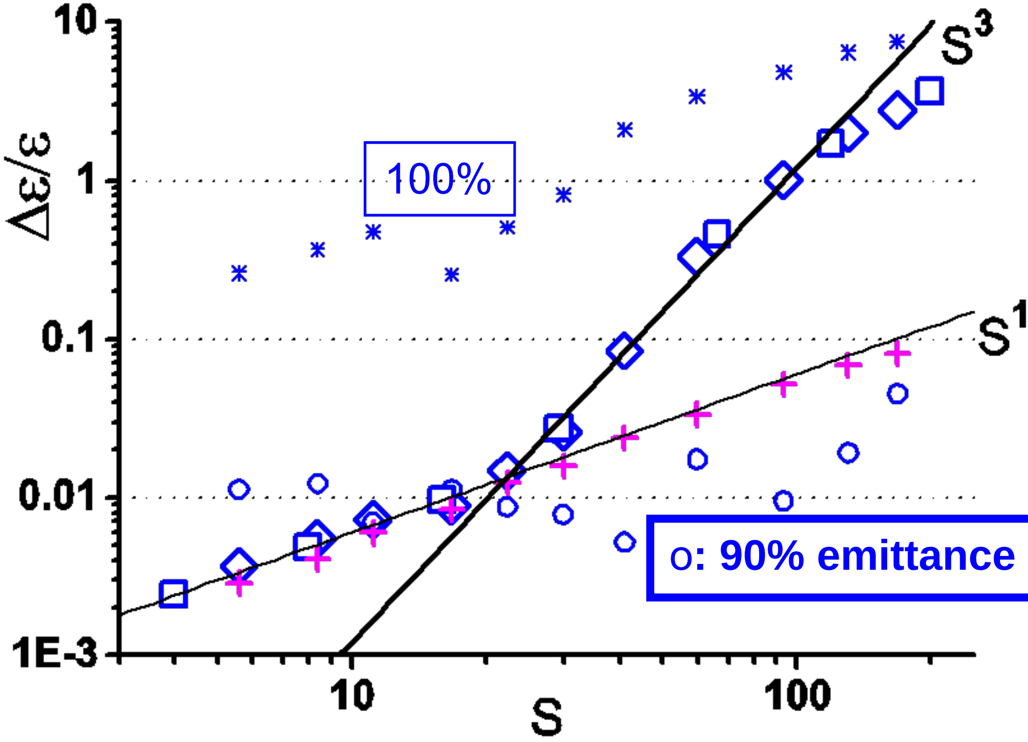
leaving stop-band



end of ramp

Negligible quantitative emittance growth only

few % of particles in ring halo → rms emittance growth irrelevant



rms emittance growth:

$$\frac{\Delta\varepsilon}{\varepsilon} = 1.2\times10^{-6} S^{3}$$

$$\frac{\Delta\varepsilon}{\varepsilon} = 6\times10^{-4}\ S^{1}$$

Conclusion

- Purely space charge driven resonances may occur in rings and linacs
- Found that emittance growth depends only on similarity parameter $S \equiv (\Delta Q)^2 / dQ/dn$ common to all problems studied
- → critical tune rate $dQ/dn \sim (\Delta Q)^2$
- Scaling laws $\Delta\varepsilon/\varepsilon \sim S^n$ found with
  - n=1 if only scattering on nonlinearity
  - n=2 or higher, if trapping in resonance islands
  - 3D effects need to be studied carefully: trapping might be suppressed by additional "fast" synchrotron motion as in linac → talk by D. Jeon on Thursday
- *Plan to extend scaling laws to "mixed" scenarios: resonances, where strength from magnets, but width dominated by space charge*