

Coherent Synchro-Betatron Resonance in the FNAL Booster

A. Burov, V. Lebedev

HB 2008

Booster

• Booster is a fast cycling proton synchrotron operating at 15 Hz.

Energy	0.4 – 8 GeV
Transition energy	5.1 GeV
Total number of particles	$4.5 \cdot 10^{12}$
Circumference	474.2 m
Harmonic number, q	84
Betatron tunes, v_x/v_y	~6.8/6.8
RF voltage	0.7- 0.9 MV
Injection type	H ⁻ , 11 turns

Transverse Instability

- As it was reported at HB 2006 [1], a transverse coherent instability at injection is one of important issues for the Booster.
- To prevent it, high chromaticities are normally set, which negatively affects the dynamic aperture and the beam lifetime.
- For studies, both chromaticities were reduced, and data were taken from 4 channel digital scope with sampling time 0.4 ns and total recording time ~700 turns.
- Off-line processing of these data included:
 - marking boundaries for each bunch,
 - subtracting bunch offsets from the differential signal,
 - obtaining density and dipole moment distributions.

Observations

- As a result, an instability was detected with the following features:
 - The instability started when the RF was almost grown up to its maximum, and the beam was almost bunched. It continued about 100 turns.
 - The amplitude of oscillations grew up to few mm.
 - All the bunches showed the same structure; they were in phase.
 - Turn-by-turn phase shifts were almost zero, as if the tunes were integers (but they were far from that!)
 - Amplitude growth was not sensitive to the beam intensity.

Structure of the oscillations



Oscillations of the same bunch every 5 turns, starting from turn 93.

- Turn-by-turn phase shifts are extremely small, $\ll 2\pi$ (7-v)
- Formally calculated growth time is extremely short, ~ 10 turns.

The problem

- This instability is more than order of magnitude faster than what can be expected from the impedance calculations.
- It was also found that this instability is not sensitive to the beam intensity.
- Zero phase advances of the coherent motion can be interpreted as an excitation of a coherent synchro-betatron resonance (CSBR).

• How that all can be explained?



• A condition of dipole CSBR is $v_b + lv_s = n$. If so, any dipole perturbation drives the resonant mode.

• CSBRs were first observed Besnier, Brandt and Zotter in simulations (1984), [2]. It was claimed CSBRs are driven by non-zero dispersion inside the cavities (no proofs were presented).

• Model of CSBR was suggested in our PR-ST paper [3], briefly followed below.

Perturbations

• Due to the non-zero dispersion, a particle gets a betatron kick every time it passes through a cavity:

$$\Delta x = -D(s_j)\sin(kz)\Delta p_{\max} / p_0$$

$$\Delta p_x = -(D'(s_j) + \alpha_x D(s_j) / \beta_x)\sin(kz)\Delta p_{\max} / p_0$$

• It can be treated as a localized periodical perturbation of a betatron motion:

$$\widetilde{x}' = \Delta x \delta_P (s - s_j)$$

$$\widetilde{p}'_x = \Delta p_x \delta_P (s - s_j)$$

$$\delta_P (s) \equiv C^{-1} \sum_{m = -\infty}^{\infty} \exp(-2\pi i m s / C)$$

Vlasov equation

• These perturbations have to be substituted into the Vlasov equation

$$\frac{\partial \psi}{\partial s} + \frac{\omega_b}{c} \frac{\partial \psi}{\partial \theta} + \frac{\omega_s}{c} \frac{\partial \psi}{\partial \phi} + \widetilde{x}' \frac{\partial \psi}{\partial x} + \widetilde{p}'_x \frac{\partial \psi}{\partial p_x} = 0$$

Transverse:

$$x = q\cos(\theta)$$
$$p_x = -(q / \beta_x)\sin(\theta)$$

Longitudinal:

$$z = r_0 \cos \phi$$
$$z' = -(r_0 \omega_s / c) \sin \phi$$

Perturbations: \widetilde{x} , \widetilde{p}

Linearization

• Linearization for $\psi = \psi_0 + \widetilde{\psi}$, with the air-bag (hollow-beam) unperturbed distribution

$$\psi_0 = f_0(q)\delta(r - r_0)$$

and the perturbation

$$\widetilde{\psi} = A(s)\sqrt{\beta_x} f_0'(q)\delta(r - r_0) \exp(i\theta + il\phi + i\chi z / r_0 - i\Omega_l s / c)$$

$$\Omega_l = \omega_b + l\omega_s \approx n\omega_0$$

$$\chi = 2\pi \frac{\xi r_0}{C\eta}; \quad \xi \equiv \frac{dv_b}{d(\Delta p / p_0)}; \quad \eta \equiv \frac{1}{\gamma_t^2} - \frac{1}{\gamma^2}$$

leads after averaging to the following equation on the slow-growing mode amplitude A(s):

Resonant mode amplitude growth



Here summation over all the cavities at $s = s_j$; j = 1, 2, ... N is performed.

The equation describes a linear growth of the mode amplitude driven by the external resonant force.

Paraphasing

 In the Booster, there are 18 cavities, located by tight pairs, with no free space in between. To prevent electric breakdown, RF is almost always the same in the cavities, and growth of the total RF is achieved by paraphasing, when the phases inside the pairs start from +-Pi/2, and then go to 0. If a cavity is out of phase, then:

$$J_{l}(\chi - kr_{0}) - J_{l}(\chi + kr_{0}) \rightarrow J_{l}(\chi - kr_{0})e^{i\phi} - J_{l}(\chi + kr_{0})e^{-i\phi}$$

• After that, this term has to go under the sum operator.

Crossing a resonanse

• In a vicinity of the resonance, with $\Delta \Omega \equiv \Omega_l - n\omega_0 \neq 0$, the RHS of the above amplitude equation

dA/ds = F

has to be multiplied by an oscillating factor :

$$dA/ds = F \exp(i\int_{-\infty}^{s} \Delta\Omega(s')ds'/c) = F \exp(i\Delta\dot{\Omega}s^{2}/2c^{2})$$

Integrating this equation, a total increase in the mode amplitude follows:

$$\Delta A = \int \frac{dA}{ds} \, ds = F(1+i) \sqrt{\frac{\pi}{l\dot{\omega}_s T_0^2}}$$

where T_0 is the revolution time.

• For the Booster parameters (details are presented in [1]), this gives:

 $\Delta A \sqrt{\beta_{\rm bpm}} \cong 1\,{\rm mm}$

• The result is very sensitive to the lattice symmetry. For the design lattice, the contributions of different cavity pairs almost cancel each other:



Contributions of 9 pairs of cavities in the total complex amplitude increase: the net result is equal to a single pair contribution. In practice, this good luck may easily not happen, so the resulting growth be 2-4 times higher.

Is Landau Damping needed?

- Calculated above CSBR growth does not depend of the intensity. The calculations assume no Landau damping of the coherent mode. It is correct, when the space charge is so strong, that the Landau damping vanishes. At low intensities, incoherent SBRs should take place instead, leading directly to emittance growth.
- A possible danger from CSBR is providing high starting value for then impedance-driving instability. However, this appears do not happen in the Booster. Until so, there is no reason to care about Landau damping of CSBR (Since Landau Damping only transfers coherent oscillations into incoherent, what eventually occurs anyway).

Suppression of CSBR

- Even if exponential grow does not happen, CSBR is still detrimental:
 - Emittance growth
 - Possible particle loss
- CSBR can be suppressed by several means:
 - To optimize tunes and chromaticities;
 - To cross CSBR faster, if possible;
 - To re-distribute RF between the cavities so that contributions of different cavities cancel each other.

<u>Summary</u>

- A model of CSBR is developed.
- It is in a agreement with observations in the Booster.
- CSBR can lead to emittance growth and particle loss
- Ways to suppress CSBR are discussed.

References

[1] V. Lebedev, A. Burov, W. Pellico, X. Yang, "Instabilities in the FNAL Booster", Proc. HB 2006, Tsukuba (2006).

[2] G. Besnier, D. Brandt, B. Zotter, CERN/LEP-TH/84-11 (1984).

[3] A. Burov, V. Lebedev, "Coherent synchrobetatron resonance", Phys Rev ST-AB, 10, 054202 (2007).