SIMULATION STUDIES OF THE INTERPLAY BETWEEN SPACE-CHARGE AND IMPEDANCE EFFECTS OF THE FERMILAB MAIN INJECTOR

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Abstract

Understanding the behavior of high-intensity beams in the Fermilab Main Injector is crucial for the future physics program at the lab. Simulations of the Main Injector including collective effects are a crucial part of this understanding. We are building up a set of integrated simulations of collective effects using the Synergia accelerator simulation framework. As a step in this work we present simulations of space-charge effects combined with impedance effects in the Main Injector.

INTRODUCTION

Virtually all current proposals for future experiments at Fermilab involve high-intensity beams. Because collective effects are most often the limiting factor(s) when increasing beam intensity, the simulation of collective effects is of greater importance than ever before. As collective effects become more important in general, the interplay between multiple collective effects in a single machine and/or simulation also becomes more important.

Fortunately, as the need for complex simulations is growing, so is the available technology for performing complex computer simulations. Such simulations now require taking advantage of (massively) parallel architectures. This paper describes progress in simulations of multiple collective effects in the Fermilab Main Injector utilizing Synergia, a framework for simulating collective effects on parallel machines.

THE FERMILAB MAIN INJECTOR

The Fermilab Main Injector is central to the Fermilab accelerator complex. It is a 3.3 km accelerator with harmonic number 588, accelerating protons from 8 GeV to 120 GeV. The Main Injector has a strong transverse resistive wall instability that was first diagnosed in the Fermilab Main ring[1]. The instability is damped by an active damping system[2].

Simulations of the Main Injector are complicated by the cross-sectional geometry of its beam pipe, as shown in Fig. 1. The pipe is approximately elliptical, with a height and width of 5.31 cm and 12.3 cm, respectively. Under vacuum, the pipe deforms to 5.08 cm in height.

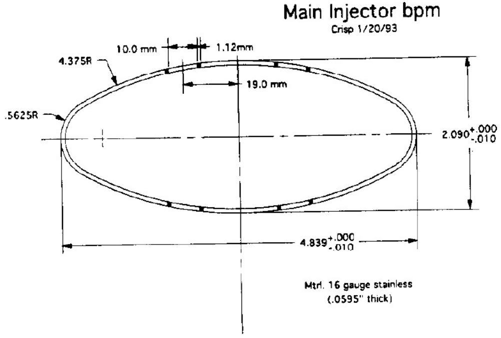

Figure 1: Main injector beam pipe cross section.

SYNERGIA

Our goal is to have a single simulation framework capable of modeling all the relevant collective effects for a given machine. We have developed Synergia[3] to be such a framework. One of the principles of the Synergia development has been to use state-of-the-art existing code wherever possible, including basic computer infrastructure such as the GNU Scientific Library and PETSc, as well accelerator physics domain-specific software such as IMPACT[4]. Synergia was designed to be modular, starting with non-linear optics and space charge module, with modules for effects such as resistive wall, electron cloud, and beam-beam to be added at a later date. This work contains the first results from the combination of Synergia's existing non-linear optics and space charge and new resistive wall modules.

Non-linear Optics

The non-linear optics in Synergia are provided by CHEF[5]. Chef provides a complete package for single-particle tracking either via arbitrary-order maps or via its less approximate internal tracking. Synergia can not only take advantage of both tracking modes, it can do so within the same simulation. Typically, we use map-based tracking almost all of the time, resorting to the slower internal tracking only for RF cavities in beam-capture mode, where it is important to be able to track the entire longitudinal phase space accurately. CHEF also provides many more features such as MAD, (and, soon, MADX) file input, a GUI for beamline visualization, etc. CHEF has been in use for many years and validated against an number of other optics implementations.

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Resistive Wall

Synergia's new resistive wall module was originally developed for Tevatron beam-beam studies utilizing BeamBeam3D[6]. It contains a simple dipole approximation for resistive-wall wakefields in the thick wall limit[7]

$$\frac{\Delta\vec{p}_\perp}{p} = \frac{2}{\pi b^3}\sqrt{\frac{c}{\sigma}}\,\frac{N_i<\vec{r}_i>}{\beta\gamma}\,\frac{L}{\sqrt{z_i}} = W_0 L <\vec{r}> . \tag{1}$$

The longitudinal charge density is calculated in a 3D Particle-in-cell (PIC) approach using cloud-in-cell charge deposition. Although this module is new to Synergia, it has been validated in its original BeamBeam3D context in Ref. [8]. There we demonstrated the validity of the resistive wall simulation with respect to synchro-betatron coupling behavior, growth rate of dipole motion as a function of intensity, and growth rate as a function of head-tail phase, showing predicted linear growth near 0 and near-universality near -1[7].

Space charge

Synergia's space charge module has been extensively validated[3]. To date, its primary application to Fermilab machines has been simulations of the Booster. The existing validations have utilized a module based on the IMPACT solver suite[4]. We have recently developed a new suite of solvers, Sphyraena. The Sphyraena suite allows the simulation of an arbitrary number of loosely-coupled bunches. The previous Synergia space charge implementation allowed the simulation of multiple bunches, but only in a strongly coupled manner that would not scale to more than a few bunches. The Sphyraena suite contains solvers for open boundary conditions and perfectly conducting cylindrical boundary conditions. A finite-difference solver capable of arbitrary boundary conditions is under development.

We have used a novel analytic technique to validate the Sphyraena solvers. Typically, Poisson solvers are validated using solvable physical cases. For example, the conducting cylindrical pipe case would be validated using the well-known solution for a uniform cylinder of charge. However, the solvable physical cases always contain a high degree of symmetry, which makes them a less-than-comprehensive test for all degrees of freedom. The novel approach we take is to postulate a field solution with non-trivial structure, then differentiate to obtain the corresponding charge density. This charge density is then used as an input to the solver. The solution can then be compared against the postulated analytical result. For our cylindrical solver, we use

$$\begin{aligned}\rho(r,\theta,z) \quad = \quad & \Big[\Big(\left(18\,{r_0}^2-14\,r^2\right)\,\sin^2\left(3\,\theta\right)+\\ & \left(18\,r^2-18\,{r_0}^2\right)\,\cos^2\left(3\,\theta\right)\Big)\times\\ & \cos^2\left(\frac{\pi\,z}{z_0}\right)\,{z_0}^2+\\ & \left(2\,\pi^2\,r^4-2\,\pi^2\,r^2\,{r_0}^2\right)\times\\ & \sin^2\left(3\,\theta\right)\,\sin^2\left(\frac{\pi\,z}{z_0}\right)+\\ & \left(2\,\pi^2\,r^2\,{r_0}^2-2\,\pi^2\,r^4\right)\times\\ & \sin^2\left(3\,\theta\right)\,\cos^2\left(\frac{\pi\,z}{z_0}\right)\Big]\\ & \Big/\left(r^2\,{r_0}^2\,{z_0}^2\right)\end{aligned} \tag{2}$$

(Fig. 2.), with the solution

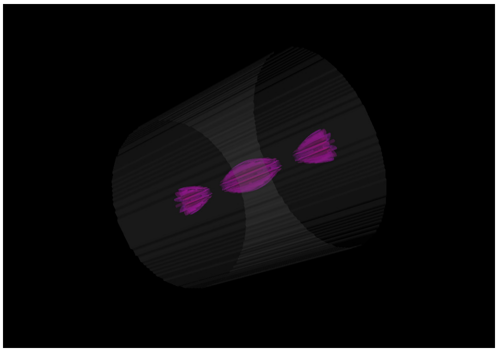

Figure 2: Charge density.

$$\phi(r,\theta,z) = \left(1-\frac{r^2}{r_0^2}\right)\sin^2\left(3\theta\right)\cos^2\left(\pi\frac{z}{z_0}\right) \tag{3}$$

(Fig. 3.)

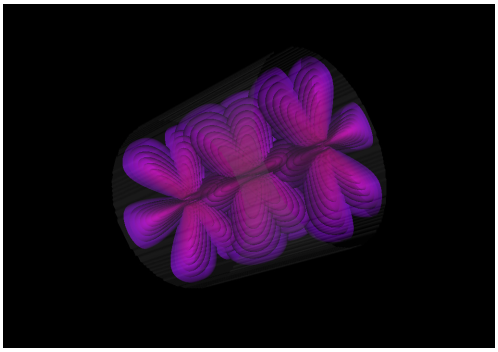

Figure 3: Field solution corresponding to the charge density of Fig. 2.

PRELIMINARY RESULTS

We have performed preliminary simulations of the Fermilab Main Injector using Synergia's non-linear optics, space charge and resistive wall modules. Our objective is to reproduce the well-studied transverse resistive wall instability in the Main Injector. The parameters of the simulation are summarized in Table 1. For the space charge module, we used perfectly conducting cylindrical boundary conditions, with the cylinder diameter corresponding to the average of the vertical and horizontal beam pipe dimensions. The resistive wall module also assumes a cylindrical pipe, but we are able to specify distinct horizontal and vertical radii. We use the half the true width and height, respectively.

Table 1: Simulation parameters

energy	8.9 GeV
longitudinal RMS bunch size	0.75 m
intensity	$6 \times 10^{(10,11)}$ protons/bunch
number of bunches	8
total macroparticles	524288
x- and y-emittances	0.26 mm-mr
x offset	1 mm
space charge grid	$32 \times 32 \times 32$
resistive wall slices	32

Figs. 4 and 5 show the simulated effects of the instability on the mean beam position for 6×10^{10} and 6×10^{11} protons/bunch, respectively. Fig. 6 shows the corresponding emittance growth.

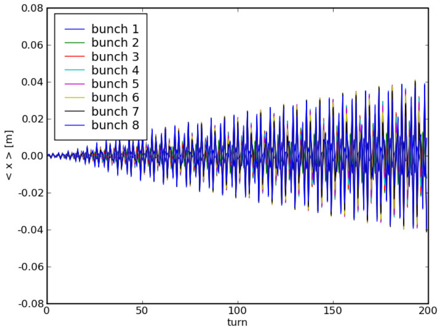

Figure 4: Simulated mean beam position.

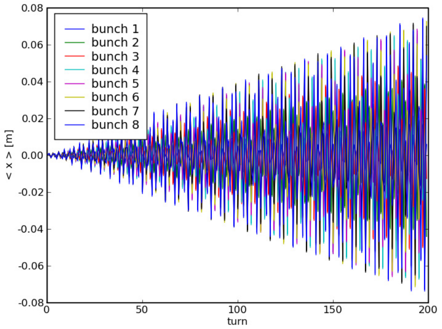

Figure 5: Simulated mean beam position.

NEXT STEPS

We have plans to run more realistic Main Injector simulations in the future. The next step will be to include the active damping system currently used in the Main Injector into our simulation. It will then be logical to simulate other portions of the Main Injector cycle, particularly transition. As soon as the arbitrary-geometry finite-difference solver Sphyraena component is complete, we will be able to do a more realistic simulation of the beampipe, allowing is to determine conclusively whether the cylindrical approximation for the Main Injector beampipe is sufficient for these simulations.

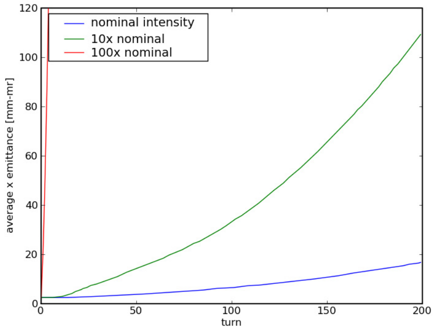

Figure 6: Simulated emittance growth.

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