Transforming the FEL: Coherence, Complex Structures, and Exotic Beams

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SLAC National Accelerator Laboratory

FEL Prize Session
37th International Free Electron Laser (FEL) conference
August 24, 2015, Daejeon, Korea

Outline

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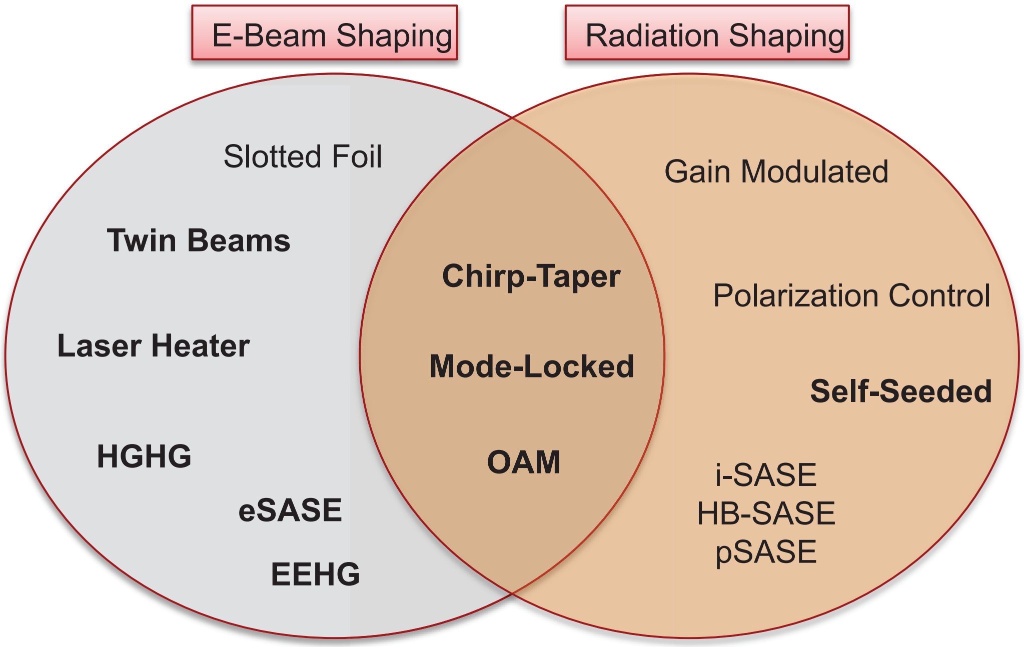
- Transforming the FEL
  - FEL pulse tailoring schemes
- Complex Structures
  - a brief introduction to OAM light
- OAM Generation in FELs
  - Mode description
  - FEL tailoring for OAM
- OAM Experiments
- Future Possibilities
- Conclusions

FEL Pulse Tailoring

- **Many** techniques for transforming the FEL output by
  - Directly shaping electron beam or
  - Exploiting features of the emitted radiation
- "Beam by Design" now possible where light can be precisely tailored to suit wide-ranging needs



Bold: Laser-based

FEL Pulse Tailoring

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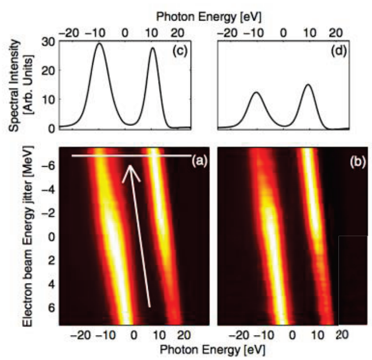
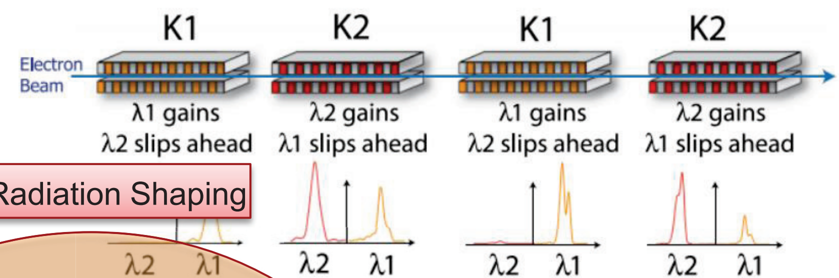
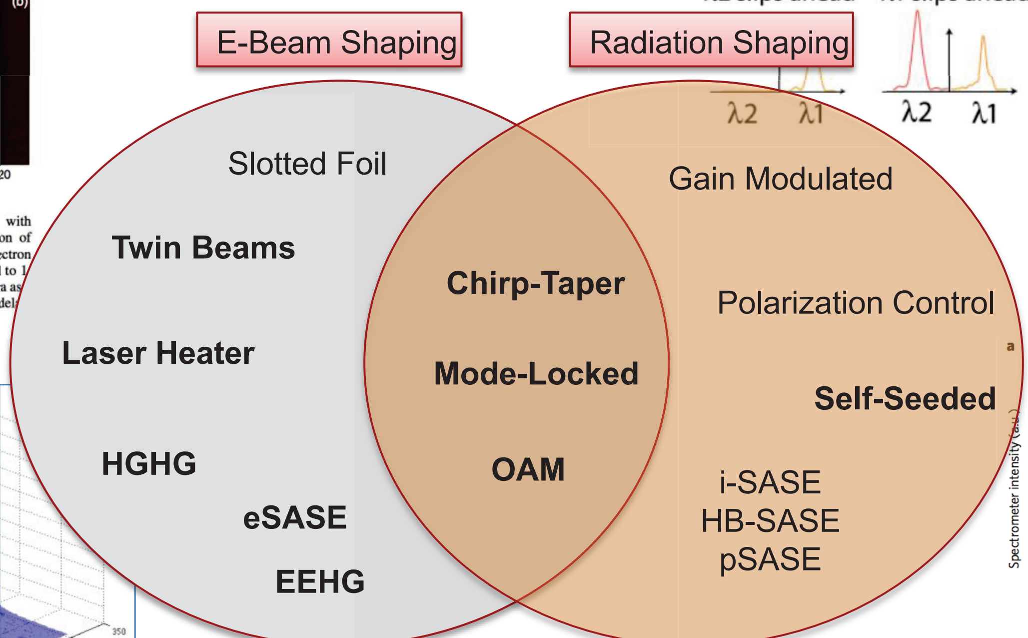
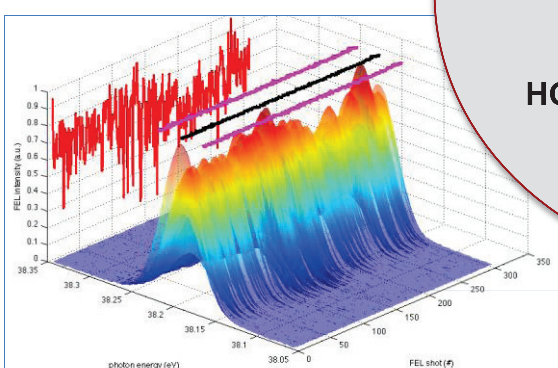
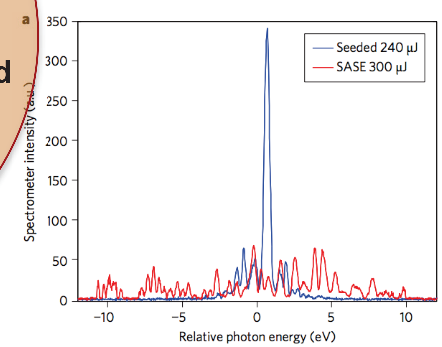


FIG. 3 (color online). Results for two-color beams with scheme I. (a),(b) Average spectral intensity as a function of the electron beam energy and photon energy. For each electron beam energy, the maximum intensity has been normalized to 1 (a) 0 fs delay. (b) 25 fs delay. (c,d) Average realigned spectra as function of the photon energy offset from 1.5 keV. (c) 0 fs del (d) 25 fs delay.









Bold: Laser-based

Exotic light

Virtually all e-beam manipulation and improved FEL lasing techniques modify the longitudinal phase space (with good results!)

… But the transverse phase space of high-brightness beams can also be manipulated through laser based interactions.

Can we generate higher-order light? Specifically, light that carries Orbital Angular Momentum?

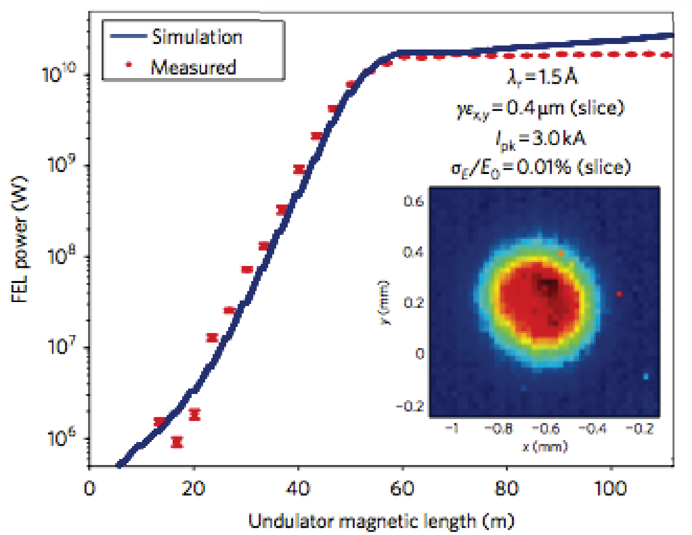
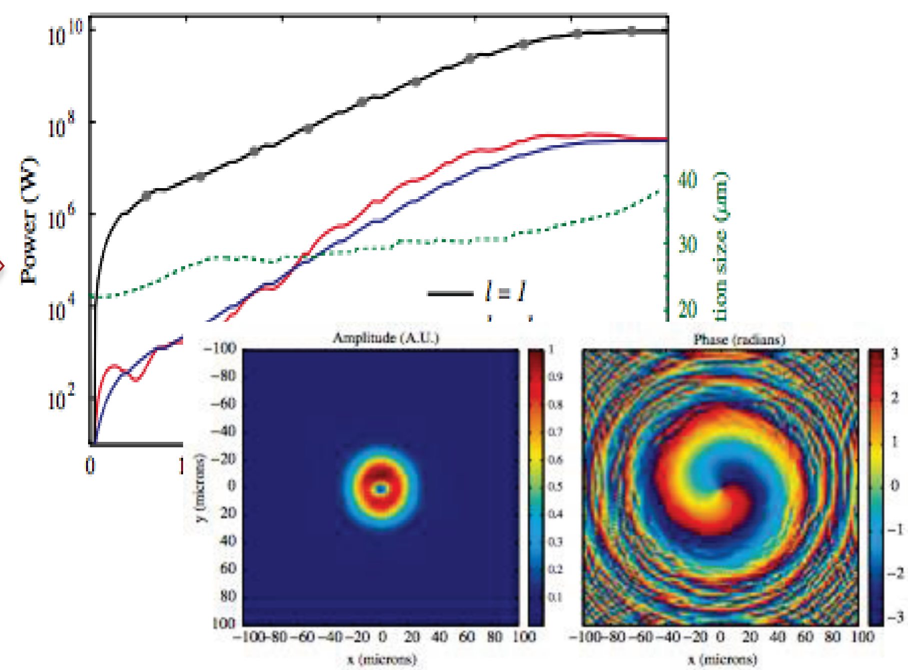


Figure 4 | FEL gain length measurement at 1.5 Å. Measured FEL power (re

??



P. Emma, et al, Nature Photonics 4, 641 - 647 (2010)

Angular Momentum of Light

- 1909 - J. H. Poynting. Circularly polarized fields must carry AM by virtue of rotating energy flux
- 1936 – R. A. Beth measures torque imparted to quarter wave plates. First direct measurement of **spin angular momentum (SAM)**

JULY 15, 1936 PHYSICAL REVIEW VOLUME 5

Mechanical Detection and Measurement of the Angular Momentum of Light

RICHARD A. BETH,* *Worcester Polytechnic Institute, Worcester, Mass. and Palmer Physical Laboratory, Princeton Universit*

(Received May 8, 1936)

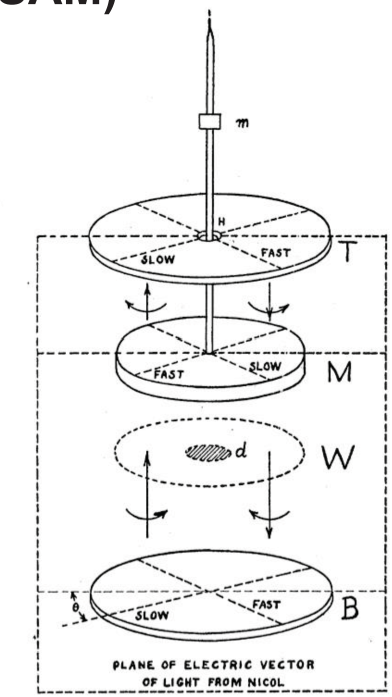


FIG. 3. Wave plate arrangement.

Angular Momentum of Light

- 1909 - J. H. Poy … st carry AM by virtue of rotating …
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in the medium. The torque per unit volume produced by the action of the electric field on the polarization of the medium is

$$\mathbf{l}=\mathbf{P}\times\mathbf{E}=(\mathbf{D}\times\mathbf{E})/4\pi. \quad (1)$$

Following Sadowsky and Epstein[1] we may calculate this torque for a simple case as follows: Assume a doubly refracting medium of permeability unity and take as x, y, and z axes the principal axes of the tensor $\mathbf{K}$ for the light frequency in question. Denote the principal values of $\mathbf{K}$ by n_x^2, n_y^2 and n_z^2 so that n_x, n_y and n_z will be the principal indices of refraction. Let the electric components of a plane light wave

* Member of the Physics Department, Worcester Polytechnic Institute. Preliminary work and final calculations were carried on at Worcester. Experimental work was done on leave-of-absence from Worcester as Research Associate at Palmer Physical Laboratory, Princeton University. Additional measurements were made by Mr. W. Harris at Princeton, as will be explained later.

[1] A. Sadowsky, Acta et Commentationes Imp. Universitatis Jurievensis **7**, No. 1–3 (1899); **8**, No. 1–2 (1900). P. S. Epstein, Ann. d. Physik **44**, 593 (1914). The derivation is repeated here because the first articles are in Russian and relatively inaccessible, while in the last there seems to be a misprint in the result. I wish to thank Dr. Boris Podolsky for translating parts of Sadowsky's articles for me from the Russian. I am also deeply indebted to Professor A. Einstein not only for his advice in checking Professor Epstein's calculation but also for several interesting discussions about the experimental part of the work here reported.

JULY 15, 1936 … E 5…

Mechanical Detection and M… t

Richard A. Beth,* *Worcester Polytechnic Insti… versit…*

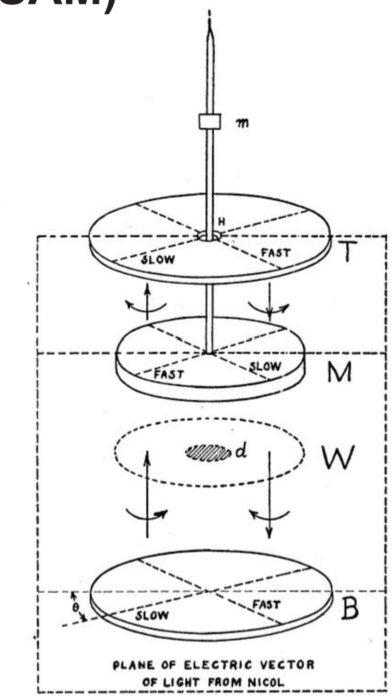

Fig. 3. Wave plate arrangement.

Origin of optical Orbital Angular Momentum

SAM, ok. Can the EM field carry OAM?

Angular momentum density

$$\vec{\mathbf{M}} = \epsilon_0 \vec{\mathbf{r}} \times (\vec{\mathbf{E}} \times \vec{\mathbf{B}})$$

Total angular momentum (Lorentz gauge)

$$\vec{\mathbf{J}} = \epsilon_0 \int d^3r \Big[\underbrace{\vec{\mathbf{E}} \times \vec{\mathbf{A}}}_{spin} + \underbrace{\sum_j^3 E_j (\vec{\mathbf{r}} \times \nabla) A_j}_{orbital} \Big]$$

For a plane wave E & A are strictly transverse, so:

$$\sum_j^3 E_j \underbrace{(\vec{\mathbf{r}} \times \nabla)}_{i\vec{\mathbf{L}}} A_j = 0$$

HOWEVER, the E,B fields of non-infinite TEM laser modes are **not strictly transverse.**

OAM in Laguerre Gaussian Modes

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PHYSICAL REVIEW A VOLUME 45, NUMBER 11 1 JUNE 1992

Orbital angular momentum of light and the transformation of Laguerre-Gaussian laser modes

L. Allen, M. W. Beijersbergen, R. J. C. Spreeuw, and J. P. Woerdman
Huygens Laboratory, Leiden University, P.O. Box 9504, 2300 RA Leiden, The Netherlands
(Received 6 January 1992)

Laser light with a Laguerre-Gaussian amplitude distribution is found to have a well-defined orbital angular momentum. An astigmatic optical system may be used to transform a high-order Laguerre-Gaussian mode into a high-order Hermite-Gaussian mode reversibly. An experiment is proposed to measure the mechanical torque induced by the transfer of orbital angular momentum associated with such a transformation.

PACS number(s): 42.50.Vk

I. INTRODUCTION

It is well known from Maxwell's theory that electromagnetic radiation carries both energy and momentum. The momentum may have both linear and angular contributions; angular momentum has a spin part associ-

larized by a fixed quarter-wave plate, passed through the plate which transformed right-handed circularly polarized light into left-handed circularly polarized light and transferred $2\hbar$ of spin angular momentum for each photon to the birefringent plate. It was found that the measured torque agreed in sign and magnitude with that pre-

2800+ Citations (Google Scholar)

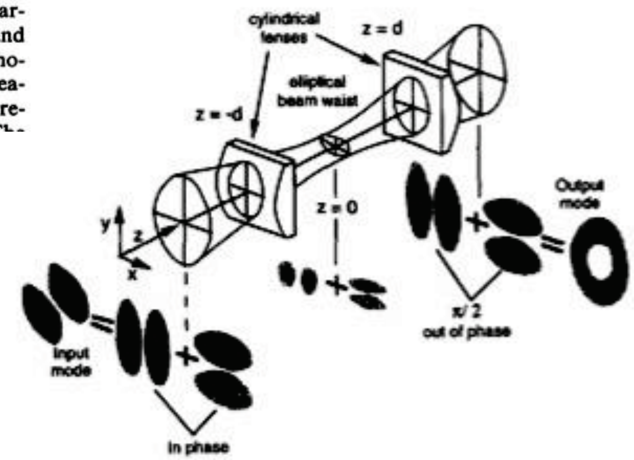


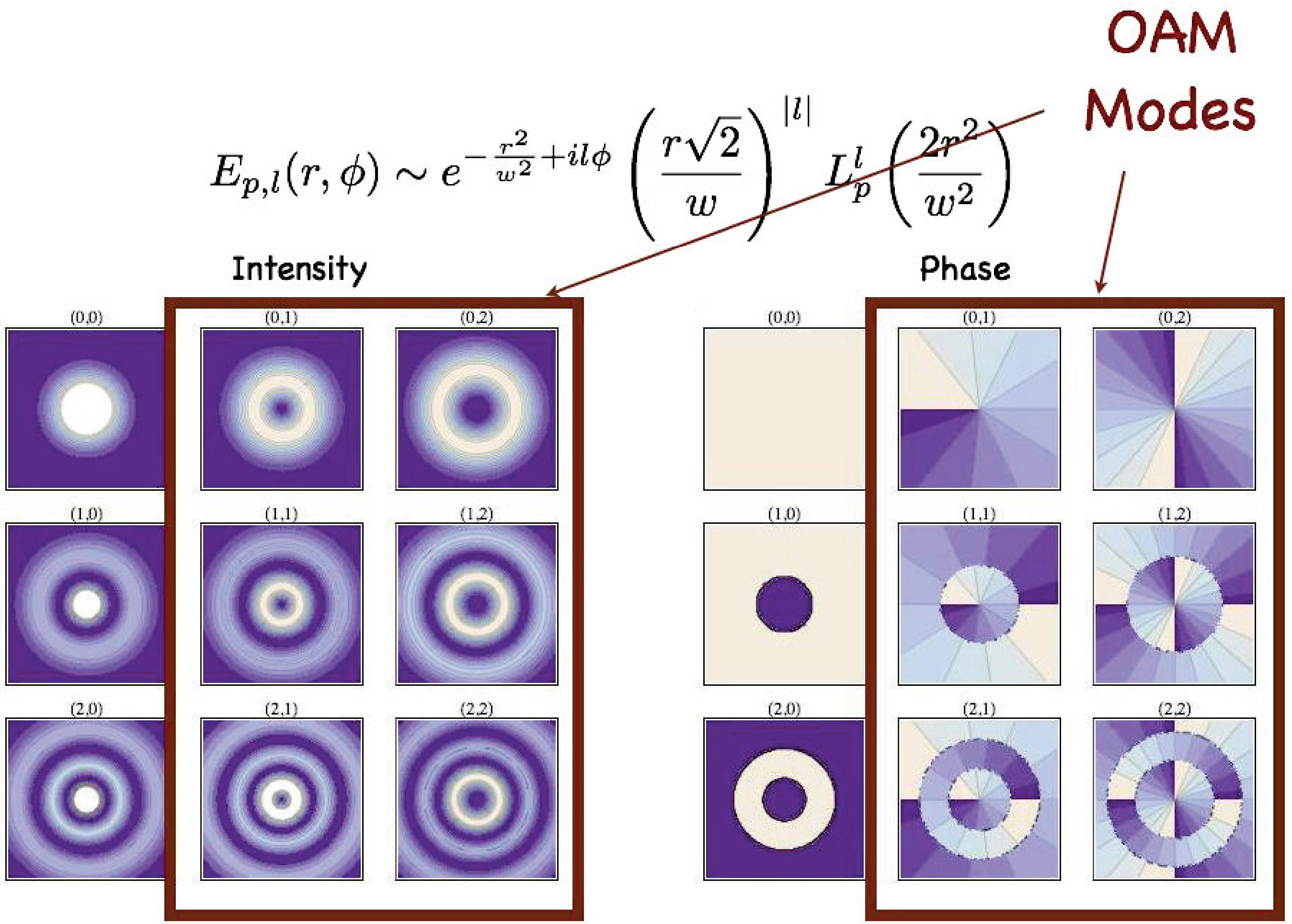
Fig. 4. The cylindrical lens mode converter. If the input $HG_{1,0}$ mode is oriented at 45° with respect to the cylinder axis of lens the mode is converted into the LG_0^1 mode.

Light Modes with Orbital Angular Momentum

Appear as solutions to the paraxial wave equation:

$$\nabla_\perp^2 E + 2ik\frac{\partial}{\partial z}E = 0$$

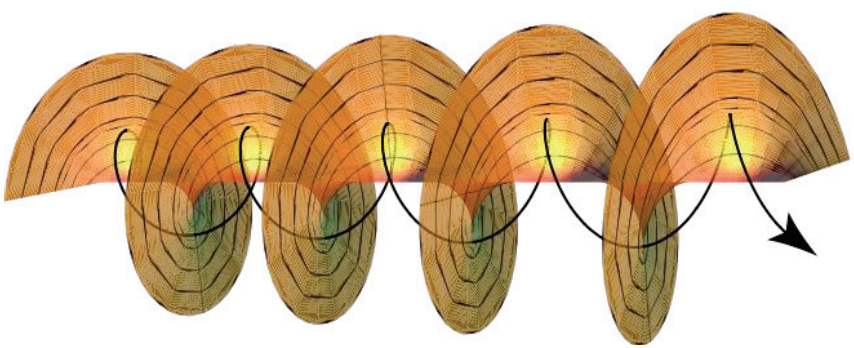
$$E_{p,l}(r,\phi) \sim e^{-\frac{r^2}{w^2}+il\phi}\left(\frac{r\sqrt{2}}{w}\right)^{|l|} L_p^l\left(\frac{2r^2}{w^2}\right)$$



OAM Light Carries Helical Phase

Photon momentum spirals about the axis

$l=1$ $l=2$ $l=3$



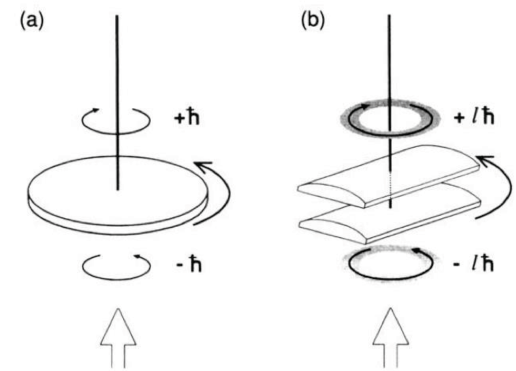


FIG. 1. (a) A suspended $\lambda/2$ birefringent plate undergoes torque in transforming right-handed into left-handed circularly polarized light. (b) Suspended cylindrical lenses undergo torque in transforming a Laguerre-Gaussian mode of orbital angular momentum $-l\hbar$ per photon, into one with $+l\hbar$ per photon.

OAM SAM

$$J_z = (l \pm 1)\hbar$$

$$L_z = lE/\omega \qquad \tau = lP/\omega$$

Applications for OAM modes

- *Mechanical micro manipulation:* precision transfer of quantized torque to samples (proteins, molecules, BECs, etc)
- *Photon Science:* Enhanced resolution and contrast in microscopy and diffraction imaging, two mode pump-probe, X-ray magnetic dichroism and angle-resolved spectrometry
- *Quantum States:* Quantum entanglement and encryption in OAM state (n-dimensional storage space), BECs

http://www.physics.gla.ac.uk/Optics/projects/spinningAndOrbiting/

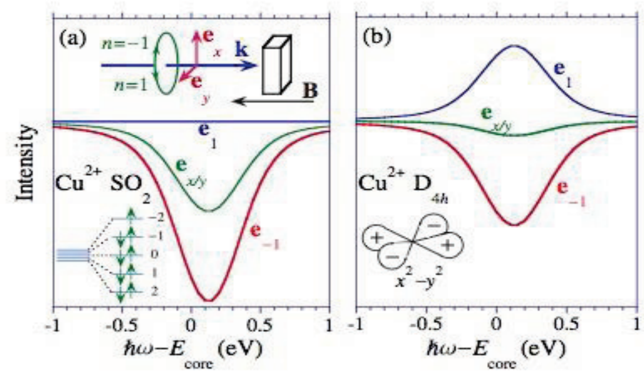
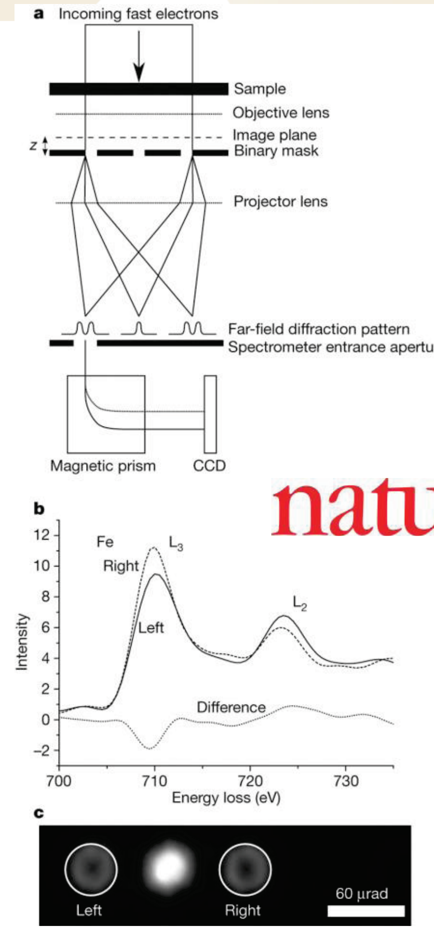


FIG. 1 (color online). (a) The dichroic OAM signal for circular ($\mathbf{e}_1$ and $\mathbf{e}_{-1}$) and linear ($\mathbf{e}_{x/y}$) polarization of the incoming x-rays for a divalent copper ion in spherical symmetry with a magnetic field along the z axis. (b) The same, but for a Cu^{2+} atom in a crystal field with D_{4h} symmetry.

Veenendaal and McNulty, PRL **98, 157401 (2007)**



nature

Application of the vortex beam to energy-loss spectrometry.

J Verbeeck *et al. Nature* **467**, 301-304 (2010)

Conventional OAM light generation

- spiral phase plate
- diffraction mask
- mode convertor optics
- …

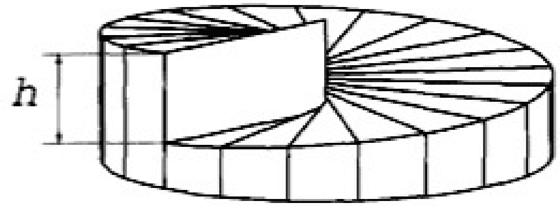
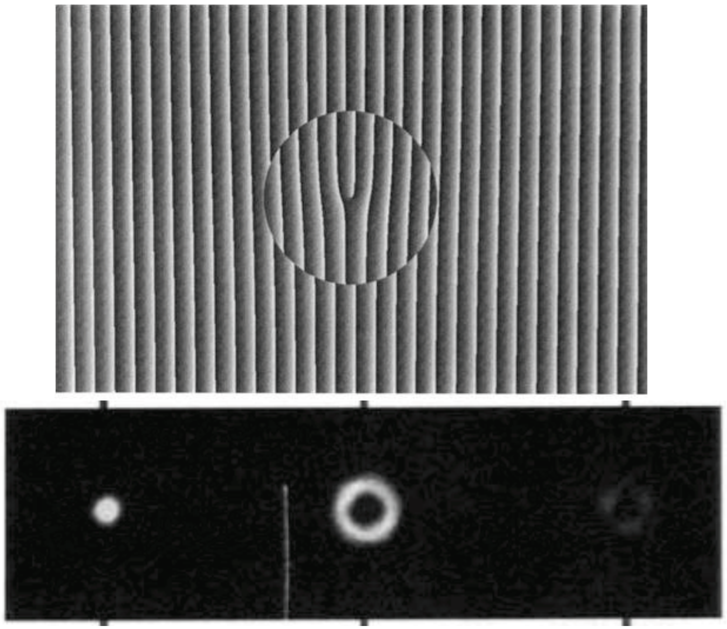


Fig. 1. Sketch of the spiral phaseplate.



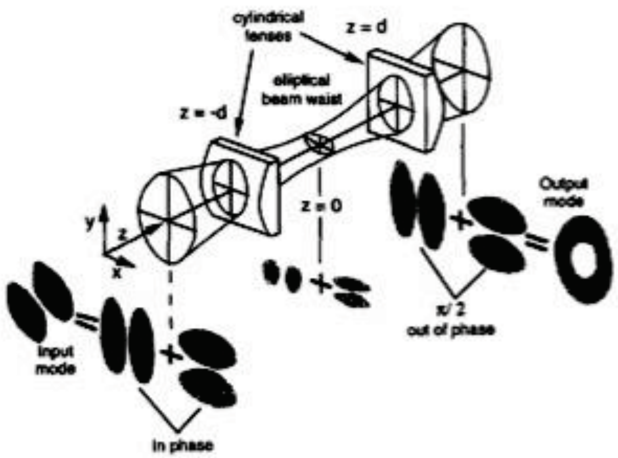


Fig. 4. The cylindrical lens mode converter. If the input $HG_{1,0}$ mode is oriented at 45° with respect to the cylinder axis of lens the mode is converted into the LG_0^1 mode.

M. W. Beijersbergen, R. P. C. Coerwinkel, M. Kristensen, J. P. Woerdman, Opt Comm, 112, 5-6, (1994)

Conventional OAM light generation

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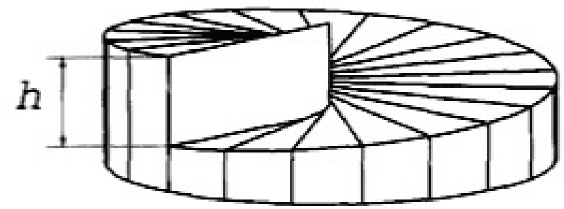

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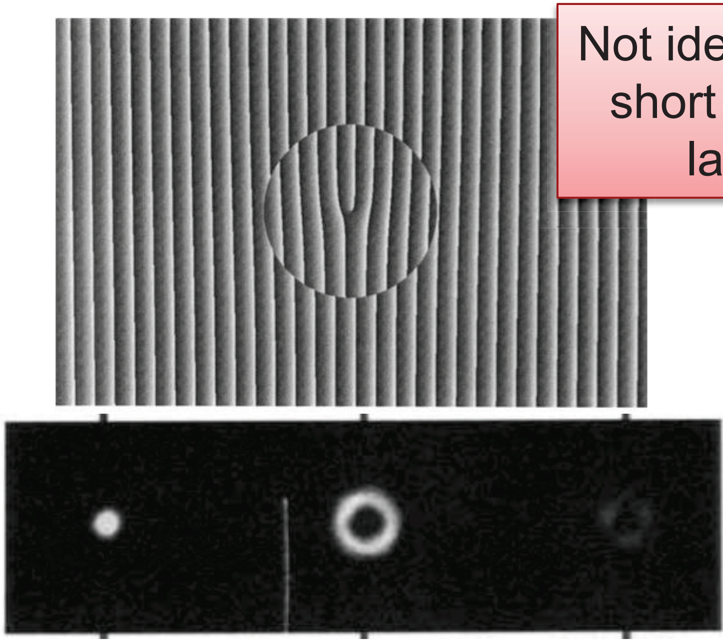
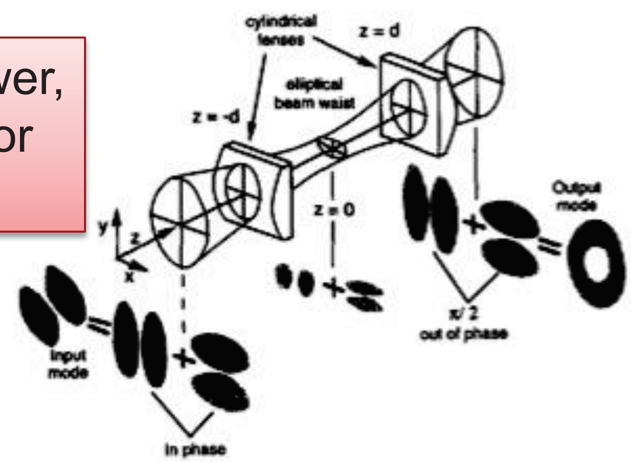


Not ideal for high-power, short wavelengths, or large flexibility



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Initial impetus for OAM in FELs

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- **Initial spark:** To understand, and determine origin of, “exotic” mode structures observed in transverse intensity pattern of VISA FEL.

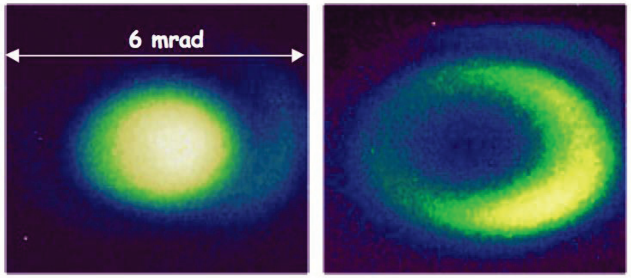
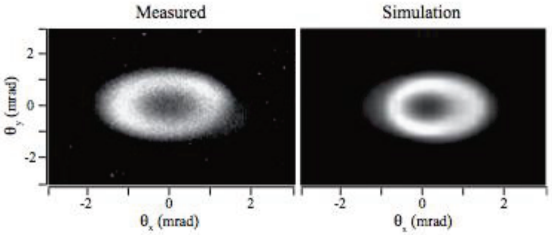
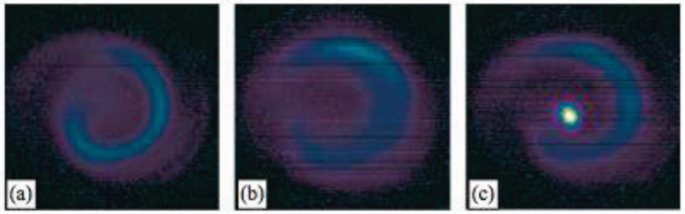


Figure III.33: Angular distribution of SASE signal at low gain (**left**) and in the high gain regime (**right**).

A. Murokh, PhD Dissertation, UCLA, 2002





G. Andonian, PhD Dissertation, UCLA, 2006

- **Approach:** Describe the guided radiation in a general, efficient way through a natural Laguerre Gaussian mode basis of paraxial optics

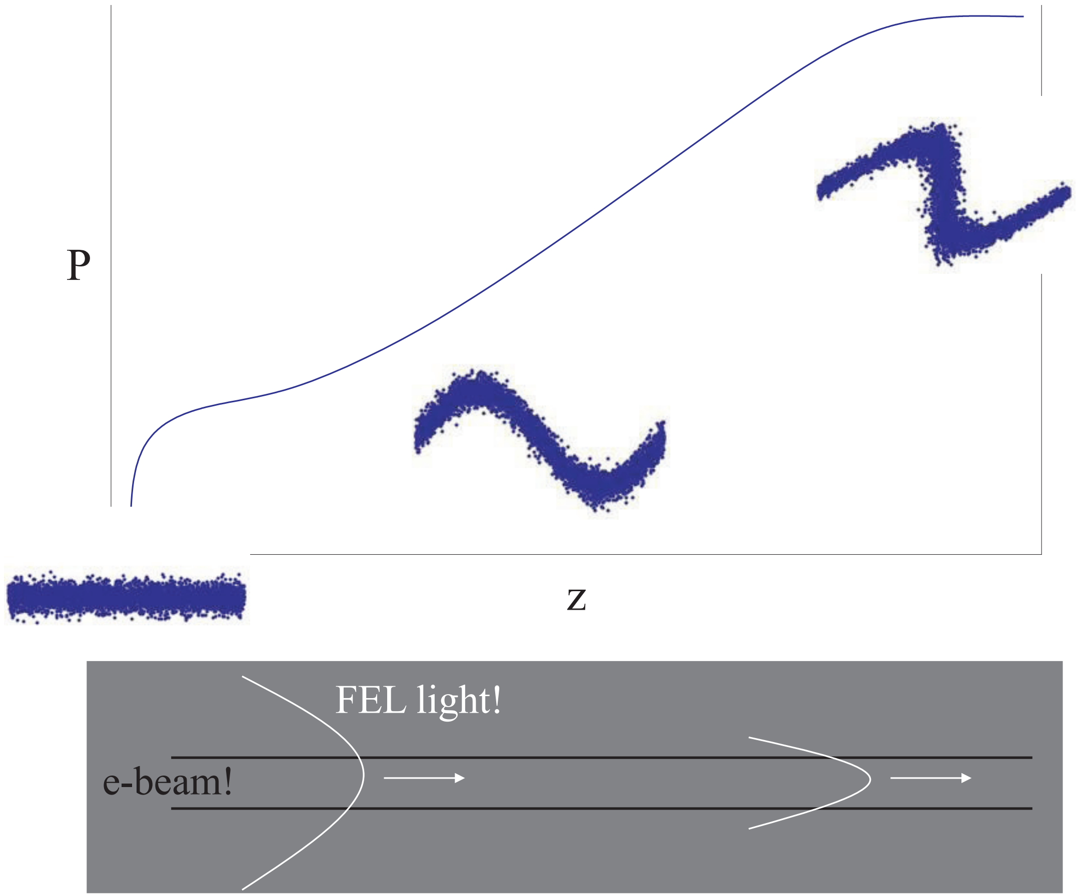
FEL Gain Guiding

THE HIGH-GAIN REGIME OF THE FREE ELECTRON LASER

(NIM A 239 (1985) 19-28)

Gerald T. MOORE

Institute for Modern Optics, Department of Physics and Astronomy, University of New Mexico, Albuquerque, New Mexico 87131, USA



- FEL electrons emit radiation, which acts back on the beam to drive microbunching
- The field grows exponentially and is **gain-guided**
- Radiation field evolves to **fixed profile** described by a single **eigenmode of the system** - usually the fundamental Gaussian(-like) mode.

Virtual Dielectric Waveguide Mode Expansion

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Approach: Expand the FEL EM fields in terms of the fixed profile LG modes of interest:

Want:

$$u_{q=p,l}(r,\phi) \propto \exp\left[il\phi - \frac{r^2}{w^2}\right]\left(\frac{r\sqrt{2}}{w}\right)^{|l|} L_p^{|l|}\left(\frac{2r^2}{w^2}\right)$$

Write:

$$\tilde{\mathbf{E}}_\perp(\mathbf{x}) = E_0 \sum_q c_q(z)\mathbf{u}_q(\mathbf{x}_\perp)e^{ik_{zq}z}$$

Modes satisfy *dielectric waveguide equation*:

$$\nabla_\perp^2 \mathbf{u}_q(\mathbf{x}_\perp) + [n_I(\mathbf{x}_\perp)^2 k^2 - k_{zq}^2]\mathbf{u}_q(\mathbf{x}_\perp) = 0$$

…where index is quadratic index medium:

$$n_I^2(r) = 1 - \left(\frac{2r}{kw^2}\right)^2,$$

PHYSICAL REVIEW A **77**, 063830 (2008)

Virtual dielectric waveguide mode description of a high-gain free-electron laser. I. Theory

Erik Hemsing,[1] Avraham Gover,[2] and James Rosenzweig[1]
[1]*Particle Beam Physics Laboratory, Department of Physics and Astronomy, University of California Los Angeles, Los Angeles, California 90095, USA*
[2]*Faculty of Engineering, Department of Physical Electronics, Tel-Aviv University, Ramat-Aviv 69978, Tel-Aviv, Israel*
(Received 15 February 2008; published 20 June 2008)

PHYSICAL REVIEW A **77**, 063831 (2008)

Virtual dielectric waveguide mode description of a high-gain free-electron laser. II. Modeling and numerical simulations

Erik Hemsing,[1] Avraham Gover,[2] and James Rosenzweig[1]
[1]*Particle Beam Physics Laboratory, Department of Physics and Astronomy, University of California Los Angeles, Los Angeles, California 90095, USA*
[2]*Faculty of Engineering, Department of Physical Electronics, Tel-Aviv University, Ramat-Aviv 69978, Tel-Aviv, Israel*
(Received 15 February 2008; published 20 June 2008)

Virtual Dielectric Waveguide Mode Expansion

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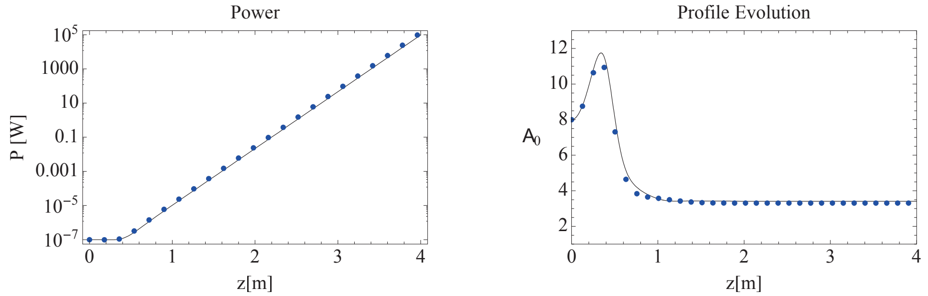
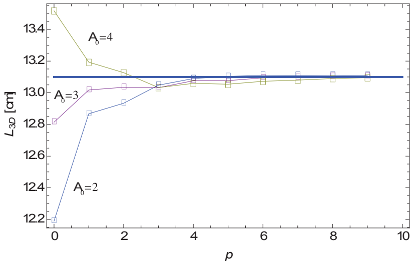
Expanding the FEL field into guided LG modes allows a connection with physical paraxial modes of the same form, while preserving the efficient guided description over many Rayleigh lengths.

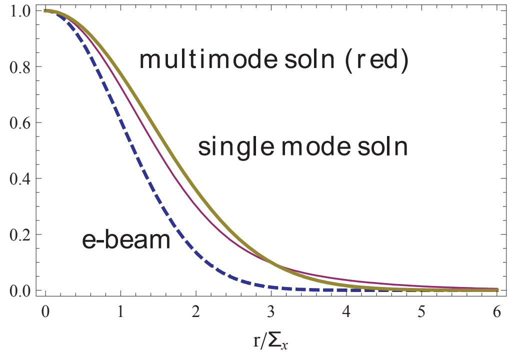
- Natural description of coupling to realistic Gaussian light fields (seed laser coupling, in situ fields)
- Easy transition from guided modes in FEL to free-space modes exiting (or entering) undulator
- Straightforward exploration of coupling to higher-order spatial modes
    - **OAM studies in FEL light**
- Analytic solutions available for coupling to simple transverse e-beam distributions

Benchmark with Genesis



Only a few modes needed





Coupling between e-beam and field modes

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e-beam density: $$n(\mathbf{x},t) = n_0\left[f(\mathbf{x}_\perp) + \mathrm{Re}\left\{\delta_n(\mathbf{x})e^{i\omega(z/v_0-t)}\right\}\right]$$

e-beam profile **density modulation**

TRICK: ALSO expand the density modulation in terms of modes of interest:

$$\delta_n(\mathbf{x}) = \sum_q a_q(z)u_q(\mathbf{x}_\perp)$$

$$\mathbb{F}_{q,q'} = \frac{\langle u_q|f(\mathbf{x}_\perp)|u_{q'}\rangle}{\langle u_q|u_q\rangle}$$

Coupling between e-beam and fields

Coupling between e-beam and field modes

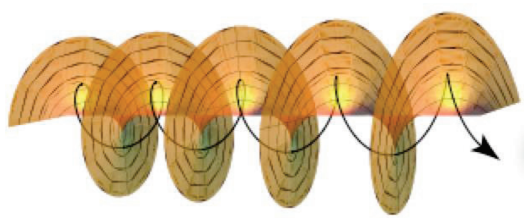
with LG modes... $$u_{q=p,l}(r,\phi) \propto \exp\left[il\phi - \frac{r^2}{w^2}\right]\left(\frac{r\sqrt{2}}{w}\right)^{|l|} L_p^{|l|}\left(\frac{2r^2}{w^2}\right)$$

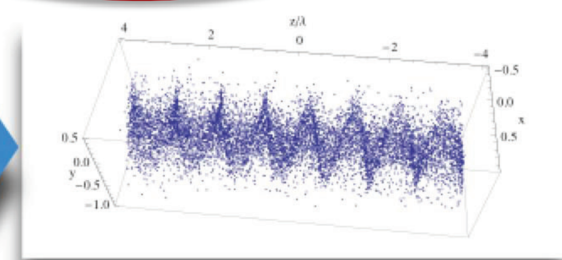
...and simple round beam... $$f(\mathbf{x}_\perp) = f(r)$$

… azimuthal modes in EM field (helical phase) couple directly to azimuthal modes in e-beam (helical density modulation):

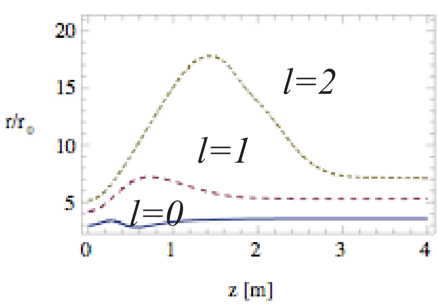
$$\mathbb{F}_{q,q'} \propto \delta_{l,l_b}$$

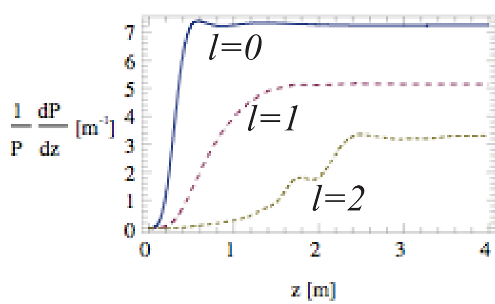
So: if they are excited in the e-beam, they are amplified in the optical field AND vice-verse.





EH, et al, NIMA, 593, (2008) 98-102.





Orthogonality between azimuthal modes forbids coupling to dominant fundamental and thus **enables direct amplification of pure OAM modes**

Full OAM FEL theory with modes

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Formalism extended to include emittance and energy spread effects

$$(\overline{\delta k} - \overline{\Delta k}_q)s_q = -\sum_{q'} s_{q'}\left[\mathbb{K}_{q,q'} - \left(\frac{2}{\sqrt{3}}\right)^3 \int_0^\infty \overline{\tau}\mathrm{d}\overline{\tau} e^{-i\overline{\tau}(\overline{\delta k}-\overline{\theta})-2\overline{\tau}^2\eta_\gamma^2} \mathbb{F}^\epsilon_{q,q'}(\overline{\tau})\right]$$

"supermode solutions"

Generalized coupling factor

Orthogonality still holds in presence of averaged betatron motion and energy spread

$$\mathbb{F}^\epsilon_{q,q'}(\overline{\tau}) \propto \delta_{l,l_b}$$

Obtained M. Xie-esque fitting formula for gain length of OAM modes

$$L_{1D}/L_{3D} = \frac{1}{1+\Lambda_{p,l}}$$

$$\begin{aligned}\Lambda_{0,\pm1} =& 1.1\eta_d^{0.57} + 3.0\eta_\gamma^2 + 0.60\eta_\epsilon^{1.56} + 950\eta_d^{1.5}\eta_\gamma^{3.7} \\ &+ 5.5\eta_d^{1.10}\eta_\epsilon^{0.5} + 11\eta_d^{0.7}\eta_\epsilon^{1.2} \\ &+ 1.14\eta_\gamma^{5.1}\eta_\epsilon^{1.6} + 20300\eta_d^{2.3}\eta_\gamma^{1.75}\eta_\epsilon^{2.1},\end{aligned}$$

E. Hemsing, PhD Dissertation, UCLA, 2011

Helical Phase in Harmonics of Helical Undulators

SLAC

Clearly an EM OAM seed can be amplified. BUT what if no EM seed is available?

CLUE from Sasaki and McNulty: Harmonics of helical undulators have helical phase!

Sasaki and McNulty, Phys. Rev. Lett., 100, 124801 (2008)

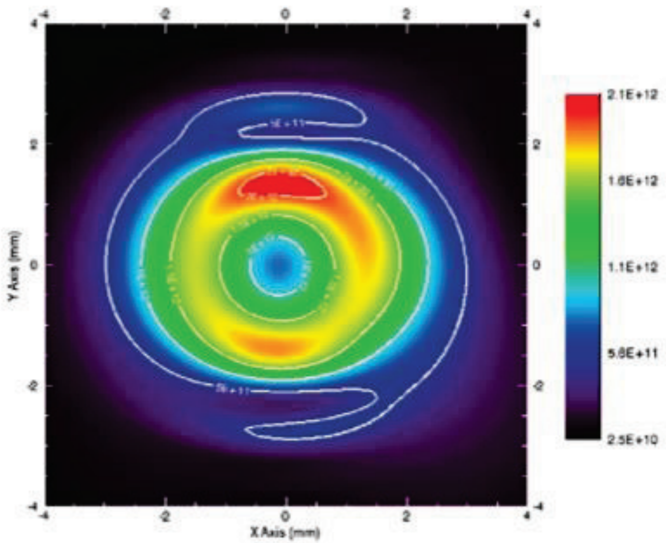


FIG. 5 (color). Calculated intensity distribution of the second harmonic from the APS CPU for $K = 2.77$, a photon energy of 830 eV, at a distance of 50 m.

Result from revisited mode theory: harmonic interaction couples helical modes in e-beam to different helical modes in EM field:

$$l_b = l \pm (h-1)$$

PRL **102,** 174801 (2009) PHYSICAL REVIEW LETTERS week ending 1 MAY 2009

Helical Electron-Beam Microbunching by Harmonic Coupling in a Helical Undulator

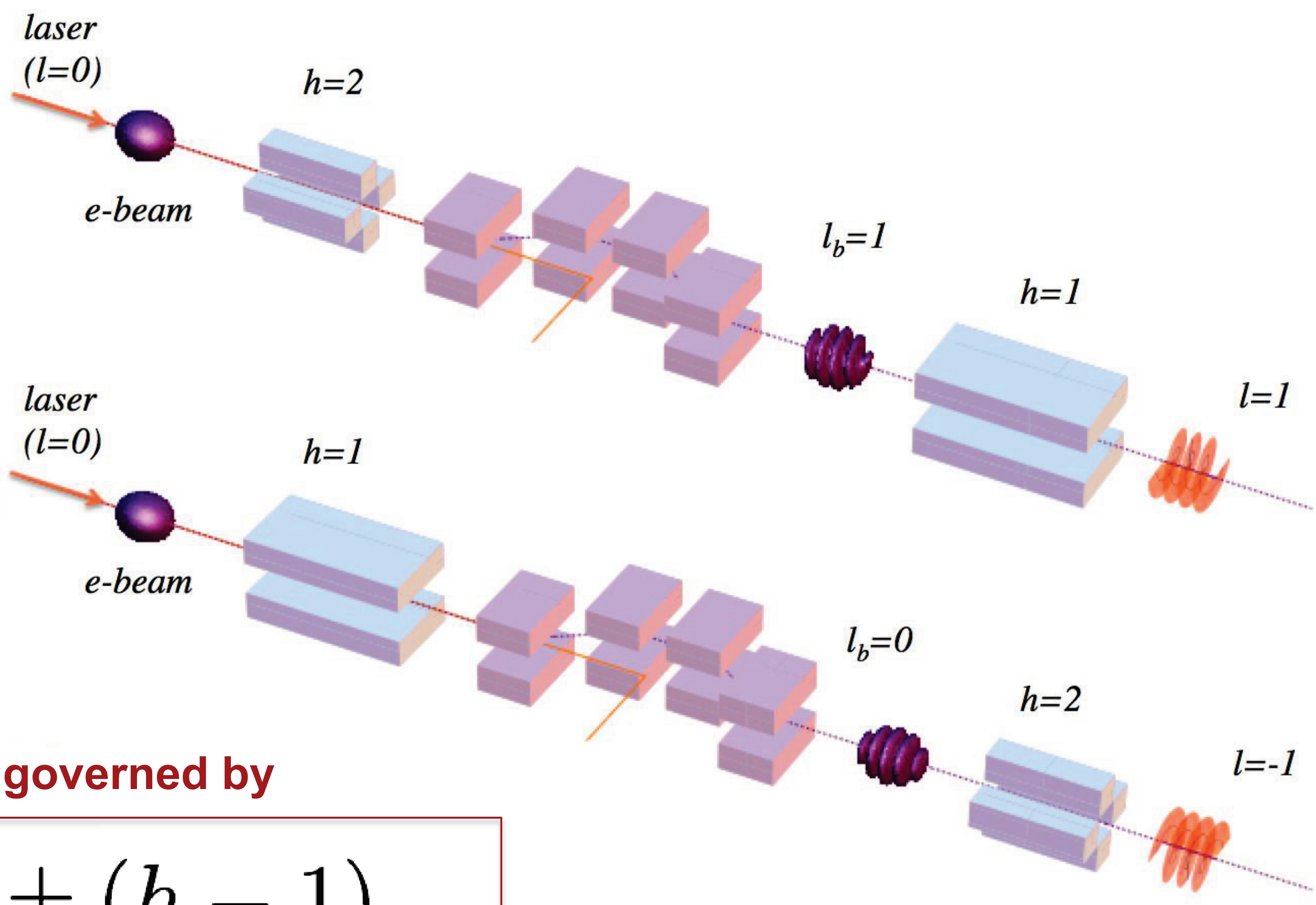
E. Hemsing,[1] P. Musumeci,[1] S. Reiche,[1] R. Tikhoplav,[1] A. Marinelli,[1,2] J. B. Rosenzweig,[1] and A. Gover[3]

Now: Two paths to OAM

e-beam Shaping

OR

Radiation Shaping

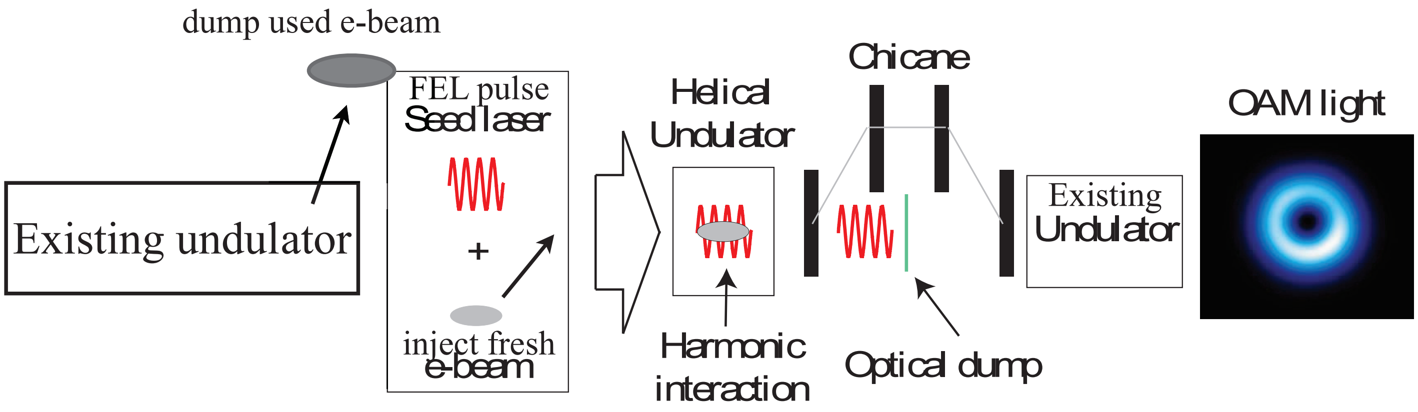


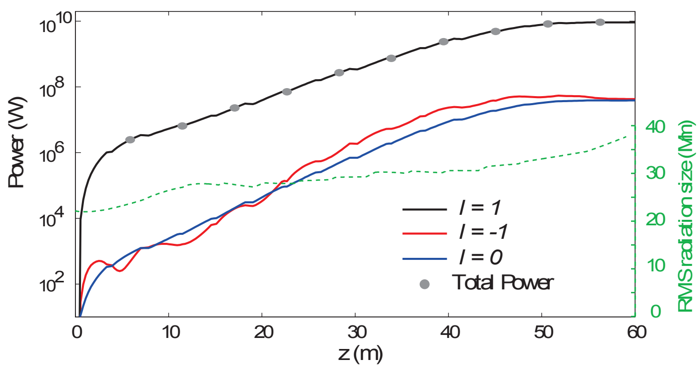
Both governed by

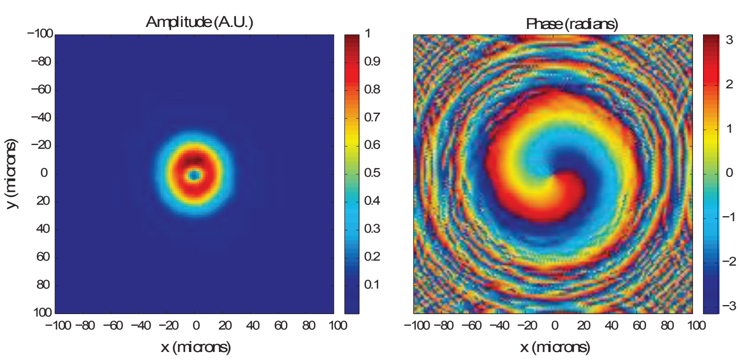
$$l_b = l \pm (h-1)$$

Proposed OAM @ 1.5Å

Two beam or self-seeded mode to drive helical bunching

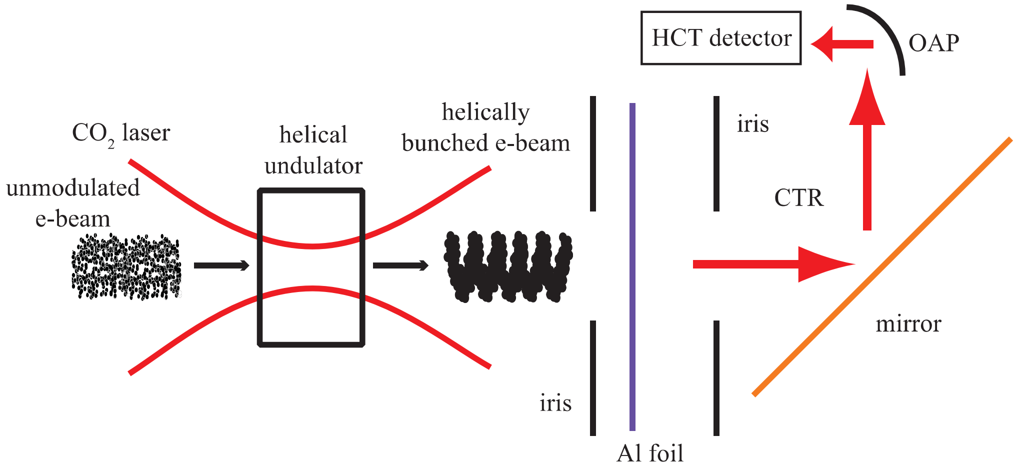


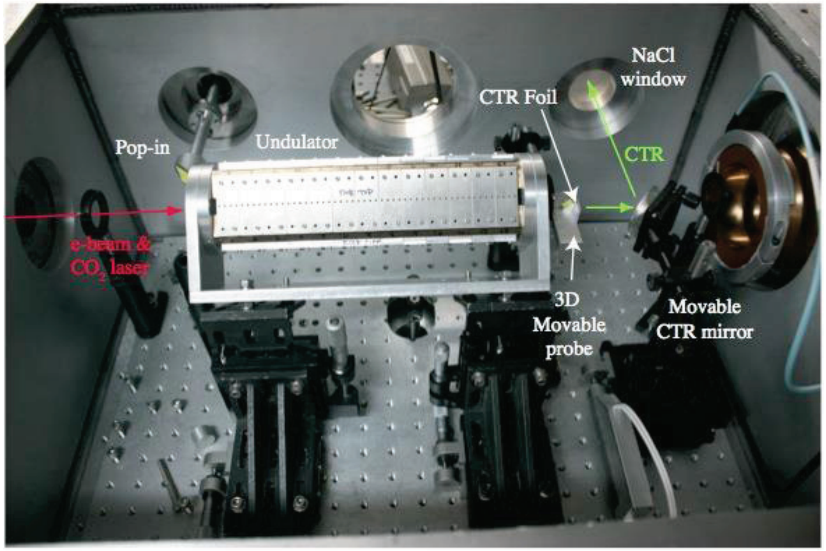


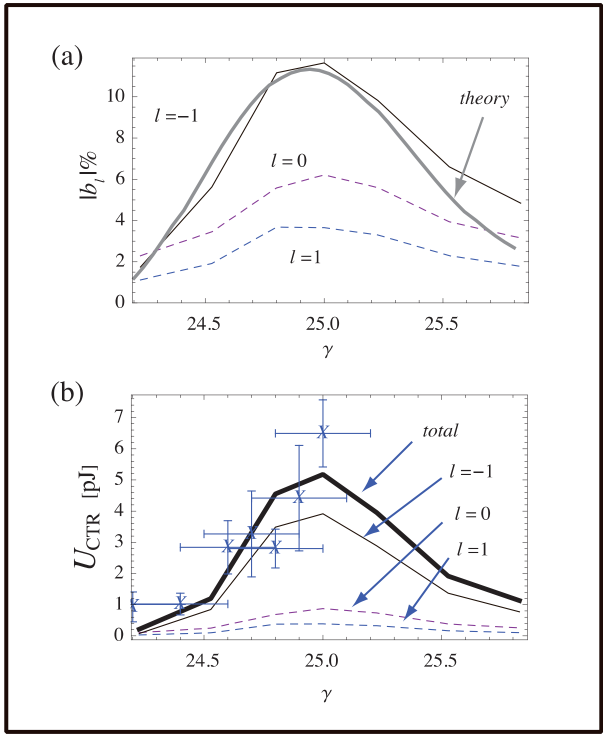


E. Hemsing, A. Marinelli, J. B. Rosenzweig, PRL 106, 164803 (2011)

First Helical Microbunching Experiment @ UCLA Neptune Lab





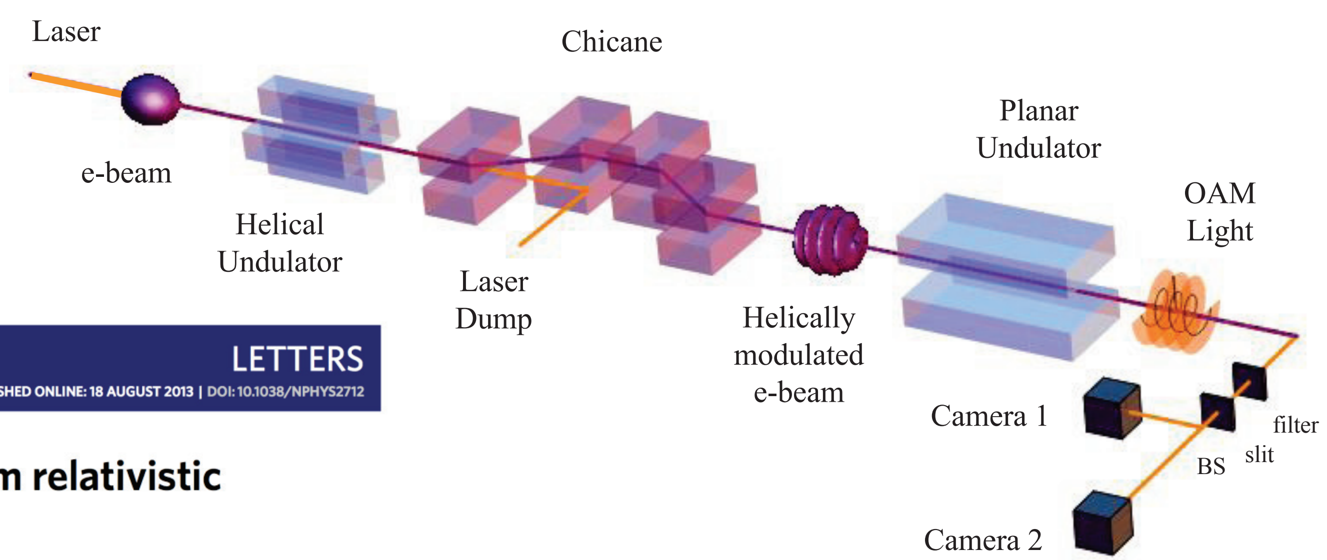


Good evidence of helical bunching observed in COTR, but direct measurement at 10.6um elusive

E. Hemsing, A. Knyazik, F. O'Shea, A. Marinelli, P. Musumeci, O. Williams, S. Tochitsky, J. B. Rosenzweig. Applied Physics Letters, 100(9):164803, 2012.

Beam Shaping OAM Experiment at SLAC-NLCTA

- 800 nm Gaussian laser profile modulates e-beam in helical undulator at 2^{nd} harmonic resonance (tuned to 1.6 um fundamental)
- Chicane dumps seed laser and converts beam energy modulation into 800 nm helical density modulation
- e-beam emits coherently in planar undulator tuned to emit 800 nm light at fundamental frequency
- Two cameras image CUR at two different focal planes



nature physics
LETTERS
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Coherent optical vortices from relativistic electron beams

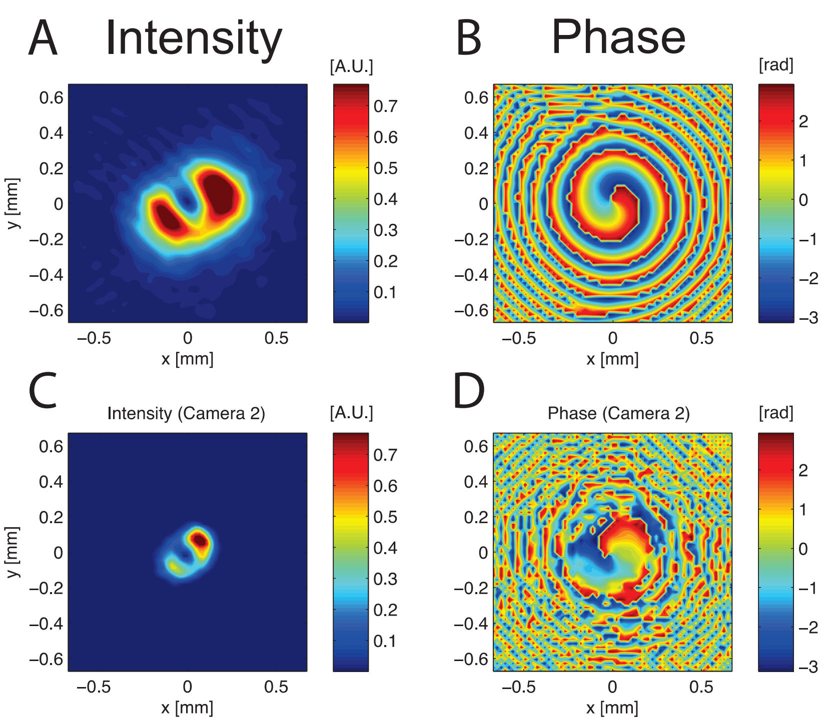
Erik Hemsing[1]*, Andrey Knyazik[2], Michael Dunning[1], Dao Xiang[1], Agostino Marinelli[1], Carsten Hast[1] and James B. Rosenzweig[2]

Determining OAM mode output

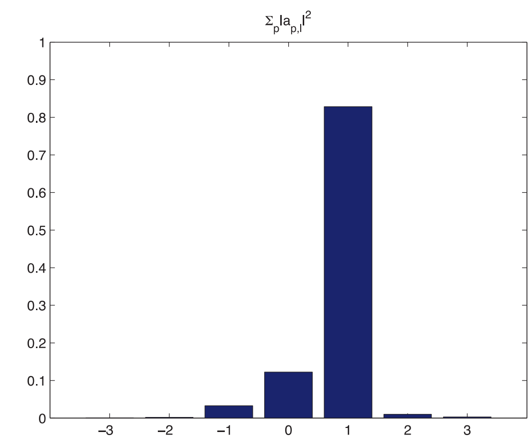
- Iterative phase retrieval algorithm reconstructs transverse phase from intensity images recorded simultaneously on both cameras
- Rapid convergence to dominant l=1 OAM mode observed after ~200 iterations
- Mode content quantified by expansion of complex fields in OAM mode basis: 85% of power in OAM mode.

Camera 1

Camera 2

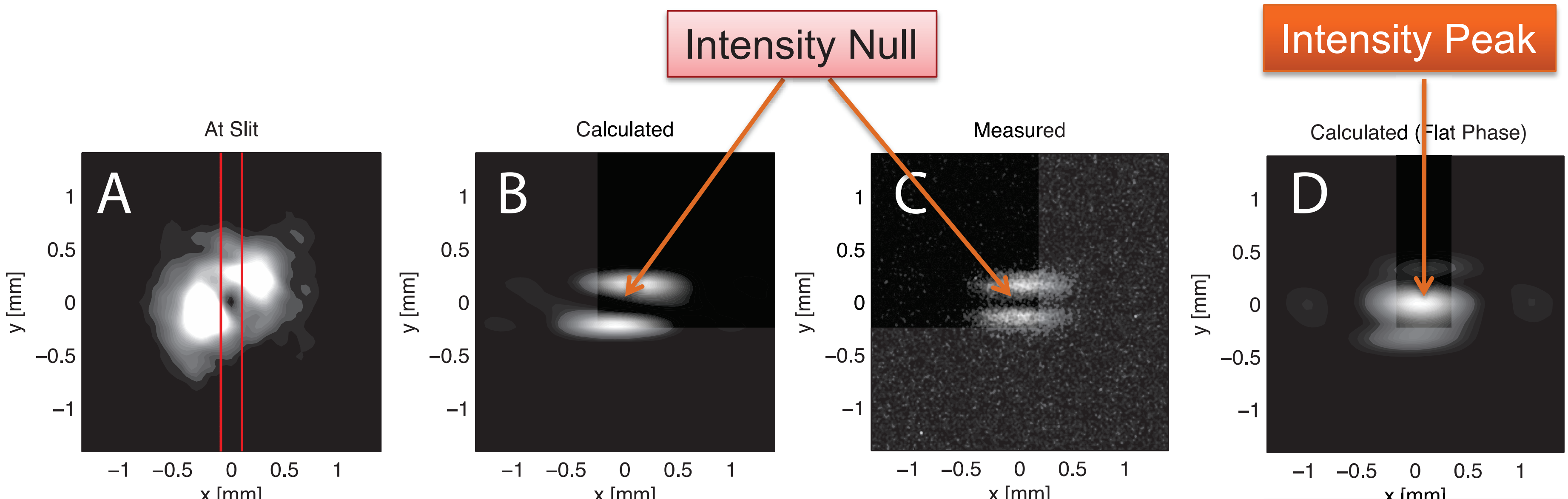


OAM mode power distribution shows 85% in l=1



Determining OAM mode output

• Narrow vertical slit (200um width) placed in path of radiation confirms presence of vortex phase via measured diffraction pattern



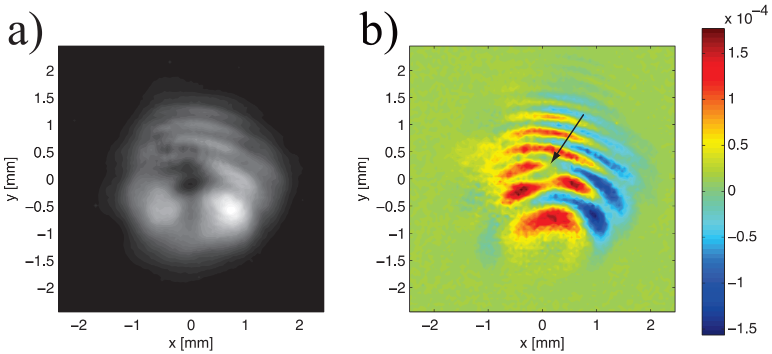
Radiation Profile at Slit

Calculated Intensity of diffraction pattern

Measured Intensity of diffraction pattern

Calculated diffraction pattern with helical phase artificially removed

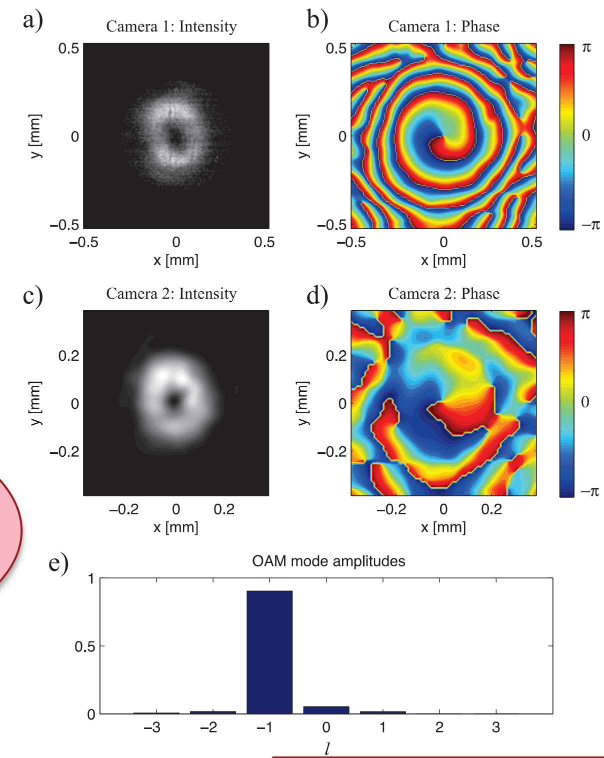
Radiation Shaping OAM Experiment at SLAC-NLCTA



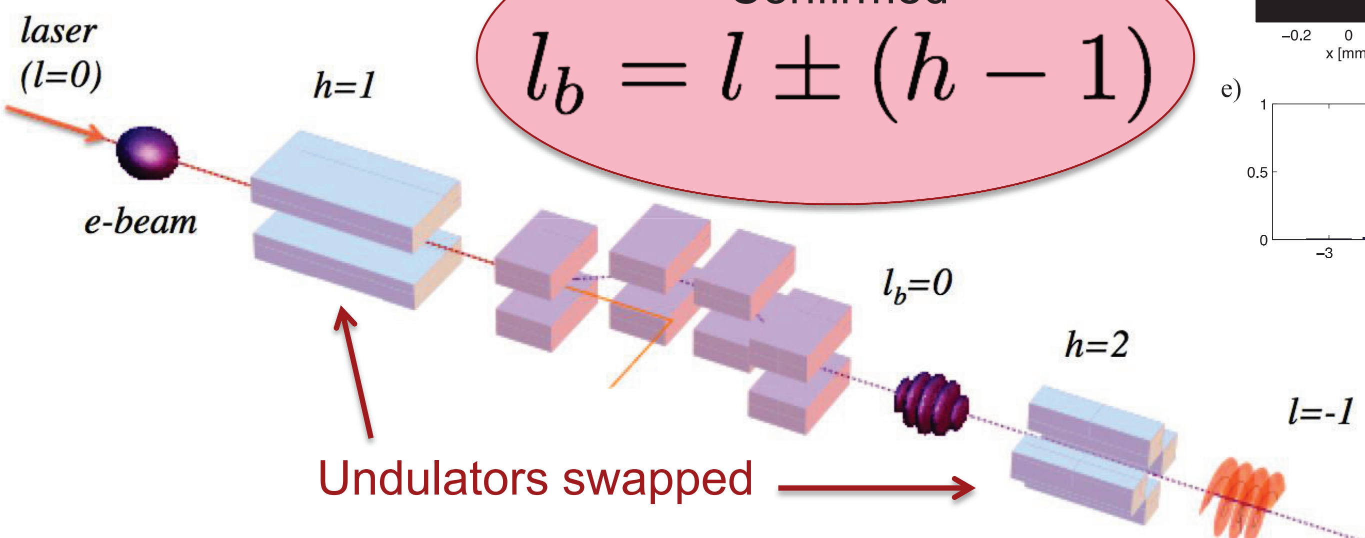
Interference between CUR and off-axis CTR reveals signature fork of a *l*=-1 vortex

Confirmed

$$l_b = l \pm (h-1)$$



OAM mode power distribution shows 91% in *l*=-1



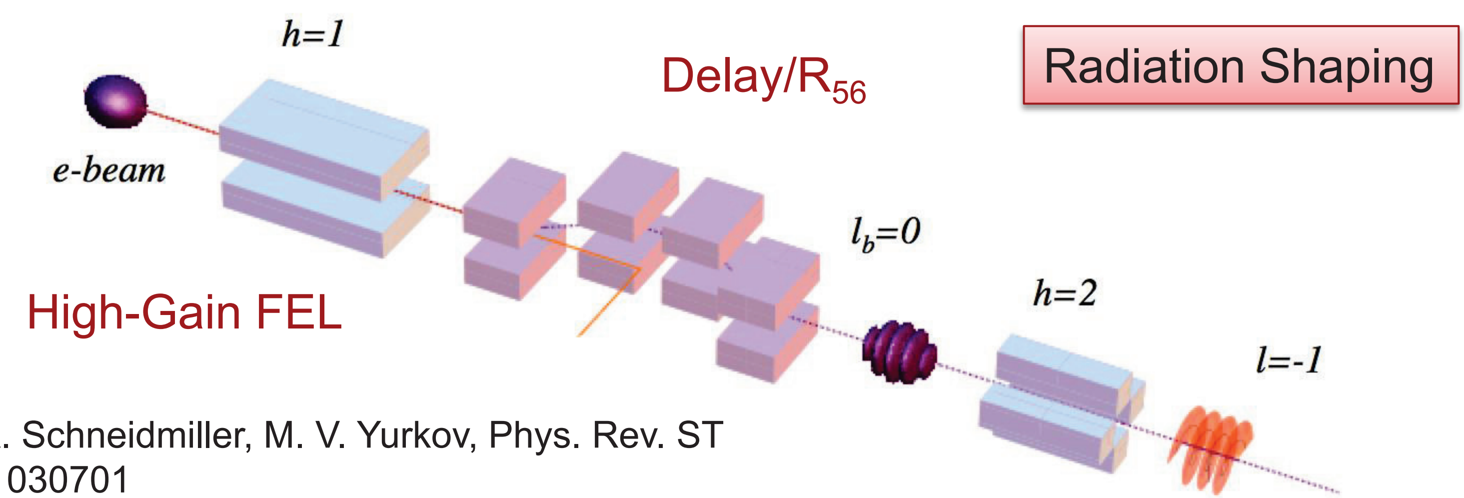
Undulators swapped

E. Hemsing, M. Dunning, C. Hast, T. Raubenheimer, and D. Xiang, PRL 113, 134803, (2014)

Future Possibilities: OAM FEL afterburner*

Chicane and helical undulator placed downstream of high gain x-ray FEL

- Single OAM mode (x-ray)
  - Lasing in FEL produces *l*=0 light and *l*=0 density modulation in beam
  - FEL light is dumped in chicane delay
  - Beam radiates coherent **x-ray** OAM at harmonic of helical undulator
- Two mode pump/probe (same frequency)
  - Lasing in FEL produces *l*=0 light and *l*=0 density modulation in beam
  - Delay allows FEL light out first
  - Beam radiates coherent **x-ray** OAM at harmonic of helical undulator with fixed delay
- Two mode pump/probe (xrays + optical OAM)
  - Lasing produces *l*=0 optical scale energy modulations by SASE process
  - R_{56} converts to *l*=0 optical density modulations
  - Beam radiates coherent **optical** OAM at harmonic of helical undulator with fixed delay



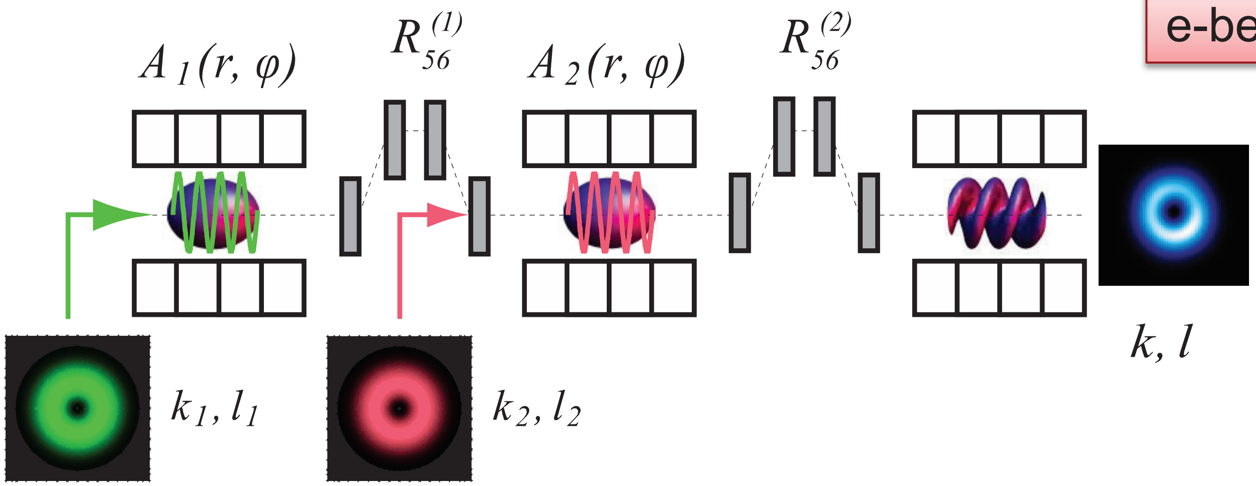
*E. L. Saldin, E. A. Schneidmiller, M. V. Yurkov, Phys. Rev. ST Accel. Beams 13, 030701

Future Possibilities: EEHG with a Twist

SLAC

Cascaded helical microbunching in EEHG for frequency and mode up-conversion

e-beam Shaping



Frequency up-conversion

$$k \equiv ak_1 = nk_1 + mk_2,$$

OAM mode up-conversion

$$l = nl_1 + ml_2.$$

E. Hemsing and A. Marinelli, PRL 109, 224801 (2012)

Summary

SLAC

- Many exciting techniques exist, or have been proposed, for tailoring the FEL output by either beam shaping, radiation shaping or combinations of both.
- Such "beam by design" can be extended to the transverse coordinates as well for producing OAM light
- A couple of proof-of-principle experiments have demonstrated in situ "mode conversion" to generate OAM light, and are in agreement with predictions from mode expansion theory
- Represent new techniques for producing intense, coherent OAM light over all FEL wavelengths, for access to new kinds of scientific R&D
- EEHG and similar seeding schemes can be extended to produce OAM at high-frequencies and high mode numbers
- OAM afterburner might be simplest scheme to implement
- FELs are versatile! We should continue to exploit it.

Thank you to friends and colleagues for numerous contributions to this work.

Thank you to the 2014 FEL Prize committee for the Young Scientist FEL Award.

And thank you for your attention!