# REVERSIBLE ELECTRON BEAM HEATER WITHOUT TRANSVERSE DEFLECTING CAVITIES

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#### Abstract

In Ref. [1] a technique to suppress the instability with the help of a reversible electron beam heater was proposed. It employs transverse deflecting cavities synchronized in a way that one of the cavities, located before a bunch compressor, generates a slice energy spread, while the other one removes it after the beam passes through the bunch compressor. In this paper we demonstrate that a reversible heater equivalent to that of [1] can be designed using much simpler elements: bend magnets and quadrupoles in combination with the energy chirp of the beam.

### **INTRODUCTION**

The performance of modern free electron lasers is often limited by the microbunching instability in the electron beam that develops during its acceleration and transportation to the undulator. A search of new methods for effective suppression of the instability is crucial for future FELs, especially the ones that use external seeding. One of the promising approaches to this problem is the idea of a reversible electron beam heater proposed in Ref. [1]. It employs transverse deflecting cavities (TDS) synchronized in a way that one of the cavities, located before a bunch compressor, introduces a slice energy spread, and the other one removes it after the bunch compressor. Being an attractive concept, it however imposes extremely tight tolerances on the synchronization of the cavities. It also adds a considerable cost of the RF structures and an additional RF power system to the accelerator.

An ideal reversible heater has to perform two actions. First, it should increase the slice energy spread in the beam before the bunch compressor to the level that suppresses generation of microbunching instability due to coherent synchrotron radiation in the compressor, and to remove it afterwards restoring the beam to the original energy spread (amplified by the compression factor). Second, it is also desirable that the heater destroys energy and density modulations in the beam which are accumulated before the compressor—these are usually associated with the longitudinal space charge impedance and earlier compressing stages. Of course, the heater should not noticeably increase the transverse emittance of the beam.

In this paper we show how a reversible heater can be implemented using DC magnets without TDS. We simplify our analysis by assuming thin elements and neglecting the optics associated with drifts between them.

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# **REVERSIBLE HEATER USING TDS**

Here we give a brief description of the reversible heater based on transverse deflecting cavities and analyze, following [1], its effect on the energy spread of the beam and the beam emittance. While our analysis can be easily extended for a broader range of distribution functions, to evaluate the effect of the heater, for specificity, we consider an initial 4D Gaussian distribution function of the beam

$$f_0(x, x', z, \eta) = A \exp\left[-\frac{1}{2}\left(\frac{x^2}{\sigma_{x0}^2} + \frac{x'^2}{\sigma_{x'0}^2} + \frac{z^2}{\sigma_{z0}^2} + \frac{\eta^2}{\sigma_{\eta0}^2}\right)\right],\tag{1}$$

where *x* and *x'* are the transverse coordinate and angle, *z* is the longitudinal coordinate in the beam,  $\eta = \Delta E/E$  is the relative energy deviation, and  $\sigma_{x0}$ ,  $\sigma_{x'0}$ ,  $\sigma_{z0}$  and  $\sigma_{\eta0}$  are the standard deviations for the corresponding variables. In the distribution function (1) we ignore the transverse coordinates *y* and *y'*. The normalization constant *A* is such that  $\int f_0 dx dx' dz d\eta = 1$ .

When the beam with the initial distribution function (1) passes through a system under consideration the distribution function is transformed,  $f_0(x, x', z, \eta) \rightarrow f_1(x, x', z, \eta)$ , where  $f_1$  is the distribution function at the exit. To find  $f_1$ , we use the formalism of linear optics with 4×4 beam transport matrices that act on vector  $(x, x', z, \eta)$ . To take into account the acceleration before the BC that generates an energy chirp *h* (that is a linear correlation between the energy  $\eta$  and the longitudinal coordinate *z*) we will use the matrix  $R_c(h)$ ,

$$R_{\rm c}(h) = \begin{pmatrix} 1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & h & 1 \end{pmatrix}.$$
 (2)

For a bunch compressor (BC) we will use the matrix

$$R_{\rm BC} = \begin{pmatrix} 1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & r\\ 0 & 0 & 0 & 1 \end{pmatrix},$$
(3)

with r being the {5, 6} element of the 6-dimensional transport matrix of the compressor.

In linear optics, the *R*-matrix for a short transverse deflecting cavity is given by

$$R_{\text{TDS}}(K) = \begin{pmatrix} 1 & 0 & 0 & 0\\ 0 & 1 & K & 0\\ 0 & 0 & 1 & 0\\ K & 0 & 0 & 1 \end{pmatrix},$$
 (4)

where the parameter *K* (*K* has dimension of inverse length) defines the strength of the transverse deflection,  $K = e\omega_{\rm rf}V_{\rm cav}/\gamma mc^3$  with  $\omega_{\rm rf}$  the cavity resonant frequency and  $V_{\rm cav}$  the cavity voltage. The {4, 1} element of this matrix introduces an energy spread in the beam that is proportional to the beam transverse size. As a result, after passing through the cavity the beam energy spread increases from  $\sigma_{\eta 0}$  to  $\sigma_{\eta}$  where

$$\sigma_{\eta}^{2} = \sigma_{\eta 0}^{2} + K^{2} \sigma_{x 0}^{2}.$$
 (5)

This is a reversible "heating" of the beam that can be used to suppress the microbunching instability and can be removed by a second TDS with opposite polarity. In addition to the slice heating, there is also another unavoidable side effect of the TDS: it tilts the beam in the x'-z plane and increases the beam projected transverse emittance. Calculations show that the beam emittance  $\epsilon_x$  after the cavity is given by

$$\epsilon_x = \epsilon_{x0} \sqrt{1 + K^2 \sigma_{z0}^2 / \sigma_{x'0}^2}, \qquad (6)$$

where  $\epsilon_{x0} = \sigma_{x0}\sigma_{x'0}$  is the initial emittance before the cavity (we remind that there is no initial correlations in the beam between *x* and *x'*).

In the reversible heater [1], the needed slice energy spread is generated by a first TDS placed before the bunch compressor, and then removed from the beam by a second TDS after the BC. The full transport matrix  $R_{\rm rh}$  of such reversible heater, from the entrance to the RF section that generates the energy chirp to the exit from the second TDS, is obtained as a product of several matrices [1]

$$R_{\rm rh} = R_{\rm TDS} \left( -CK \right) \cdot R_{\rm BC} \cdot R_{\rm TDS}(K) \cdot R_{\rm c}(h) \tag{7}$$

where  $C = (1 + hr)^{-1}$  is the compression factor. Matrix  $R_c(h)$  on the right hand side accounts for the beam energy chirp *h* needed for compression, and matrix  $R_{\text{TDS}}(-CK)$  removes the energy spread introduced by  $R_{\text{TDS}}(K)$ . Multiplying the matrices we find<sup>1</sup>

$$R_{\rm rh} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -CK^2r & 1 & 0 & -CKr \\ Kr & 0 & \frac{1}{C} & r \\ CKhr & 0 & h & 1 \end{pmatrix}.$$
 (8)

Note that the second TDS also compensates the emittance growth (6) introduced by the first TDS. While this compensation is not complete, the residual emittance growth after the reversible heater for the parameters of Ref. [1] is small,  $\Delta \epsilon_x / \epsilon_{x0} \approx 1.3\%$ .

Finally, as is shown through computer simulations in [1], the TDS reversible heater satisfies the second condition outlined in the introduction: it smears out initial energy and density modulations accumulated in the beam before the compression. In Appendix we demonstrate the smearing in an analytical model of a cold beam.

### **BEAM HEATING WITHOUT TDS**

A reversible energy spread generated by TDS is not a unique property of deflecting cavities—it can also be obtained in passive systems that do not require RF power (assuming that the beam has already an energy chirp). To illustrate this statement we begin from a simple observation: a combination of a beam energy chirp and a bend magnet introduces a slice energy spread in a cold beam.

Let us assume that the beam with the distribution function (1) passes through an accelerating system that generates an energy chirp *h* and then through a *thin* bend magnet with a bending angle  $\theta$ . For simplicity, we consider the case of a cold beam,  $\sigma_{\eta 0} = 0$ . The transport matrix for the bend,  $R_{\rm b}$ , is

$$R_{\rm b}(\theta) = \begin{pmatrix} 1 & 0 & 0 & 0\\ 0 & 1 & 0 & \theta\\ -\theta & 0 & 1 & 0\\ 0 & 0 & 0 & 1 \end{pmatrix}.$$
 (9)

The combined effect of the chirp and the bend are given by the product  $R_b(\theta) \cdot R_c(h)$ . It transforms the initial variables to the final (indicated by the subscript 1):  $x_1 = x$ ,  $\eta_1 = \eta + hz$ ,  $x'_1 = x' + \theta(\eta + hz)$ ,  $z_1 = z - \theta x$ . Expressing from these equations  $x, x', z, \eta$  through  $x_1, x'_1, z_1, \eta_1$  and substituting them into (1) gives the distribution function after the bend  $f_1(x_1, x'_1, z_1, \eta_1)$  (hereafter, we drop the indices in the arguments of  $f_1$ ). To find the beam distribution over  $\eta$  and z we integrate  $f_1$  over x, x'. A straightforward calculation gives

$$\int dx \, dx' f_1(x, x', z, \eta) = A_1 \exp\left(-\frac{\eta^2}{2\sigma_{\eta}^2} + a\eta z - bz^2\right).$$
(10)

with the slice energy spread after the bend as

$$\sigma_{\eta} = \frac{|h\theta|\sigma_{z0}\sigma_{x0}}{\sqrt{\theta^2 \sigma_{x0}^2 + \sigma_{z0}^2}} \approx |h\theta|\sigma_{x0}, \tag{11}$$

and parameters *a* and *b* that are of no interest for what follows. In the last equation we assumed  $|\theta|\sigma_{x0} \ll \sigma_{z0}$ . We can see from (10) that each slice in the beam with a given value of coordinate *z* has now a Gaussian distribution in energy with the rms spread proportional to the product of the bending angle and the energy chirp.

The mechanism behind this heating is illustrated in Fig. 1. An initial cold beam distribution of a chirped beam in the longitudinal *z*- $\eta$  phase space is a narrow line shown in the left part of the figure. After passage through the bend, particles with different horizontal coordinates, *x*, in each slice of the beam are shifted along *z* by  $-\theta x$ , as shown by the horizontal arrow, due to the {31} element of matrix *R*<sub>b</sub>, see Eq. (9). This widens the original thin-line distribution of the beam into an ellipse, and introduces slice energy spread  $\sigma_{\eta}$  shown in the right plot.

A unavoidable side effect of the slice heating in this process is increasing of the beam transverse emittance through

<sup>&</sup>lt;sup>1</sup> Matrix  $R_{\rm rh}$  differs from the corresponding matrix in [1]. We believe that this is due to an error in the original publication—the matrix in [1] is not symplectic.



Figure 1: Illustration of beam heating.

tilting the phase space x-z. A direct calculation of the beam emittance  $\epsilon$  with the distribution function  $f_1$  gives the following expression for  $\epsilon$ :

$$\epsilon = \sqrt{\epsilon_0^2 + h^2 \theta^2 \sigma_{z0}^2 \sigma_{x0}^2},\tag{12}$$

where  $\epsilon_0 = \sigma_{x0}\sigma_{x'0}$  is the initial emittance. Note a similarity with the TDS case, Eq. (6).

To illustrate the magnitude of the heating and the emittance increase, we consider the following numerical example: the beam energy E = 100 MeV,  $\sigma_{z0} = 1$  mm, h = 0.01/mmcorresponding to 1% of the total energy,  $\sigma_{x0} = 0.2$  mm,  $\theta = 50$  mrad. Using Eq. (11) we find

$$\sigma_n E \approx |h\theta| \sigma_{x0} E \approx 10 \text{ keV}.$$
 (13)

In addition, assuming the initial horizontal projected normalized emittance  $\epsilon_{x0} = 1 \ \mu m$  we find from (12) that the final beam emittance, after the chirp-bend system, will be increased to about 20  $\mu m$ , that is twenty times.

It is important to emphasize that the heating and the emittance increase due to the mechanism discussed above are reversible. Adding two bending magnets of opposite polarity before and after the bunch compressor would lead to an effective increase of the slice energy spread (11) ("beam heating") prior to the compressor, and removing the energy spread after the passage through the system ("beam de-heating"). This should result in the suppression of the beam microbunching due to the coherent synchrotron radiation of the beam inside the BC [2-5]. Note that in this arrangement, due to the bends, the beam line becomes tilted at an angle with respect to the direction of the beam line before (and after) the auxiliary bending magnets. Note also that while this arrangement is similar to the setup proposed in Ref. [6], it is however not the same: in particular, the beam energy chirp in Ref. [6] is generated between the bends, and in our case it is outside of the bends.

A simple reversible heater using the two bend magnets is not completely satisfactory: it does not suppress the incoming energy and density modulations in the beam. Indeed, if we calculate the *R*-matrix of the sequence "bend–bunch compressor–bend of opposite polarity" we find that this matrix is equal to the original  $R_{\rm BC}$  matrix,

$$R_{\rm b}(-\theta) \cdot R_{\rm BC} \cdot R_{\rm b}(\theta) = R_{\rm BC}.$$
 (14)

This means that while suppressing microbunching due to CSR in the BC, the system converts an energy modulation, accumulated by the beam in the linac before the compression, into a density modulation through the r matrix element exactly the same way as without the auxiliary bends. In the next section we will show how this can be improved with the help of additional accelerating cavities that modify the beam chirp after the first bending magnet.

#### **REVERSIBLE HEATER WITHOUT TDS**

We first show that the TDS matrix itself can be implemented as a combination of energy chirps, bends and focusing. In this analysis we use the following thin-quad matrix with the focal length F:

$$R_{\rm q}(F) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -1/F & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$
 (15)

With the matrices defined earlier, it is a matter of simple matrix multiplication to show the product of the matrices

$$R_{q}(f) \cdot R_{b}(-\theta) \cdot R_{chirp}(-h) \cdot R_{b}(\theta) \cdot R_{chirp}(h), \quad (16)$$

is identical to the matrix  $R_{\text{TDS}}(K)$  given by Eq. (4) if parameters *F* and *h* and  $\theta$  are chosen such that

$$F = -\frac{1}{K\theta}, \qquad h = \frac{K}{\theta}.$$
 (17)

For a given *K*, the bending angle  $\theta$  in these relations can be considered as a free parameter.

Eq. (16) proves that the TDS matrix can be implemented as a sequence "energy chirp–bend–opposite energy chirp– opposite bend–quad". With this result, it is not surprising that full matrix (8) of the reversible heater can be implemented as a sequence of the following elements: "energy chirp–thin bend–energy de-chirp–bend of opposite sign–thin quad" (as above, we keep ignoring the transverse optics effects due to the finite length of each element). Indeed, as can be verified by direct matrix multiplication, the following combination  $R_{\rm rh}^{(1)} = R_{\rm b}(\theta_1) \cdot R_{\rm BC} \cdot R_{\rm chirp} (-h(1 + \theta/\theta_1)) \cdot R_{\rm b}(\theta) \cdot R_{\rm chirp}(h)$  gives the matrix

$$R_{\rm rh}^{(1)} = \begin{pmatrix} 1 & 0 & 0 & 0\\ h\theta(\theta + \theta_1) & 1 & 0 & \theta + \theta_1\\ -C^{-1}(\theta + \theta_1) & 0 & \frac{1}{C} & r\\ \frac{h\theta(\theta + \theta_1)}{\theta_1} & 0 & -\frac{h\theta}{\theta_1} & 1 \end{pmatrix}, \quad (18)$$

where the compression factor now is  $C^{-1} = 1 - rh\theta/\theta_1$ . It is easy to check that selecting the angles  $\theta$  and  $\theta_1$  such that

$$\theta = -\frac{C^2 K r}{C - 1}, \qquad \theta_1 = \frac{C K r}{C - 1}, \tag{19}$$

reduces  $R_{\rm rh}^{(1)}$  to matrix (8). Note that because angles  $\theta$  and  $\theta_1$  have different absolute values, the result of the system (18) is the rotation of the beam line by the angle  $\theta_1 - |\theta|$ , which

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imposes a certain constrain on the geometry of the beam line.

To illustrate that the system with the matrix (18) offers a practical approach, we calculate the bending angles  $\theta$  and  $\theta_1$  that make the reversible heater without TDS equivalent to the one with TDS from Ref. [1] with the parameter *K* of the first TDS equal to  $K = 9.6 \times 10^{-4}$  cm. Following [1] we take the beam energy E = 360 MeV, the compression factor C = 13, r = -138 mm. Using Eqs. (18) we find the bending angles of the dipole magnets in our system

$$\theta_1 = 10.8^\circ, \qquad \theta = -0.83^\circ.$$
 (20)

The heating inside the bunch compressor will be

$$\sigma_n E \approx |h\theta_1| \sigma_{x0} E \approx 10 \text{ keV}.$$
 (21)

The projected emittance growth inside the BC will be  $\epsilon_x/\epsilon_{x0} \approx 32$ ; it will be almost completely removed at the exit from the heater.

## BUILDING BLOCKS FOR REVERSIBLE HEATER

Using a bend magnet as an elementary building block of a reversible heater has an evident disadvantage that, if not compensated by a subsequent bend of opposite polarity, it might require the downstream beam line to be angled relative to the direction at the entrance to the heater. As pointed out above, matrix (18), results in rotation of the downstream beam line by an angle  $\theta_1 - |\theta|$ . In this regard, it might be advantageous to replace bends as elementary building blocks with a sequence of magnetic elements that neither rotate not displace the beam line. One of such examples is provided by the following combination:

$$R_{\rm b}\left(\frac{1}{4}\theta\right) \cdot R_{\rm d}(l) \cdot R_{\rm q}(F) \cdot R_{\rm d}(l) \cdot R_{\rm b}\left(-\frac{1}{2}\theta\right)$$
$$\cdot R_{\rm d}(l) \cdot R_{\rm q}(F) \cdot R_{\rm d}(l) \cdot R_{\rm b}\left(\frac{1}{4}\theta\right) \cdot R_{\rm q}\left(\frac{F}{2}\right), \qquad (22)$$

where  $R_d(l)$  is the *R*-matrix of a drift of length *l*. Choosing an arbitrary *F* and l = 2F and multiplying the matrices gives matrix (9). Since the total bending angle in (24) is zero, replacing bends in our previous analysis by this sequence eliminates beamline tilts and offsets for the price of additional complexity of the beam line.

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# APPENDIX: EFFECT OF THE REVERSIBLE HEATER ON INITIAL MODULATION

To analyze the effect of the reversible heater on an initial beam modulation, we make two simplifying assumptions. First, we assume a cold beam,  $\sigma_{\eta} = 0$ . Second we consider a long bunch, formally corresponding to the limit  $\sigma_z \to \infty$ . The latter is justified if the period of the modulation  $\lambda_0$  is much smaller than  $2\pi\sigma_z$ .

We consider two cases: an initial energy modulation and an initial density modulation of the beam. In the first case the initial beam distribution function is

$$f_0(x, x', z, \eta) = \frac{n_0}{2\pi\sigma_x \sigma_{x'}} \exp\left(-\frac{x^2}{2\sigma_{x0}^2} - \frac{x'^2}{2\sigma_{x'0}^2}\right) \\ \times \delta(\eta - \Delta\eta \sin k_0 z),$$
(23)

and in the second case we have

$$\Delta f_0(x, x', z, \eta) = \frac{\Delta n_0 \cos(k_0 z)}{2\pi \sigma_x \sigma_{x'}} \exp\left(-\frac{x^2}{2\sigma_{x0}^2} - \frac{x'^2}{2\sigma_{x'0}^2}\right) \\ \times \delta(\eta).$$
(24)

In these equations,  $k_0 = 2\pi/\lambda_0$  is the wavenumber of the modulation,  $\Delta \eta$  is the amplitude of the energy modulation,  $\Delta n_0$  is the amplitude of the density modulation, and  $n_0$  is the beam density (number of particle per unit length). Note that in the second case we consider only the part  $\Delta f$  of the distribution function responsible for the density perturbation.

To evaluate the effect of the heater on modulations we will compute the Fourier component of the density perturbation  $\Delta \hat{n}(k)$  before and after the heater. For the first case, we do not have an initial density perturbation and hence  $\Delta \hat{n}_0(k) = 0$ . For the second case we initially have

$$\begin{aligned} \Delta \hat{n}_0(k) &= \int_{-\infty}^{\infty} dz \, dx \, dx' d\eta \, e^{-ikz} \Delta f_0(x, x', z, \eta) \\ &= \Delta n_0 \int_{-\infty}^{\infty} dz \, e^{-ikz} \cos(k_0 z) \\ &= \frac{1}{2} \Delta n_0(\delta(k - k_0) + \delta(k + k_0)). \end{aligned}$$
(25)

To calculate  $\hat{n}(k)$  after the system we need to transform the distribution function from the initial (Eq. (23) or (24)) and then to calculate the Fourier integrals again. The transformation from the initial to final coordinates according to the R-matrix (8)

$$x_1 = x, \qquad z_1 = Krx + \frac{1}{C}z + r\eta,$$
  

$$\eta_1 = CKhrx + hz + \eta, \qquad (26)$$

where the subscript 1 indicates the variables after the heater and the variables without the subscript are before the heater and  $C = (1 + hr)^{-1}$ . Instead of expressing the initial coordinates in terms of the final ones and substituting then into the distribution function, one can, equivalently, integrate the initial distribution function replacing the final coordinate in  $e^{-ikz_1}$  by  $e^{-ik(Krx+z/C+r\eta)}$ . Hence, one needs to compute

$$\Delta \hat{n}(k) = \int_{-\infty}^{\infty} dz \, dx \, dx' d\eta \, e^{-ik(Krx+z/C+r\eta)} f_0(x, x', z, \eta).$$
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In the case of the energy modulation, Eq. (23), we have

$$\Delta \hat{n}(k) = \frac{n_0}{2\pi\sigma_{x0}\sigma_{x'0}} \int_{-\infty}^{\infty} dz \, dx \, dx' d\eta \, e^{-ik(Krx+z/C+r\eta)} \\ \times \exp\left(-\frac{x^2}{2\sigma_{x0}^2} - \frac{x'^2}{2\sigma_{x'0}^2}\right) \delta(\eta - \Delta\eta \sin k_0 z). \quad (27)$$

For comparison, consider first the effect of the bunch compressor, that is when there are no TDS cavities, K = 0. Integrating over dx and dx' and using the identity

$$e^{-ia\sin\zeta} = \sum_{m=-\infty}^{\infty} e^{im\zeta} J_m(-a)$$
(28)

we find

$$\Delta \hat{n}(k) = n_0 \sum_{m=-\infty}^{\infty} J_m(-kr\Delta\eta) \int_{-\infty}^{\infty} dz e^{-iz(k/C - mk_0)}$$
$$= 2\pi C n_0 \sum_{m=-\infty}^{\infty} J_m(-mCk_0r\Delta\eta)\delta(k - mCk_0).$$
(29)

This is the standard effect of conversion of the energy modulation into the density one through r (augmented by the compression factor C). We see that  $\Delta \hat{n}(k)$  consists of infinitely many harmonics of the initial wavenumber  $k_0$  with the harmonic number given by the integer m.

Returning to the original equation (27) with nonzero *K* we find that  $\Delta \hat{n}(k)$  given by (29) is now multiplied by an additional suppression factor

$$\frac{1}{\sqrt{2\pi}\sigma_x} \int_{-\infty}^{\infty} dx \, e^{imCk_0 Krx} \exp\left(-\frac{x^2}{2\sigma_x^2}\right)$$
$$= e^{-(mCk_0 r K\sigma_{x0})^2/2}.$$
(30)

We see that if the exponent in this equation is large enough the final density modulation converted from the initial energy modulation is exponentially suppressed by the heater. For the initial density modulation, Eq. (24), we have

$$\Delta \hat{n}(k) = \frac{\Delta n_0}{2\pi\sigma_{x0}\sigma_{x'0}} \int_{-\infty}^{\infty} dz \, dx \, dx' d\eta \, e^{-ik(Krx+z/C+r\eta)} \\ \times \cos(k_0 z) \exp\left(-\frac{x^2}{2\sigma_{x0}^2} - \frac{x'^2}{2\sigma_{x'0}^2}\right) \delta(\eta)$$
(31)
$$= \frac{1}{2} C \Delta n_0 e^{-(Ck_0 r K \sigma_{x0})^2/2} (\delta(k - Ck_0) + \delta(k + Ck_0)).$$

We see that it is compressed by a factor of *C* and suppressed by the same factor (30) with m = 1.

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