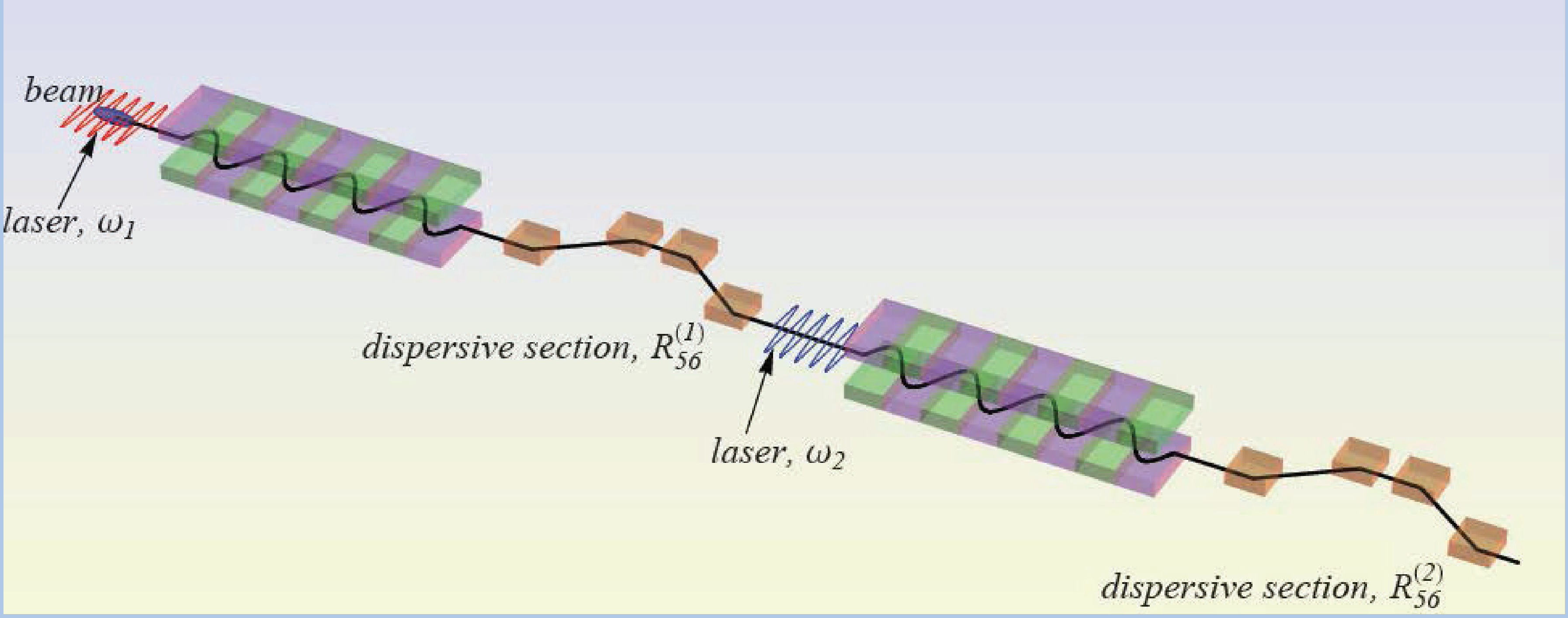
Bunching Coefficients in EEHG and Coulomb Diffusion

G. DATTOLI AND E. SABIA

ENEA-FRASCATI

Echo Scheme
G. Stupakov, PRL 102, 074801 (2009), Xiang-Stupakov, PRST 12, 030702 -2009

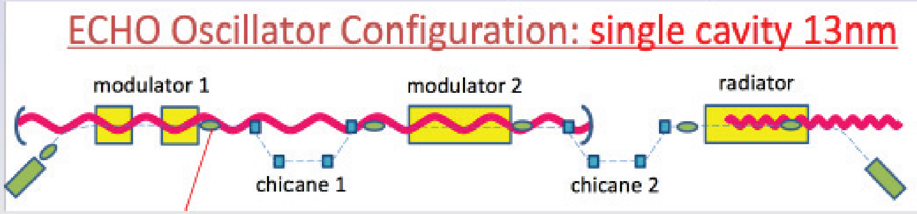
- EEHG for FEL seeding employs 2-undulators & 2 chicanes
- To induce a fine structure in e-beam phase space which at the end turns into HHM of the beam current.



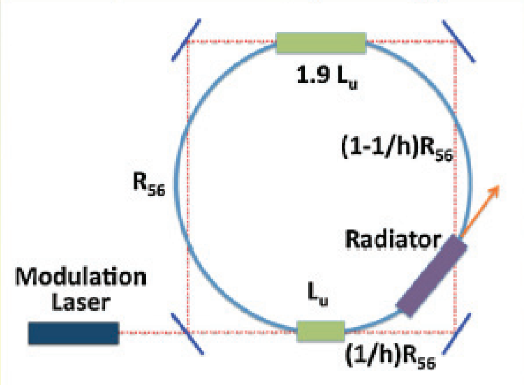
Other Schemes (further development)

Further development of the echo idea:

- J. Wurtele: using echo oscillator (FEL 2010).

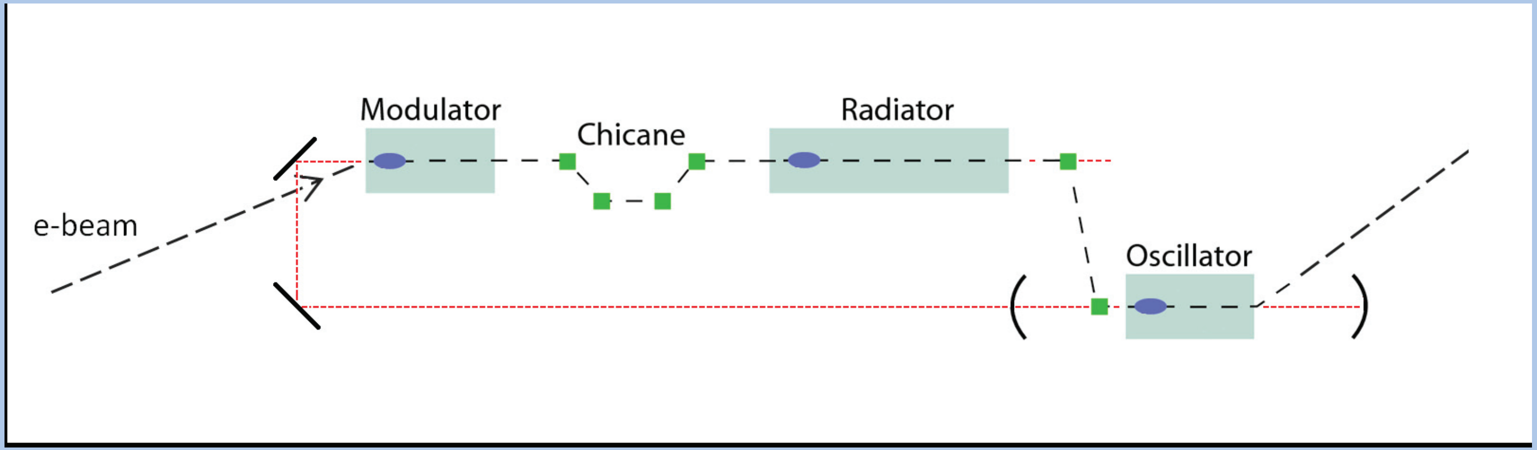


- D. Ratner and A. Chao: steady state microbunching in storage ring (PRL, **105**, 154801, (2010)).



The Future!!!

- Gandhi, Penn, Reinshe, Wurtele, Fawley PRST 16,020703 (2013)



The Future has necessarily a past
Jurassik Suggestions

- F. Ciocci et al IEEE J. Quantum Electr. 31, 1242 (1995).
- G. Dattoli et al IEEE J. Quantum Electr. 31,1584 (1995)
- R. Barbini et al. “80 nm FEL Design in an Oscillator Amplifier Configuration”, Proceedings of the Workshop on Prospects for a 1 Angstrom Free-Electron Laser, Sag Harbor, NY, 1990, edited by J.C. Gallardo, BNL Report 52273 (1991)

Effect of Coulomb Diffusion on Bunching

- The reduction of the bunching efficiency may be due to different mechanisms associated with: beam quality, CSR, early saturation…
- These effects are now quite well understood ,
- In 1-D models they can be explained in terms of an equivalent energy spread inducing an increasing suppression with the order of the harmonic

$$b_n \propto e^{-n^2 \sigma_\varepsilon^2}$$

Coulomb

In Refs-

- G. Stupakov, Phys. Rev. Lett. 102, 074801 (2009). G. Stupakov, in Proceedings of the FEL2011 Conference, Shanghai, China, 2011 [http:// www.jacow.org]
- A further mechanism has been considered, namely: the intra bunch Coulomb diffusion

..

- Very roughly speaking the paradigm is always the same

$$\text{Coulomb Interaction} \rightarrow \text{Coulomb Diffusion} \rightarrow$$
$$\rightarrow \text{Induced Energy Spread} \rightarrow \text{bunching reduction}$$

- But «Very Roughly»

General Criteria

- The strategy:
- Merge Coulomb diffusion and longitudinal dynamics using algebraic techniques symilar to symplectic methods for beam transport

- ***A. J. Dragt, Lie methods for Non linear dynamics…(2013)***
- ***G. D., P. L. Ottaviani, A. Torre, L. Vazquez (1997)***
- ***G. D., M. Migliorati, A. Schiavi, M. Venturini, Collective effects in accelerators (2009)***
- ***R. Warnock, J. Ellison SLAC (2000)***
- ***…***

- The main step will be the inclusion of diffusion (heat type) contributions

Heat equation

- Weierstrass Transform

$$\partial_t F(x,t) = K\,\partial_x^2 F(x,t),$$

$$F(x,0) = f(x).$$

$$F(x,t) = e^{tK\partial_x^2} f(x)$$

$$F(x,t) = \frac{1}{2\sqrt{\pi K t}} \int_{-\infty}^{+\infty} e^{-\frac{(x-\xi)^2}{2Kt}} f(\xi) \equiv \text{Weierstrass - Transform}$$

$$f(x) = e^{-x^2} \rightarrow F(x,t) = \frac{1}{\sqrt{1+4Kt}} e^{-\frac{x^2}{1+4Kt}}$$

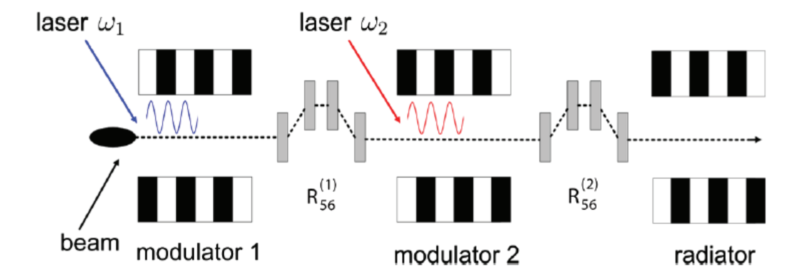
Heat Type equations and Coulomb diffusion

- Diffusion equation due to Coulomb interaction with an initially density modulated beam

$$\partial_s f(p,\zeta;s) = D\,\partial_p^2 f(p,\zeta;s),$$

$$f(p,\zeta;0) = f_0(p,\zeta) \to f(p,\zeta;s) = e^{sD\partial_p^2} f_0(p,\zeta) =$$

$$= \frac{1}{2\sqrt{\pi D s}} \int_{-\infty}^{\infty} e^{-\frac{(p-\eta)^2}{4Ds}} f_0(\eta,\zeta)\,d\zeta,\ \ p = \frac{E-E_0}{\sigma_E}$$



- Bunching coefficients

$$f(p,\zeta;s) = \frac{1}{2\sqrt{\pi D s}} \int_{-\infty}^{\infty} e^{-\frac{(p-\eta)^2}{4Ds}} f_0(\eta,\zeta)\, d\zeta,$$

$$f(p,\zeta,s) = \sum_{n=-\infty}^{\infty} b_n(p,s)\, e^{in\zeta} \rightarrow$$

$$\rightarrow b_m = \frac{1}{2\sqrt{\pi D s}} \int_{-\infty}^{\infty} e^{-\frac{(p-\eta)^2}{4Ds}} b_m(\eta)\, d\eta \rightarrow$$

Bunching and diffusion
G. D., E. Sabia, PRST July (2013)

- .

$$f_0(p,\zeta)=\frac{1}{\sqrt{2\pi}}e^{-\frac{(p-A_1\sin(\zeta-B_1 p))^2}{2}},$$

$$b_m(p)=\frac{1}{\sqrt{2\pi}}e^{-\frac{p^2}{2}}e^{-i\,m\,B_1 p}J_m(-i\,A_1 p)$$

$$A_1=\frac{\Delta E}{\sigma_E},B_1\propto R_{5,6},$$

$$b_m\cong e^{-\frac{(m\,B_1)^2 D\,s}{1+2D\,s}}\Phi_m,$$

$$\Phi_m\propto\frac{1}{2\sqrt{\pi}\sqrt{1+2\,D\,s}}e^{-\frac{p^2}{2(1+D\,s)}}e^{-i\frac{m\,B_1 p}{2(1+2D\,s)}}J_m\left(\frac{A_1(m\,B_1 D\,s-i\,p)}{1+2\,D\,s}\right)$$

…

- Dispersion 2Ds
- Suppression of the higher order harmonics

$$b_m \cong e^{-\frac{(mB_1)^2 Ds}{1+2Ds}} \Phi_m,$$

$$\Phi_m \propto \frac{1}{2\sqrt{\pi}\sqrt{1+2Ds}} e^{-\frac{p^2}{2(1+Ds)}} e^{-i\frac{mB_1 p}{2(1+2Ds)}} J_m\left(\frac{A_1(mB_1 Ds - ip)}{1+2Ds}\right)$$

Effect on a distribution

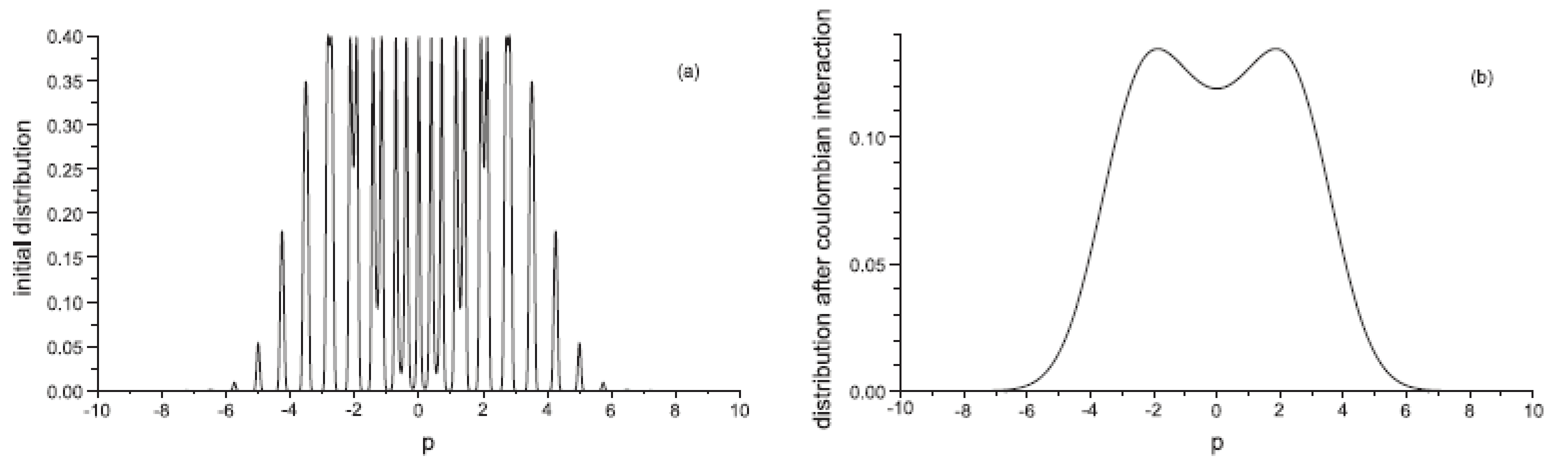
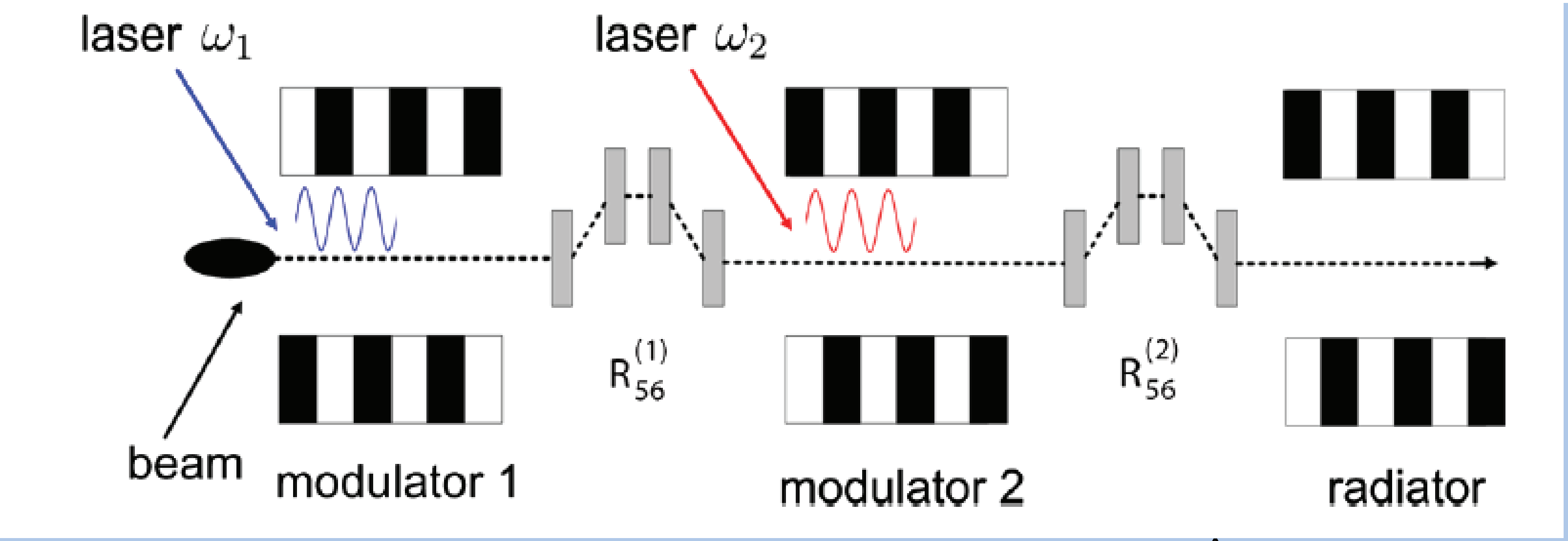


FIG. 1. Evolution of the distribution function $f(p, \zeta, s)$ undergoing the Coulombian diffusion. (a) $f(p, 0, 0)$, $A_1 = 3$, $B_1 = 8.47$, and (b) $f(p, 0, 30)$, $A_1 = 3$, $B_1 = 8.47$, $D = 7 \times 10^{-3}$.

$$f_0(p,\zeta)=\frac{1}{\sqrt{2\pi}}e^{-\frac{(p-A_1\sin(\zeta-B_1p))^2}{2}}$$



$$H=B\frac{p^2}{2}+AV(\zeta)\to$$

$$\to\hat{L}=-Bp\partial_\zeta+AV'(\zeta)\partial_p,$$

$$V(\zeta)=\cos(\zeta)$$

$$\partial_s f(p,\zeta,s)=\hat{L}f(p,\zeta,s),$$

$$f(p,\zeta,0)=e^{-\frac{p^2}{2}}$$

$$f(p,\zeta,s)=e^{s\hat{L}}f_0(p)\to$$

$$\to f_0(p-As\sin(\zeta-Bsp)),$$

$$A\propto\frac{\Delta E_1}{\sigma_E},B\propto R_{5,6}\frac{k_L\sigma_E}{E_0},$$

$$\zeta=k_Lz$$

Bunching coefficients

- The equation satisfied by the bunching coefficients is

$$\partial_s b_n = -i\,B\,p\,n\,b_n + \frac{A}{2i}\left[b_{n-1} - b_{n+1}\right],$$

$$b_n(p,0) = e^{-\frac{p^2}{2}}\delta_{n,0}$$

Liouville & Diffusion Fokker-Planck

- The inclusion of diffusion in the previous «Liouvillian» can be done (almost irresponsably) as it follows

$$\hat{L} \to \hat{V} = D\partial_p^2 - Bp\partial_\zeta + AV'(\zeta)\partial_p = D\partial_p^2 + \hat{L},$$

$$D = 1.55\frac{I[kA]}{\varepsilon_x[\mu m]\sigma_x[100\,\mu m](\sigma_E[keV])^2},$$

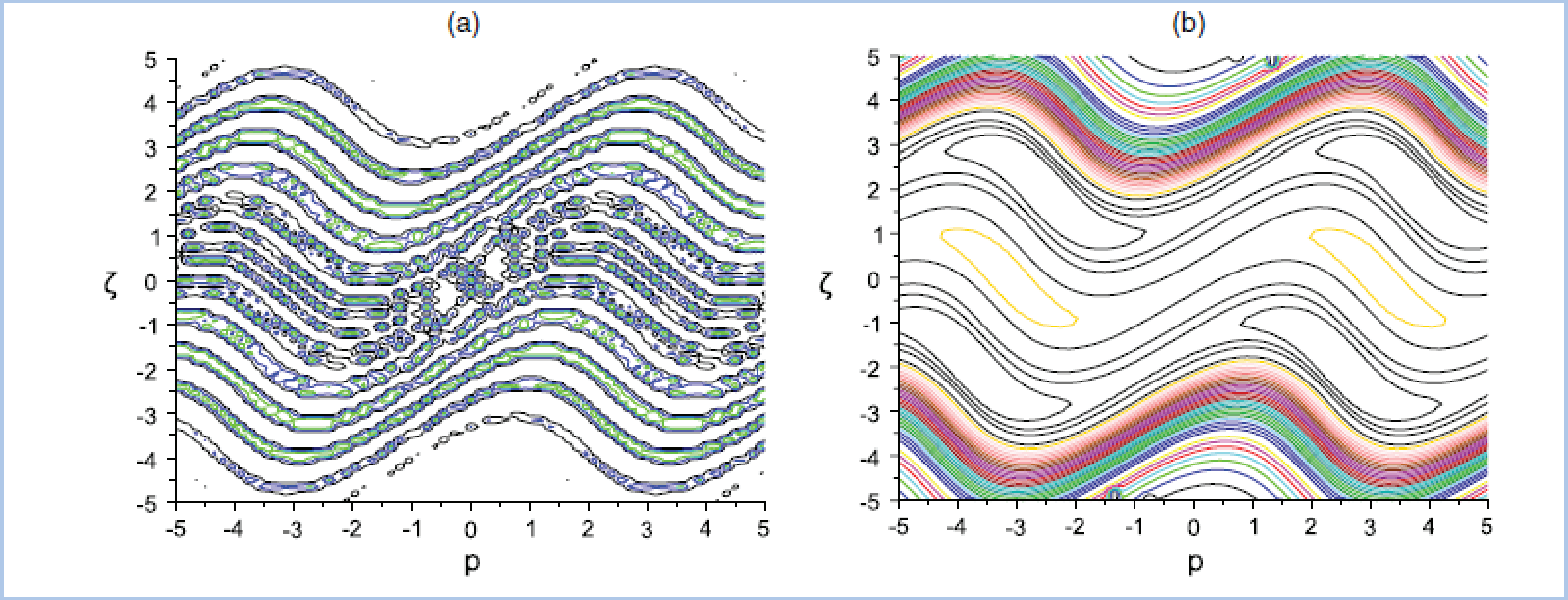
$$\partial_s f(p,\zeta,s) = \hat{V}\, f(p,\zeta,s)$$

$$f(p,\zeta,0) = f_0(p,\zeta)$$

$$f(p,\zeta,s) = e^{s\left[D\partial_p^2 + \hat{L}\right]} f_0(p,\zeta)$$

Liouville distribution contour plots: a) without coulombian diffusion, b) with coulombian diffusion D = 0.7.

- Phase space contour plots



Phase space plots

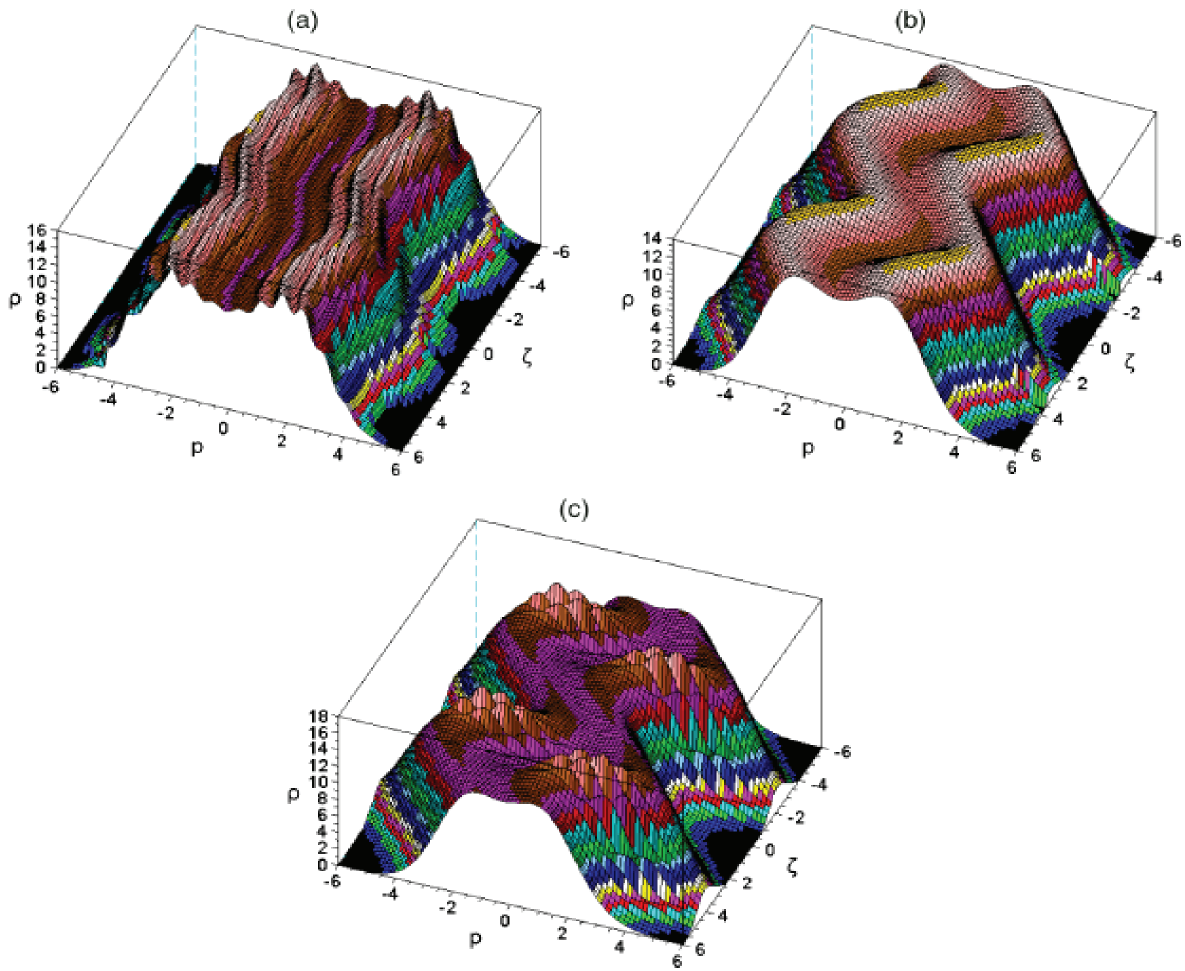


FIG. 3. Liouville distribution under the action of the Coulombian diffusion ($D = 7 \times 10^{-3}$) for different s values: (a) $s = 6$, (b) $s = 20$, (c) $s = 30$.

Bunching factor suppression

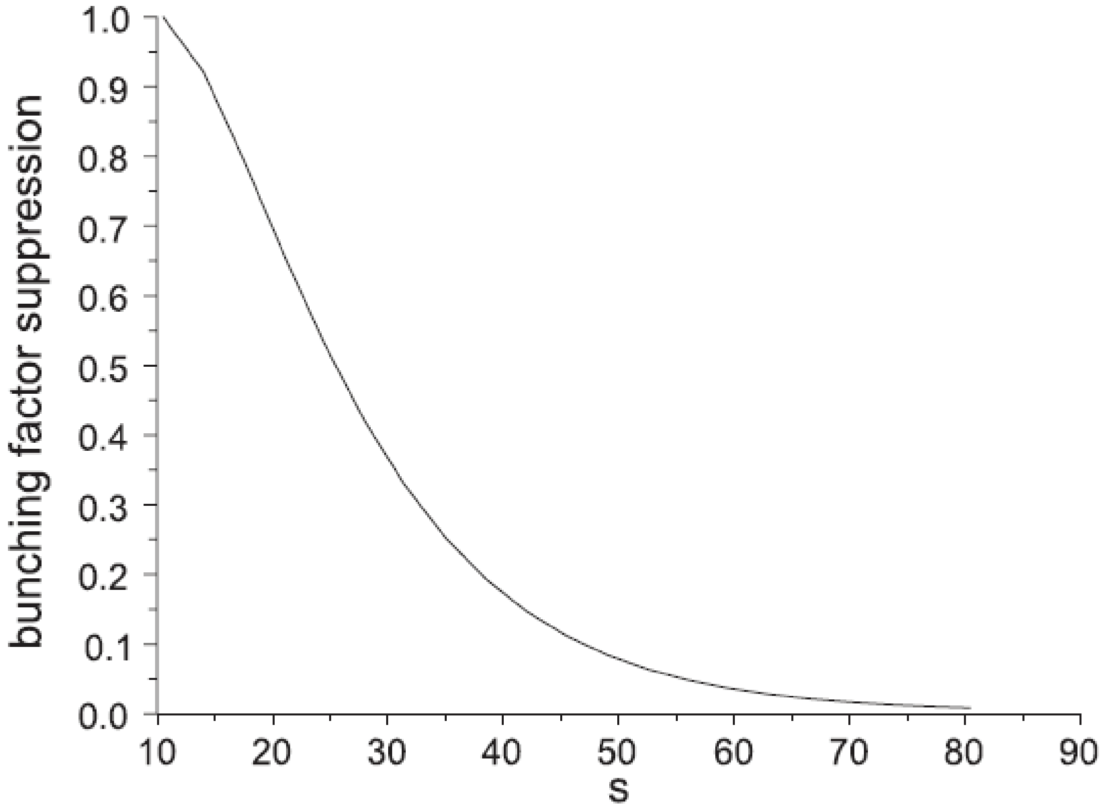


FIG. 4. Effect of the Coulombian diffusion ($D = 3.5 \times 10^{-4}$) on the bunching coefficient ($m = 9$) $b_9(D)/b_9(0)$ vs s.

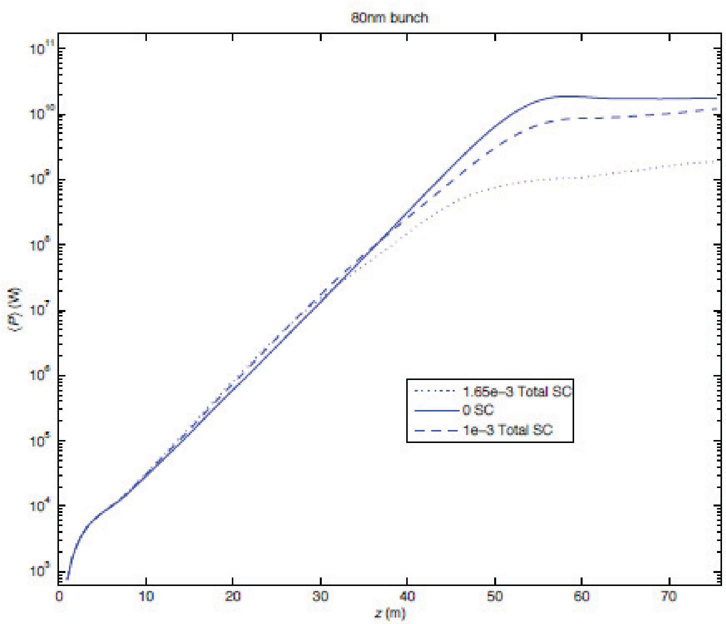
…Very Roughly

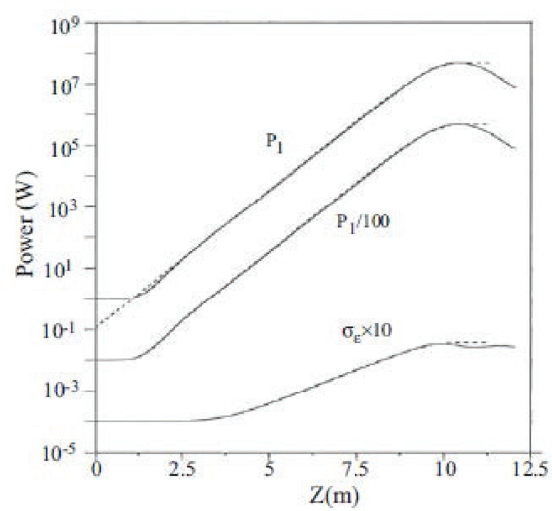
- The effect of Coulomb induced energy spread is provided by the FEL interaction itself when the density modulation increases.
- The model to be developed should include self consistently the interplay between density modulation-Coulomb diffusion-bunching…

FEL SASE & Coulomb effects

- D. Ratner (Too Much Ado about…Stanford 2011)

Figure 8.5: 1D FEL simulation for an 80nm bunch (LCLS parameters) when space charge produces a relative energy spread of $\sigma_\delta/\delta = 10^{-3}$ at saturation (dashed line). The resulting power is slightly lower than the result without space charge γ (solid line). When the space charge effect increases to $\sigma_\delta/\delta = 1.65 \times 10^{-3}$ (dotted line), the power diminishes considerably.





…

- The effect cannot be simply evaluated as due to an incoherent energy spread using a logistic map scheme (G. D., P.L. OTTAVIANI)

$$P_L(z) = P_0 \frac{A(z)}{1 + \frac{P_0}{P_F} A(z)},$$

$$A(z) = \left[\cosh\left(\frac{z}{L_g}\right) - \left(e^{-\frac{z}{2L_g}} \cos\left(\frac{\pi}{3} - \frac{\sqrt{3}z}{2L_g}\right) + e^{\frac{z}{2L_g}} \cos\left(\frac{\pi}{3} + \frac{\sqrt{3}z}{2L_g}\right)\right)\right],$$

$$L_g = \chi L_g^{(o)}, \chi \cong 1 + 0.185 \frac{\sqrt{3}}{2} \tilde{\mu}_\varepsilon^2, \tilde{\mu}_\varepsilon = 2\frac{\sigma_\varepsilon}{\rho}$$

Power vs. length matched and non matched beam

- G. D., E. Dipalma, A. Petralia, M. Quattromini (IEEE-JQE (2013)
- L. L. Lazzarino et al. (SPARC collaboration)
- @SPARC models have been developed to include the effect of tansverse dynamics on the SASE power evolution