Transverse gradient undulators for a storage ring-based x-ray FEL oscillator

Ryan R. Lindberg

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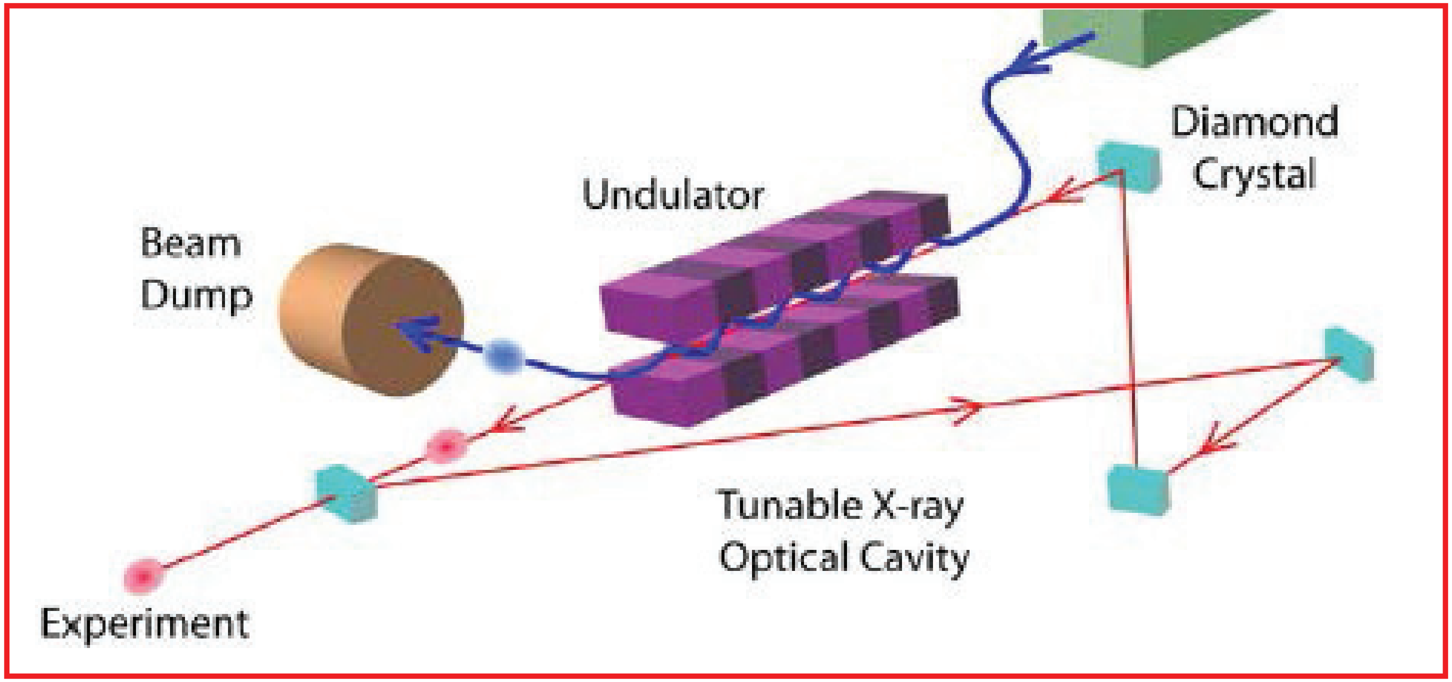
An x-ray FEL oscillator (XFELO)



- Fully coherent, tunable hard x-ray source (5 – 20 keV)
- ~1-10 meV bandwidth
- ~10^9 photons/pulse at a ~1 MHz repetition rate

K.-J. Kim, Y. Shvyd'ko, and S. Reiche, Phys. Rev. Lett. **100**, 244802 (2008)

Ultimate storage rings for FEL oscillators

- Storage rings can provide stable beams with a constant repetition rate
- Ultimate storage rings (USRs) have very small emittances such that $\varepsilon_x < \lambda/4\pi$ at hard x-ray wavelengths
- An x-ray FEL oscillator (XFELO) might be a natural fit at such a facility

Can such a scheme work?

Well-suited to x-ray FEL oscillator:
* beam energy
* peak current
* $\varepsilon_x < \lambda/4\pi$

PEP-X ultimate storage ring♣

Parameter	Description	Value
$\gamma_0 mc^2$	Beam energy	6.0 GeV
ε_x, ε_y	x, y emittances	5.2 pm-rad
I	Peak current	20 A
σ_t	Bunch length	2 ps
$\sigma_\gamma/\gamma = \sigma_\eta$	Energy spread	0.14 %

$$\lambda = \lambda_u \frac{1 + K^2/2}{2\gamma^2}$$

For reasonable FEL gain the wavelength spread must be less than the FEL bandwidth

$$\frac{\Delta\lambda}{\lambda} \sim \frac{1}{\pi N_u} \Rightarrow \frac{\Delta\gamma}{\gamma_0} < \frac{1}{2\pi N_u}$$

While a typical XFELO has

$$\frac{1}{2\pi N_u} \sim \frac{1}{2 \times 10^4} < 10^{-4}$$

Energy spread too big by an order of magnitude

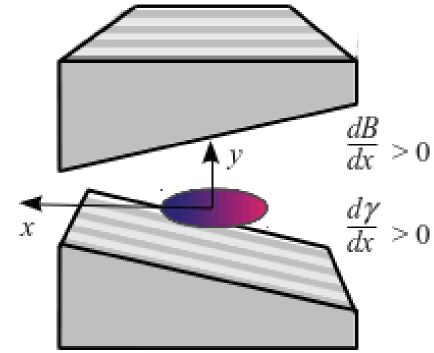
♣ Y. Cai, K. Bane, R. Hettel, Y. Nosochkov, M.-H. Wang, and M. Borland, Phys. Rev. ST-AB **15**, 054002(2012)

Mitigating the effects of energy spread using a transverse gradient undulator (TGU)

- Smith and collaborators♣ proposed employing an undulator whose magnetic field varies with transverse position (a TGU)
  $\Rightarrow$ variation in the resonance condition with x
- By correlating the electron energy with position such that higher energy particles see larger field the resonance condition can be (approximately) satisfied for the entire beam
- Early low-gain analysis by Kroll, Rosenbluth and others♠ applies to a different regime when focusing due to gradient is important

Low gain x-ray FELs have $1 < k_n L_u \equiv \frac{K_0 k_u}{\sqrt{2}\gamma_0} \ll \frac{K_0 \alpha}{\sqrt{2}\gamma_0}$ α is the TGU field gradient

- Inspiration came from recent work in the high gain regime applied to laser wakefield accelerators♦ and USRs♥



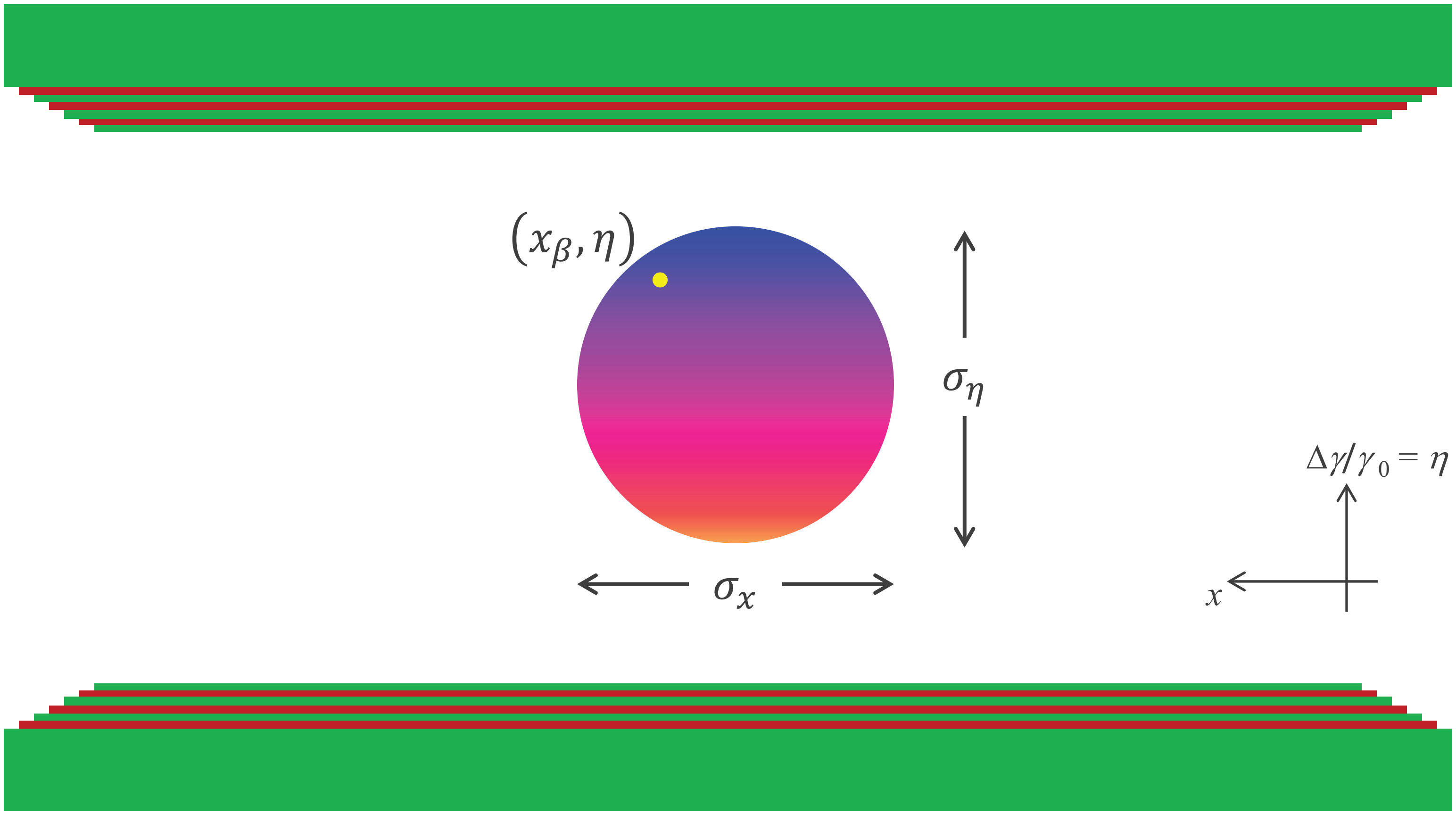
♣ T.I. Smith, L.R. Elias, J.M.J. Madey, and D.A.G. Deacon, J. Appl. Phys. **50**, 4580 (1979).

♠ N.M. Kroll, P.L. Morton, M.N. Rosenbluth, J.N. Eckstein, and J.M.J. Madey, IEEE J. Quantum Electron. **17**, 1496 (1981); N.M. Kroll and M.N. Rosenbluth, J. de Physique **44**, C1-85 (1983).

♦ Z. Huang, Y. Ding, and C.B. Schroeder, Phys. Rev. Lett. **109**, 204801 (2012) .

♥ Y. Ding, P. Baxevanis, Y. Cai, Z. Huang, and R. Ruth, IPAC'13, Shanghai, China, May 2013, WEPWA075.

Illustration of the transverse undulator gradient FEL with a large energy spread beam



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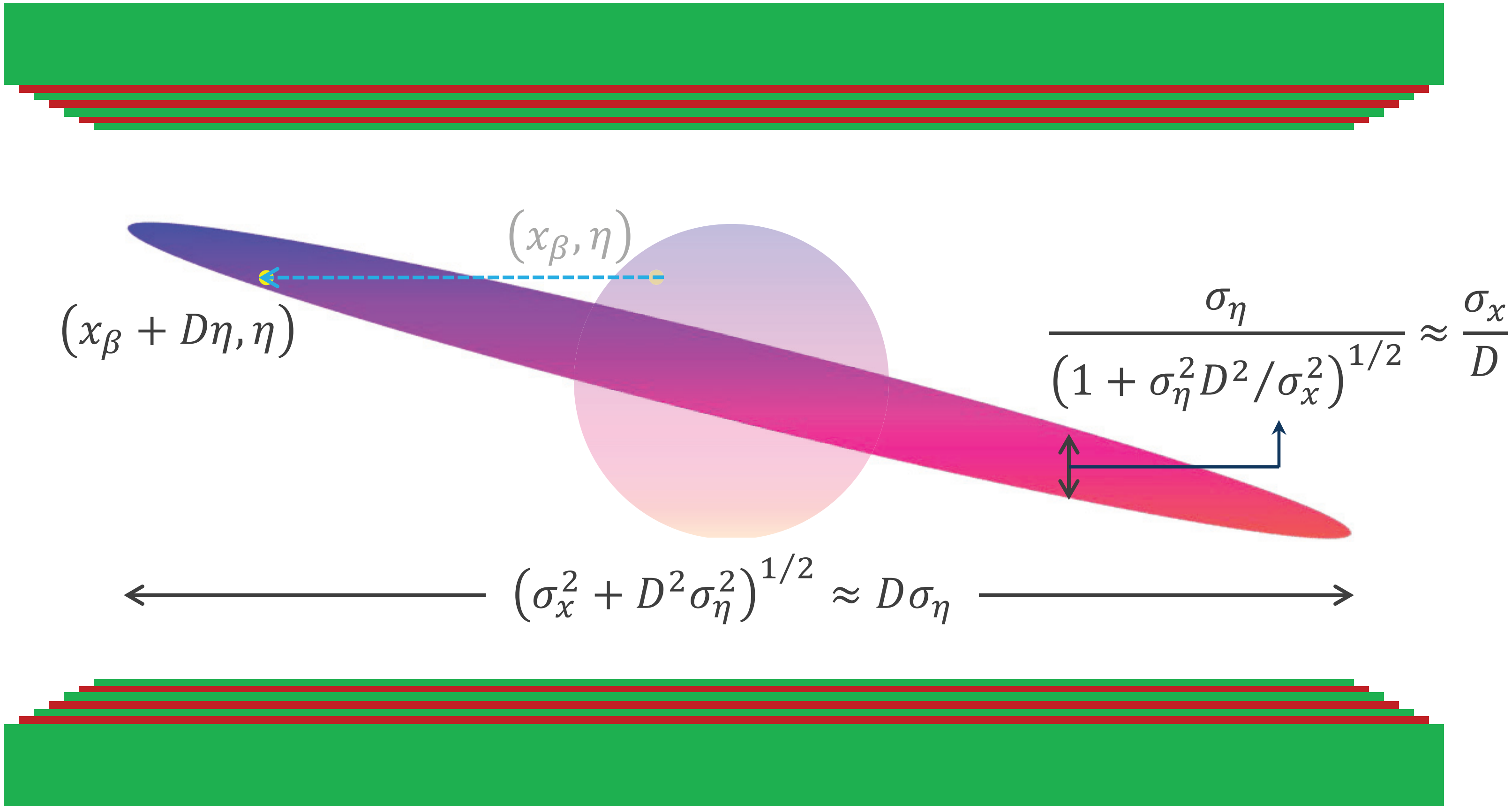

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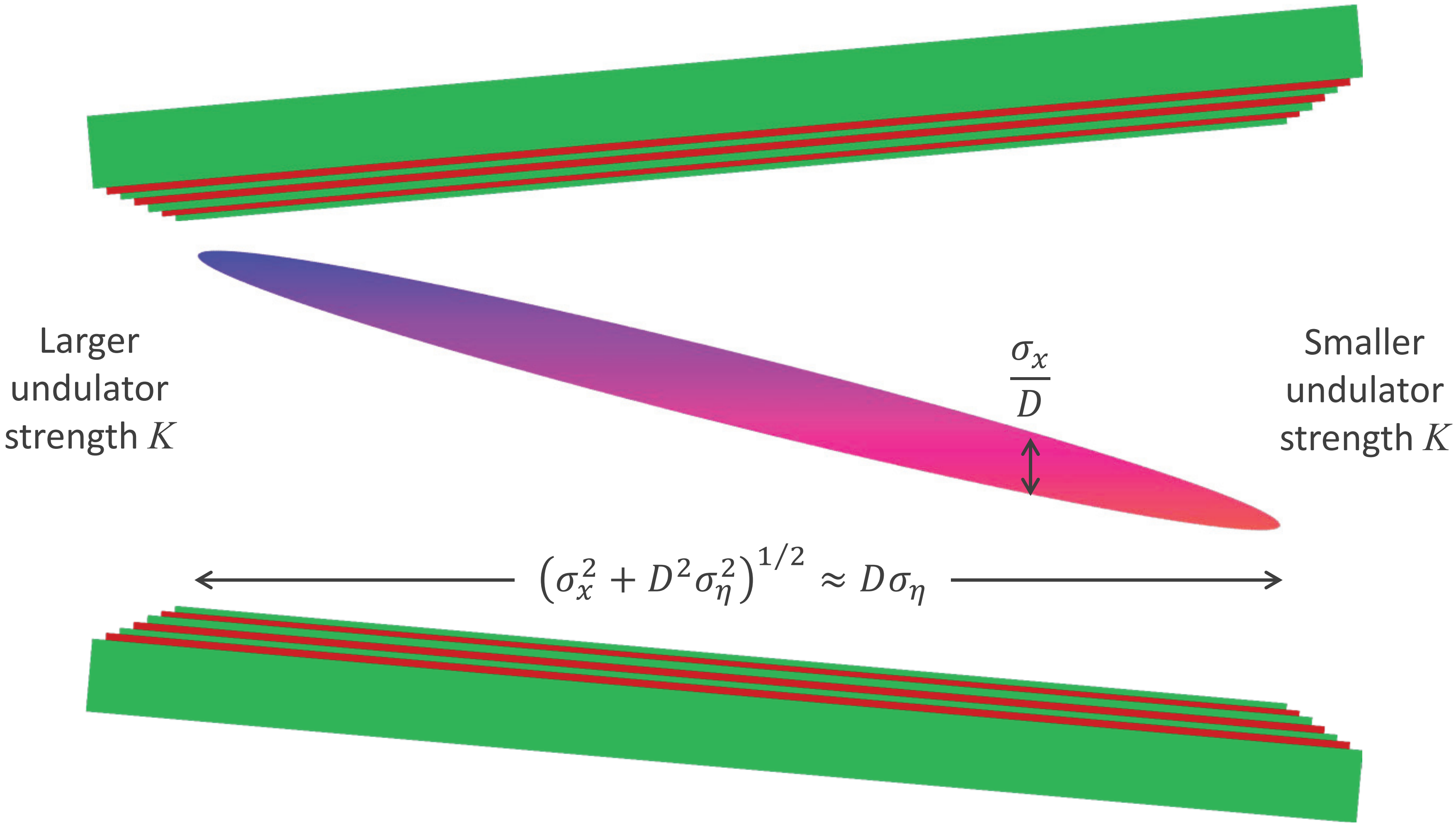

TGU-effect on the resonance condition

The resonance condition depends on the transverse position x_j in a transverse gradient undulator (TGU):

$$\lambda = \lambda_u \frac{1 + \left[K\left(x_j\right)\right]^2/2}{2\gamma_0^2\left(1+\eta_j\right)^2} \rightarrow \lambda_u \frac{1 + \left[K\left(x_{\beta,j} + D\eta_j\right)\right]^2/2}{2\gamma_0^2\left(1+\eta_j\right)^2}$$

$$\approx \lambda_u \frac{1+K_0^2}{2\gamma_0^2}\left[1 + \frac{K_0^2\alpha}{1+K_0^2/2}\left(x_{\beta,j} + D\eta_j\right) - 2\eta_j\right]$$

Assuming linear dependence
$K(x) = K_0 + \alpha x$

Cancel by choosing gradient α such that

$$D\alpha = \frac{2+K_0^2}{K_0^2}$$

$$\Rightarrow \lambda \approx \lambda_u \frac{1+K_0^2}{2\gamma_0^2\left(1 + x_{\beta,j}/D\right)^2}$$

Variation in resonance replacing $\sigma_\eta \Rightarrow \sigma_x/D$

1D FEL gain including energy spread

$$G = -\frac{2\pi^2}{\gamma}\frac{I}{I_A}\frac{K_0^2[\mathrm{JJ}]^2}{1+K_0^2/2}\frac{N_u^3\lambda_1^2}{2\pi\Sigma_x\Sigma_y}\int_{-1/2}^{1/2} dsdz\,(z-s)e^{-2[2\pi N_u(z-s)\sigma_\eta]^2}\sin[2\pi N_u\Delta\nu(z-s)]$$

Transverse beam area

$\Sigma_{x,y}^2 \equiv \sigma_{x,y}^2 + \sigma_{r_{x,y}}^2$

Related to convolution of gain for single energy with beam whose rms energy spread is σ_η

1D FEL gain including energy spread

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For optimal mode matching, we set $\Sigma_x = \Sigma_y = \sqrt{\sigma_r^2+\sigma_x^2} \approx \sqrt{2}\sigma_r = \sqrt{\frac{\lambda_1 Z_R}{2\pi}} \approx \frac{\sqrt{\lambda_1 L_u}}{2\pi}$

$$G = -\frac{4\pi^3}{\gamma}\frac{I}{I_A}\frac{K_0^2[\mathrm{JJ}]^2}{1+K_0^2/2}\frac{N_u^2\lambda_1}{\lambda_u}\int_{-1/2}^{1/2} dsdz\,(z-s)e^{-2[2\pi N_u(z-s)\sigma_\eta]^2}\sin[2\pi N_u\Delta\nu(z-s)]$$

1D FEL gain including energy spread

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Small energy spread: $(2\pi\sigma_\eta)^2 \ll 1/N_u^2$

the integral can be evaluated

$$\Rightarrow G \propto N_u^2\frac{d}{d\nu}[\mathrm{sinc}(2\pi N_u\Delta\nu)]^2$$

Derivative of the spontaneous emission spectrum (Madey's theorem)

Large energy spread: $(2\pi\sigma_\eta)^2 \gg 1/N_u^2$

the integral is negligible unless $|z-s| \lesssim 1/N_u\sigma_\eta$, and is maximized when $\Delta\nu \approx \sigma_\eta$

$$\Rightarrow G \propto \frac{1}{\sigma_\eta^2}$$

1D gain for a TGU enabled FEL

A fitting formula for the 1D FEL gain is $G_{\text{FEL}} = g_0 \dfrac{N_u^2}{1 + \left(5.46 N_u \sigma_\eta\right)^2}$

The TGU-FEL results in the replacements

$$\sigma_\eta \rightarrow \sigma_x / D \qquad \Sigma_x \rightarrow \left(\sigma_{r,x}^2 + \sigma_x^2 + D^2 \sigma_\eta^2\right)^{1/2} \approx D\sigma_\eta$$

$$G_{\text{TGU}} \approx g_0 \frac{\sqrt{2}\sigma_x}{D\sigma_\eta} \frac{N_u^2}{1 + (5.46 N_u \, \sigma_x / D)^2}$$

Larger size

Decrease in effective energy spread

G_{TGU} is maximized when $D/\sigma_x = 5.46 N_u$

$$\Rightarrow \frac{G_{\text{TGU}}}{G_{\text{FEL}}} \approx \frac{1}{\sqrt{2}} \frac{D\sigma_\eta}{\sigma_x} \gg 1$$

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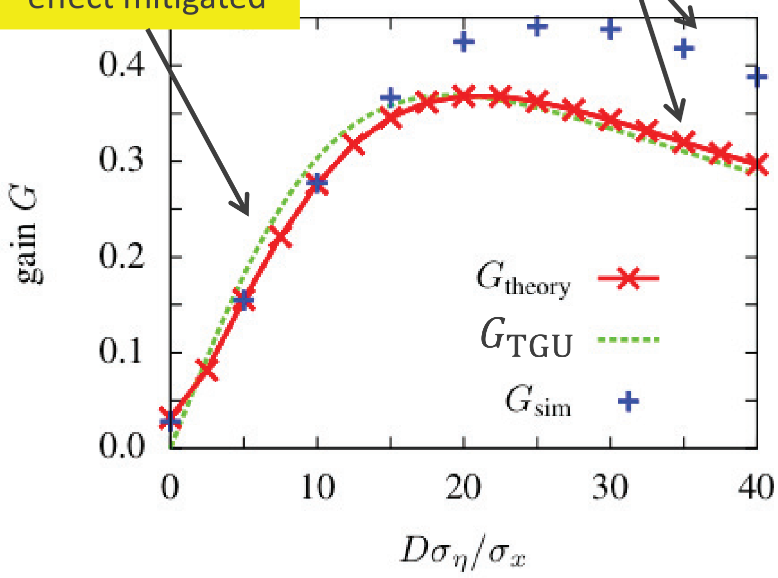
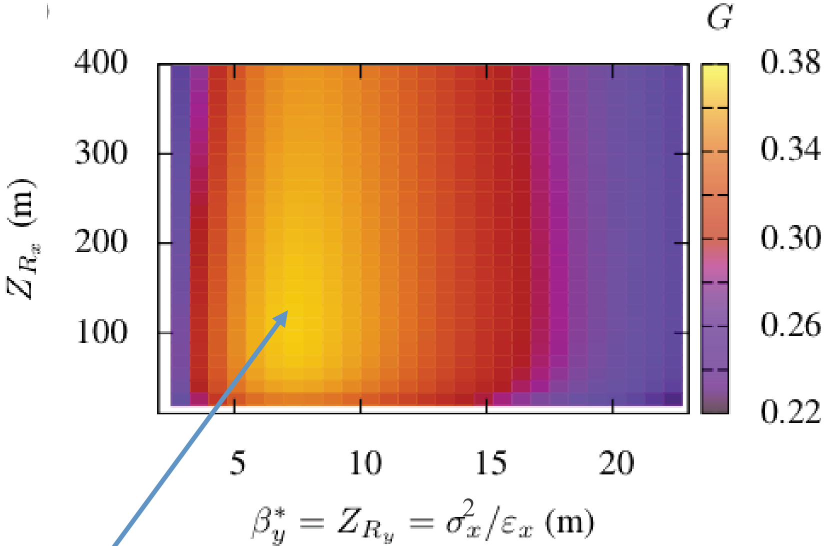
Gain increases as energy spread effect mitigated

Gain decreases due to larger e-beam size



3D gain and mode shape in a TGU enabled FEL

Parameter	Description	Value
$\gamma_0 mc^2$	Beam energy	6.0 GeV
$\varepsilon_x, \varepsilon_y$	x, y emittances	5.2 pm-rad
I	Peak current	20 A
$\sigma_\gamma/\gamma = \sigma_\eta$	Energy spread	0.14 %
λ_1	Radiation wavelength	0.886 Å
λ_u	Undulator period	1.63 cm
L_u	Undulator length	40.75 m
K_0	Deflection parameter	1.0
α	Transverse gradient	34/m
D	Dispersion	8.8 cm



3D analytic theory predicts broad maximum in the FEL gain near the waist $\beta_y^* = Z_{Ry} = \sigma_x^2/\varepsilon_x = 7.3$ m $\approx L_u/2\pi$

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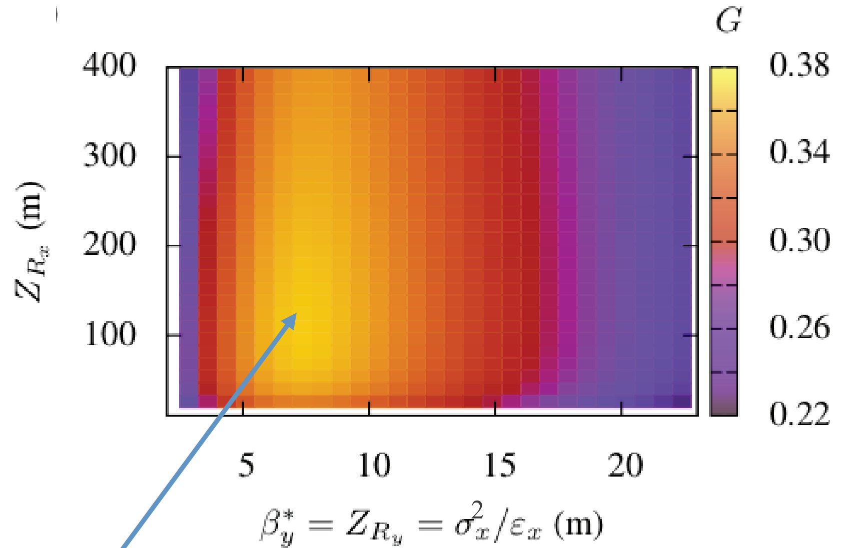
Z_{Rx}	Rayleigh range in x	105 m
Z_{ry}	Rayleigh range in y	7.3 m
G	Linear gain	0.44



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Using the theoretically obtained focusing parameters in a full TGU-FEL simulation with $\frac{D\sigma_\eta}{\sigma_x} = 20.$

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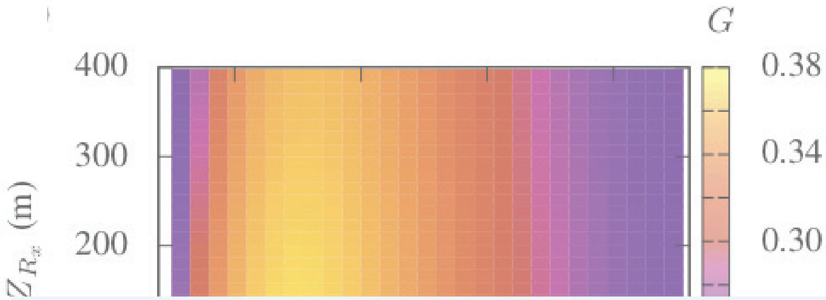
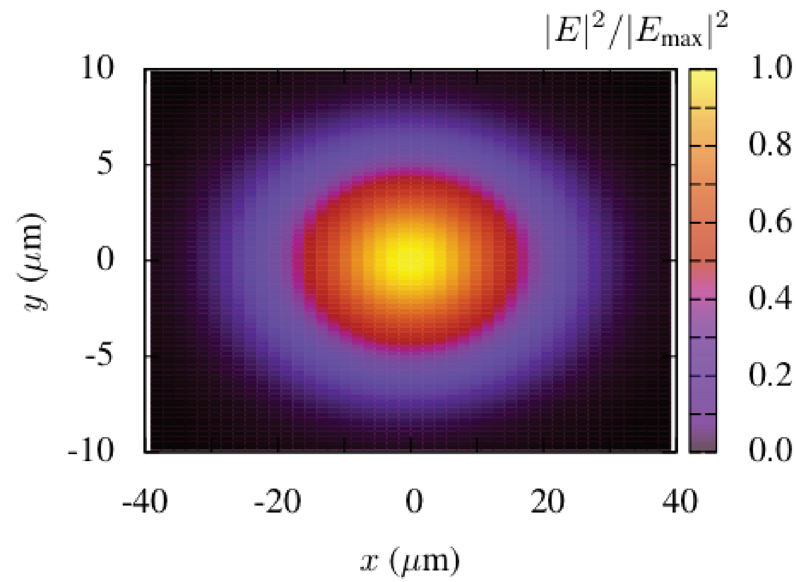


Simulated image of the mode shape at the undulator center, showing a mode size aspect ratio of about 3.7 : 1 (compare to e-beam elongation 20 : 1)



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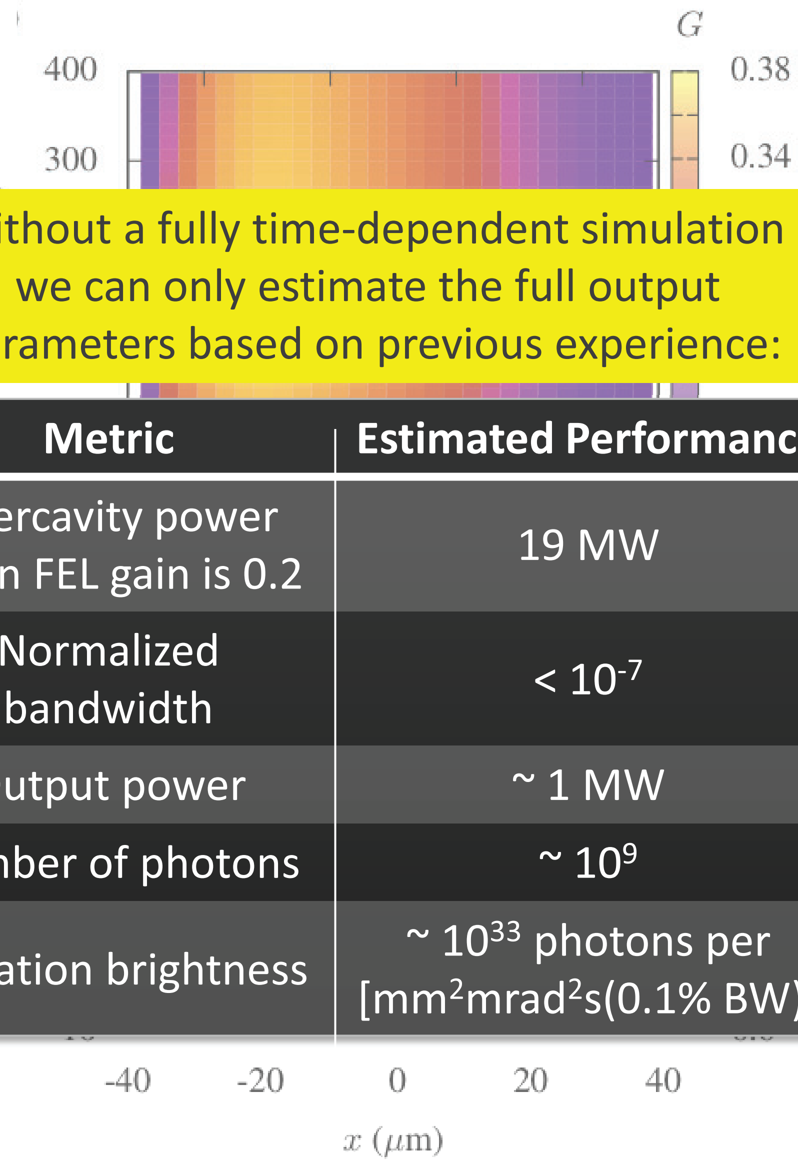
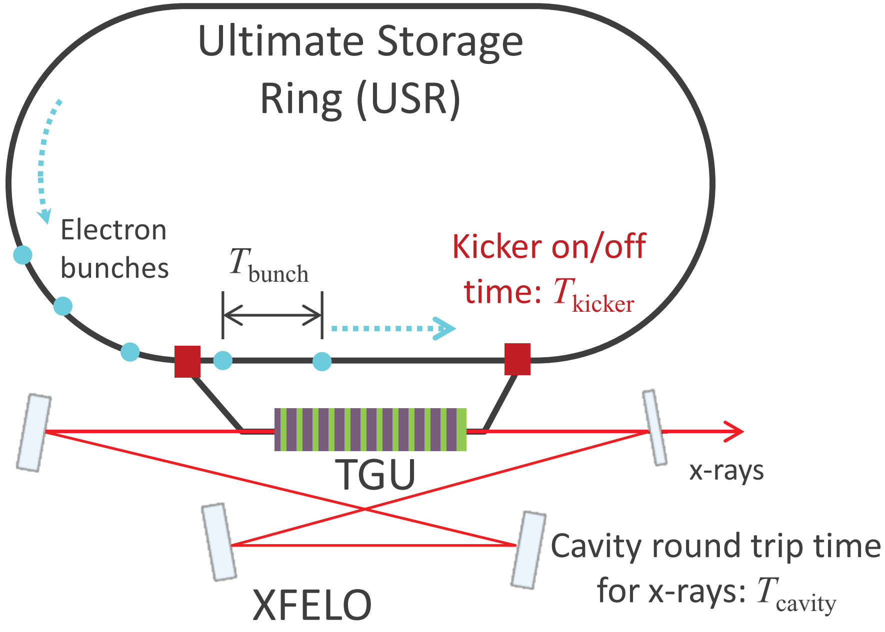


Without a fully time-dependent simulation we can only estimate the full output parameters based on previous experience:

Metric	Estimated Performance
Intercavity power when FEL gain is 0.2	19 MW
Normalized bandwidth	$< 10^{-7}$
Output power	~ 1 MW
Number of photons	~ 10^9
Radiation brightness	~ 10^{33} photons per [$\mathrm{mm^2 mrad^2 s}$(0.1% BW)]

Incorporating the XFELO into a USR

- The storage ring and insertion-device FELs can act as coupled oscillators where oscillations in FEL power are out of phase with oscillations of e-beam quality ♣
- We propose decoupling the storage ring beam dynamics with that of the XFELO by operating a pulsed FEL in a bypass of the USR:



Requirements on timing are
$$T_{\text{kicker}} < T_{\text{bunch}} < T_{\text{cavity}}$$

and T_{cavity} is an integer multiple of T_{bunch}

♣ P. Elleaume, J. de Physique **45**, 997 (1984)

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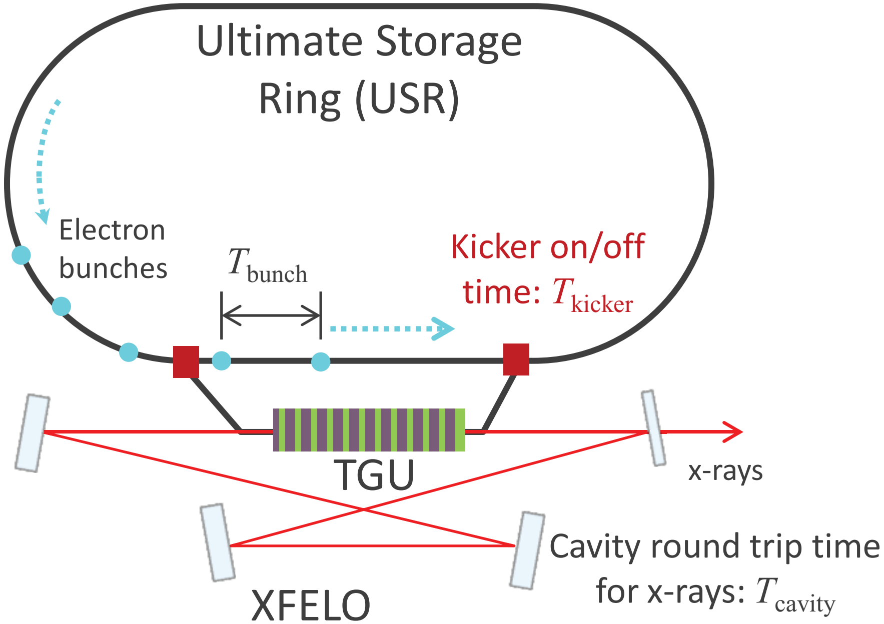

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$$T_{\text{kicker}} < T_{\text{bunch}} < T_{\text{cavity}}$$

KEK-ATF measured $T_{\text{kicker}} \leq$ 5 ns for their ILC prototype fast kicker♠

Round trip time for stable cavity of ~300 m length is $T_{\text{cavity}} \sim$ 1 μs

♣ P. Elleaume, J. de Physique **45**, 997 (1984)
♠ T. Naito, S. Araki, H. Hayano, K. Kubo, S. Kuroda, N. Terunuma, T. Okugi, and J. Urakawa, Phys. Rev. ST Accel. Beams **14**, 051002 (2011)

A TGU-enabled XFELO compatible with the PEP-X USR design♣

Parameter	Description	Value
C_{ring}	Circumference	2234 m
τ_x, τ_y, τ_z	Damping times	13, 15, 9 ms
$\gamma_0 mc^2$	Beam energy	6.0 GeV
$\varepsilon_x, \varepsilon_y$	x, y emittances	5.2 pm-rad
I	Peak current	20 A
σ_t	Bunch length	2 ps
$\sigma_\gamma/\gamma = \sigma_\eta$	Energy spread	0.14 %

We propose filling every 10th bucket
$\Rightarrow$ 1117 stored bunches spaced at
$T_{\text{bunch}} = 6.67$ ns > 5 ns $= T_{\text{kicker}}$

FEL Gain is maximized when $Z_{Ry} = 7.5$ m
$\Rightarrow$ Distance between mirrors should be ~ 75-100 m for stability

We choose an optical path length = 186 m
$\Rightarrow T_{\text{cavity}} = 647$ ns and we kick every 93rd bunch into the XFELO

Every bunch is used by XFELO exactly once after about 0.72 ms

Turn off XFELO for a time > 3 τ_y = 45 ms to reach equilibrium, and repeat
$\Rightarrow$ XFELO duty factor is 1%-2%

♣ Y. Cai, K. Bane, R. Hettel, Y. Nosochkov, M.-H. Wang, and M. Borland, Phys. Rev. ST-AB **15** (2012) 054002

Other potential USR-based XFELOs

- The parameter space is rather limited, but a TGU-enabled XFELO is compatible with the PEP-X USR design
- However, PEP-X has a comparatively large peak current > 10 A
    1. Naturally small momentum compaction
    2. Strong rf for longitudinal focusing

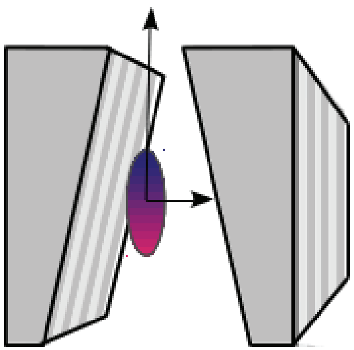
But below microwave instability threshold!

Tevatron ultimate storage ring♣

Parameter	Description	Value
C_{ring}	Circumference	6280 m
$\gamma_0 mc^2$	Beam energy	9.0 GeV
ε_x, ε_y	x, y emittances	3.1 pm-rad
I	Peak current	4 A
σ_t	Bunch length	50 ps
$\sigma_\gamma/\gamma = \sigma_\eta$	Energy spread	0.14 %

Bunch lengthening due to nonlinear and collective effects results in a small (but probably more typical) peak current and TGU-enabled FEL gain of only 10%

Potential solution:
Decrease coupling and disperse beam in vertical (small emittance) direction.
Preliminary $G \approx 30\%$



♣ M. Borland, IPAC'12, New Orleans, TUPPP033 p. 1683 (2012) http://www.JACoW.org
♠ Y. Ding, P. Baxevanis, Y. Cai, Z. Huang, and R. Ruth, IPAC'13, Shanghai, China, May 2013, WEPWA075.

Conclusions

- Good ideas have long lifetimes!
- A transverse gradient undulator (TGU) can be used to significantly increase the FEL gain in the presence of energy spreads that are nominally 10× too large
- Reasonable TGUs appear to make operation of a x-ray FEL oscillator (XFELO) compatible with beams from a ultimate storage rings (USR)
- Operation of a TGU-enabled XFELO in a USR will probably require a bypass
- Potential designs are strongly constrained by the availability of fast kickers
- While the available parameter space is small, a TGU-enabled XFELO appears to be compatible with the PEP-X USR.
- Extending a TGU-enabled XFELO to other USRs must confront the problems of small peak currents
- Tolerance and stability requirements of bypass need to be investigated

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Thanks for your attention