Suppressing the Shot Noise in Electron-Beams at Short Wavelengths

A. Nause*, A. Gover, E. Dyunin**, M. Fedurin*****

*Tel-Aviv University Dept. of Physics , Israel

**Tel-Aviv University Dept. of Physical Electronics, Israel

***BNL, Advanced Test Facility, USA

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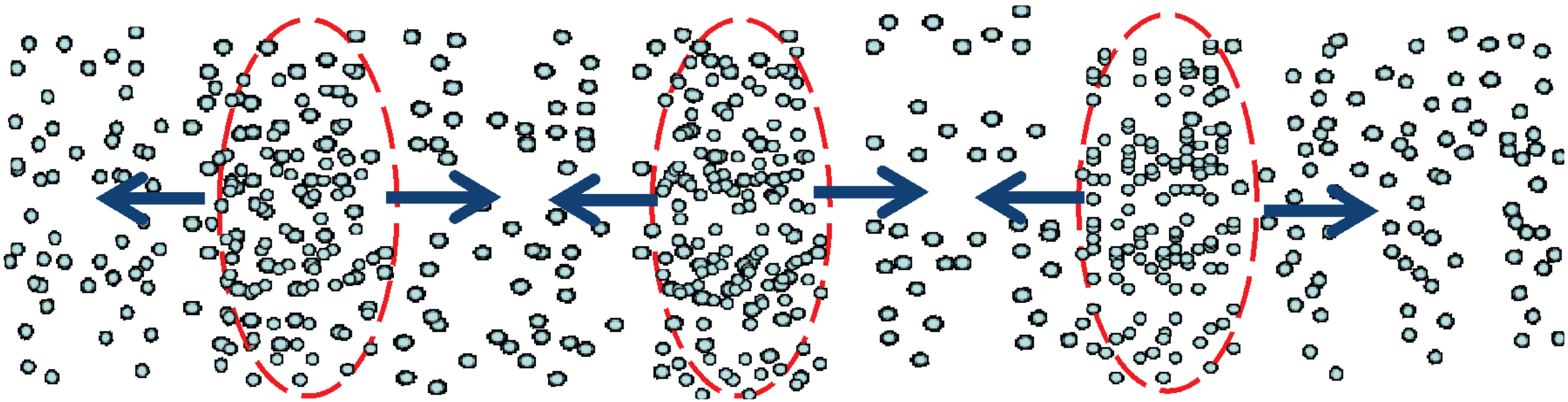
Random Bunching (Shot-Noise)



Shot-Noise spectral power $\overline{\left|\check{I}(\omega)\right|^2} = eI_b$ [A^2-Sec]

Plasma Oscillation in a Uniform e-Beam Drift Section

$$\breve{i}(L_d,\omega)=\left[\breve{i}(0,\omega)\cos\phi_p - i\breve{V}(0,\omega)\left(\sin\phi_p / W_d\right)\right]e^{i\phi_b(L_d)}$$
$$\breve{V}(L_d,\omega)=\left[-i\breve{i}(0,\omega)W_d\sin\phi_p + \breve{V}(0,\omega)\cos\phi_p\right]e^{i\phi_b(L_d)}$$

<u>Kinetic voltage:</u>
(axial velocity modulation)

$$\breve{V}(z,\omega)\propto\breve{\gamma}(z,\omega)\propto\breve{v}_z(\omega)$$

<u>Optical phase:</u>

$$\phi_b=\frac{\omega}{v_z}L_d$$

<u>Beam Impedance</u>

$$\boldsymbol{W_d=\frac{r_p^2\sqrt{\mu_0/\varepsilon_0}}{k\theta_{pr}A_e}}$$

<u>Plasma phase:</u>

$$\phi_p=\theta_{pr}L_d$$

<u>Plasma wavenumber:</u>

$$\theta_{pr}=r_p\frac{\omega_{pL}}{v_0}\quad,\quad\omega_{pL}=\left(\frac{e^2n_0}{m\varepsilon_0\gamma^3}\right)^{1/2}$$

[A. Gover, E. Dyunin, PRL 102, 154801 (2009)]

Current Shot-Noise Suppression

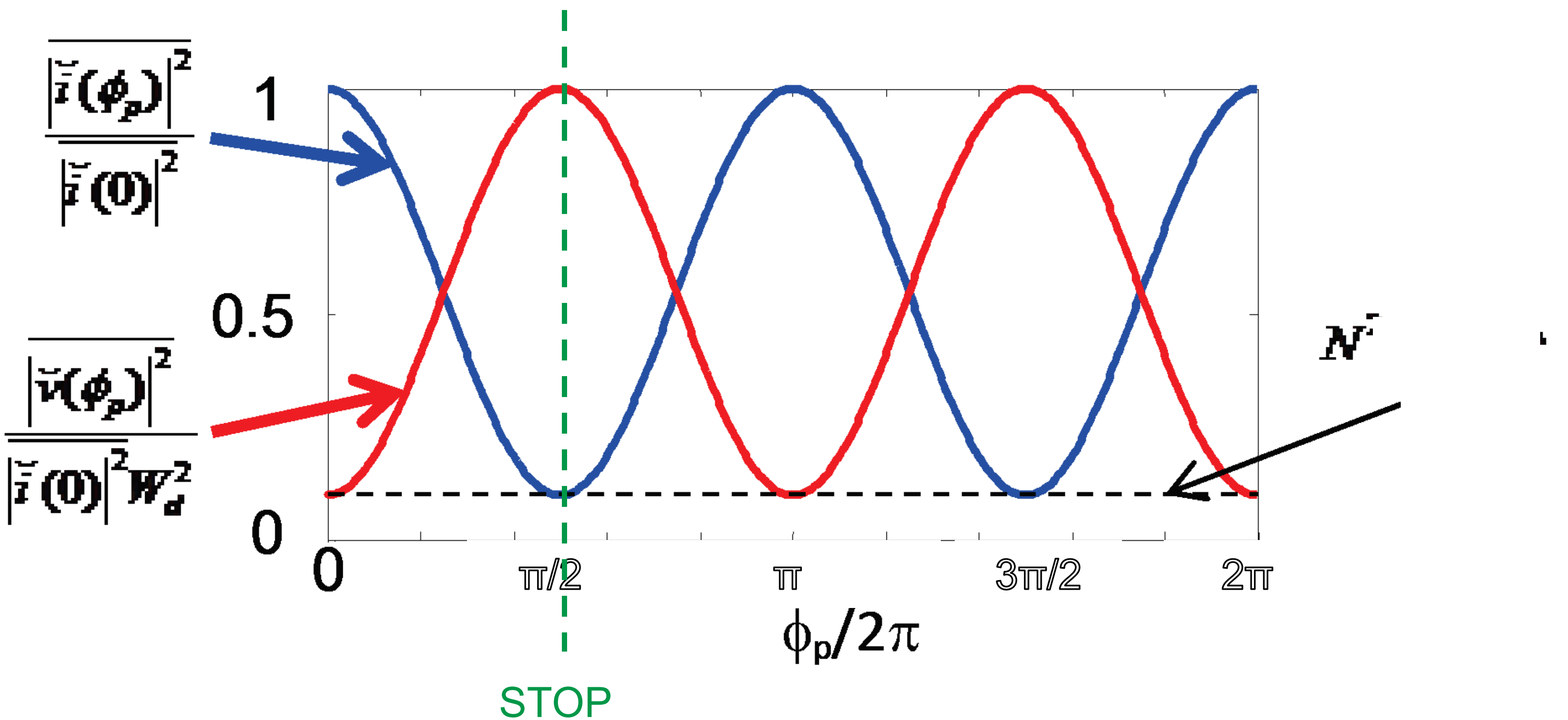
$$\boxed{gain = \frac{\overline{\left|\breve{i}(L_d, \omega)\right|^2}}{\overline{\left|\breve{i}(0, \omega)\right|^2}} = \cos^2\phi_p + N^2 \sin^2\phi_p}$$

$$\boxed{N^2 = \frac{\overline{\left|\breve{V}(0, \omega)\right|^2}}{W_d^2 \overline{\left|\breve{i}(0, \omega)\right|^2}}}$$

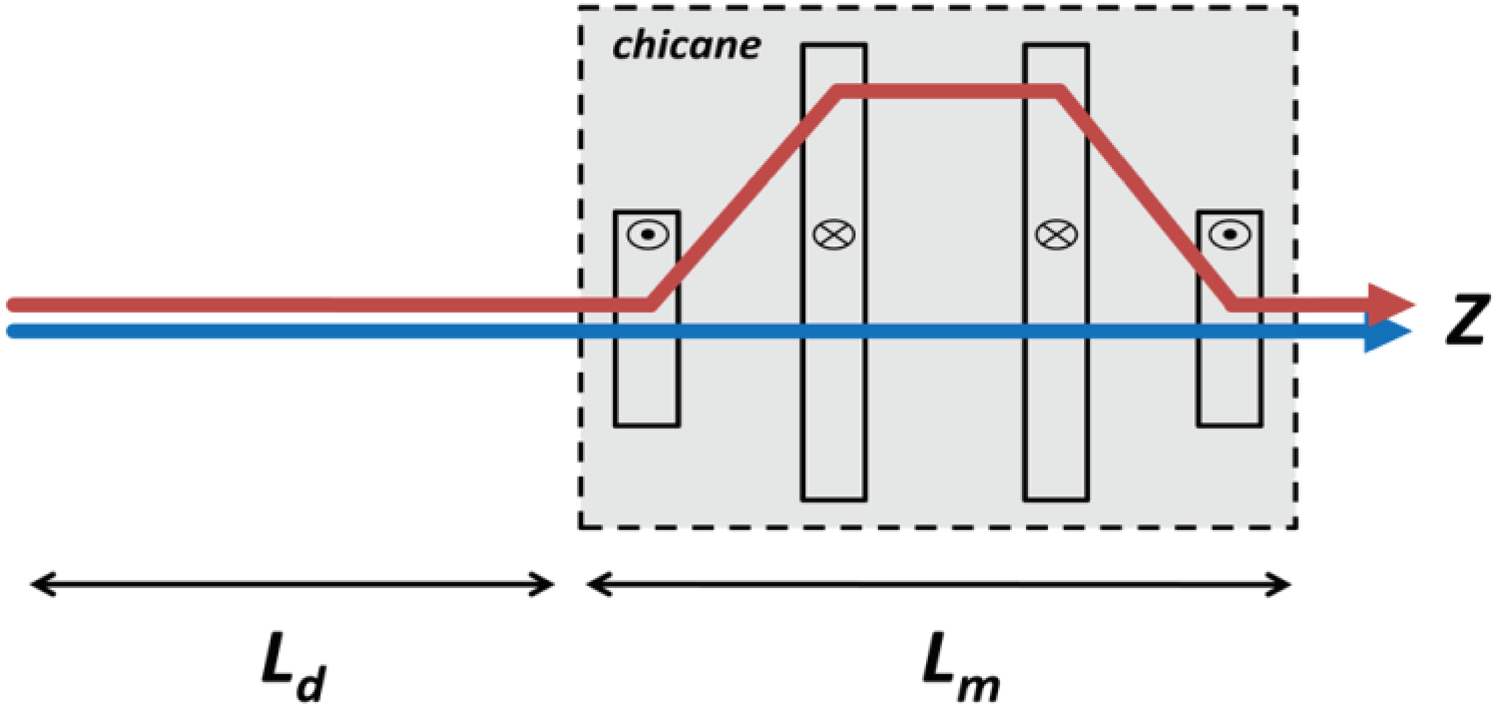
$\Rightarrow$ $$\boxed{Gain(\phi_p = \pi/2) = N^2}$$

$\langle\langle 1$ **For current noise dominated beam.**

Periodic Power Exchange of Current and Velocity Noise



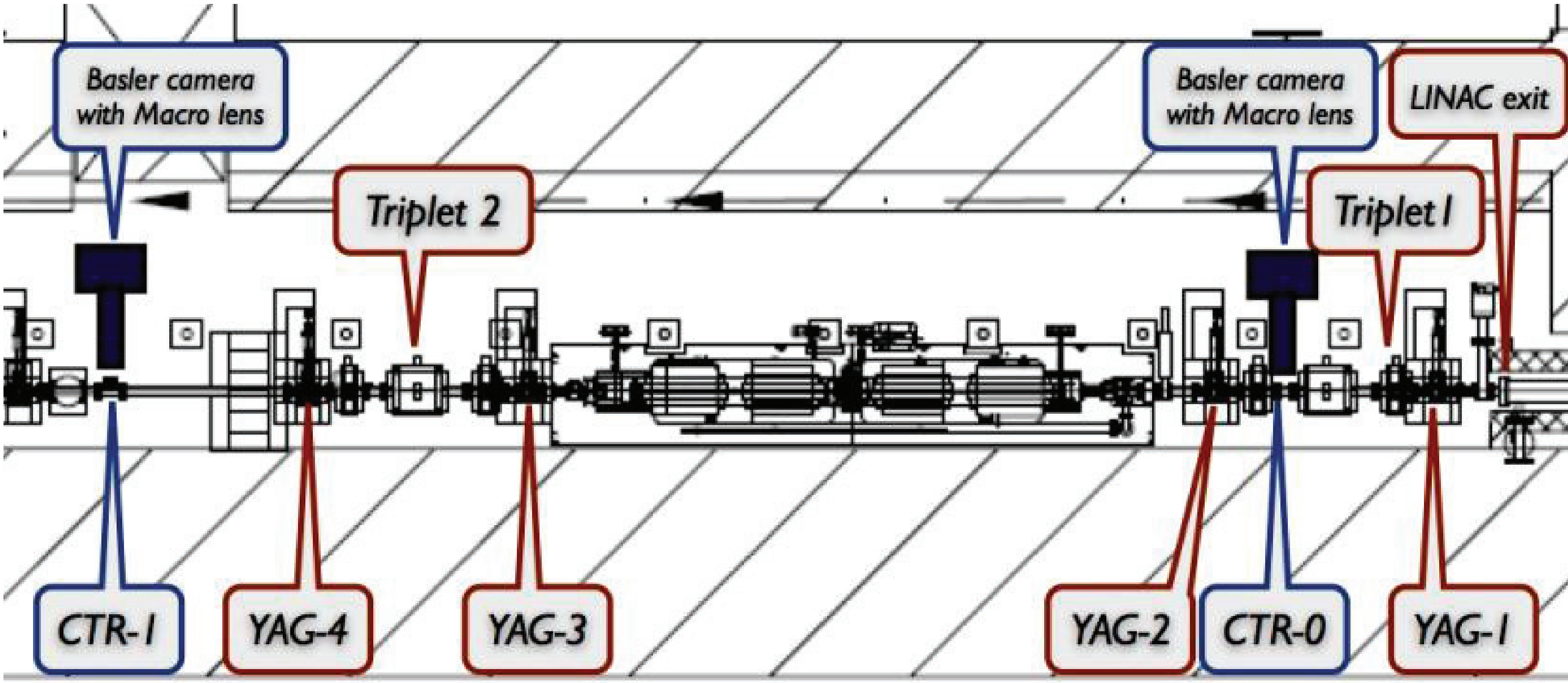
Drift / Dispersion Transport



Drift Dispersive Section

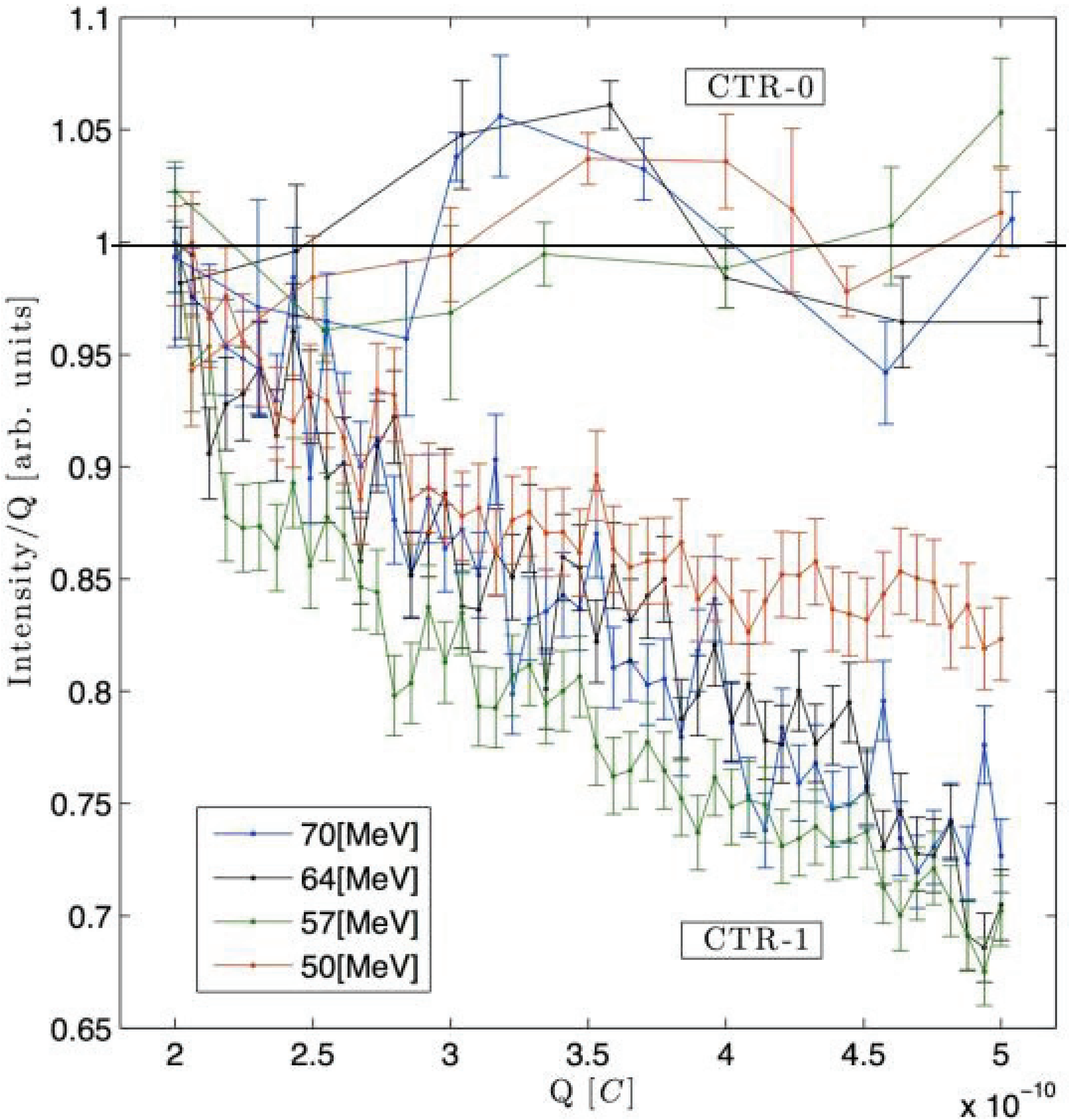
D. Ratner Z. Huang G. Stupakov, Phys. Rev. ST-AB, **14**, 060710 (2011)
A.Gover, E.Dyunin, T.Duchovni, A.Nause, *Phys. of Plasmas*, **18,** 123102 (2011).

NOISE SUPPRESSION EXPERIMENT IN ATF
OCTOBER 2011



A. Gover, A. Nause, E. Dyunin, M. Fedurin, *Nature Physics* **8**, 877–880 (2012)

Measured OTR Signal per unit charge



Before drift: linear dependence on Q

After drift: sub-linear dependence on Q

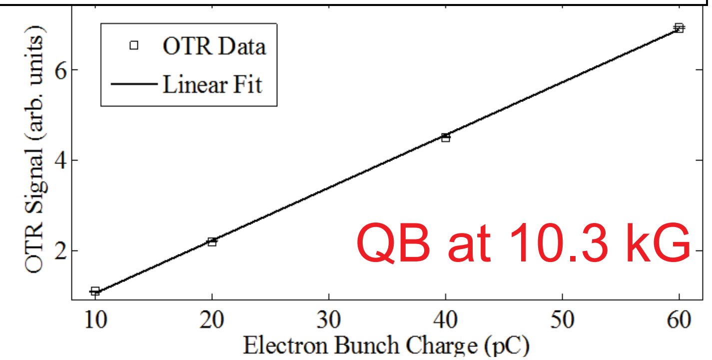
Dispersive Shot Noise Supp.

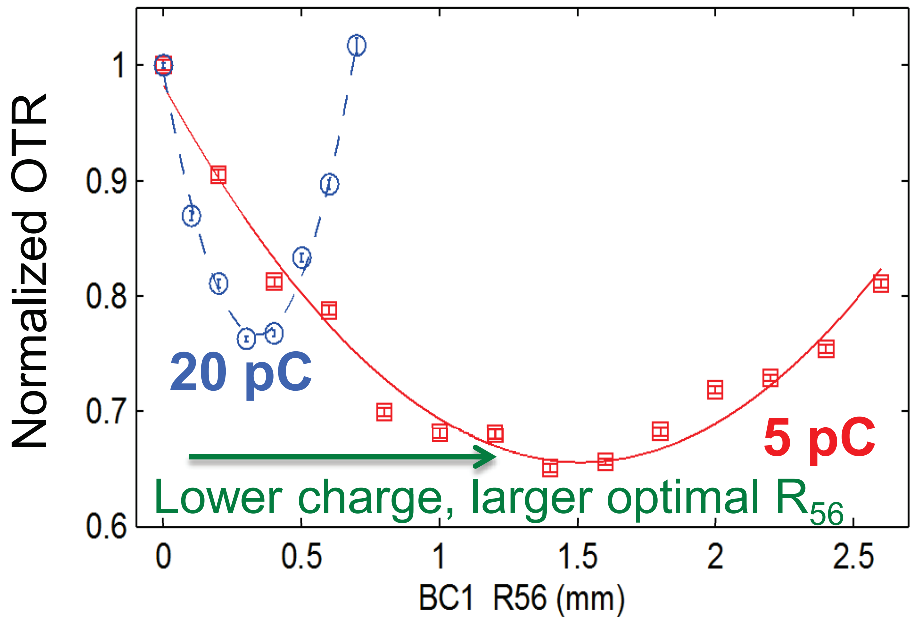
1D Dispersive Shot Noise Suppression

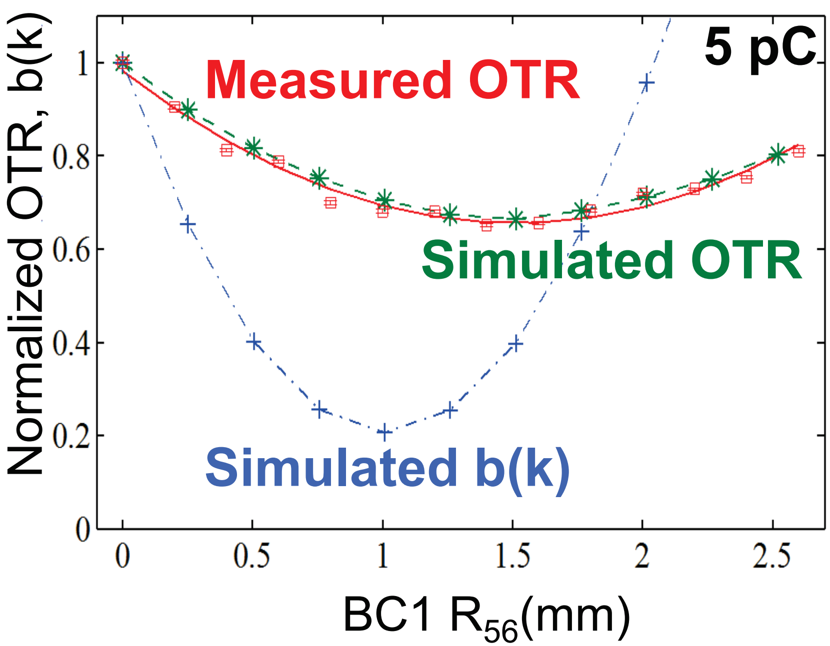
$$\boxed{N\left\langle |b(k)|^2 \right\rangle = (1-\Upsilon)^2}$$

$$\boxed{\Upsilon \equiv n_0 R_{56} A}$$

OTR proportional to charge at start







D. Ratner, G. Stupakov, Phys. Rev. Lett. 109, 034801 (2012)

Significance of N^2

Noise dominance parameter

$$N^2 \equiv \frac{\overline{\left|\breve{v}(0,\omega)\right|^2}}{\overline{\left|\breve{i}\,(0,\omega)\right|^2 W_d^2}}$$

Minimal gain factor in drift

$$gain\big|_{\phi_{bd}=\pi/2} = N^2$$

Landau-damping parameter

$$N_D = \frac{k}{k_D} \qquad \left(k_D = \frac{2\pi}{\lambda_D} = \frac{\omega_{pL}}{\delta v_z} \right)$$

$$N = \frac{N_D}{\beta_0} \approx N_D$$

Phase-spread parameter

$$\Delta\varphi_b = kL_d \frac{\Delta\beta_z}{\beta_z^2} = kL_d \frac{\Delta\beta_z c}{\omega_{pr}\beta_z^2} \frac{\omega_{pr}}{c} = \frac{k}{k_D} \frac{L_d \theta_{pr}}{\beta_z} = N\phi_{prd}$$

$$\Delta\varphi_b\big|_{\phi_{pd}=\pi/2} = \frac{\pi}{2} N$$

Conditions for noise suppression – Drift transport

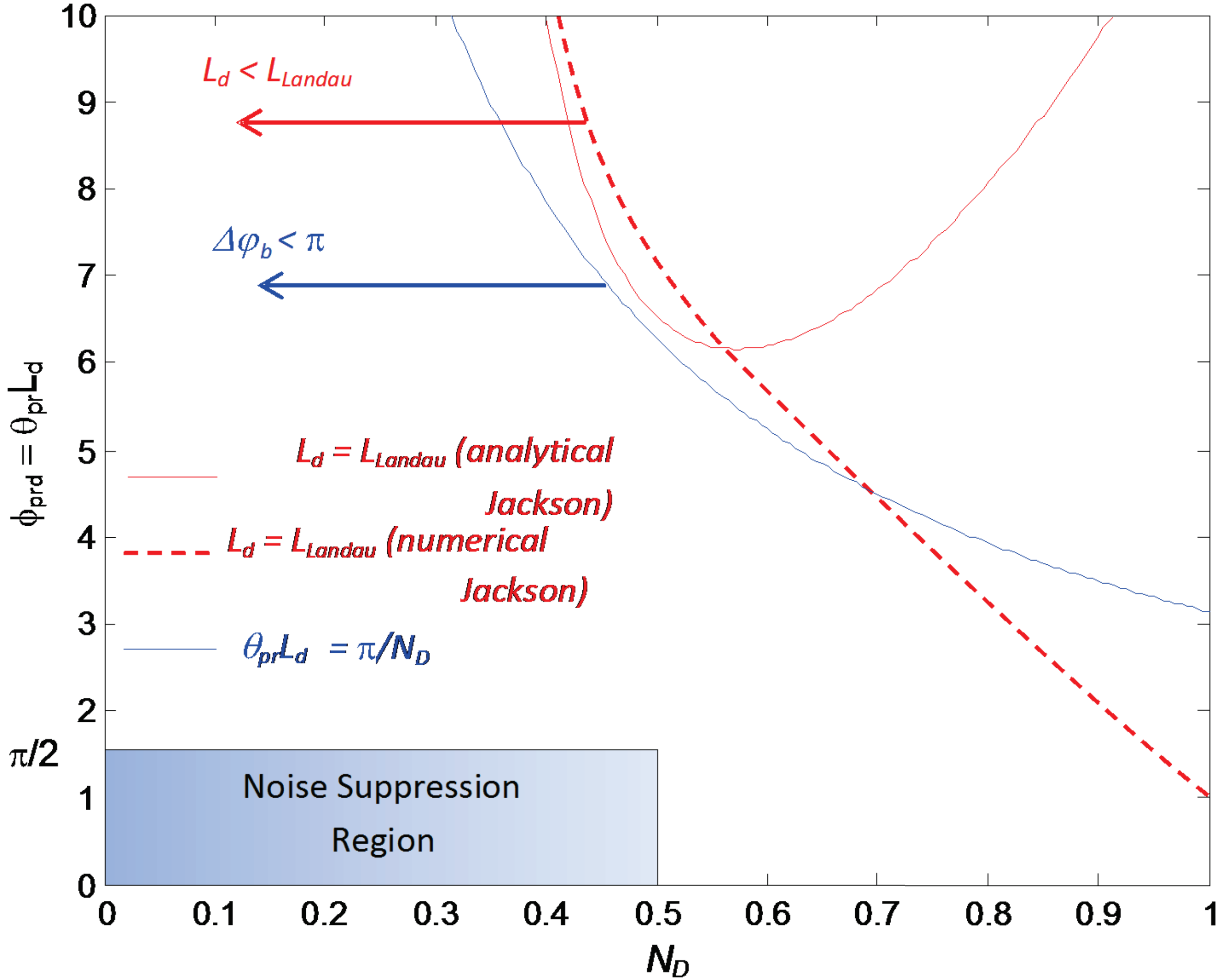
Current Shot-noise dominance $N^2 << 1$

Landau damping neglect ${N_D}^2 << 1$

Phase-spread neglect $\Delta\varphi_b\big|_{\phi_{pd}=\pi/2} = \frac{\pi}{2} N << \pi$

Significant suppression $Gain = N^2 << 1$

The Landau damping neglect region ($L_d < L_{Landau}$) and the ballistic electron optical phase spread region ($\Delta\varphi_b < \pi$). Both conditions are automatically satisfied in the region of interest for noise suppression: $\phi_{prd} < \pi/2$, $N \approx N_D < 0.5$ (current shot-noise dominance condition).



Short wavelengths limits

For significant suppression

(and negligible Landau damping):

$$N = \frac{\lambda_D}{\lambda} = k\frac{\Delta\beta_z}{\theta_p} << 1$$

Ballistic condition

(same as Landau for $L_d=\pi/2\theta_p$):

$$\Delta\phi_p = kL_d\Delta\beta_z << 1$$

SPARC:
Current 50 A
Beam Energy 176 MeV
Beam Radius 150 um
Sliced Energy Spread 10^{-4}
Emittance 1 mm mrad

$L\pi/2 = 14m$

$$\begin{cases} \dfrac{k}{\theta_p}\dfrac{\Delta\gamma}{\gamma^3} << 1 \\ \dfrac{k}{\theta_p}\left(\dfrac{\varepsilon_n}{\gamma\sigma_x}\right)^2 << 1 \end{cases}$$

λ >> 46 nm

λ >> 21 nm

**TUPD17, Proceedings of FEL2012, Nara, Japan*

Granularity condition:

$$n_0 A_e \lambda = \frac{I_0}{ec}\lambda >> 1$$

10,000

(for λ = 10 nm)

Dispersive Transport Noise Suppression

$$gain = \frac{\overline{\left|\breve{i}(L,\omega)\right|^2}}{\overline{\left|\breve{i}(0,\omega)\right|^2}} = \left(\cos\phi_{pd} + \gamma_0^2\theta_{pd}R_{56}\sin\phi_{pd}\right)^2 + N^2\left(\sin\phi_{pd} - \gamma_0^2\theta_{pd}R_{56}\cos\phi_{pd}\right)^2$$

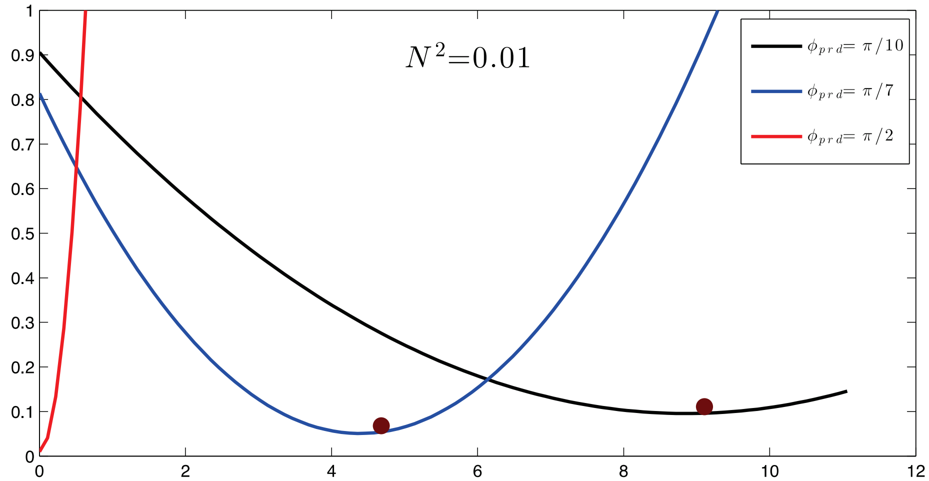
$$\boxed{K_d = \frac{\gamma_0^2\left|R_{56}\right|}{L_d}} \qquad N \ll 1 \qquad \phi_{pd} \ll 1$$

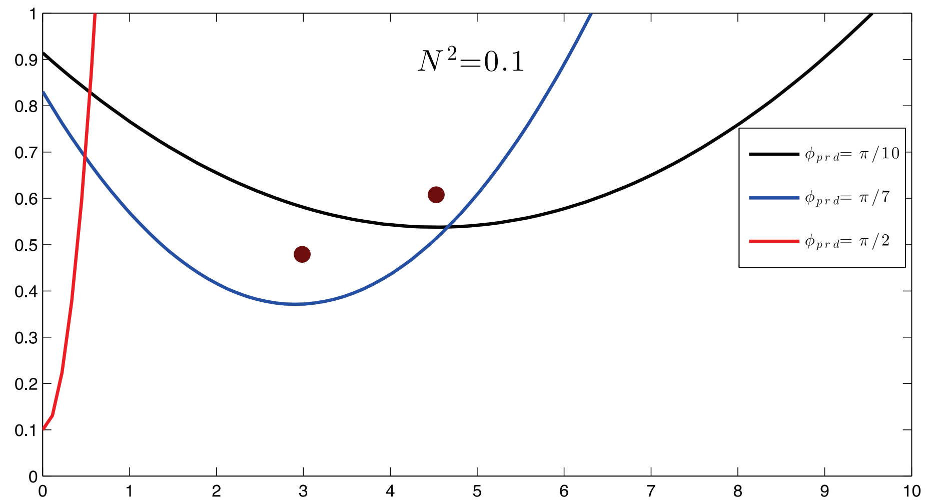
$$gain = \left(1 - K_d\phi_{pd}^2\right)^2 + N^2\phi_{pd}^2\left(1 + K_d\right)^2$$

<u>For maximal suppression:</u> $N^2 \ll \phi_{pd} \ll 1$

$$\begin{cases} \left(K_d\right)_{\min} = \dfrac{1}{\varphi_{pd}^2} \\ \boxed{\left(gain\right)_{\min} = \dfrac{N^2}{\varphi_{pd}^2}} \end{cases}$$

Dispersive Transport Gain





- Maximal suppression points according to approximation:

$$N^2 << \phi_{pd}^2 << 1$$

Phase-Spread Neglect Condition - Dispersive

$$\Delta\phi_b = k \int_0^{L_d+L_m} \Delta\left(\frac{1}{\beta_z}\right) dz << \pi$$

For $\beta \approx 1$ $\left(\Delta\beta = \frac{1}{\gamma^3}\Delta\gamma\right)$

$$\Delta\phi_b = k\left(L_d \Delta\beta_z + R_{56}\frac{\Delta\gamma}{\gamma}\right) << \pi \Rightarrow \Delta\varphi_b = \frac{kL_d}{\gamma^2}\frac{\Delta\gamma}{\gamma}\left(1+K_d\right) << \pi$$

Use $N = \frac{k}{\theta_p}\frac{\Delta\gamma}{\gamma^3} \Rightarrow$ $N\phi_{pd}\left(1+K_d\right) << \pi$

But $(K_d)_{\min} = \frac{1}{\varphi_{pd}^2} >> 1 \Rightarrow$ $\boxed{N << \phi_{pd} << 1}$

Conclusion

- It is possible to adjust the e-beam current shot- noise level by controlling the longitudinal plasma oscillation dynamics.
- Suppression was demonstrated experimentally at optical frequencies with a scheme of quarter plasma oscillation in drift and with a scheme of dispersive transport.
- Suppression at X-UV wavelengths seems feasible.
- Dispersive transport scheme can be realized with shorter length, but suppression is smaller and the short wavelength limit is tighter.
- E-beam noise control can be used to enhance FEL coherence and relax seeding power requirement (Gover, Dyunin, JQE-46, 1511, 2010) .