About Accelerators for X-ray FELs

challenges: energy, energy spread, gain length

nm … Å …

overlap electron – photon beam → emittance

$$\lambda_l = \frac{\lambda_u}{2(\gamma_0 + \delta\gamma)^2}\left(1 + \frac{K^2}{2}\right) + \frac{\lambda_u}{2}\left(x'^2 + y'^2\right)$$

resonance energy — energy spread — emittance & optics

$$L_g \propto \sqrt[3]{\frac{2mc}{\mu_0 e} \cdot \frac{\gamma^3 \lambda_u}{K^2} \cdot \frac{\sigma_r^2}{\hat{I}}}$$

current density

(1d theory)

typical numbers:	1 … 10 GeV …	energy
	0.0001 … 0.001	relative energy spread
	0.1 … 1 µm	normalized emittance
	1pC … 1nC	bunch charge
	1 … 10 kA …	bunch current

About Accelerators for X-ray FELs

1 Remarks

this is not about all possible guns, accelerators,
bunch compression systems …

some aspects are about accelerators in general, some
about acc. for FELs in particular (f.i. bunch compression)

most examples are close to European XFEL or FLASH
parameters are not necessarily design (they are sometimes chosen to provoke effects)

from general to particular: from a layout of a complete facility down to effects (as wakes) and tolerance estimations

about accelerators from the view point of simulations

this is not about particular simulation tools and programs but about approaches and effects

from particular to general: about the design of bunch
compression systems (tolerances)

references on last slide

2 Gun to Undulator Tracking

2.1 X-FELs Overview

2.2 Some Important Components - Gun

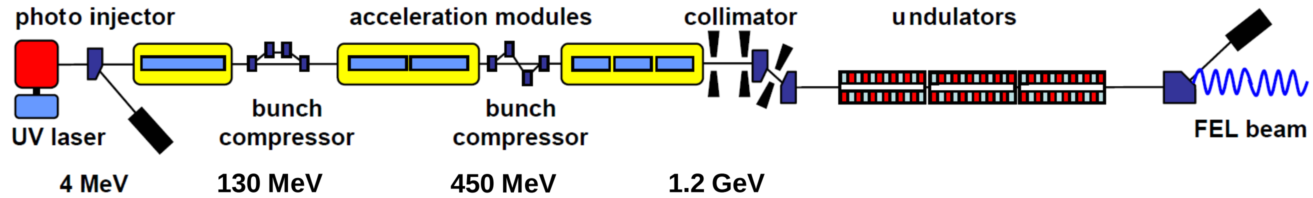
2.3 Some Important Components – Bunch Compressor

2.4 Some Important Components - Accelerator

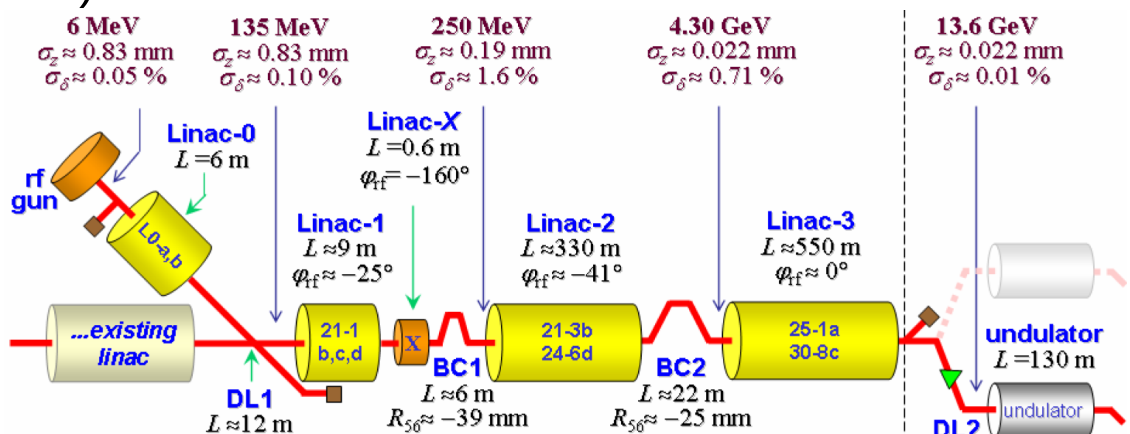
2.5 Simulation Procedure

FLASH (2010: 1.2 GeV, 4 nm)

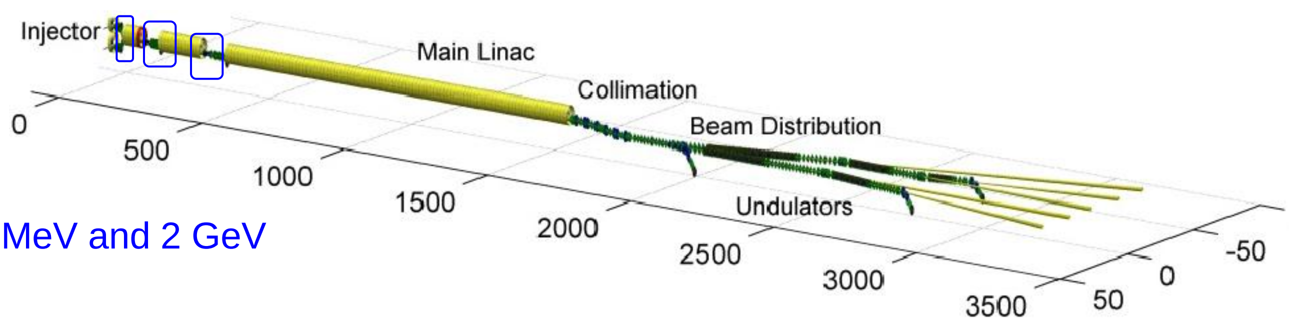
FLASH 2009:



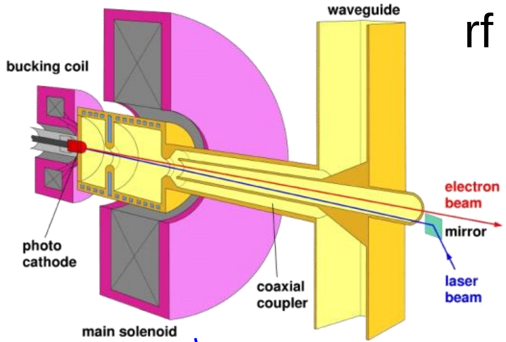
LCLS (14 GeV, 0.15 nm) [1]



European XFEL (0.1 nm)



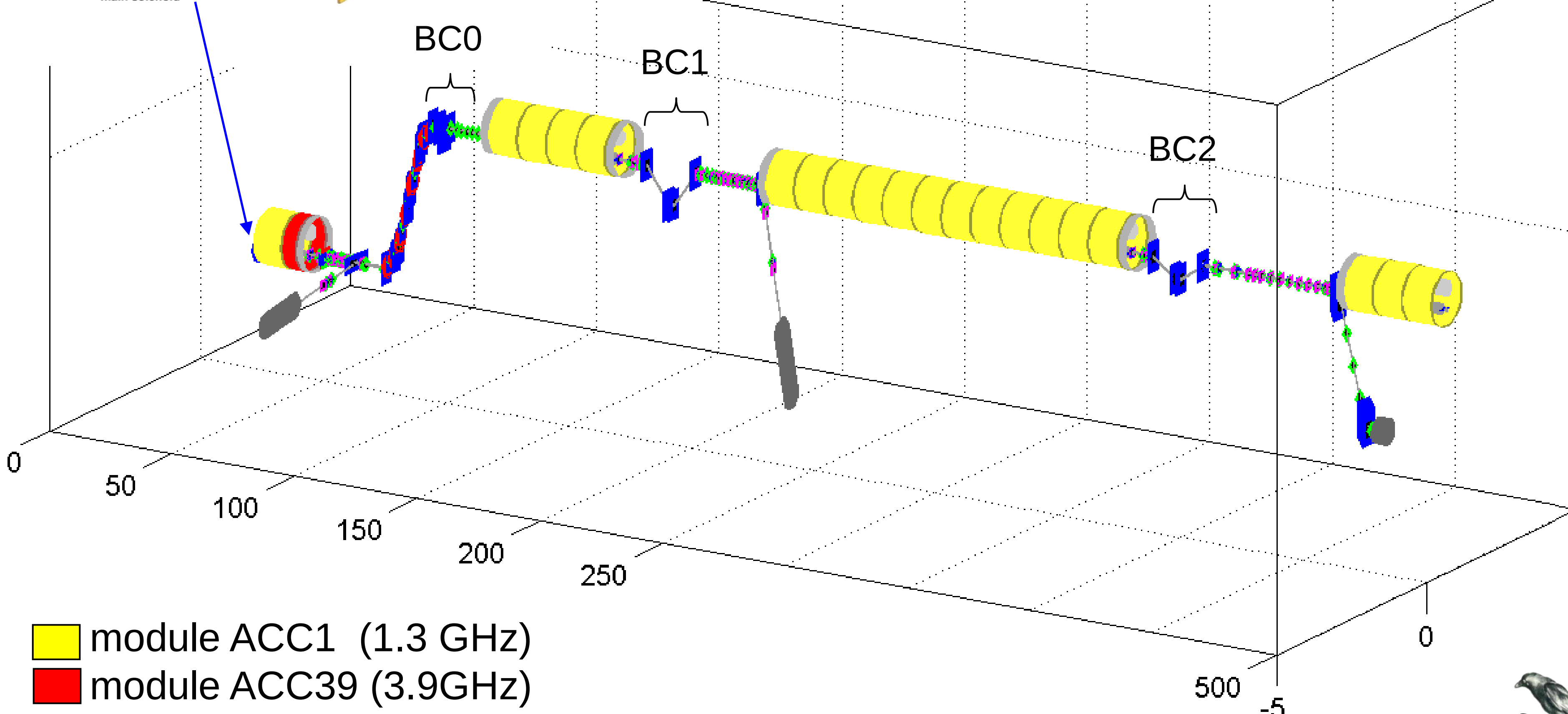
BCs at 130 MeV, 500 MeV and 2 GeV



rf gun

3 stage bunch compression system:

5 MeV	gun 1nC,	50 A
130 MeV	BC0	→ ~ 100 A
500 MeV	BC1	→ ~ 1 kA
2 GeV	BC2	→ ~ 5 kA

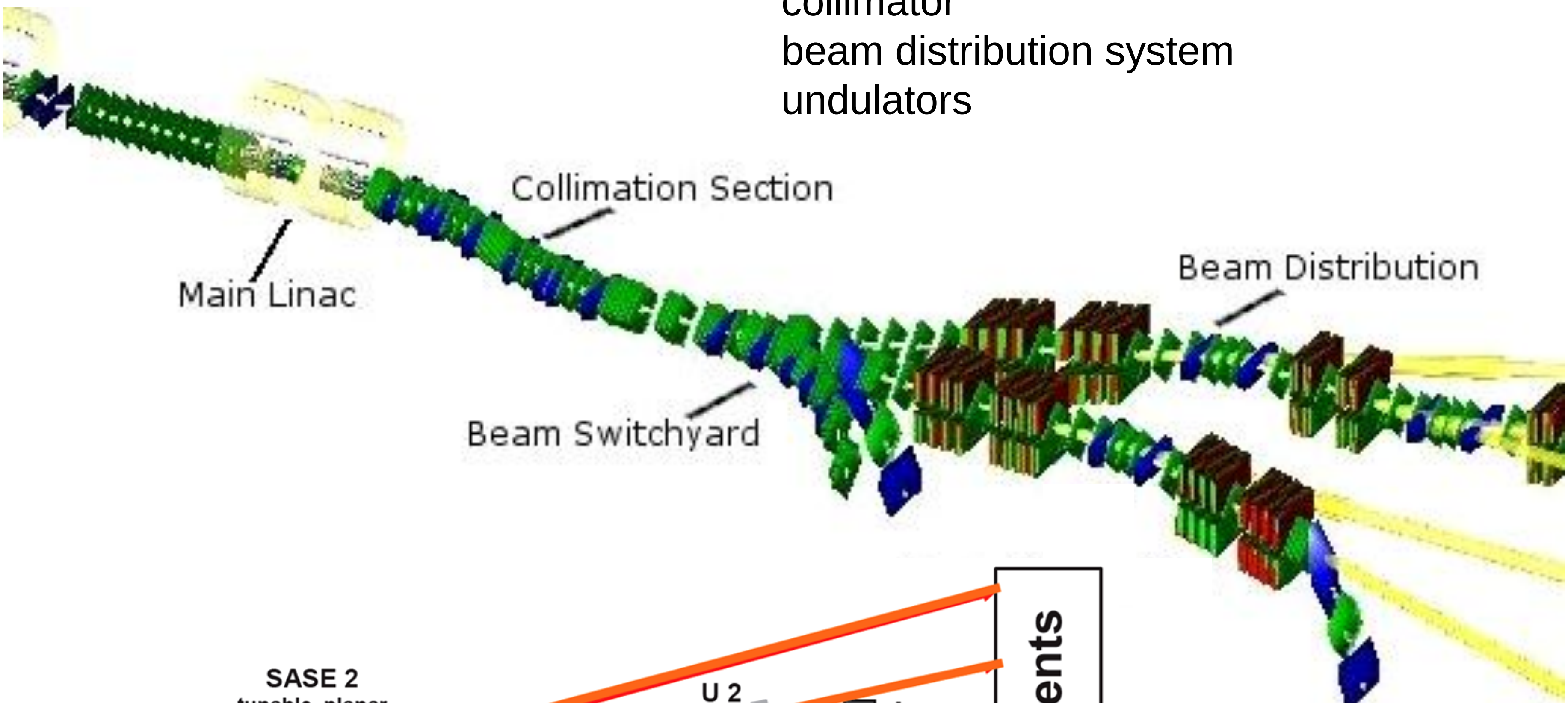


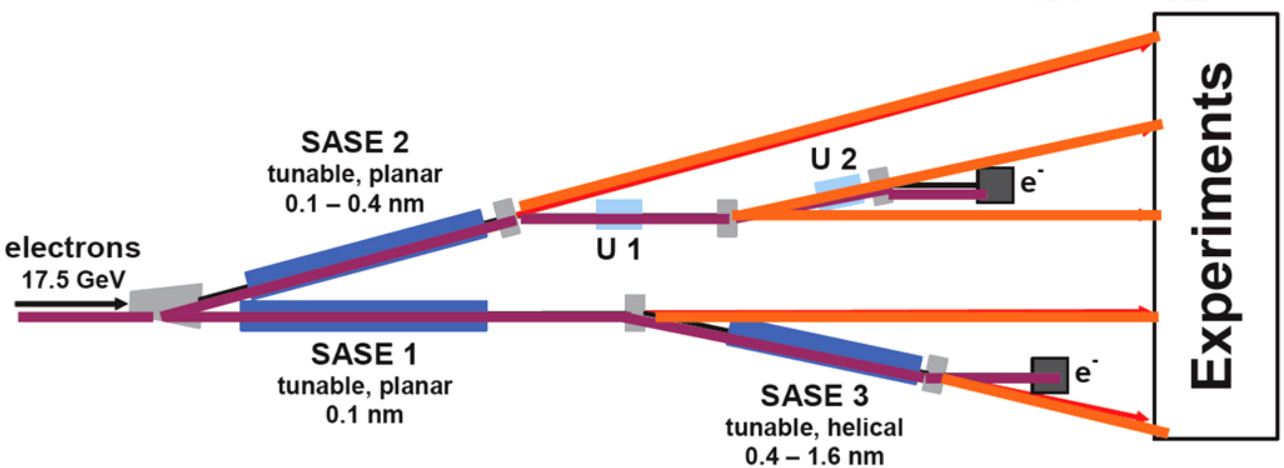
module ACC1 (1.3 GHz)
module ACC39 (3.9GHz)

2.1 X-FELs Overview

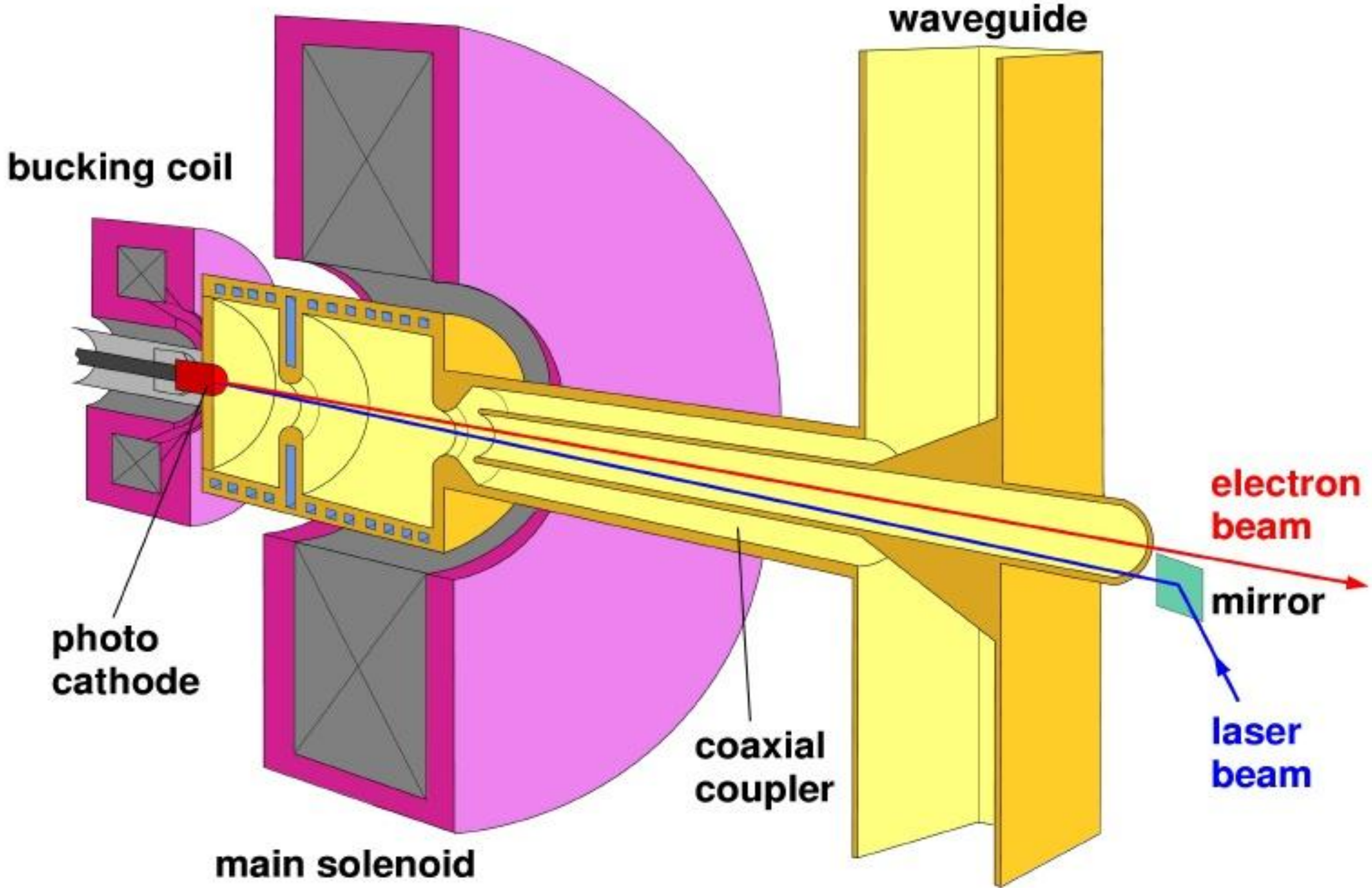
European XFEL – 2km more

main LINAC
collimator
beam distribution system
undulators



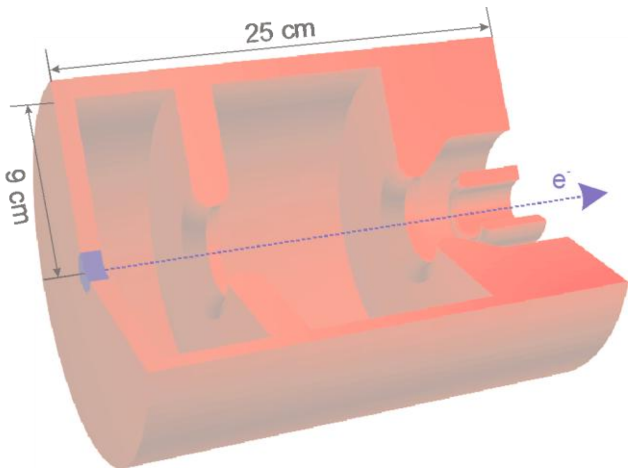


2.2 Some Important Components - Gun

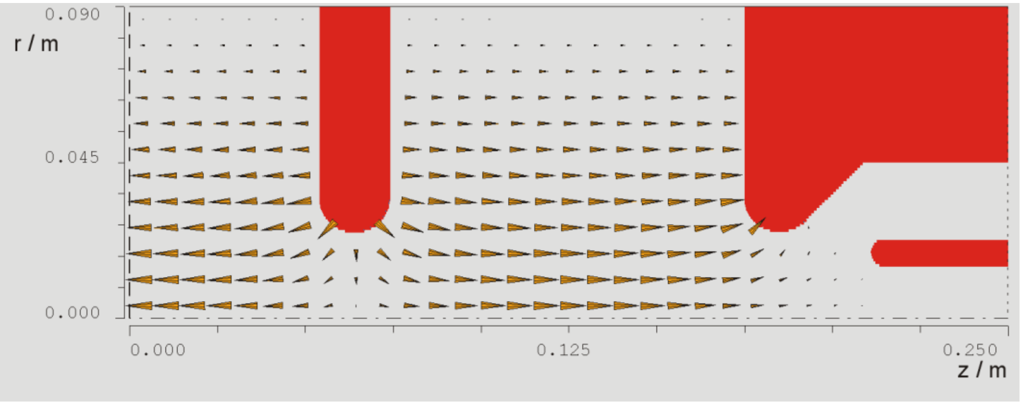


2.2 Some Important Components - Gun

external fields:

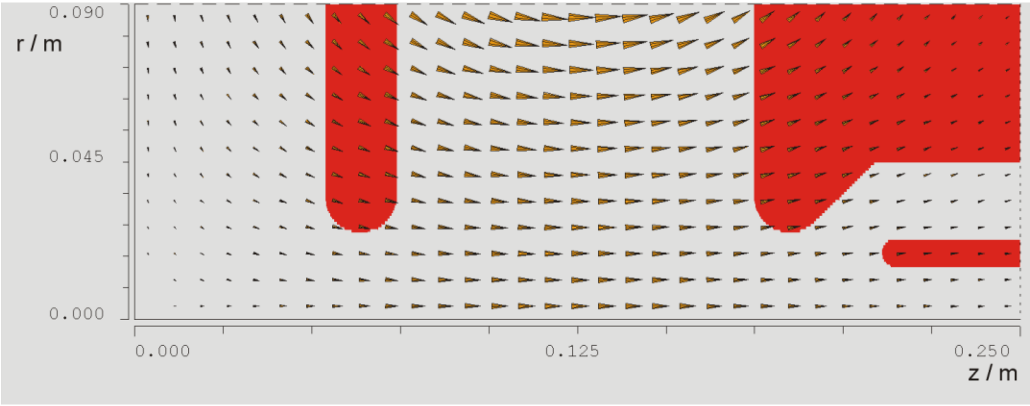


RF E-field (E_{01}-π mode)



(f=1.3GHz, E0~50MV/m, laser launch phase -40deg)

Static B-field



(B_{zmax}=0.2T)

[2]

2.2 Some Important Components - Gun

0th order description: particles are accelerated in **external** rf field

→ initial conditon (emission model) +
time dependent acceleration +
time dependent focussing

1st order description: with **self** fields (0th order)

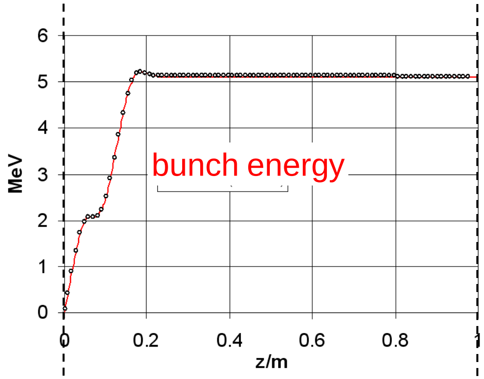
→ space charge effects
longitudinal self field depends on long. & transverse position in bunch; source of uncorrelated energy spread
focusing depends on longitudinal position: “different optics” along bunch; causes growth of **projected** emittance

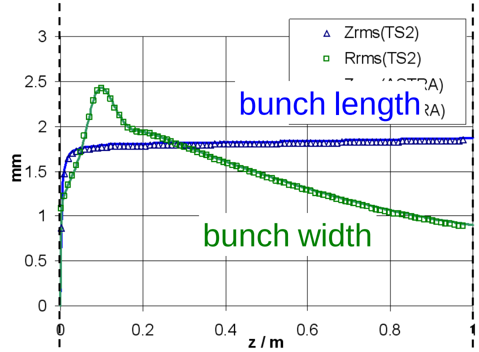
an other **external field: solenoid**
compensates (to some extend) different slice optics

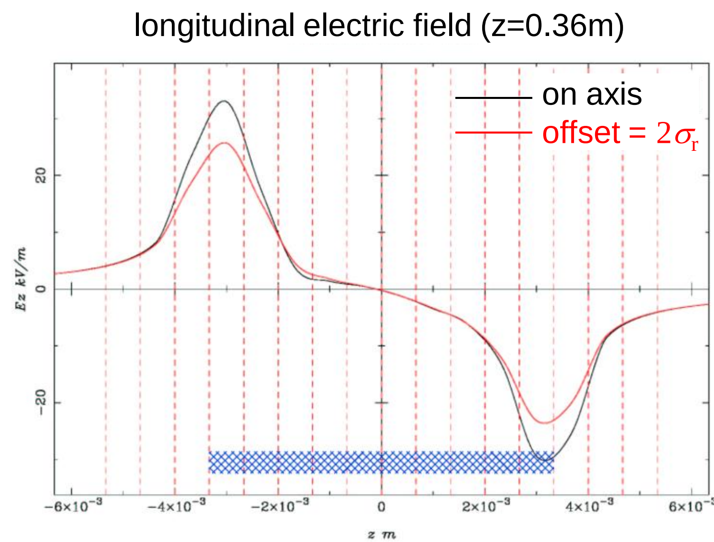
different slice optics (may) cause projected emittance growth
optimal setting for external fields depends on self effects (and Q!)

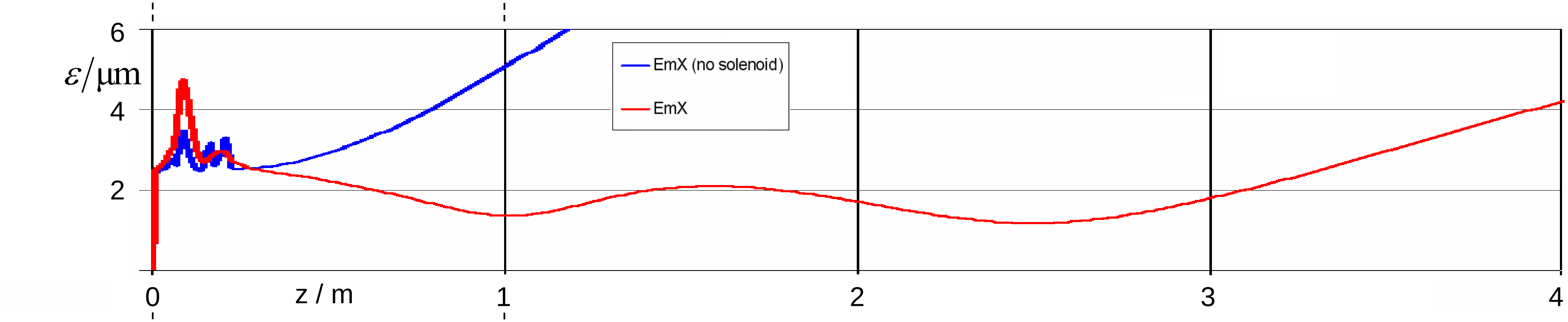
beam dynamics must include SC effects

2.2 Some Important Components - Gun







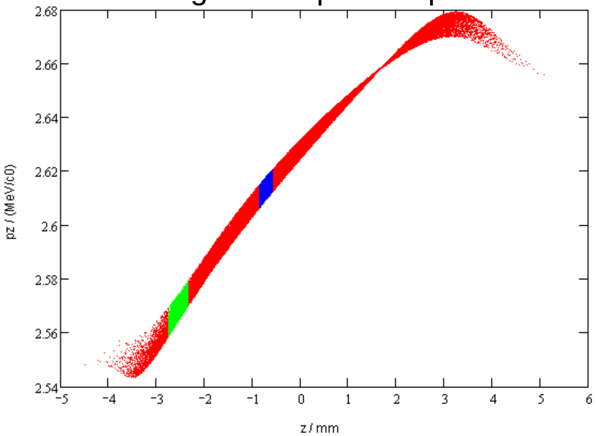


[2]

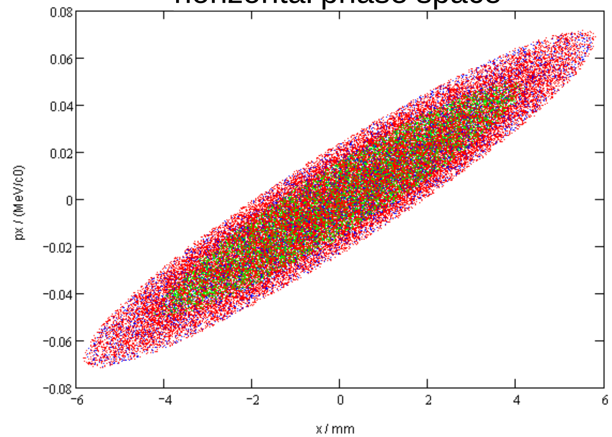
2.2 Some Important Components - Gun

phase space pictures at z=0.1m (in the gun)

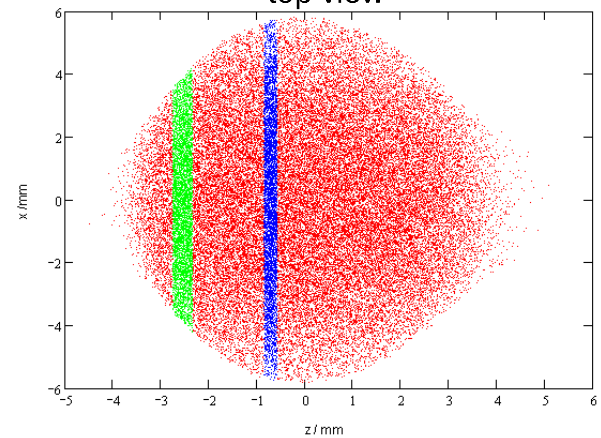
longitudinal phase space



horizontal phase space

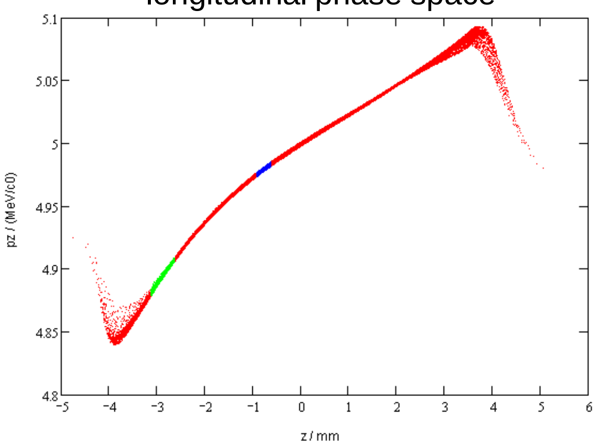


top view

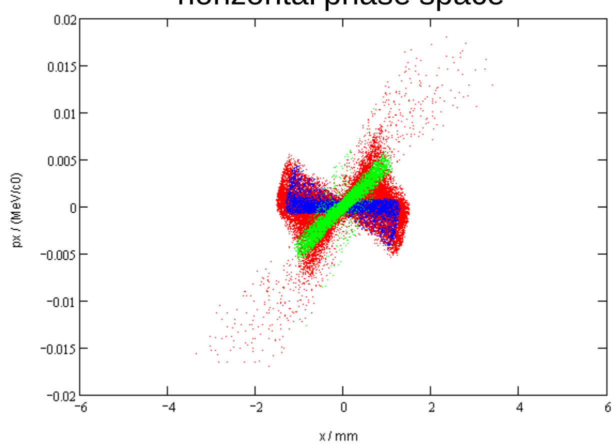


phase space pictures at z=2.6m (in the gun)

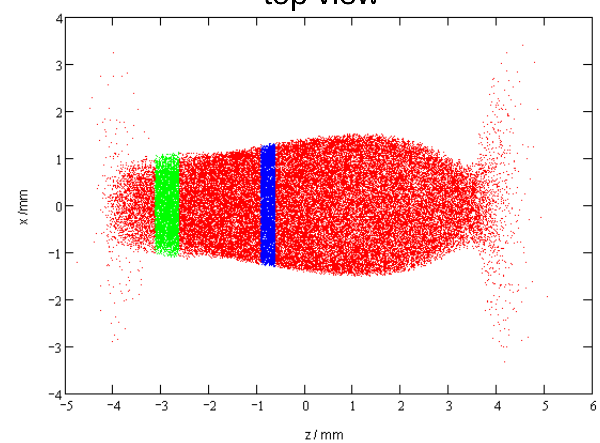
longitudinal phase space



horizontal phase space



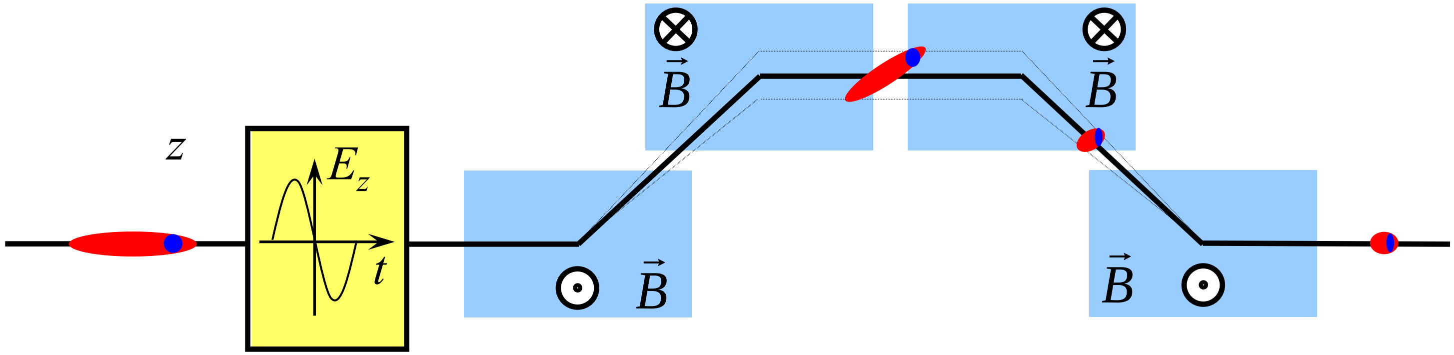
top view



2.3 Some Important Components – Bunch Compressor

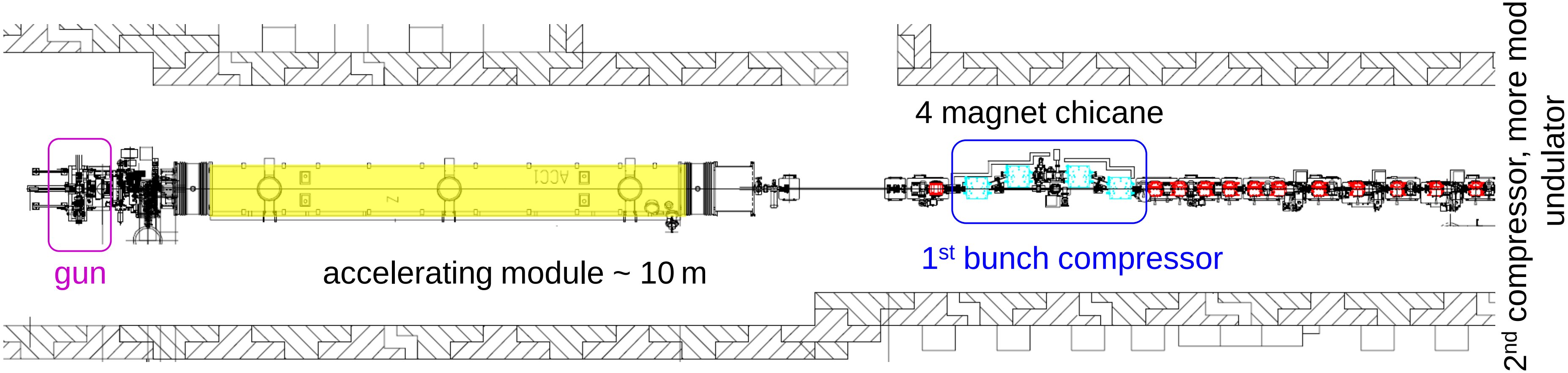
$\gamma >> 1 \rightarrow$ velocity differences are too small for effective compression

magnetic compression: path length depends on energy



acceleration "off crest" $\rightarrow$
head particle with less energy than tail

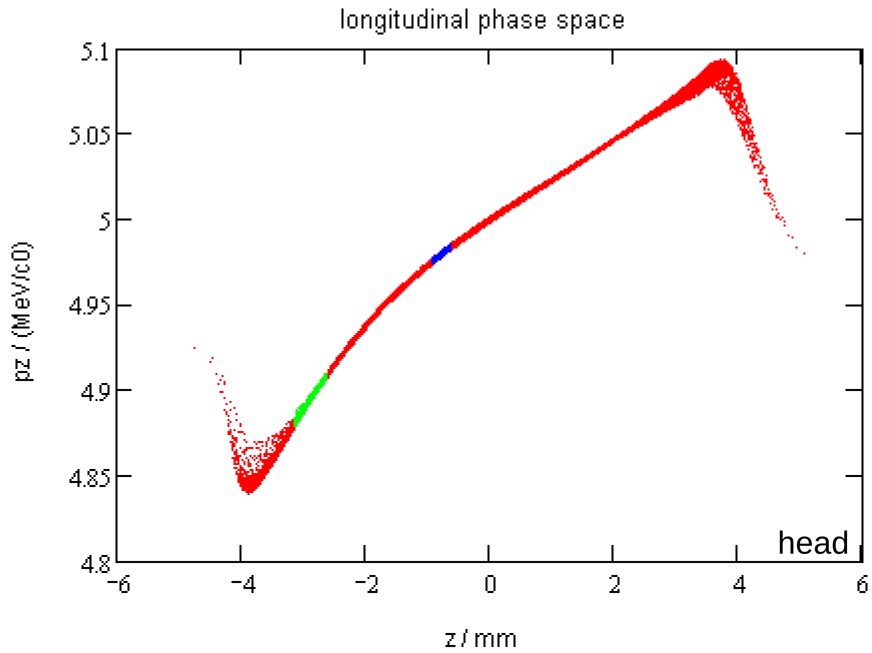
FLASH:



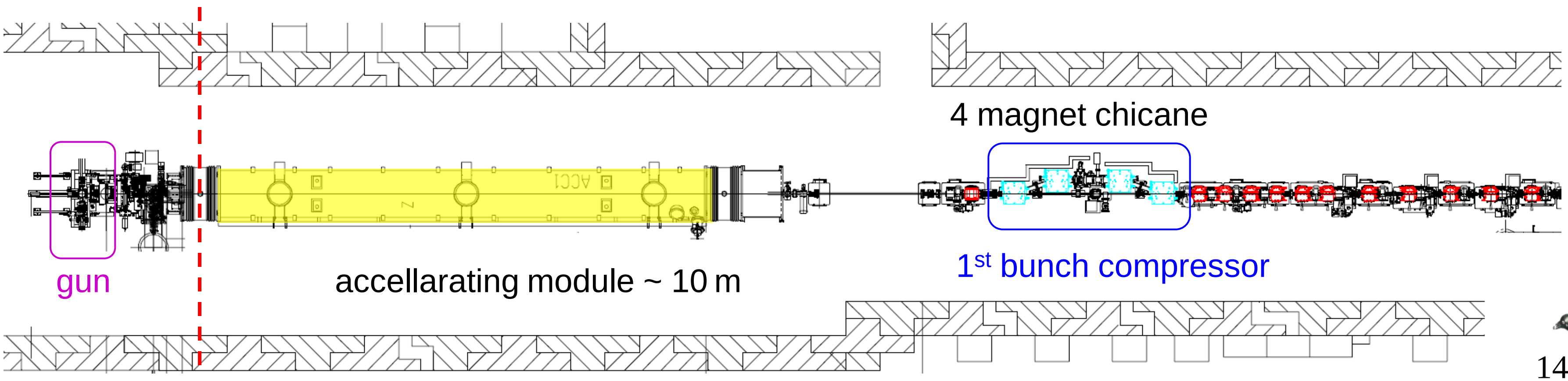
beam dynamics in BC must include CSR effects

2.3 Some Important Components – Bunch Compressor

longitudinal phase space after rf gun (before accelerating module):

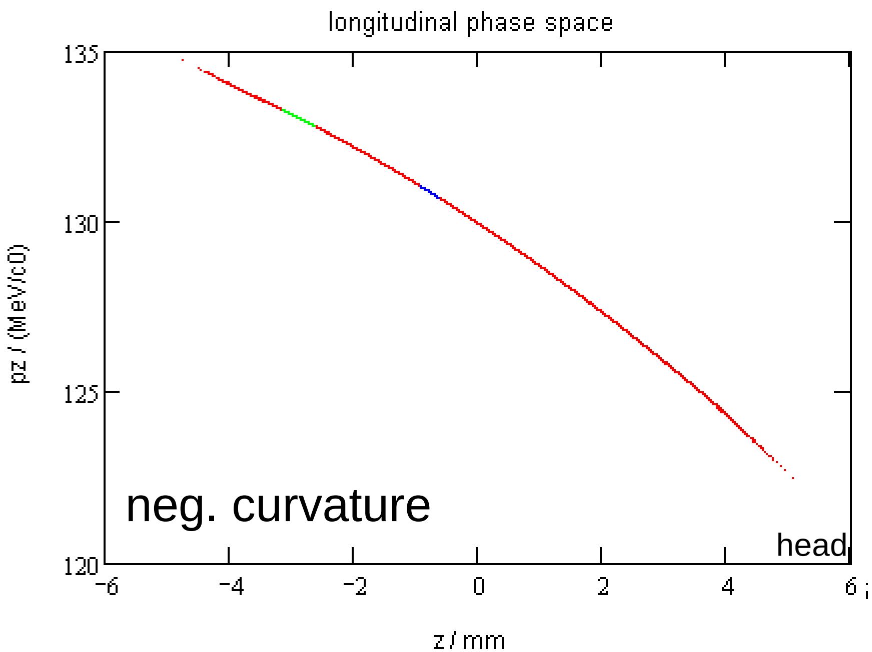


slice energy spread ~ 1 keV
total energy spread ~ 50 keV



2.3 Some Important Components – Bunch Compressor

longitudinal phase space after accelerating module (before compressor):



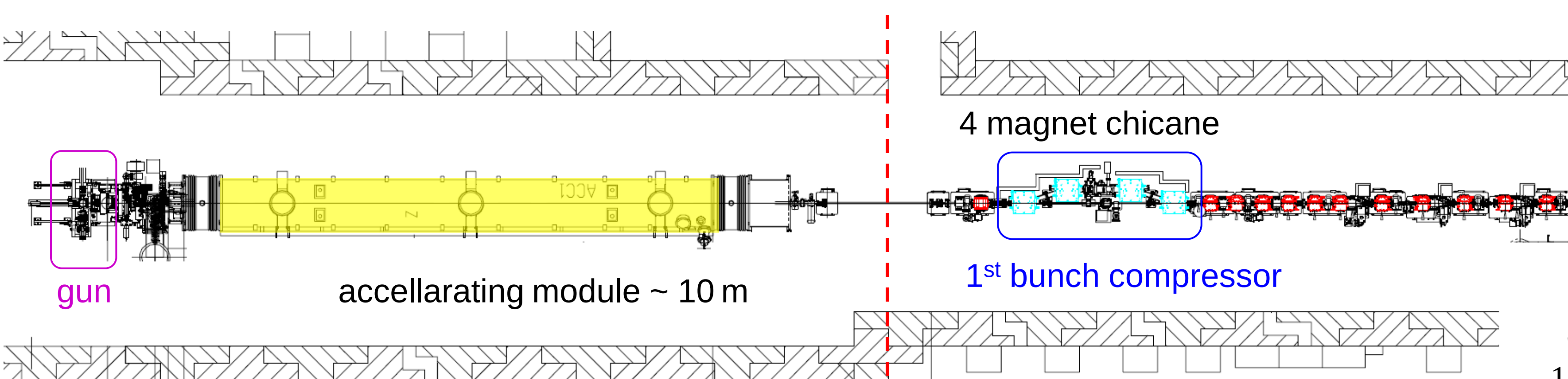
amplitude & phase of cavity

$E0 := 133 \cdot 10^{6}$ $\qquad \phi := 20 \cdot \frac{\pi}{180}$ $\qquad k\_rf := \frac{\omega}{c}$

$Ez(s) := E0 \cdot \cos(k\_rf \cdot s + \phi)$ rf shape

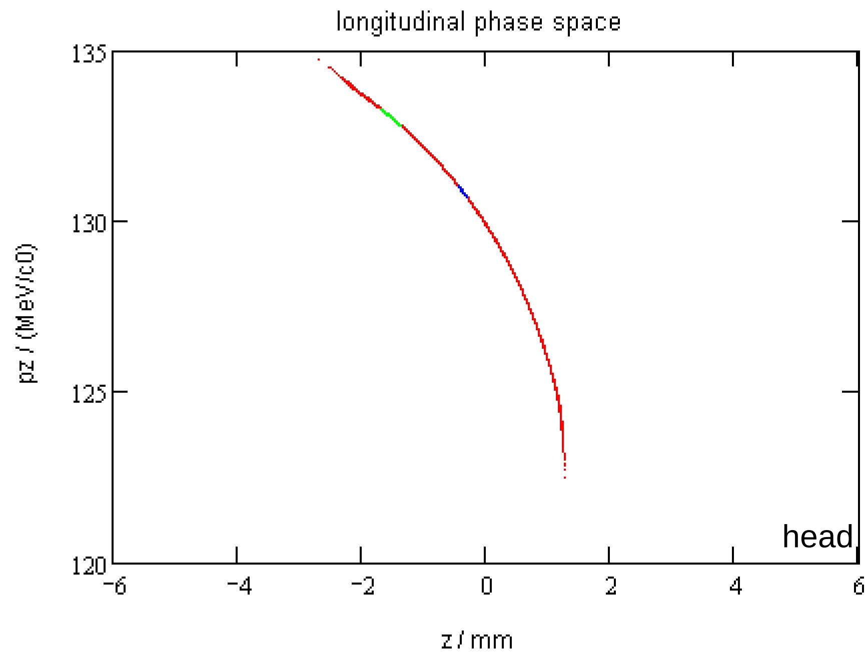
$pz_i := pz_i + Ez(z_i)$

new longitudinal momentum with chirp!



2.3 Some Important Components – Bunch Compressor

after compressor



relative momentum deviation

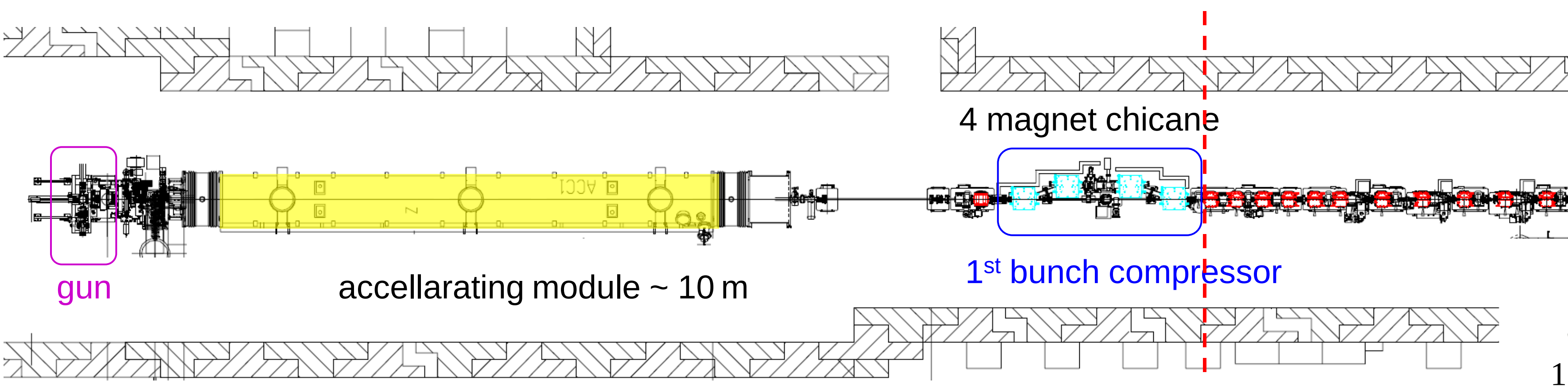
$$P\_ref := 130 \cdot 10^6 \qquad \delta p(p) := \frac{p - P\_ref}{P\_ref}$$

$$path\_length(\delta p) := -0.060 \cdot \delta p + 0.090 \cdot \delta p^2$$

path length depends on momentum!

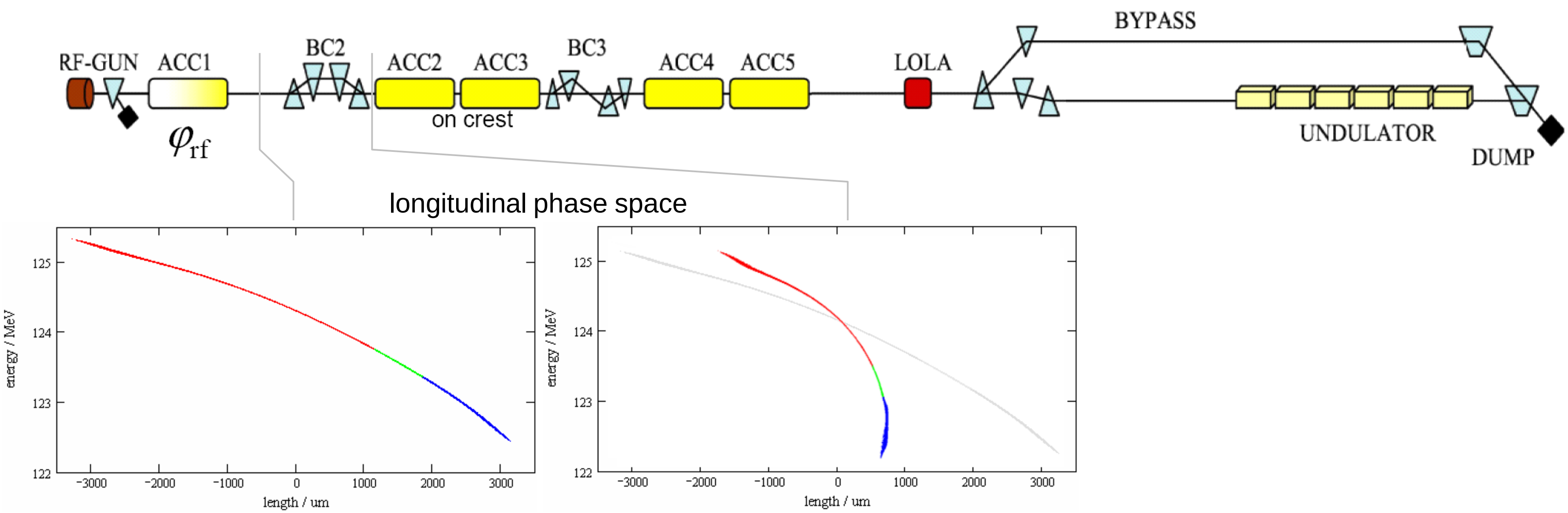
$$z_i := z_i - path\_length(\delta p(pz_i))$$

new longitudinal position



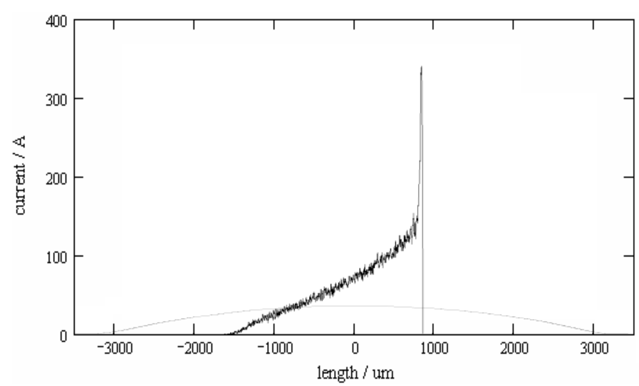
2.3 Some Important Components – Bunch Compressor

all is non linear: initial long. phase space, rf-shape, path length vs. pz
→ only weak compression or rollover



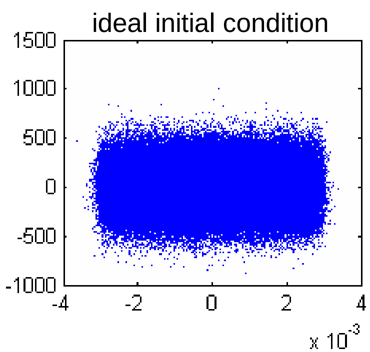
this “spike” mode was used in FLASH (≤2009)

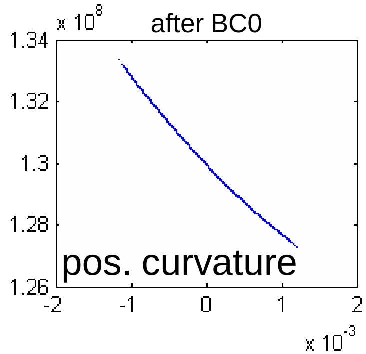
see example

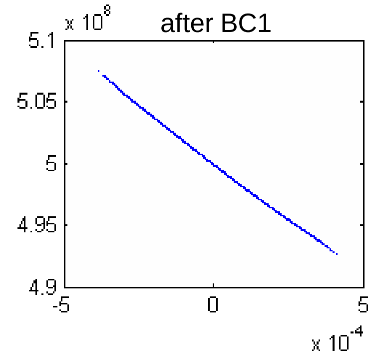


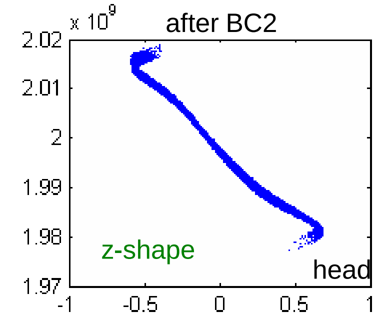
2.3 Some Important Components – Bunch Compressor

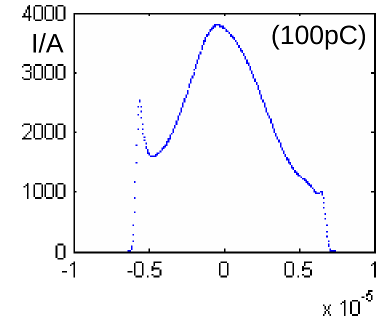
with higher harmonic rf systems it is possible to compensate non-linearities of compression and to avoid the “spike” mode





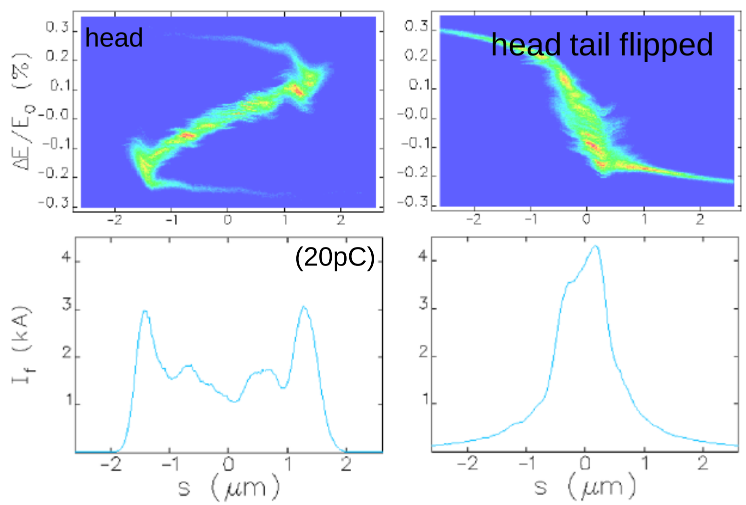






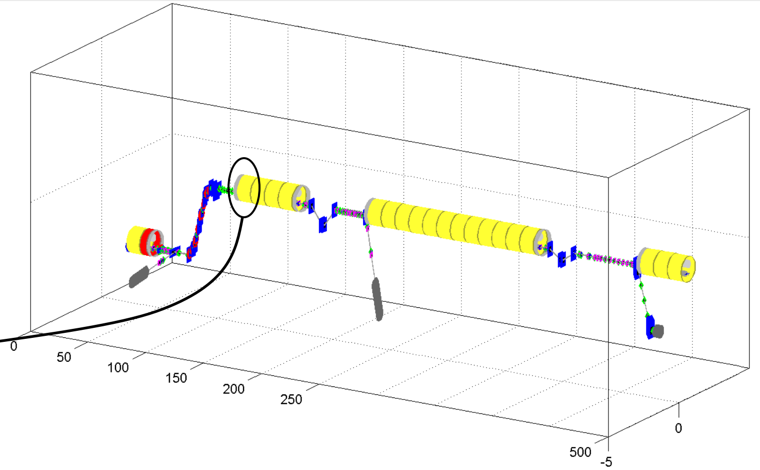
simulation
for **LCLS:** [3]

z–shape →

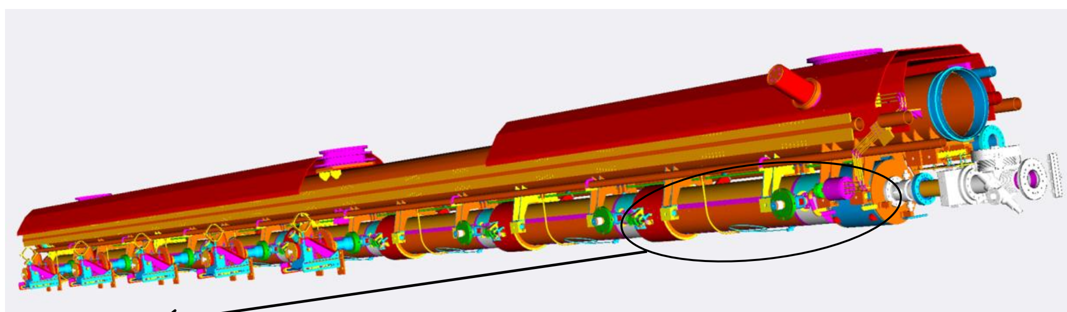


higher harmonic rf system are important components!

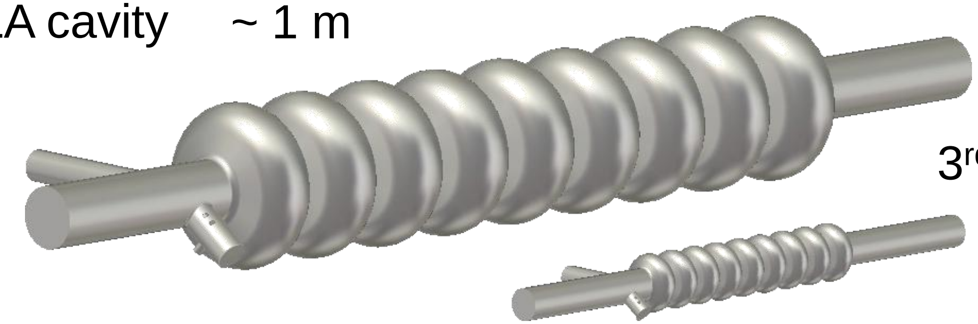
2.4 Some Important Components - Accelerator



cryo module with 8 cavities ~ 10 m



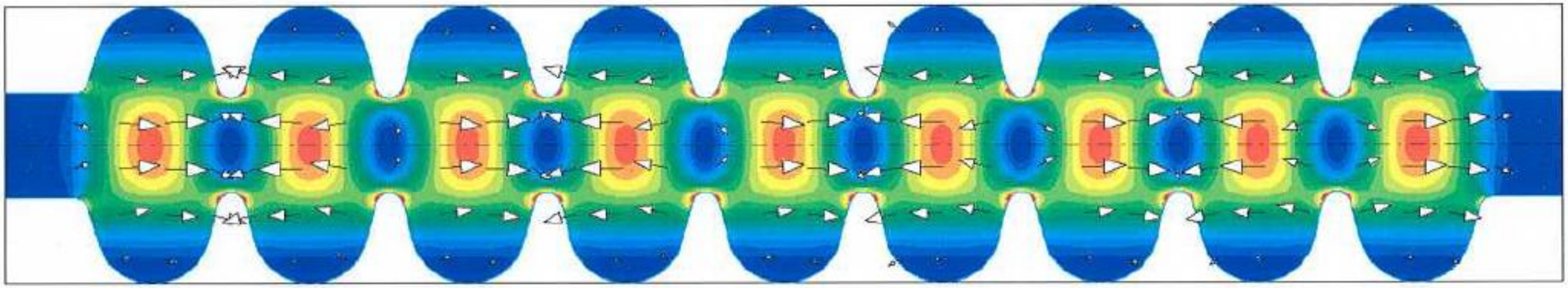
1.3GHz TESLA cavity ~ 1 m



3^{rd} harmonic cavity

2.4 Some Important Components - Accelerator

external field of cavity (driven by external source f.i. klystron)



usual cavities with symmetry of revolution, usual monopole mode

usualy the accelerating mode is a monopole mode

π mode in standing wave cavities (as shown) or

$2\pi/3$ mode in most travelling wave structures

transverse focusing (rf focusing) ~ $1/\gamma^2$ (negligible at high energy)

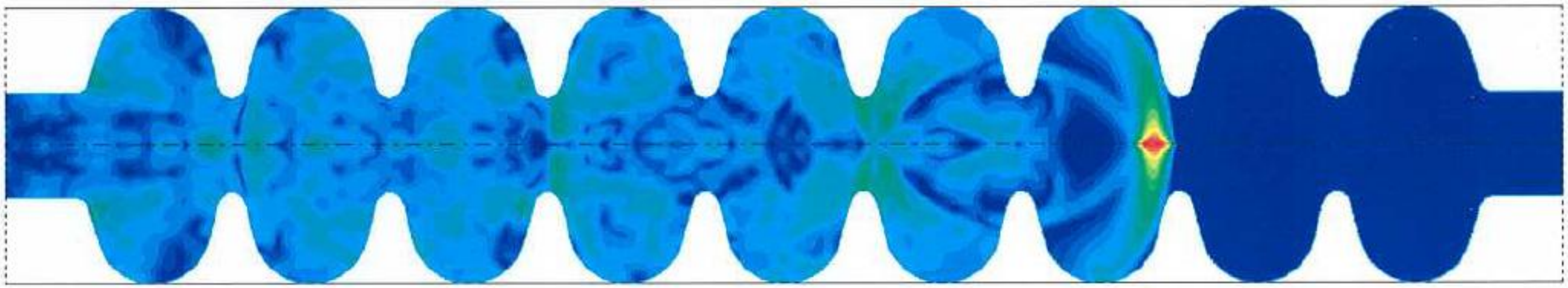
coupler (input and HOM)

causes field asymmetries and unwanted kicks

lowest order contribution is usually the time dependency of the transverse kick $\rightarrow$ different kick for head and tail, growth of projected emittance

2.4 Some Important Components - Accelerator

self field of cavity (driven driven by bunches)



the concept of wake fields is used to describe the integrated kick (caused by a source particle, seen by an observer particle)

short range wakes describe interaction of particles in same bunch
long range wakes describe multi bunch interactions

important for FELs: longitudinal single bunch wakes change the energy chirp and interfere with bunch compression; “chirp compensation”

coupler (input and HOM)

not the most important effect

projected emittance growth can be avoided by symmetric design or by alternating orientation of couplers on successive cavities

---

beam dynamics must include wakes & SC effects

2.5 Simulation Procedure

simulation with “macro“ particles

particle generator, number of macro particles

different tracking programs might be used (different physical approximation)

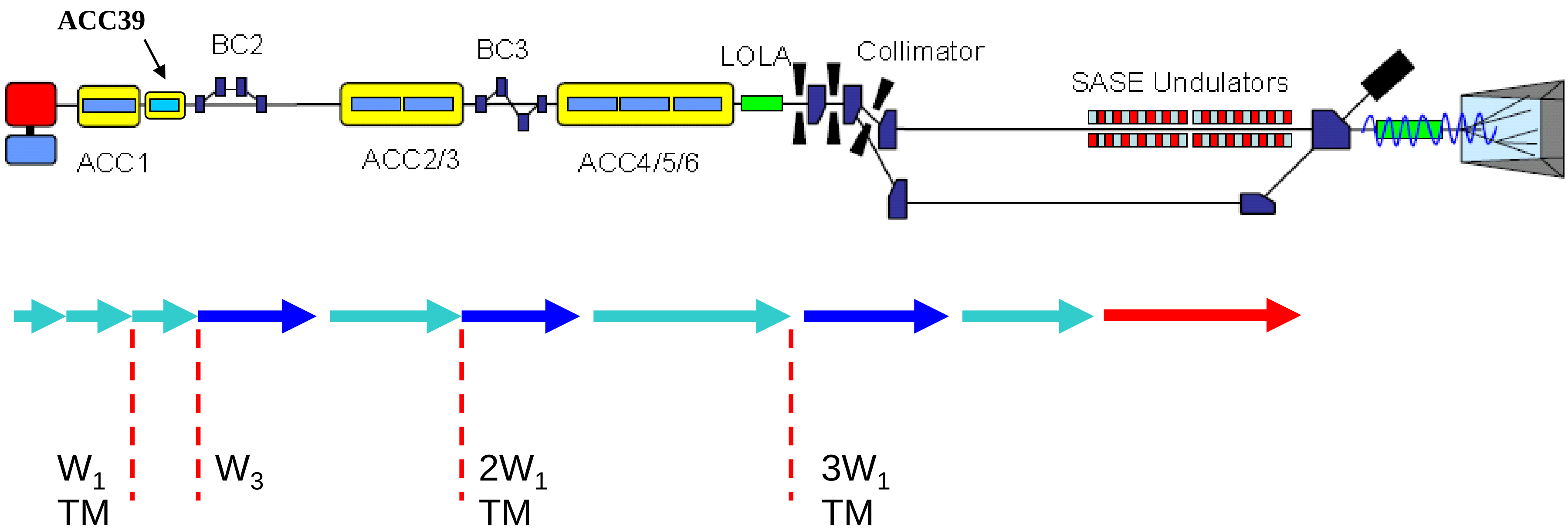
- f.i. tracker with space charge solver (Astra, Parmela, …)
  (for linear sections with cavities and optical elements)
- f.i. CSR solver (CSRtrack, Bmad, elegant, …)
  (effects from coherent synchrotron radiation)
- FEL (Alice, Genesis, …)
- interface routines, conversion tools

discrete elements (coupler kick & wake)

distributed elements $\rightarrow$ wake per length

matching and trajectory correction

2.5 Simulation Procedure



ASTRA

CSRtrack “projected” model
(sub-bunch approach)

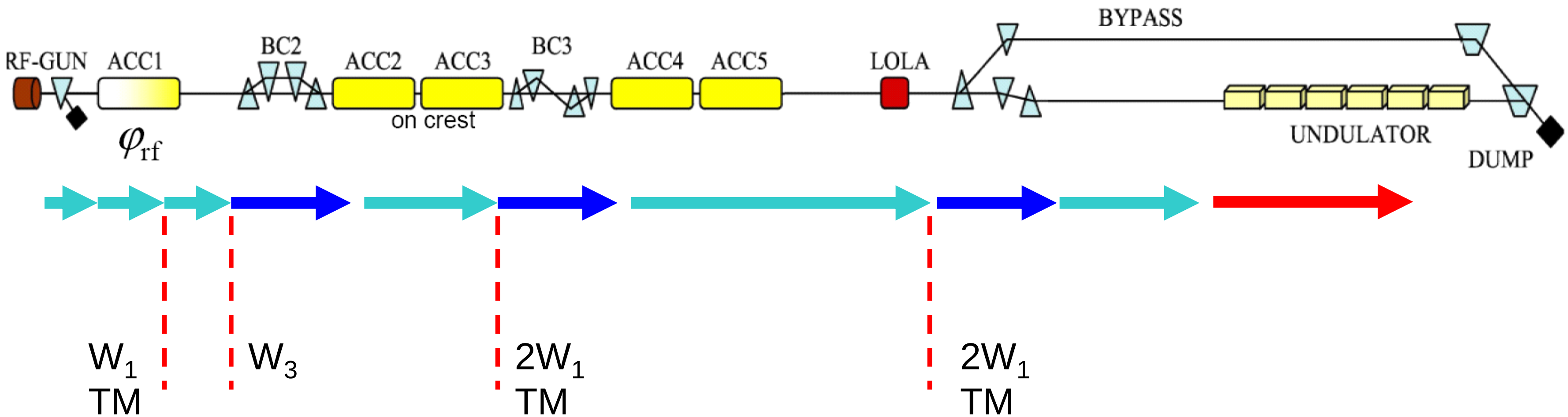
ALICE

W1 -TESLA cryomodule wake

W3 - ACC39 wake

TM- transverse matching to
design optics

2.5 Simulation Procedure – example



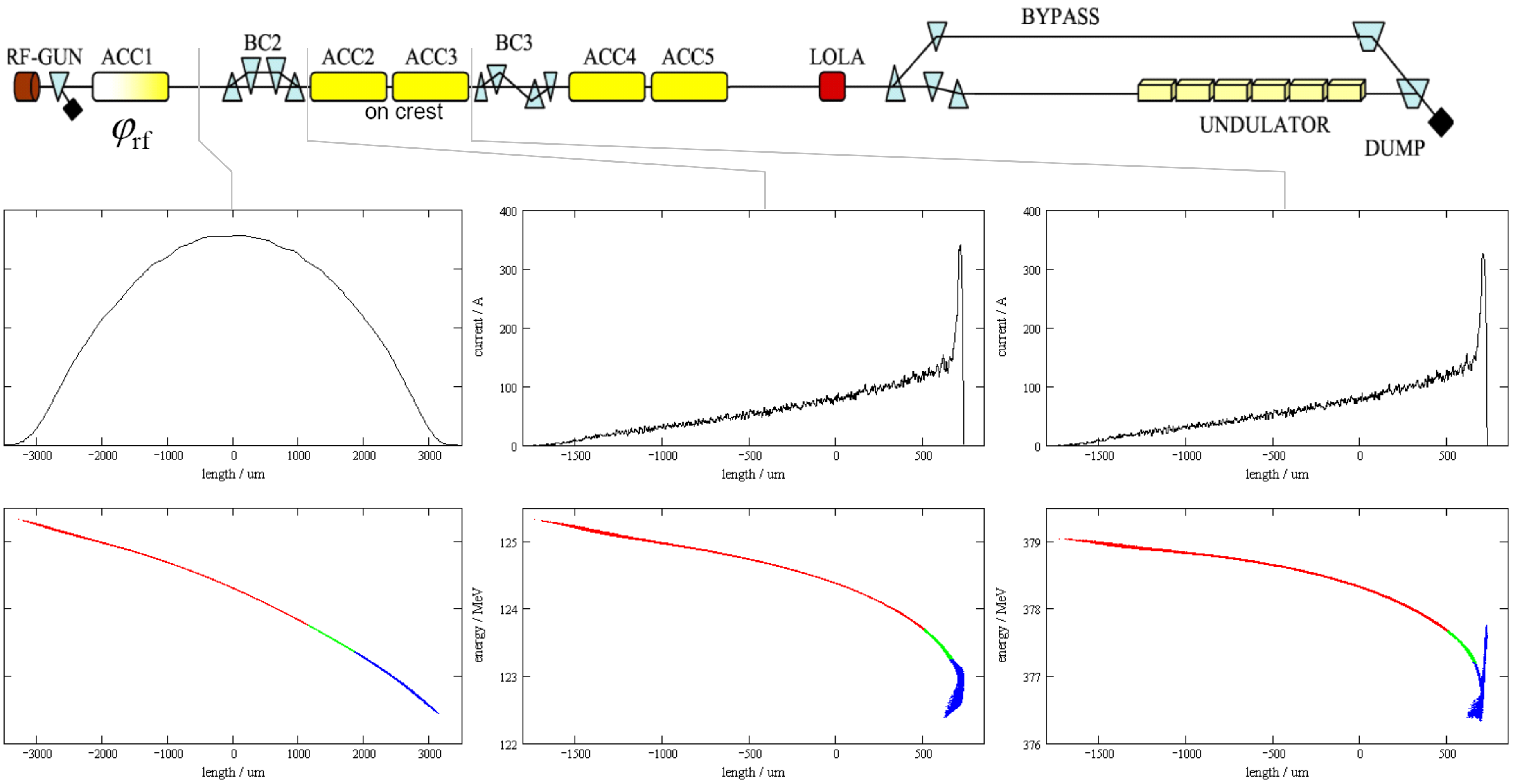
125 MeV

380 MeV

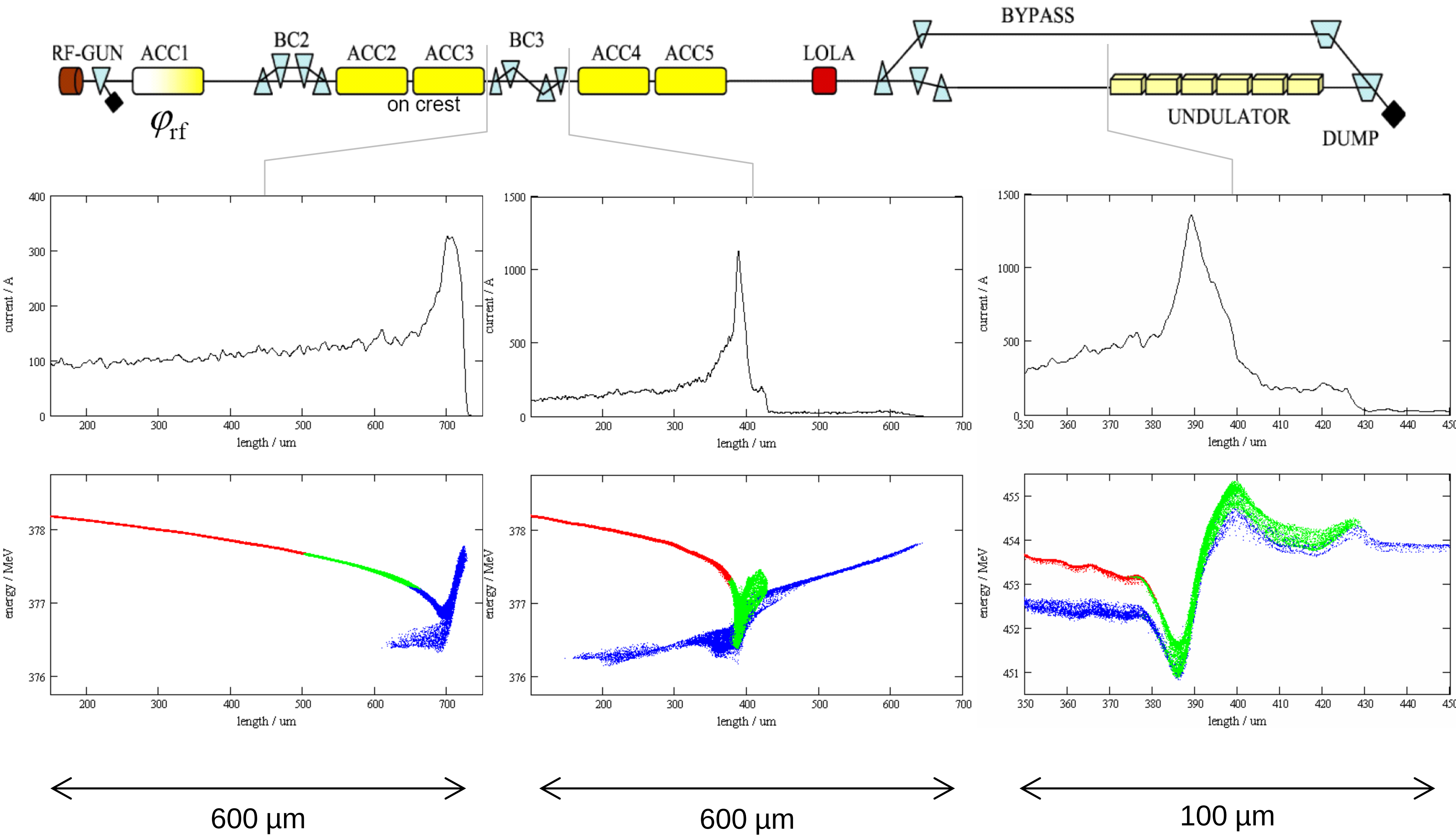
450 MeV

it is pre-FLASH called TTF (2005)

2.5 Simulation Procedure – example



2.5 Simulation Procedure – example



3 Simulation Tools

3.1 Tracking with Space Charge Effects

idea: local uniform motion

use uniform motion approach for field calculation

particles in individual uniform motion → point to point interaction

particles together in uniform motion → field calculation in rest frame with Poisson solver

3.2 Tracking with Radiative Interactions

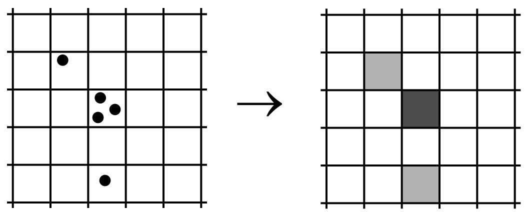
general approaches (how to solve Maxwell's equations)

the popular CSR approach: "the projected method"

3.1 Tracking with Space Charge Effects

tracking program with Poisson solver:

distribution of macro-particles $\mathbf{r}_\nu, \mathbf{p}_\nu, q_\nu \quad \rightarrow \quad \overline{\mathbf{v}}$ velocity of rest frame



$$\rho = \rho(\mathbf{r} - \overline{\mathbf{v}}t)$$

smooth charge density function
f.i. unform inside of grid cells

Lorentz transformation to rest frame
solve E static problem
transform back

$$\mathbf{E}^{(s)} = \mathbf{E}^{(s)}(\mathbf{r} - \overline{\mathbf{v}}t) \qquad \leftarrow$$

$$\mathbf{B}^{(s)} = \overline{\mathbf{v}} \times \mathbf{E}^{(s)} / c^2$$

$$\mathbf{E}^{(e)}, \quad \mathbf{B}^{(e)}$$

calculate external field f.i. analytic description in quadrupoles or field maps in cavities

integrate equation of motion f.i. with Runge Kutha method:

$$\dot{\mathbf{r}}_\nu = \mathbf{v}_\nu$$

$$\dot{\mathbf{p}}_\nu = q(\mathbf{E}_\nu + \mathbf{v}_\nu \times \mathbf{B}_\nu)$$

3.1 Tracking with Space Charge Effects

input for Poisson solver (f.i. Astra):

initial particle distribution (from particle generator or previous tracking)

external field

fieldmaps (f.i. solenoid or Ez component of cavities)

lattice (position, size and strength of quadrupoles, solenoid etc.)

rf settings (position of cavities, amplitudes & phases)

(Linac parameters)

self field

settings of Poisson solver

f.i. spatial resolution (mesh) and accuracy parameters

integrate e.o.m (tracker)

accuracy parameters as step width

end point

3.2 Tracking with Radiative Interactions

the model 'local uniform motion' is not applicable in BCs, doglegs, ...

… or one has to verify it; f.i. compare SC field with CSR field

$$E_z^{SC} \sim \frac{Z_0 \hat{I}}{\pi \sigma_s \gamma^2} \ln \frac{\gamma \sigma_s}{\sigma_r} \qquad E_z^{CSR} \sim \frac{1}{2\pi \sqrt[3]{3}} \frac{Z_0 \hat{I}}{\sigma_s^{1/3} R_c^{2/3}}$$

peak current I, long. and transverse bunch dimension σ_s, ρ_r
relativistic factor γ, rucature radius R_c

EM field calculation for general sources

solve **partial differential equation** on a mesh

severe problems with conventional field solvers (as FDTD)
bunch- and wavelengths on µm scale but interaction length in the range of meters
$\rightarrow$ special approach with paraxial approximation

integrate retarded sources

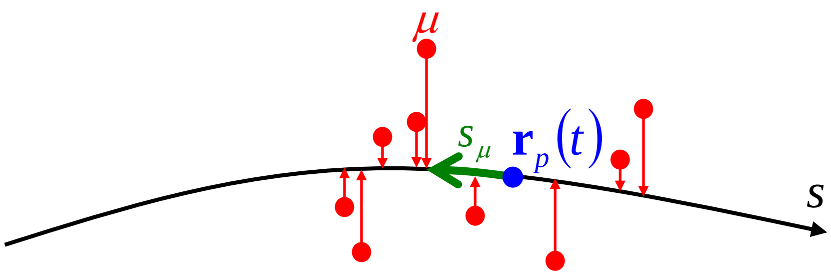
keep source distributions in time and space $\rho(\mathbf{r},t)$ $\mathbf{J}(\mathbf{r},t)$

integration of retarded sources f.i. $\mathbf{B} \sim \nabla \times \int \frac{\mathbf{J}(\mathbf{r}',t')}{\|\mathbf{r}-\mathbf{r}'\|} dV'$

large numerical effort; not practical in standard simulations

popular CSR approach: “the projected method”

neglects offset from ideal trajectory



$$\mathbf{r}_\mu(t) \to \mathbf{r}_p(t + s_\mu / v)$$

longitudinal field is calculated by Lienard-Wiechert formula for source particle on ideal trajectory and test particle at $\mathbf{r}_p(\tau = t + s/v)$

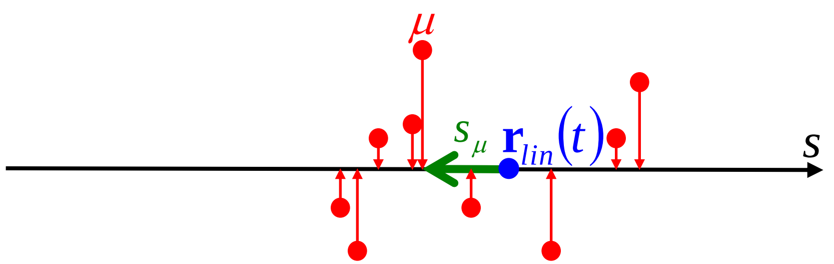
$$E(\tau,t) = \mathbf{e}_\parallel(\tau) \cdot \mathbf{E}(\mathbf{r}_p(\tau), t)$$

but the field is singular for $t \to \tau$

3.2 Tracking with Radiative Interactions

the field of particles on a linear trajectory is used to extract the singularity:

$$E_{lin}(\tau,t)=\mathbf{e}_{\parallel}\cdot\mathbf{E}_{lin}(\mathbf{r}_{lin}(\tau),t)$$



CSR kernel: (it is a definition)

$$K_{CSR}(s,t)=E(t+s/v,t)-E_{lin}(t+s/v,t)$$

the projected method:

calculate longitudinal charge density on trajectory $\lambda(s,t)\leftarrow\rho(\mathbf{r},t)$
(this involves smoothing)

calculate CSR field (of all particles): $E_{CSR}^{\lambda}(s,t)=\int\lambda(s-u,t)K(u,t)du$

solve equation of motion with $\mathbf{E}=\mathbf{E}^{(e)}+\mathbf{e}_{\parallel}E_{CSR}^{\lambda}$ and $\mathbf{B}=\mathbf{B}^{(e)}$

3.2 Tracking with Radiative Interactions

some remarks to the projected method:

- space charge part neglected
- no dependency on transverse beam size nor on transverse particle offset
- no transverse fields
- “retarded” 1D charge distribution with fixed shape
- the method is fast and efficient
- smoothing parameter is critical (μ-bunching instability)
- it is an empirical method

4.1 Some Effects and Models

4.1 Wakes and Impedances

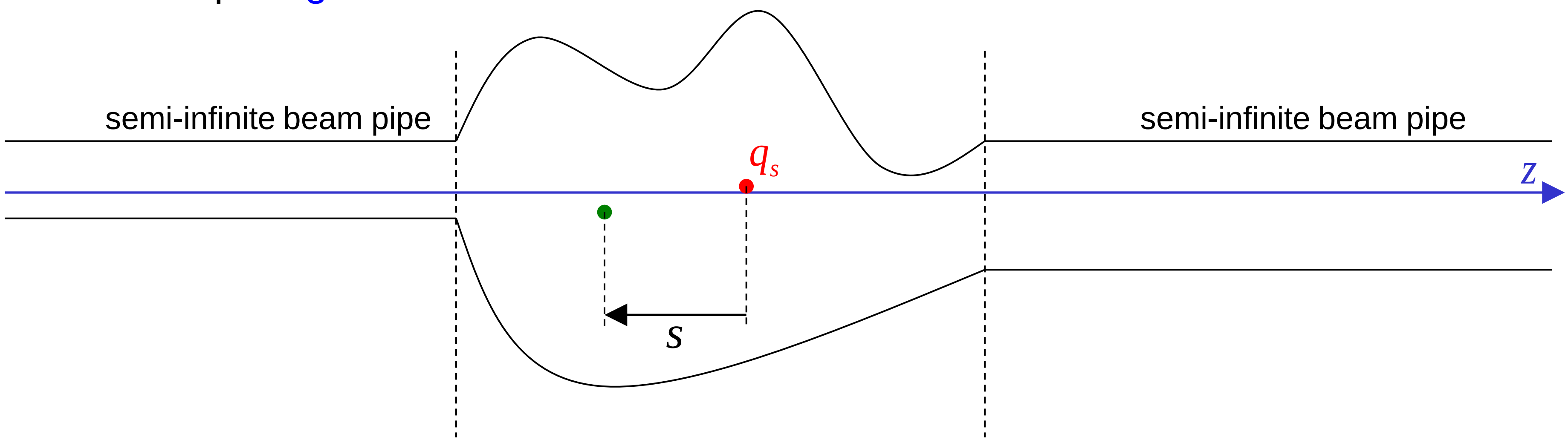
4.2 Some Longitudinal Impedances

4.3 Some CSR Effects

4.4 Transverse Space Charge Model

4.1 Wakes and Impedances

concept of geometric wake field:



source particle q_s $\quad \mathbf{r}_s(t) = x_s \mathbf{e}_x + y_s \mathbf{e}_y + c(t - t_0)\mathbf{e}_z$ (point particle, v=c)

test particle $\quad \mathbf{r}_t(t) = x_t \mathbf{e}_x + y_t \mathbf{e}_y + (c(t - t_0) - s)\mathbf{e}_z$

wake function: $\quad \mathbf{W}(x_s, y_s, x_t, y_t, s) = \frac{c}{q_s q_t} \Delta \mathbf{p} \quad \leftarrow \int_{-\infty}^{\infty} dz (\mathbf{E} + \mathbf{v} \times \mathbf{B})$

it is a discrete approach: assumption “kick can be localized”

periodic structures, resistive beam pipes, space charge effects

$\rightarrow \mathbf{W}'(x_s, y_s, x_t, y_t, s)$ “wake per length”

4.1 Wakes and Impedances

distributed 3D source distribution $\rho(x,y,s)$:

$$\Delta\mathbf{P}(x_t,y_t,s_t)=\frac{q_t}{c}\int dV_s\,\rho(x_s,y_s,s_s)\,\mathbf{W}(x_s,y_s,x_t,y_t,s_t-s_z)$$

special case 1D: symmetry of revolution, source particle on axis

$$\mathbf{W}(0,0,x_t,y_t,s)=\mathbf{e}_z W_{\|}(s) \qquad !$$

longitudinal field of 1D source distribution $\lambda(s)$:

$$E_{\|}(s)=\int\lambda(s-u)W_{\|}'(u)\,du$$

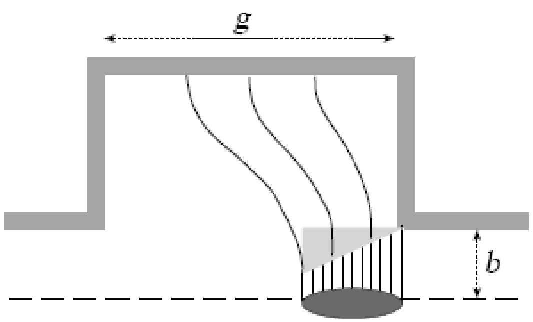
$$E_{CSR}^{\lambda}(s,t)=\int\lambda(s-u,t)K(u,t)\,du$$

the same concept is used for the CSR approach “projected”

longitudinal impedance

$$Z_{\|}(\omega)=\text{FourierTransformed}\left\{W_{\|}(ct)\right\}$$

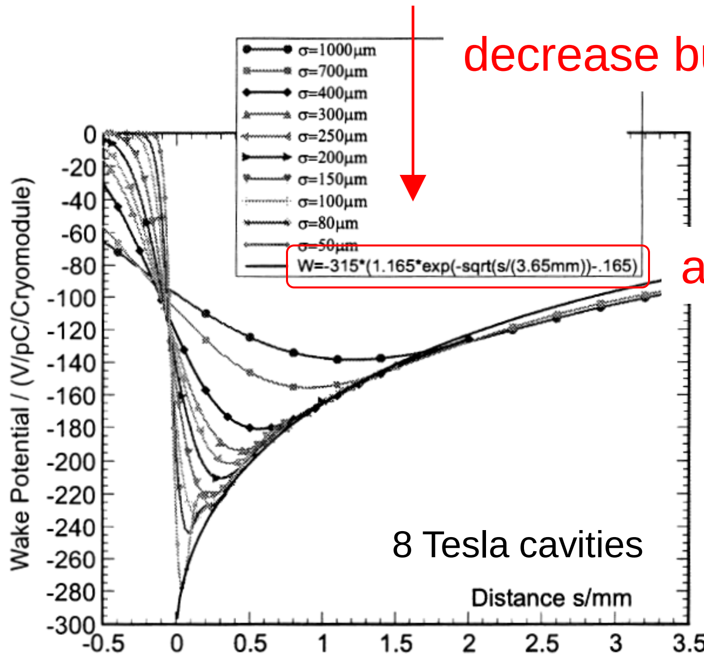
4.2 Some Longitudinal Impedances

uniform motion		
response of geometry or matter	space charge	radiation
single cavity "gap"	free space	constant curvature
$Z(\omega)=(1+i)\frac{Z_0}{2\pi b}\sqrt{\frac{gc}{\omega\pi}}$	$Z'_{sc}(\omega)=-\frac{iZ_0}{4\pi}\frac{\omega}{c\gamma^2}F_\eta\left(\frac{\omega\, r_\eta}{c\,\gamma}\right)$	$Z'_{CSR}(\omega)\approx 0.15\, Z_0\sqrt[3]{\frac{\omega}{icR_{curv}^2}}$
	$x<<1$: $F_\eta(x)\approx -2\ln(d_\eta x)$ η = transverse shape, $d_\eta \propto 1$	for $\omega << \frac{c\gamma^3}{R_{\mathrm{curv}}}$
resistive …	up to very high frequencies inductive !	

calculations of (longitudinal) wake for string of cavities:

numerical calculation for finite bunch length, extrapolation for $\sigma \to 0$

decrease bunch length σ



asymptotic expression

$$W_{\parallel}(s<0) = 0\,\frac{\mathrm{V}}{\mathrm{C}}$$

$$W_{\parallel}(s>0) = -315\cdot 10^{12}\,\frac{\mathrm{V}}{\mathrm{C}}\cdot\left(1.165\exp\left(-\sqrt{\frac{s}{3.65\,\mathrm{mm}}}\right)-0.165\right)$$

(for very short bunches ~ step function)

[5]

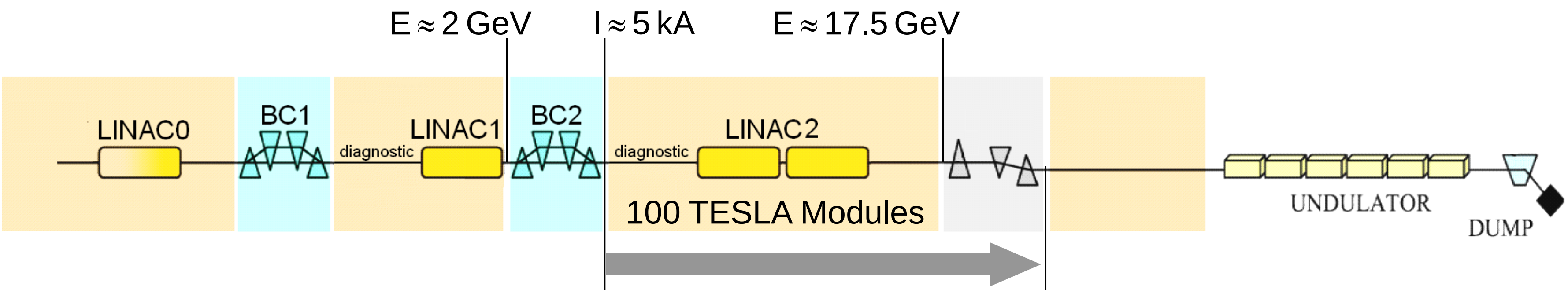
scaling of wake per length with fundamental mode frequency
(or inverse cavity size):

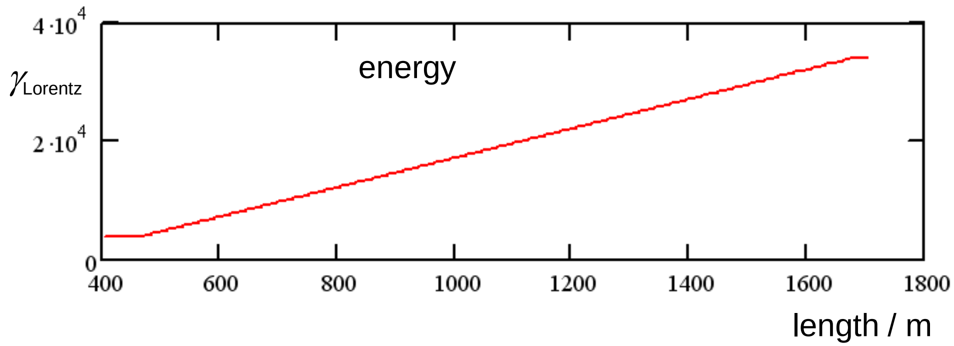
$$W_{\parallel}' \propto \omega^{-2}$$

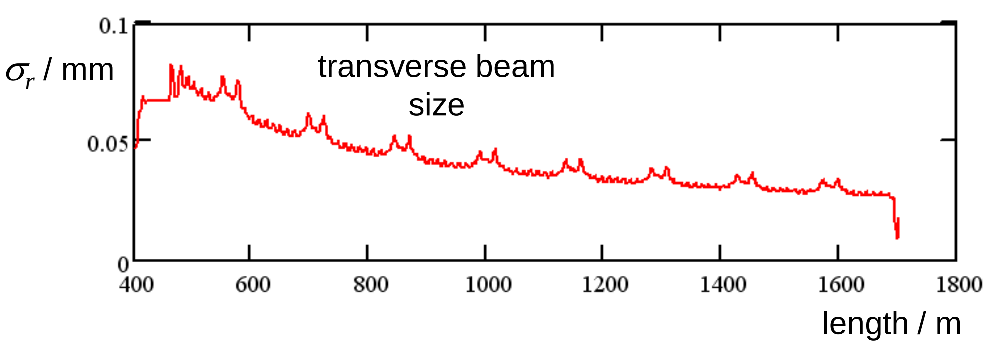
$$W_{\perp}' \propto \omega^{-3}$$

4.2 Some Longitudinal Impedances

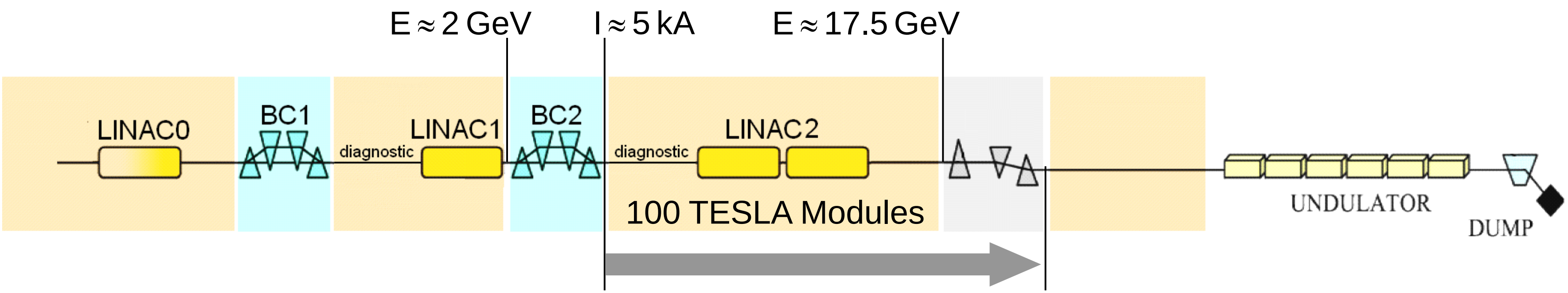
example:

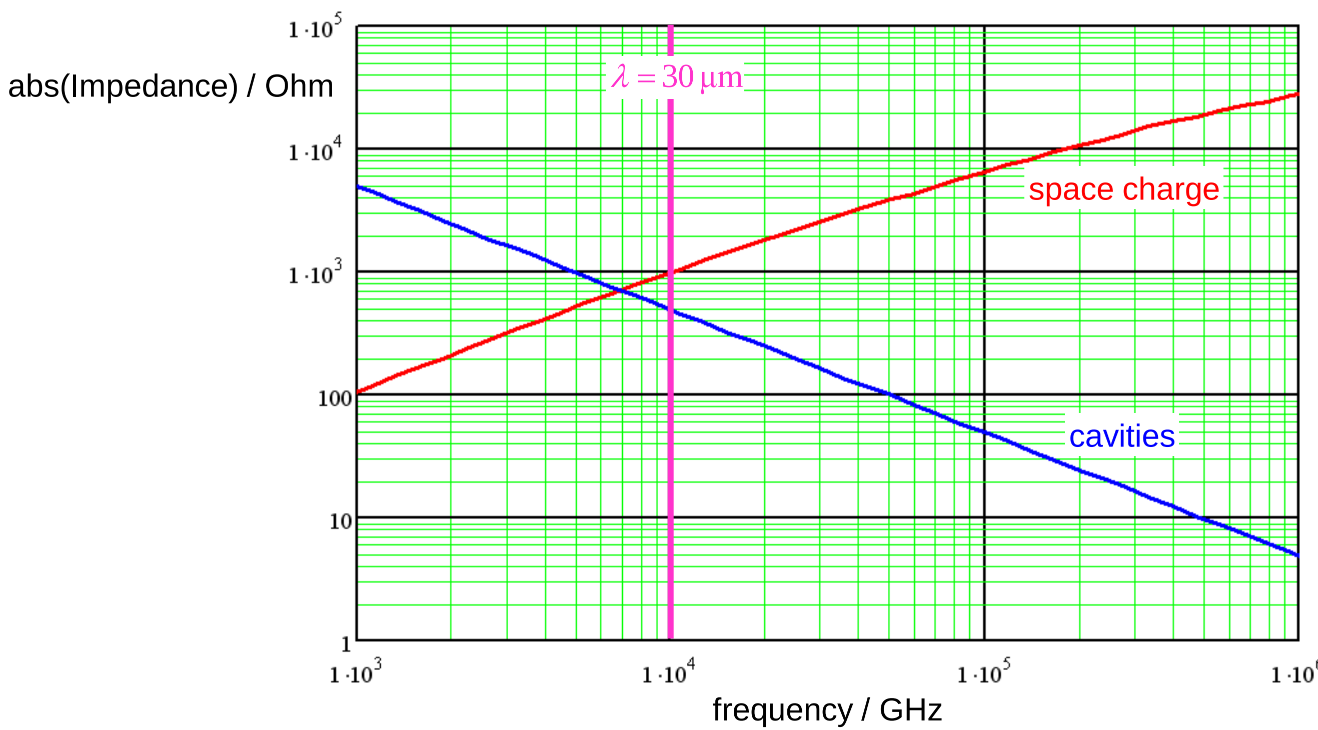






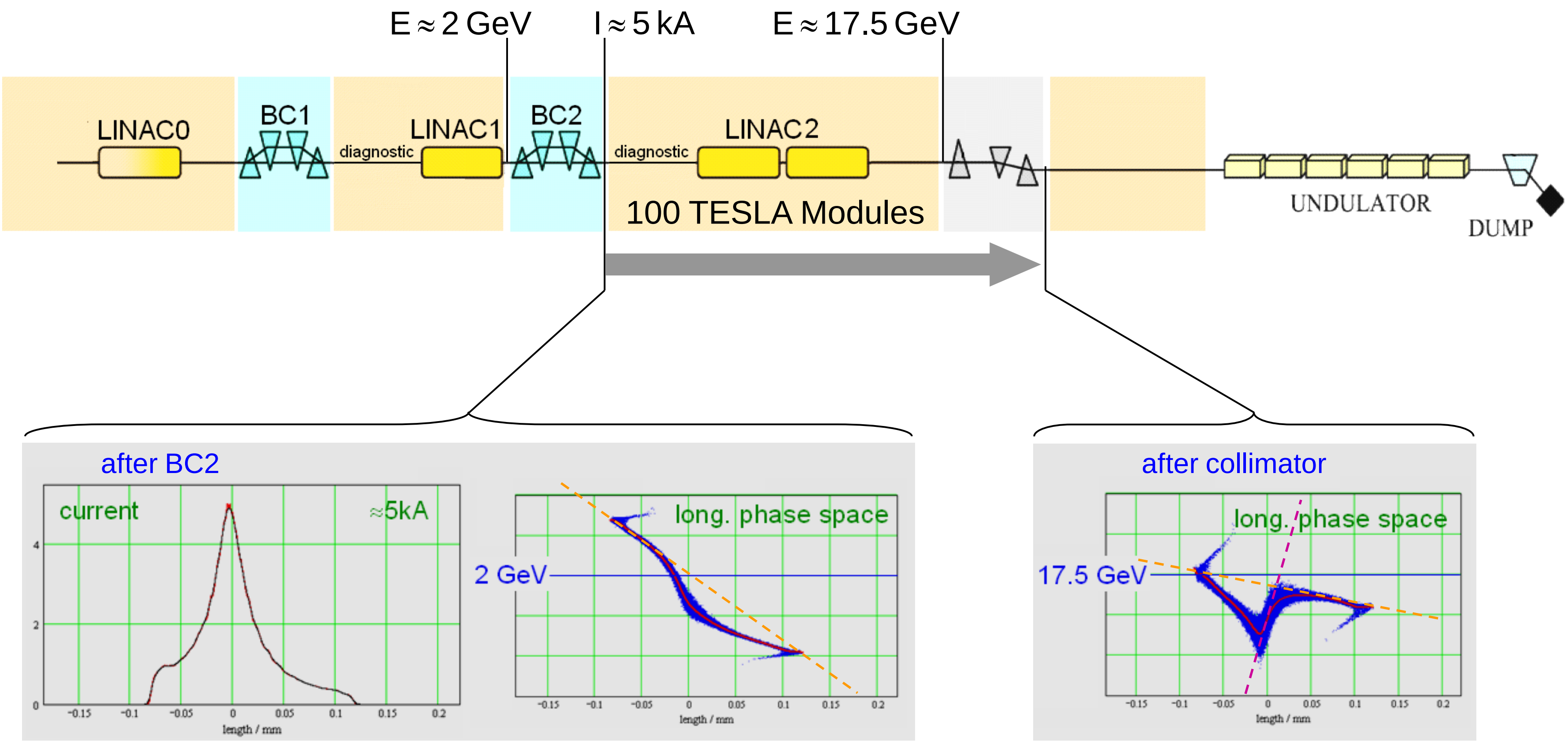
example:


E ≈ 2 GeV
I ≈ 5 kA
E ≈ 17.5 GeV
LINAC0
BC1
diagnostic
LINAC1
BC2
diagnostic
LINAC2
100 TESLA Modules
UNDULATOR
DUMP


abs(Impedance) / Ohm
1·10^5
1·10^4
1·10^3
100
10
1
1·10^3
1·10^4
1·10^5
1·10^6
λ = 30 μm
space charge
cavities
frequency / GHz

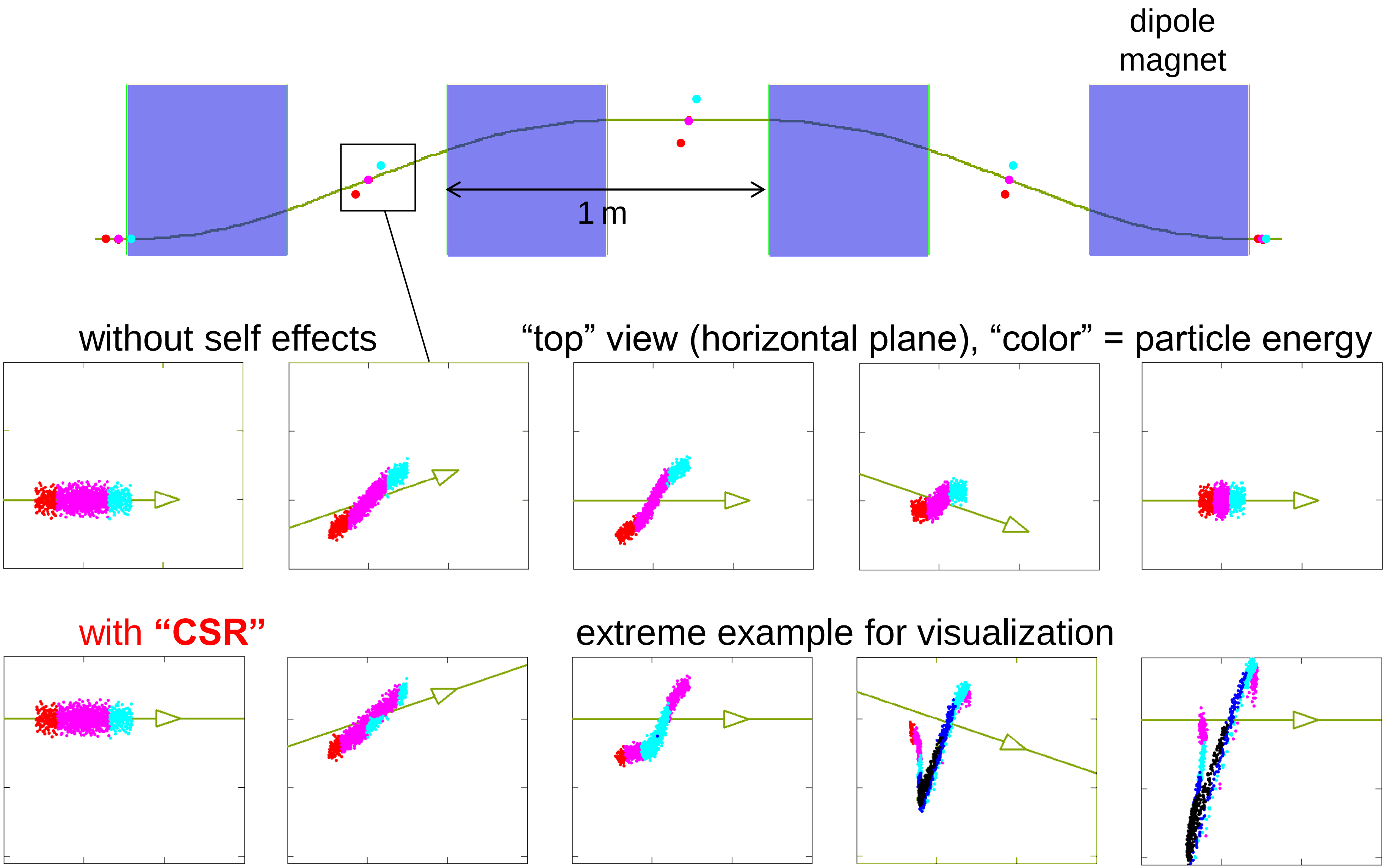
4.2 Some Longitudinal Impedances

example:



negative chirp compensated by LINAC wakes
positive chirp induced by space charge !

4.3 Some CSR Effects

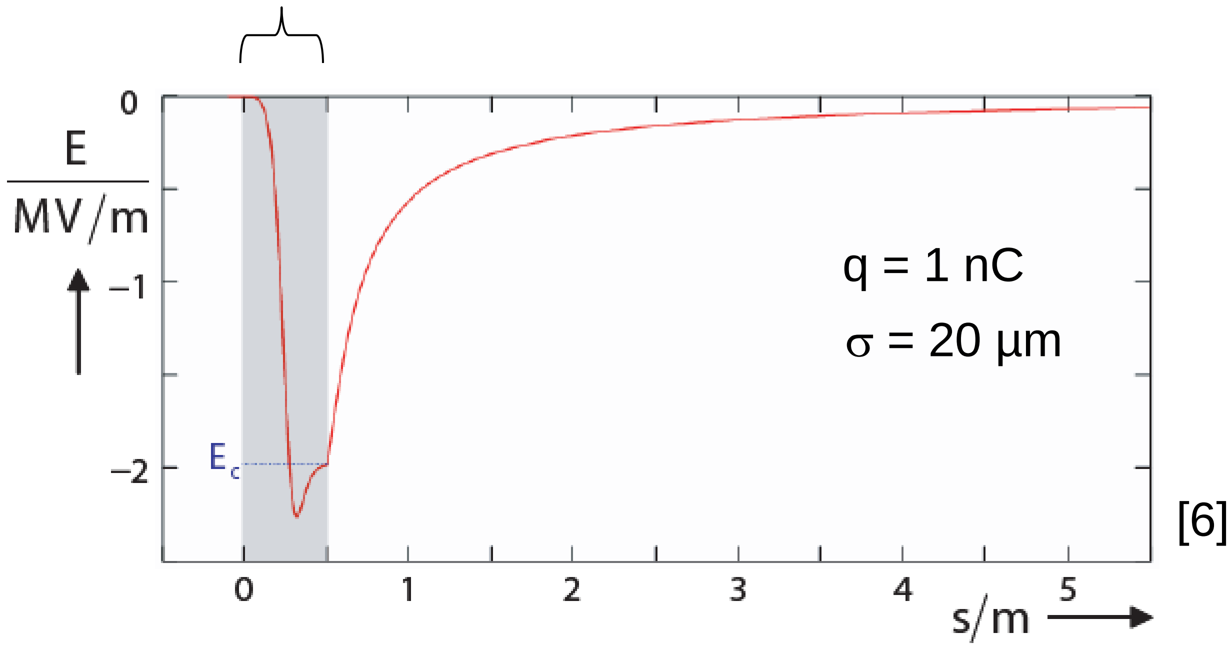


4.3 Some CSR Effects

CSR effects are not instantaneous

example: longitudinal field in center of a spherical bunch that travels through a bending magnet

magnet with bending radius R = 10 m



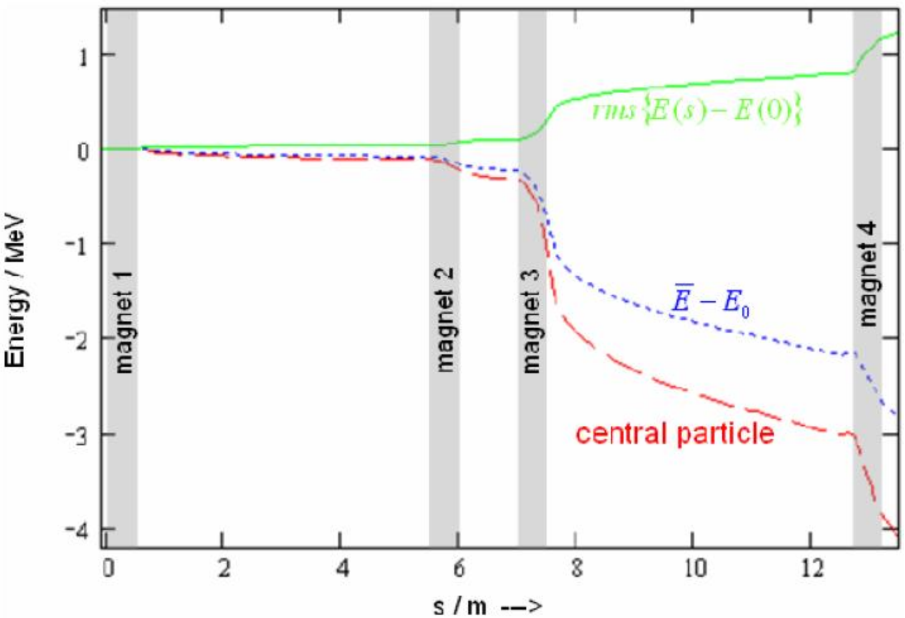
steady state field in magnet scales as $E_c = \frac{1}{\sqrt[2]{3}(2\pi)^{3/2}} \frac{1}{R^{2/3}\sigma^{4/3}} \frac{q}{\varepsilon_0}$

CSR interaction in and after bend

4.3 Some CSR Effects

example [6]: **4 magnet chicane**

compression 600A (1nC) → 6KA at 5 GeV



rms energy spread of bunch

mean energy of bunch

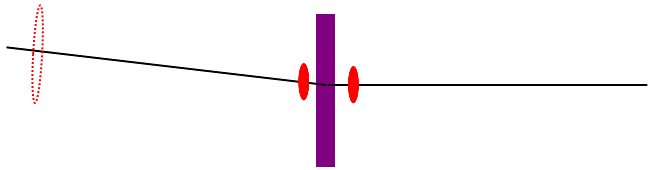
energy of particle in bunch center

significant change of energy profile in the last magnets and in the drift between

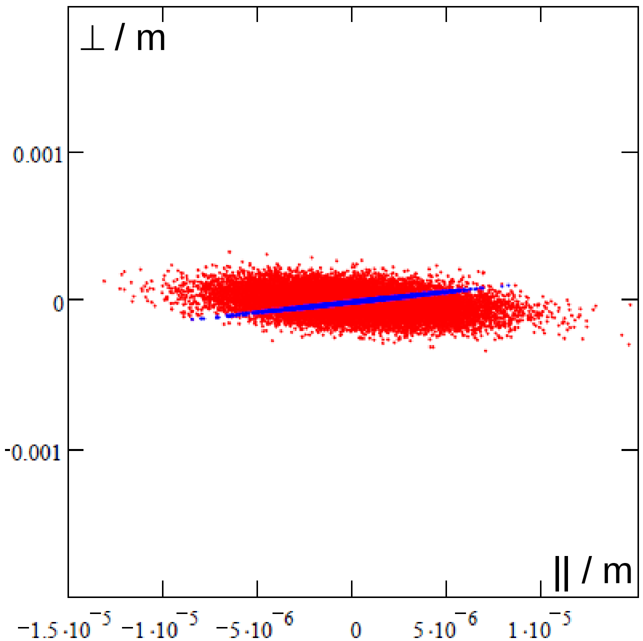
4.3 Some CSR Effects

an effect causing growth of slice emittance

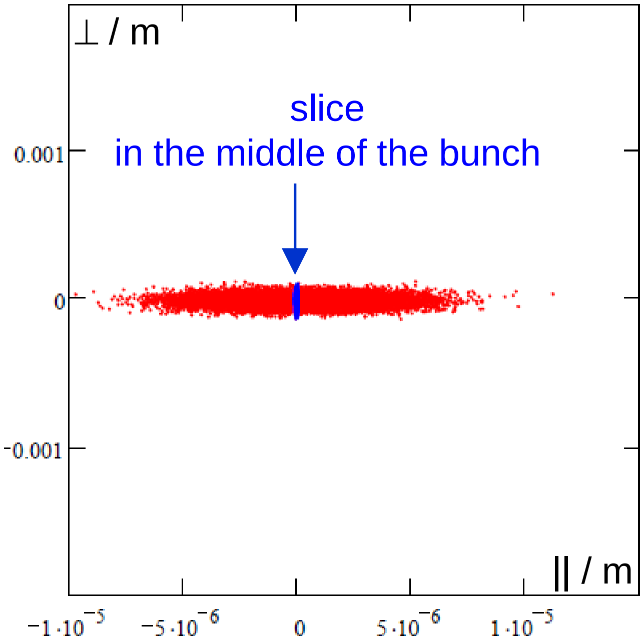
simplified picture of the last magent of a chicane: length $\rightarrow$ 0 but kick = const



top view: a bit before last magnet



directly after last magnet

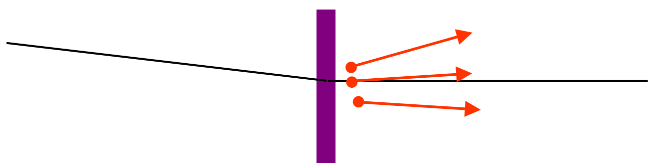


different long. field is seen by particles that come to the same slice (after compressor)

4.3 Some CSR Effects

an effect …

particles with different energy get different kick



growth of emittance $\varepsilon = \sqrt{\varepsilon_0^2 + \varepsilon_0 \beta (\phi \Delta E_{rms} / E)}$

with $\varepsilon_0, \varepsilon$ emittance before after discrete magnet

β beta function (lattice)

ϕ deflection angle

ΔE_{rms} energy spread of particle bunch (slice or full bunch)

E energy of particle bunch

ΔE_{rms} depends weak on energy

therefore: $\beta \rightarrow$ small; focus of lattice function in last magent

$E \rightarrow$ high

4.4 Transverse Space Charge Model

coasting pencil beam:

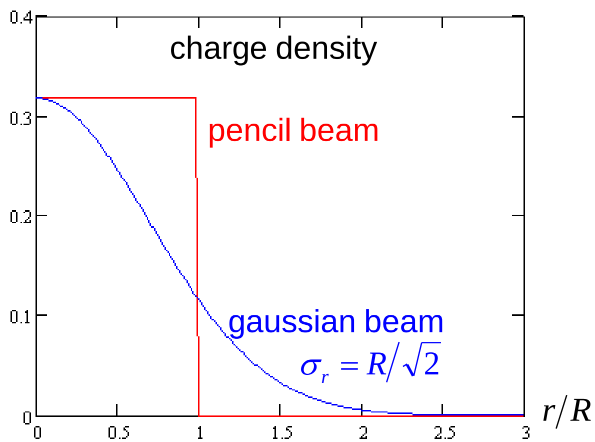
$$E_r = \frac{I/v}{2\pi\varepsilon}\frac{1}{R}\begin{cases} r/R & \text{if } r<R \\ R/r & \text{otherwise}\end{cases}$$

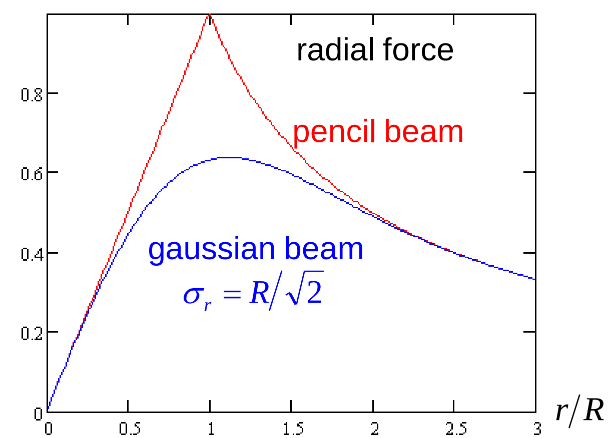
$$B_\varphi = \frac{I/c^2}{2\pi\varepsilon}\frac{1}{R}\begin{cases} r/R & \text{if } r<R \\ R/r & \text{otherwise}\end{cases}$$

radial force:

$$F_r = q\left(E_r - vB_\varphi\right)$$

$$F_r = q\frac{I/v}{2\pi\varepsilon}\underbrace{\left(1-\frac{v^2}{c^2}\right)}_{\gamma^{-2}}\frac{1}{R}\begin{cases} r/R & \text{if } r<R \\ R/r & \text{otherwise}\end{cases}$$





4.4 Transverse Space Charge Model

a slice model is used to estimate transverse self effects:

$$F_r \propto \frac{I(s)}{\gamma^2} f\left(r, \text{shape}(s)\right)$$

force depends on energy

force is usually non-linear in offset $\rightarrow$ effective linear force for rms properties

force depends on shape

force depends on bunch coordinate
$\rightarrow$ transverse optics depends on slice coordinate s

linear(ized) force

transverse EoM: $$x'' + \frac{p_r'}{p_r} x' - \left(k_x^{(0)} + k_s \right) x = 0$$

quadrupoles — space charge $\quad k_s = k_s\left(\sigma_x, \sigma_y, \gamma, I(s), \text{shape}\right)$

space charge parameter $\hat{S}$ compares these terms

4.4 Transverse Space Charge Model

simplification: $\sigma_x \approx \sigma_y \approx \sqrt{\varepsilon\beta}$

$$k_x^{(0)} \leftarrow \beta^{-2}$$

$$k_s \leftarrow k_s\left(\sqrt{\varepsilon\beta}, \cdots, \text{gaussian}\right)$$

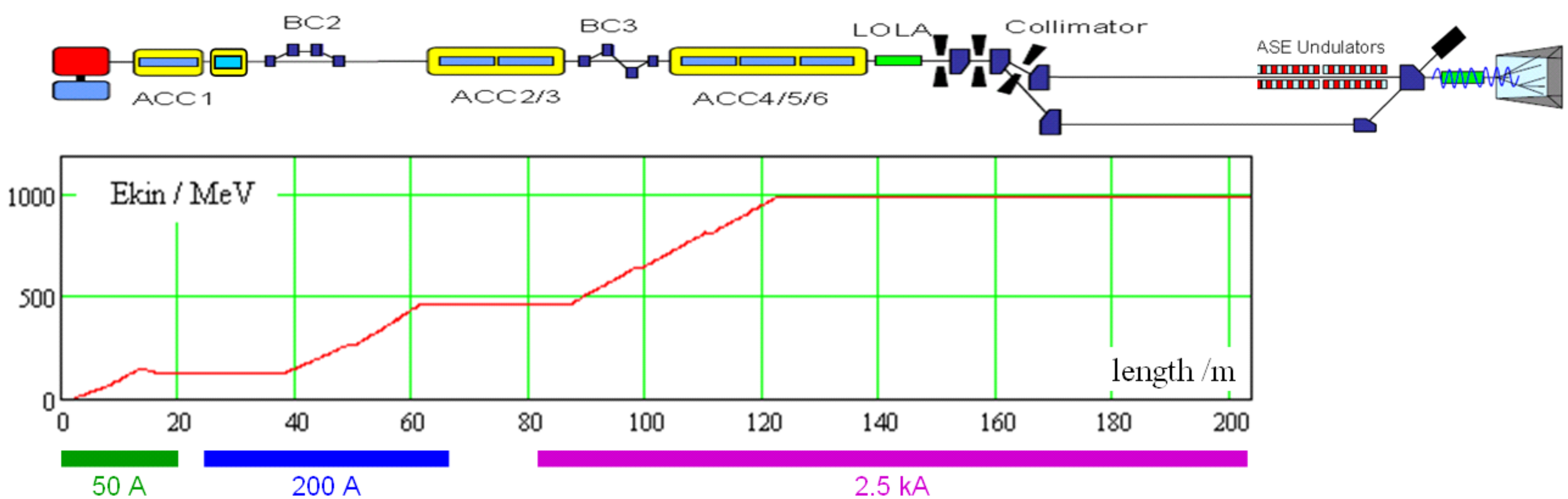
with emittance ε and averaged β -function

space charge parameter: $$\hat{S} = \frac{k_s}{k_x^{(0)}} = \frac{I(s)}{I_\mathrm{A}} \frac{\beta}{\gamma^2 \varepsilon_\mathrm{n}}$$

$\varepsilon_n \approx \gamma\varepsilon$ normalized emittance

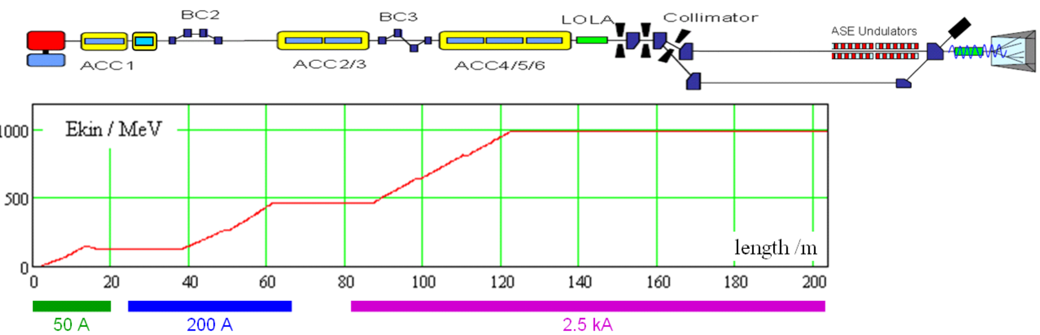
$I_\mathrm{A} = 17\ \mathrm{kA}$ Alven current

example:

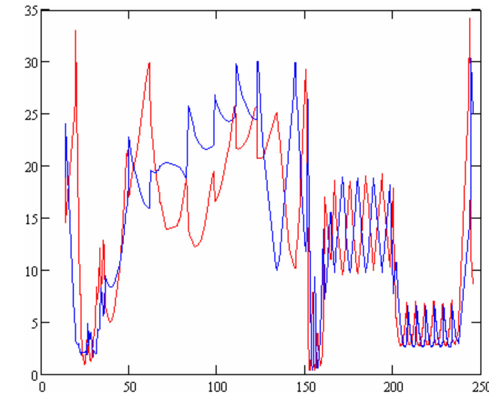


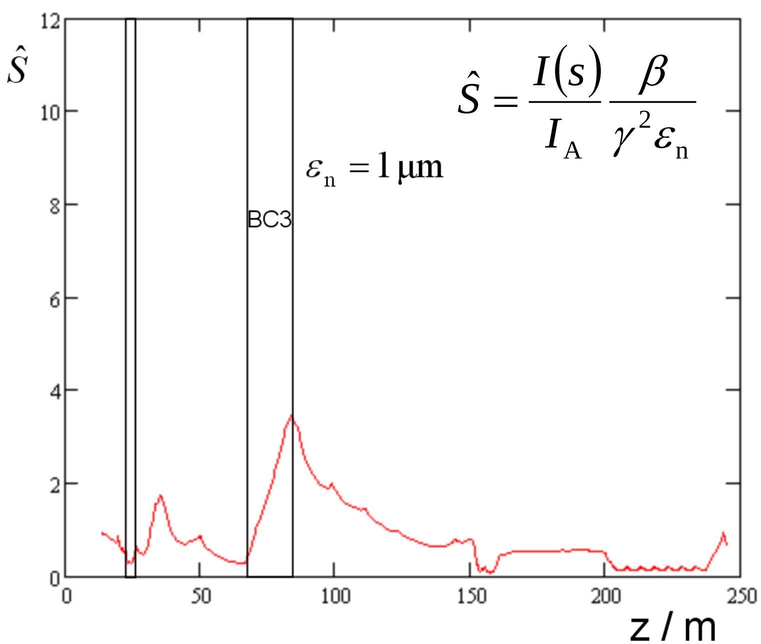
4.4 Transverse Space Charge Model

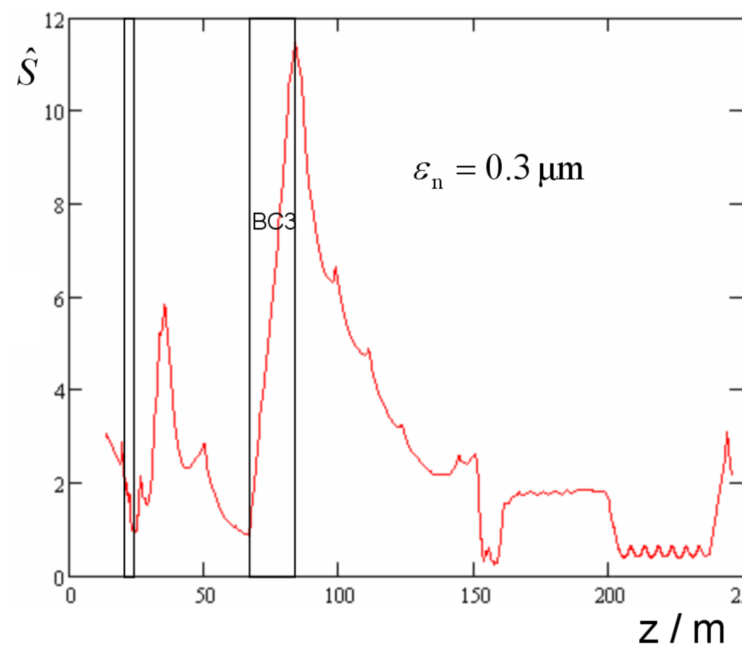
example:



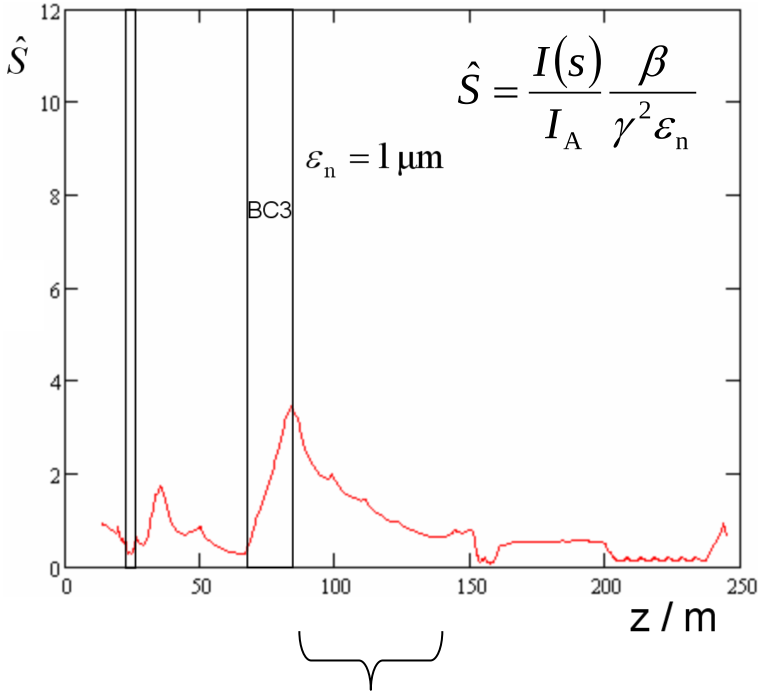
beta – functions / m







4.4 Transverse Space Charge Model



$$\gamma = \gamma_0 + \gamma' z$$

$$\int \hat{S} dz = \frac{I}{I_A} \frac{\beta}{\varepsilon_n} \int \frac{1}{\gamma^2} dz \sim \frac{I}{I_A} \frac{\beta}{\varepsilon_n} \boxed{\frac{1}{\gamma_0}} \frac{1}{\gamma'}$$

integrated effect ~ 1 / (beam energy @ compression)

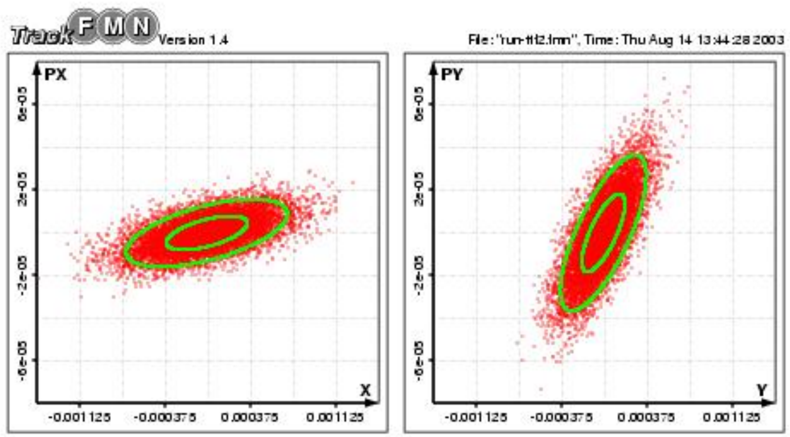
similar scaling for longitudinal SC effects

ICFA Future Light Sources Sub-Panel
Mini Workshop on
Start-to-End Simulations of X-RAY FELs

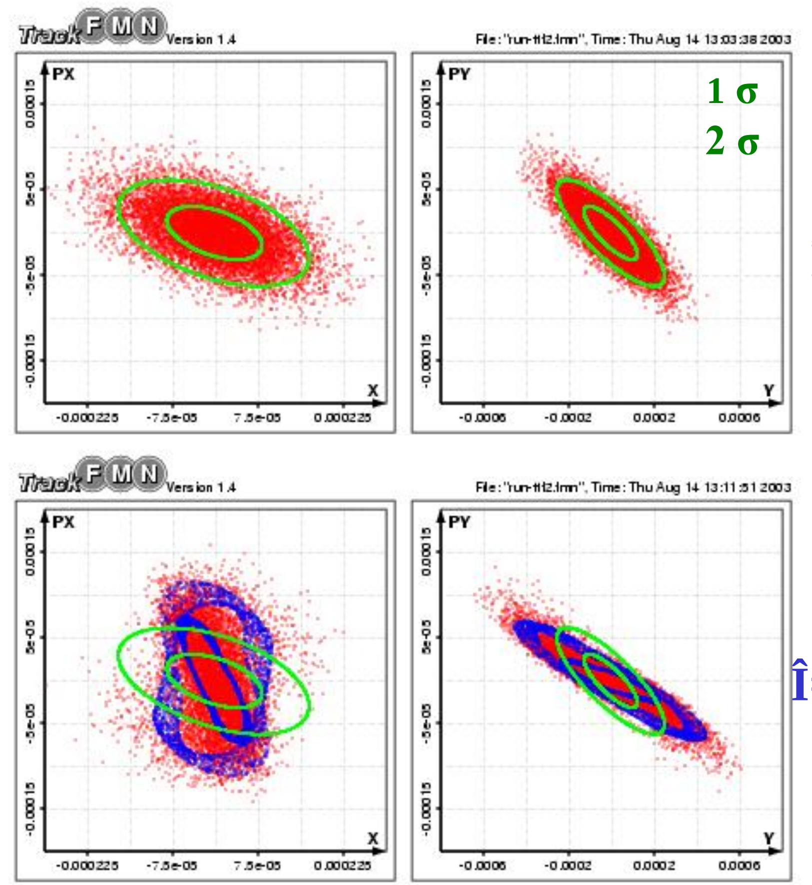
August 18-22, 2003 at DESY-Zeuthen
(Berlin, GERMANY)
[5]

verse space charge after BC3

initial: ACC4 exit



final: undulator entrance



Î=0

Î=2kA

Red: 10 000 particles, Gaussian distr.
Green (I=0) & Blue (I=2kA): test particles:
– do not contribute in space charge forces
– are tracked in space charge field of the main beam

CPU time = 90 sec (grid: 256x256, Nsp/elem=5)

5 Bunch Compression Systems

compression to kA bunches has to happen at high enough beam energy
to avoid strong space charge effects

is it possible to compress the bunch in one stage?
yes, but: rf tolerances are extremely tight
linearization by higher harmonic system at high energy level
($\rightarrow$ costs, wakes & beam loading)

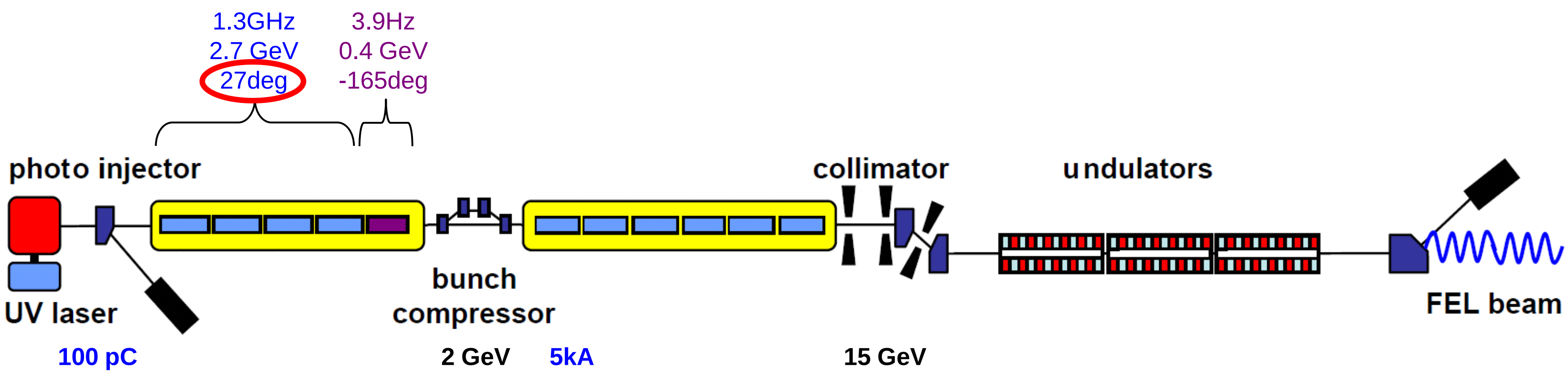
some problems increase with energy:
a) energy spread (for chirp) is limited
b) required longitudinal dispersion needs space and/or strong magnets
c) emittance growth due to incoherent synchrotron radiation

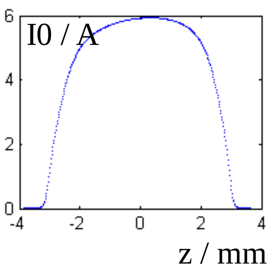
5.1 RF Tolerances in Single Bunch Compressor

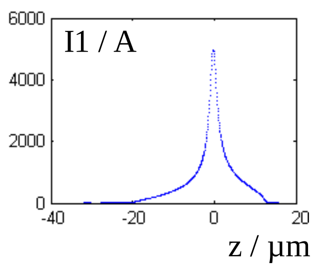
5.2 RF Tolerances in Multi-BC System

5.1 RF Tolerances in Single Bunch Compressor

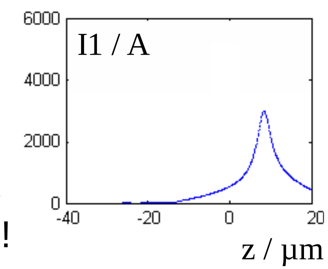
example: compression ≈ 1000

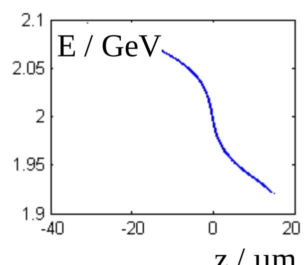






phase of 1.3 GHz LINAC
is changed by **0.01 deg** !!!





strong effect on
compression &
arrival time

5.1 RF Tolerances in Single Bunch Compressor

linear model of compression process:

coordinates z, p are deviation from nominal length Z and momentum P_z

rf system creates chirp

$$\begin{pmatrix} z^{(2)} \\ p^{(2)} \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 \\ a & 1 \end{pmatrix}}_{\text{rf matrix } \mathbf{R}} \begin{pmatrix} z^{(1)} \\ p^{(1)} \end{pmatrix}$$

chirp parameter

magnetic chicane changes z

$$\begin{pmatrix} z^{(3)} \\ p^{(3)} \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix}}_{\text{bunch compressor matrix } \mathbf{B}} \begin{pmatrix} z^{(2)} \\ p^{(2)} \end{pmatrix}$$

dispersion parameter

$$\begin{pmatrix} z^{(3)} \\ p^{(3)} \end{pmatrix} = \mathbf{BR} \begin{pmatrix} z^{(1)} \\ p^{(1)} \end{pmatrix} = \begin{pmatrix} 1+ab & b \\ a & 1 \end{pmatrix} \begin{pmatrix} z^{(1)} \\ p^{(1)} \end{pmatrix}$$

inverse compression

compression factor:

$$C = \frac{1}{(\mathbf{BR})_{1,1}} = \frac{1}{1+ab}$$

strong compression $\rightarrow ab \approx -1$

5.1 RF Tolerances in Single Bunch Compressor

rf tolerances of C are closely related to chirp tolerances

$$C = \frac{1}{1 + ab}$$

$$\frac{1}{C}\frac{\partial C}{\partial a} = -Cb = \frac{C-1}{a}$$

relative error of compression: $$\frac{\delta C}{C} = (C-1)\frac{\delta a}{a}$$

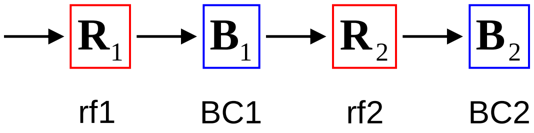
formula for cosine rf curvature and non linear dispersion (of 4 magnet chicane):

$$\frac{\delta C}{C} = (C-1)\left(3\tan\varphi + \frac{1}{\tan\varphi}\right)\delta\varphi$$

the example: $C = 1000$, $\varphi = 27\ \mathrm{deg}$ $\rightarrow$ half compression

5.2 RF Tolerances in Multi-BC System

system with two compressors:



compression factor: $$C = \frac{1}{\left(\mathbf{B}_2\mathbf{R}_2\mathbf{B}_1\mathbf{R}_1\right)_{1,1}}$$

$$C = \frac{1}{(1+a_1 b_1)(1+a_2 b_2)+b_2 a_1}$$

chirp parameter

5.2 RF Tolerances in Multi-BC System

compression factor: $$C = \frac{1}{(1+a_1 b_1)(1+a_2 b_2)+b_2 a_1}$$

chirp parameter

extreme 1: acceleration by rf2 is on crest $a_2 = 0$

$$C = \frac{1}{1+a_1(b_1+b_2)}$$

BC1 and BC2 work as one bunch compressor although they are on different energy levels

$$\frac{\delta C}{C} = (C-1)\frac{\delta a_1}{a_1}$$

no improvement with respect to tolerances!

5.2 RF Tolerances in Multi-BC System

compression factor: $$C = \frac{1}{(1+a_1 b_1)(1+a_2 b_2)+b_2 a_1}$$

chirp parameter

extreme 2: decouple both compression stages:

$$|b_2 a_1| << (1+a_1 b_1)(1+a_2 b_2)$$

small chirp in rf1

small dispersion in BC2

$$|b_2 a_1| << C^{-1}$$ (decoupling condition)

$$C = C_1 C_2$$ with $C_1 = \frac{1}{(1+a_1 b_1)}$ and $C_2 = \frac{1}{(1+a_2 b_2)}$

relative error of decoupled compression:

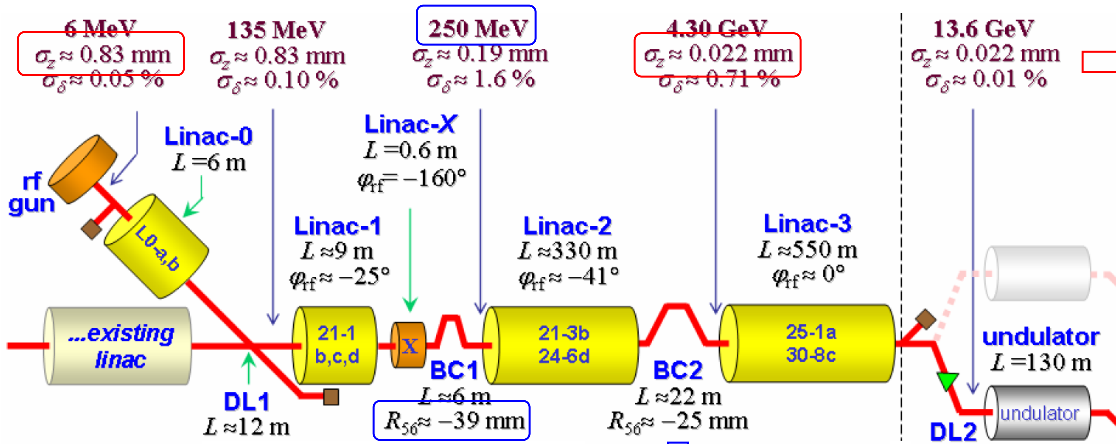
$$\frac{\delta C}{C} = (C_1 - 1)\frac{\delta a_1}{a_1} + (C_2 - 1)\frac{\delta a_2}{a_2}$$

relaxed tolerances !

but it is difficult to fulfill decoupling condition for large compression factors

5.2 RF Tolerances in Multi-BC System

last example:



$$C = \frac{0.83\ \text{mm}}{0.022\ \text{mm}} \approx 38$$

$$C_1 = \frac{0.83\ \text{mm}}{0.19\ \text{mm}} \approx 4.4$$

$$b_1 = \frac{R_{56}^{(1)}}{\mathcal{E}^{(1)}} \qquad b_2 = \frac{R_{56}^{(2)}}{\mathcal{E}^{(2)}}$$

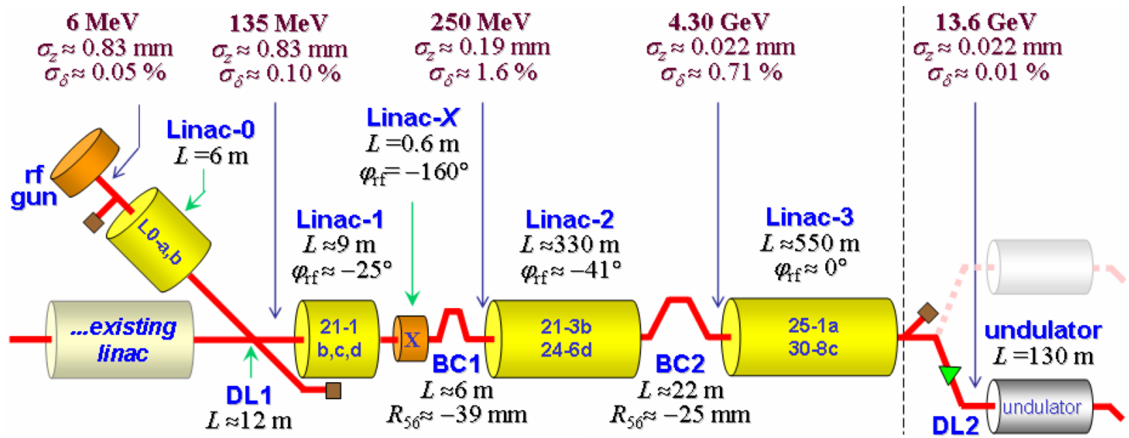
$$C_1 = \frac{1}{1 + a_1 b_1} \qquad C = \frac{1}{(1 + a_1 b_1)(1 + a_2 b_2) + b_2 a_1}$$

$$\Downarrow \qquad \Downarrow$$

$$a_1 \qquad a_2$$

5.2 RF Tolerances in Multi-BC System

last example:



$$C = \frac{0.83\,\mathrm{mm}}{0.022\,\mathrm{mm}} \approx 38$$

not completely decoupled: $|b_2 a_1| = 0.029 \leftrightarrow C^{-1} = 0.027$ (in all XFELs)

completely decoupled: $C_1 - 1 = 3.4$
not decoupled: $C - 1 = 37$

sensitivity to chirp parameter: $$\frac{\delta C}{C} \approx 8\frac{\delta a_1}{a_1} + 7\frac{\delta a_2}{a_2}$$

Linac-2: $$\frac{\delta a_2}{a_2} = \frac{\delta\varphi_2}{\tan\varphi_2}$$ $\delta\varphi_2 \approx 0.4$ deg causes 10% change of compression

($\delta\varphi_1 \approx 0.2$ deg)

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