Variable-period Permanent Magnet Undulators

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Outline

1. Introduction
2. **Splitted-pole undulator**
   a. *General idea*
   b. *Approximate formula for the hybrid PMU field*
3. **Mechanical design**
   a. *Calculation of repulsing force*
   b. *Possible design scheme*
4. **Advantages of the variable-period undulators**
   a. *Generation of spontaneous radiation*
   b. *High-gain X-ray FEL application*

One of the main FEL advantages is the ability to adjust the wavelength

$$\lambda = \lambda_u \frac{1}{2\gamma^2}\left(1+\frac{K^2}{2}\right)$$

Variation of magnetic field

Electromagnetic undulator

Variable gap undulator

Variation of beam energy

Variation of undulator period

Variable period undulator

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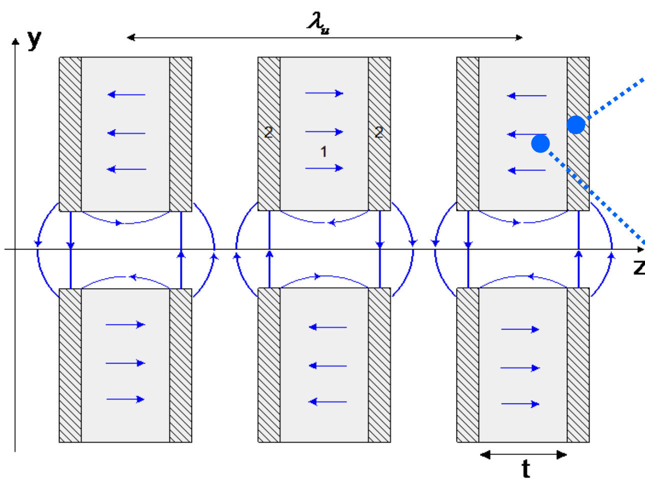
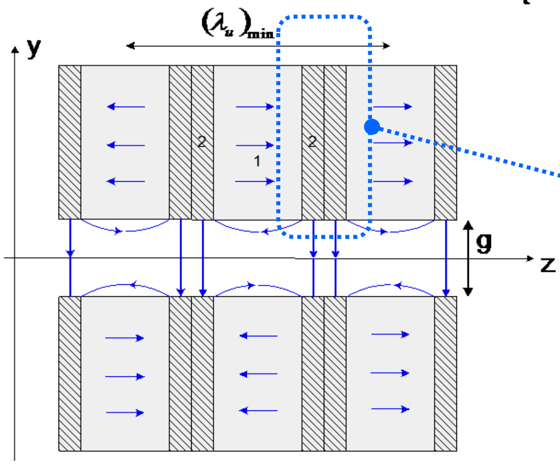
a. General idea

Hybrid permanent magnet undulator

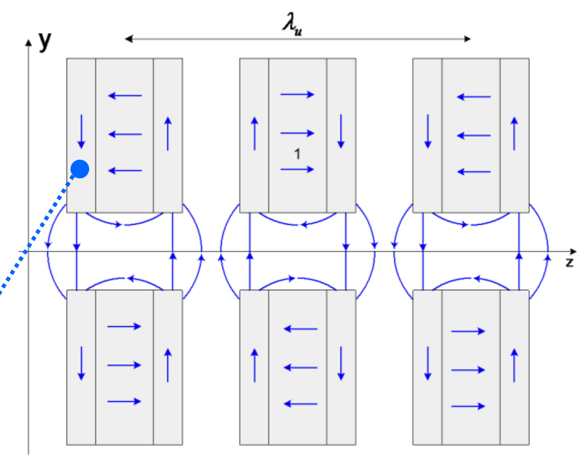




iron

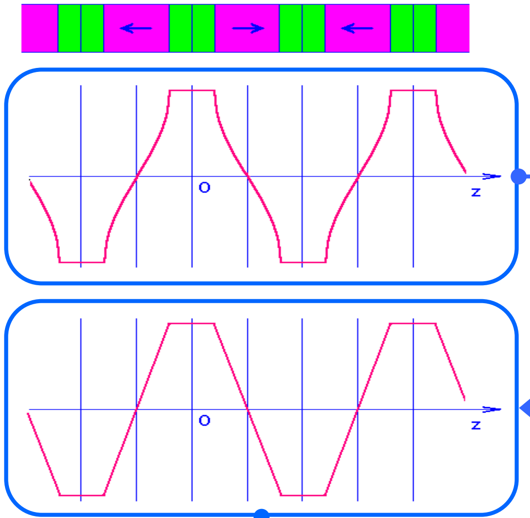
magnet blocks

Pure permanent magnet undulator



Each pole is divided to two halves

b. Approximate formula for the hybrid PMU field



Approximate scalar potential at the pole surface by trapezoidal curve

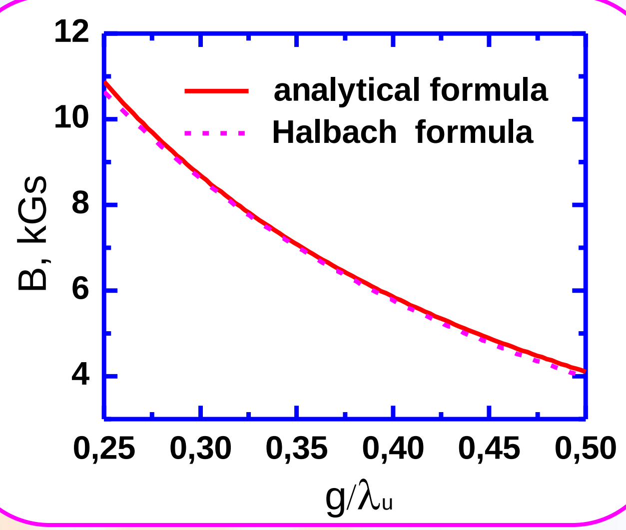
Find the first harmonic of scalar potential

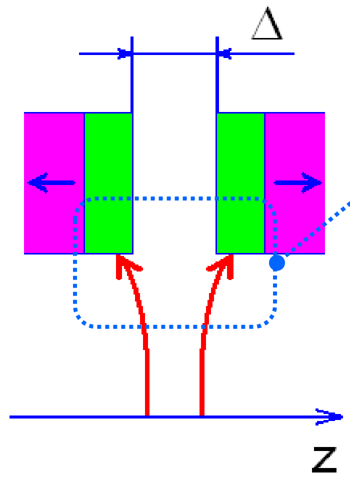
$$H_c \frac{2\lambda_u}{\pi^2} \sin\frac{\pi t}{\lambda_u} = B_0 \frac{\lambda_u}{2\pi} \sinh\frac{\pi g}{\lambda_u}$$

Find field at median plane

$$B_0 = \frac{4}{\pi} H_c \frac{\sin\frac{\pi t}{\lambda_u}}{\sinh\frac{\pi g}{\lambda_u}} \rightarrow H_c \frac{\sin\frac{\pi t}{\lambda_u}}{\sinh\frac{\pi g}{\lambda_u}}$$

Compare to the Halbach formula



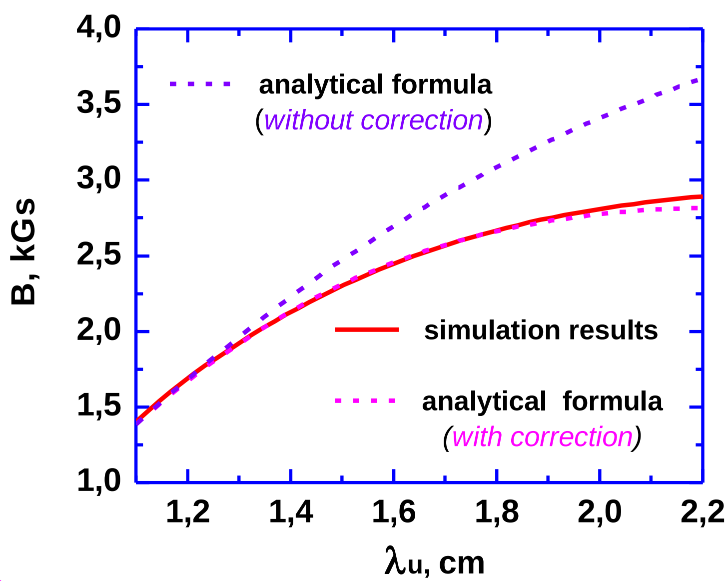


Magnetic field decreases due to the break

Introduce correction factor

$$\kappa = 1 - \frac{\pi}{\tanh\left(\frac{\pi g}{\lambda_u}\right)} \frac{1}{\mathrm{sinc}\left(\frac{\pi t}{\lambda_u}\right)} \left(\frac{\Delta}{\lambda_u}\right)^2$$

Compare resulting analytical formula to computer simulations



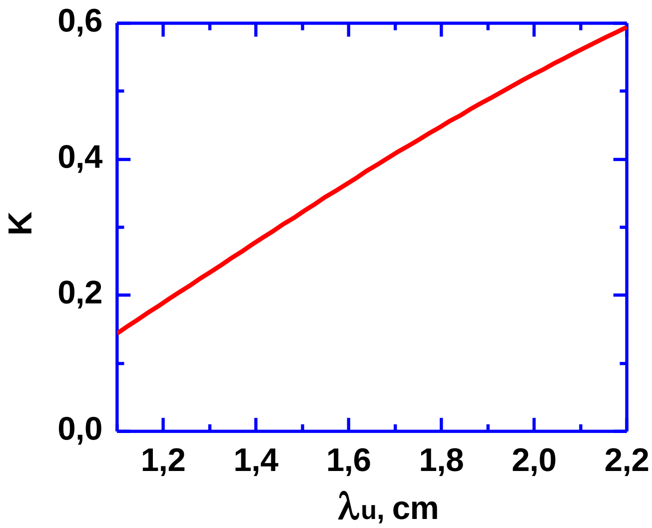
Undulator Simulation Example

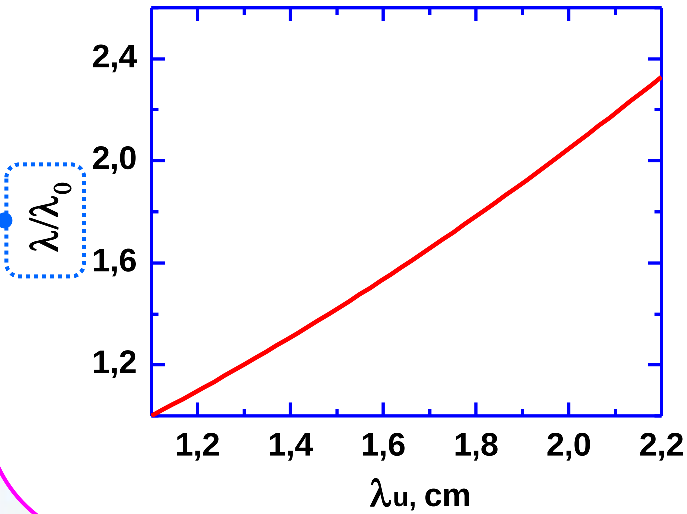
Undulator parameters

Minimal period, cm	1.1
Maximal period, cm	2.2
Undulator gap, cm	1
Magnet width along longitudinal axes, cm	0.4
Magnet height, cm	1.3

$$\frac{\lambda}{\lambda_0} = \frac{\lambda_u}{\lambda_{u0}} \frac{2+K^2}{2+K_0^2}$$

Simulation results





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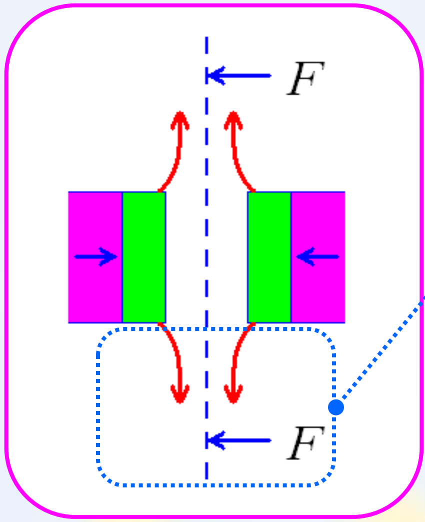
a. Calculation of repulsing force



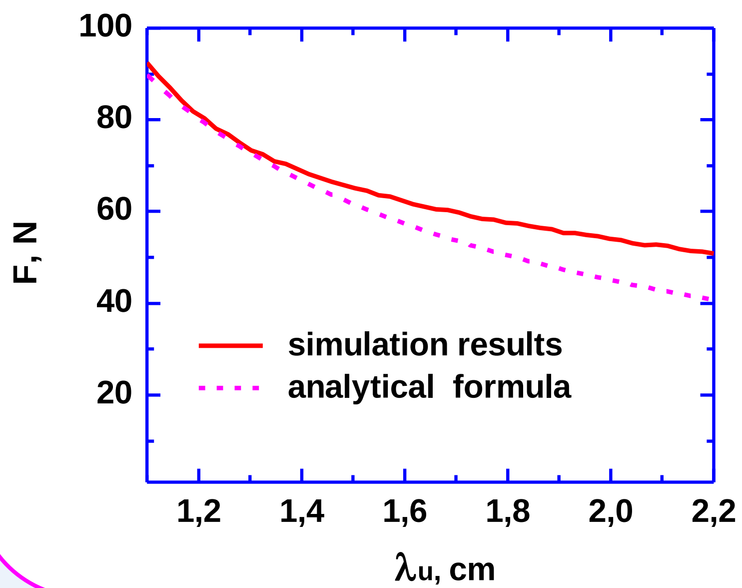
The force is caused by magnetic field pressure

$$\chi(x)=\int_0^1 \ln\left(\frac{x+s^2}{1+xs^2}\right)^2 \frac{1}{1-s^2}\,ds$$

$$F=\frac{H_c^2}{8\pi}\,\frac{\lambda_u}{2t}\,\frac{(\lambda_u)_{\min}P}{\pi^3}\,\chi\left(\frac{1-\sin\left(\frac{\pi t}{\lambda_u}\right)}{1+\sin\left(\frac{\pi t}{\lambda_u}\right)}\right)$$

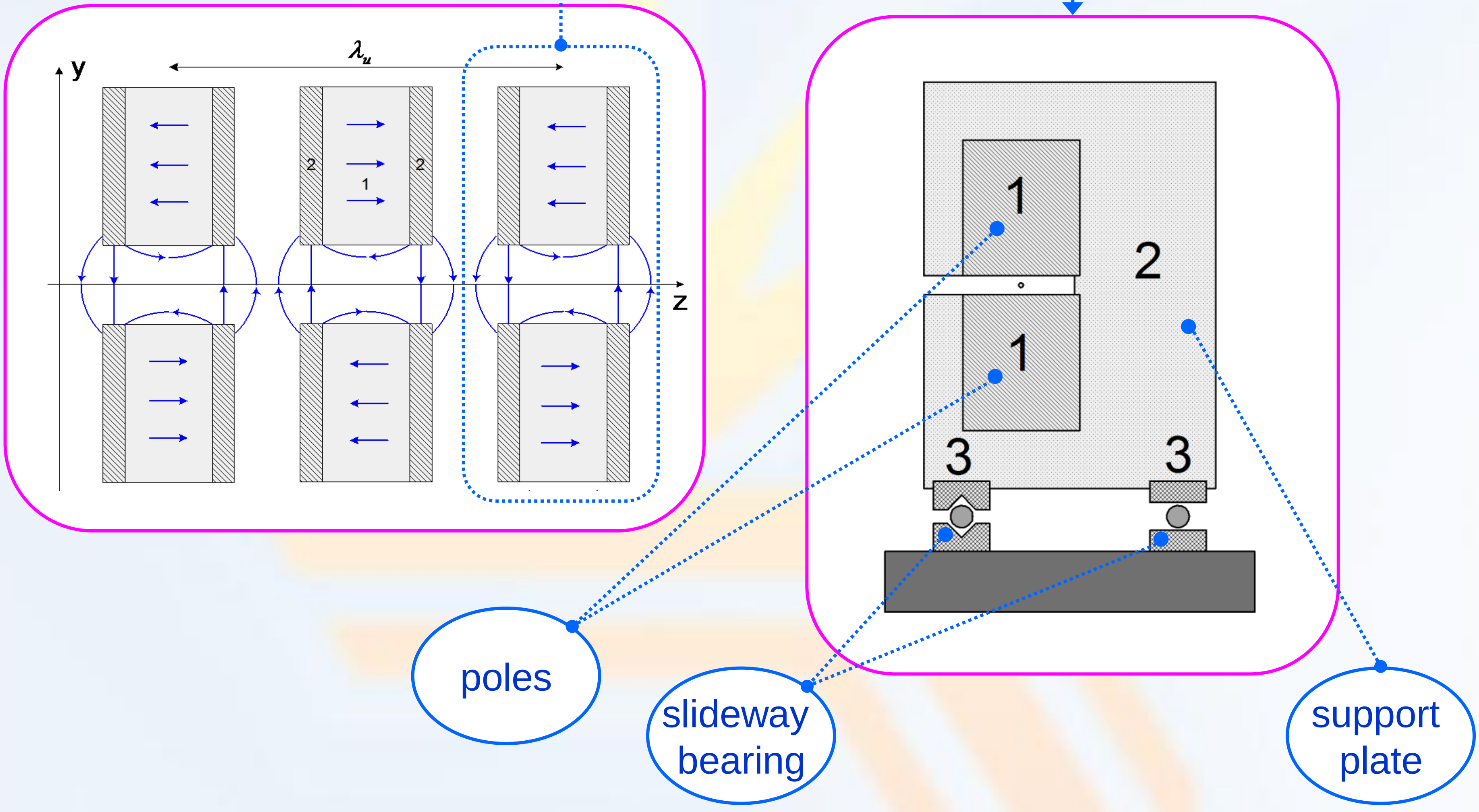
Replace by simple fitting formula

$$F=\frac{H_c^2}{8\pi}\,\frac{(\lambda_u)_{\min}P}{2\pi}\arcsin\left(\frac{2t}{\lambda_u}\right)$$

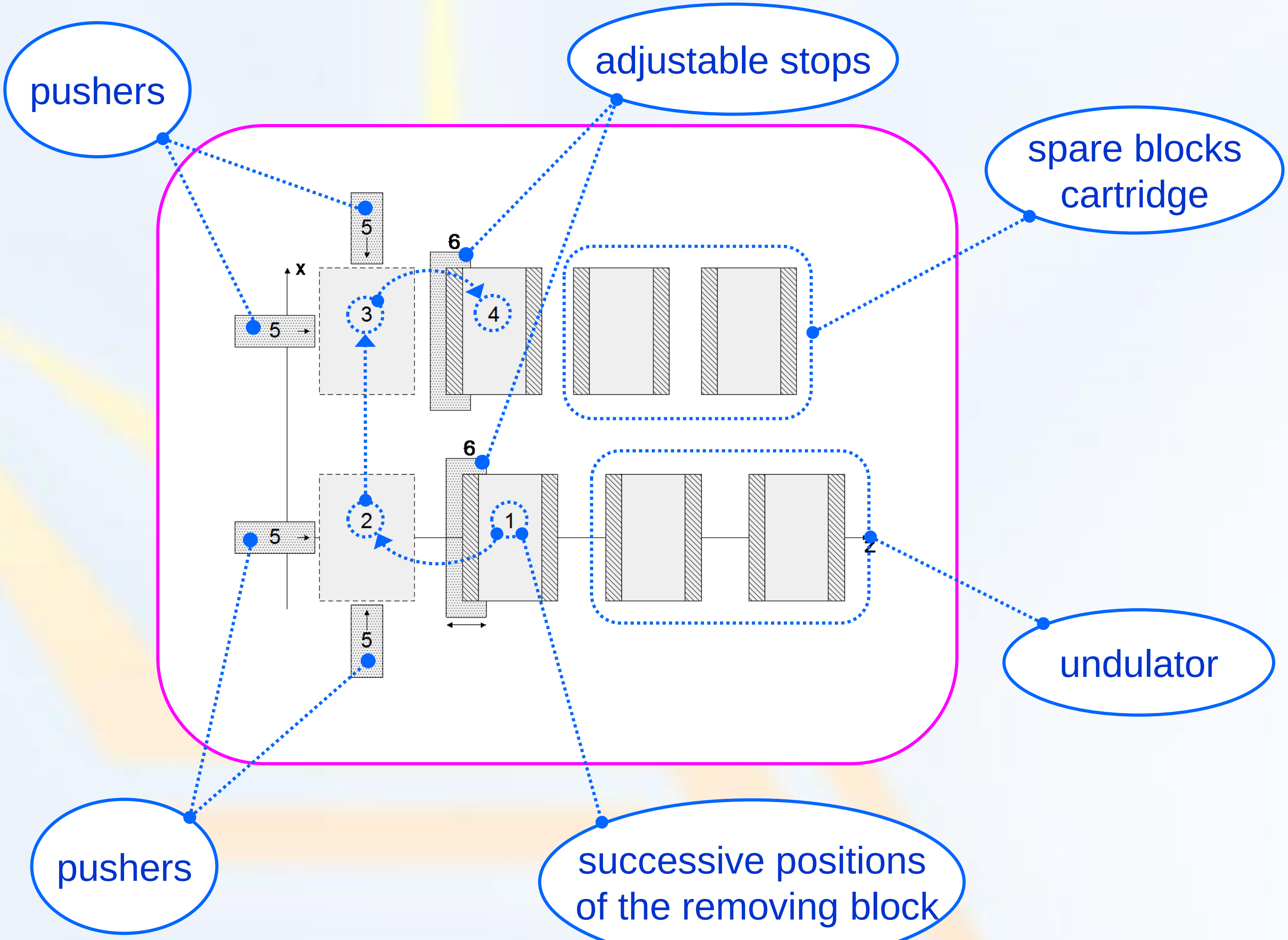


b. Possible design scheme

Side view of a half-period of the fixed-gap VPU



Variable Period Number Option



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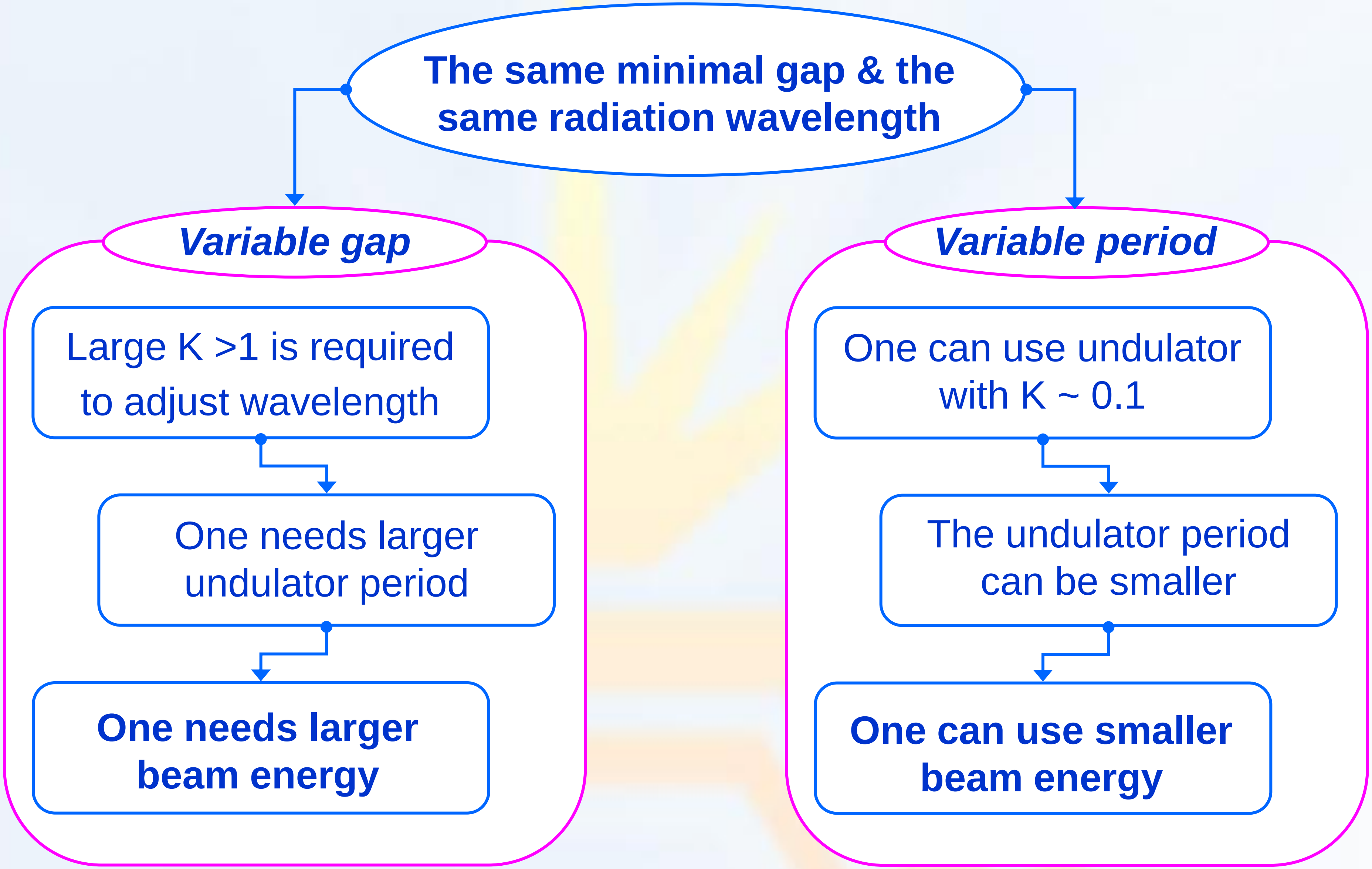
a. Generation of spontaneous radiation



a. Generation of spontaneous radiation

$$g = 1\,cm,\ \lambda_0 = 0.1\,nm$$

Variable gap

$$K_{max} = \sqrt{2}$$

$$\lambda_u = 2.4\,cm$$

$$E = 5.5\,GeV$$

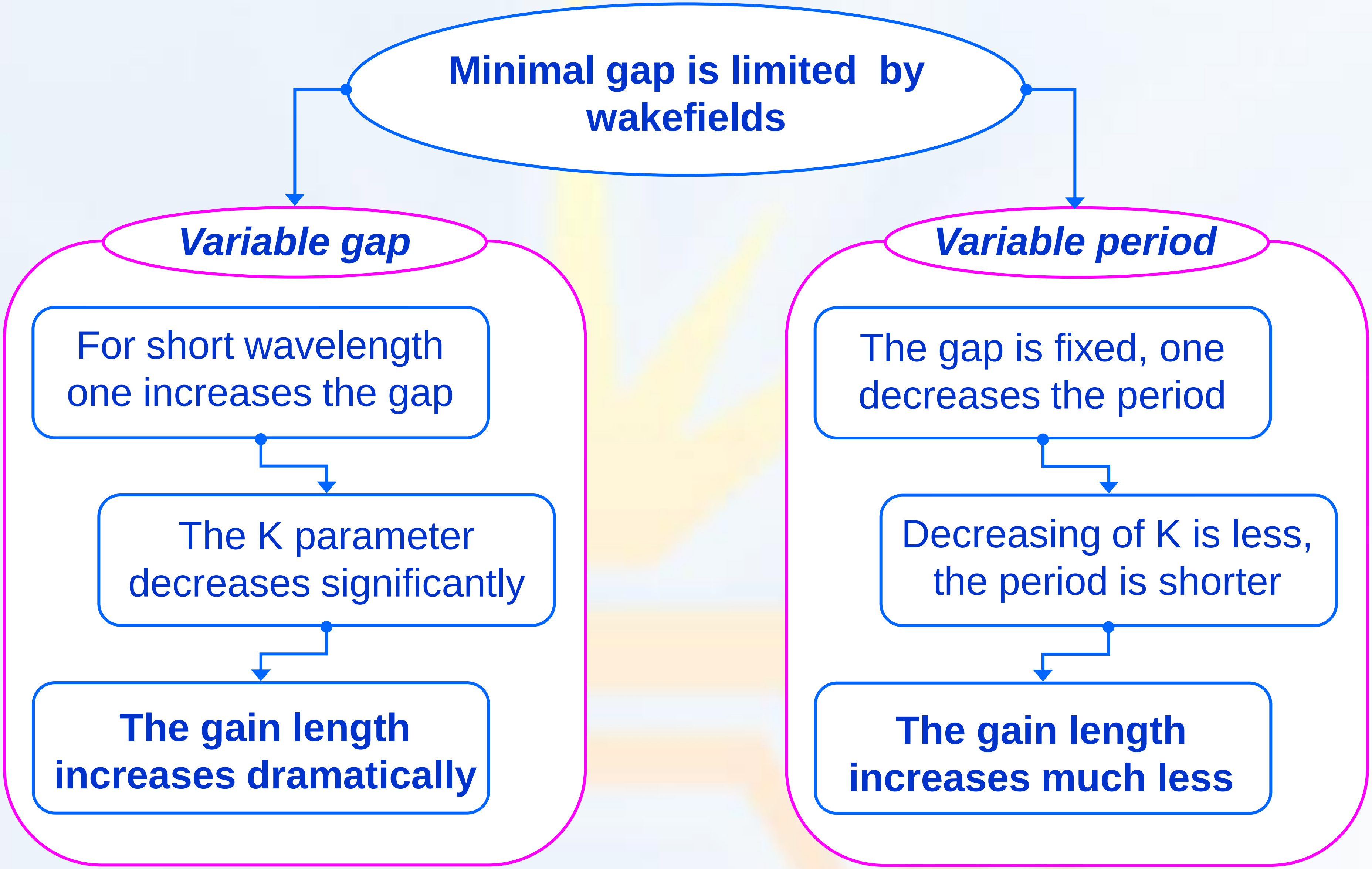
Variable period

$$K = 0.1$$

$$(\lambda_u)_{min} = 1.1\,cm$$

$$E = 3.7\,GeV$$

b. *High gain X-ray FEL application*



FEL simulation example

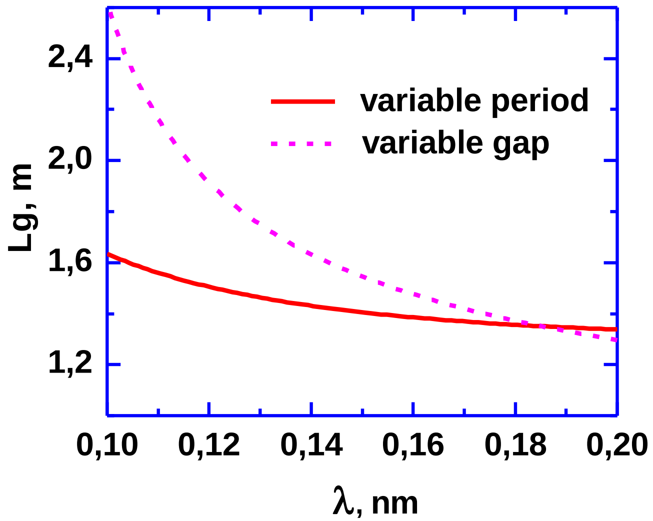
Beam parameters

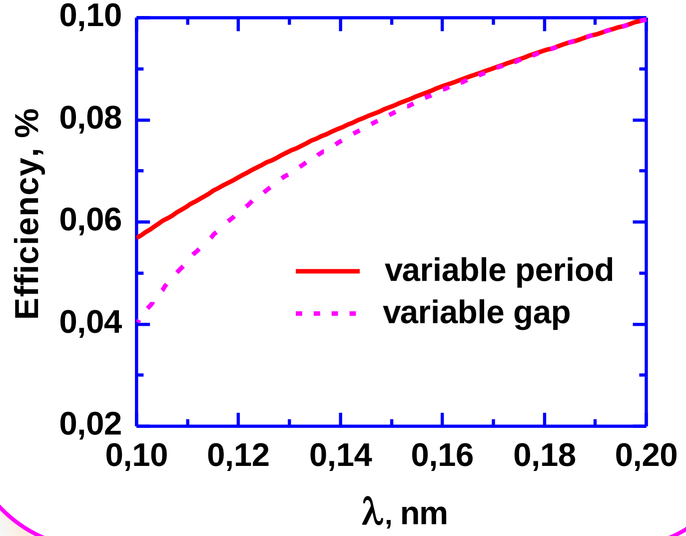
Electron energy, Gev	5.84
Beam current, kA	2
Normalized emittance, μm	0.2
Energy spread, %	0.01

Undulator parameters

VPU minimal period, cm	1.6
VPU maximal period, cm	2.14
Minimal gap, cm	0.6
VGU period, cm	2.07
VGU maximal gap, cm	1.1

Simulation results





Conclusion

1. We considered the new design of permanent magnet undulators which allows to change undulator period.

2. Variable period undulators have many advantages compared to conventional undulators.

3. Application of variable period undulators can open new prospects for further improvements of accelerator-based radiation sources.

Thank you for your attention !

The end.