

# Variable-period Permanent Magnet Undulators

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# Outline

## 1. Introduction

## 2. Splitted-pole undulator

*a. General idea*

*b. Approximate formula for the hybrid PMU field*

## 3. Mechanical design

*a. Calculation of repulsing force*

*b. Possible design scheme*

## 4. Advantages of the variable-period undulators

*a. Generation of spontaneous radiation*

*b. High-gain X-ray FEL application*

One of the main FEL advantages is the ability to adjust the wavelength

Variation of magnetic field

$$\lambda = \lambda_u \frac{1}{2\gamma^2} \left( 1 + \frac{K^2}{2} \right)$$

*Electromagnetic undulator*

*Variable gap undulator*

Variation of beam energy

Variation of undulator period

***Variable period undulator***

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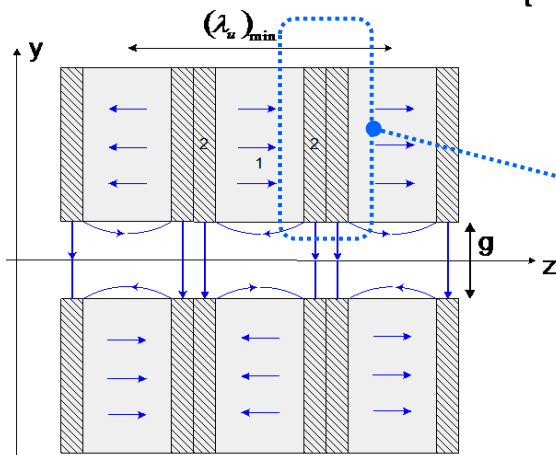
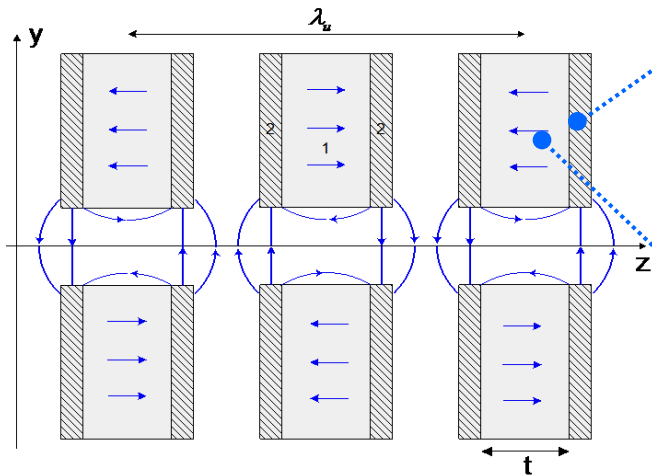
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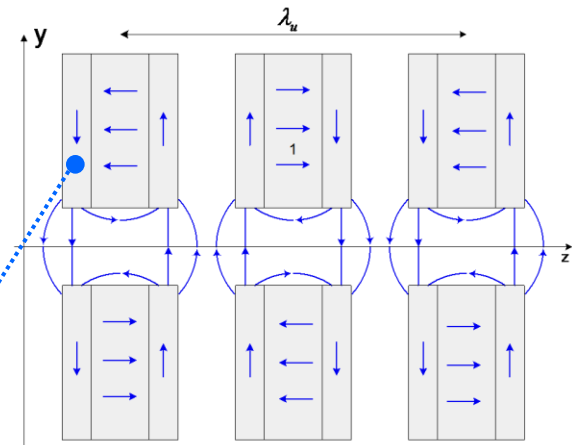
# a. General idea

## Hybrid permanent magnet undulator



iron

## Pure permanent magnet undulator

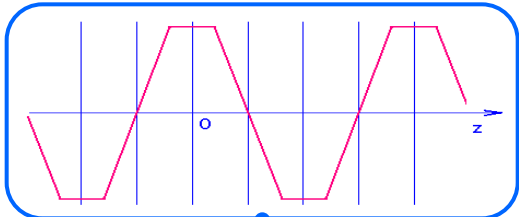
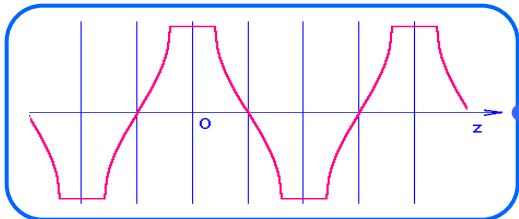


magnet blocks

Each pole is divided to two halves

## b. Approximate formula for the hybrid PMU field

Approximate scalar potential at the pole surface by trapezoidal curve



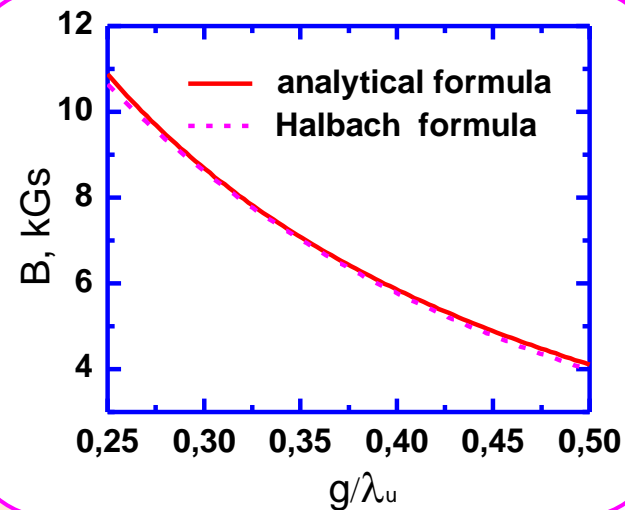
Find the first harmonic of scalar potential

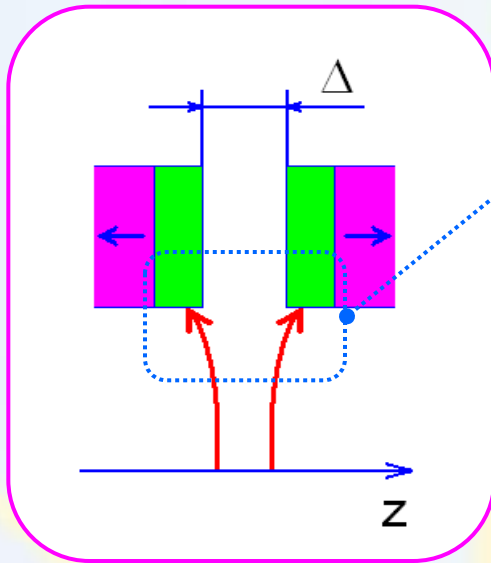
$$H_c \frac{2\lambda_u}{\pi^2} \sin \frac{\pi t}{\lambda_u} = B_0 \frac{\lambda_u}{2\pi} \sinh \frac{\pi g}{\lambda_u}$$

Find field at median plane

$$B_0 = \frac{4}{\pi} H_c \frac{\sin \frac{\pi t}{\lambda_u}}{\sinh \frac{\pi g}{\lambda_u}} \rightarrow H_c \frac{\sin \frac{\pi t}{\lambda_u}}{\sinh \frac{\pi g}{\lambda_u}}$$

Compare to the Halbach formula

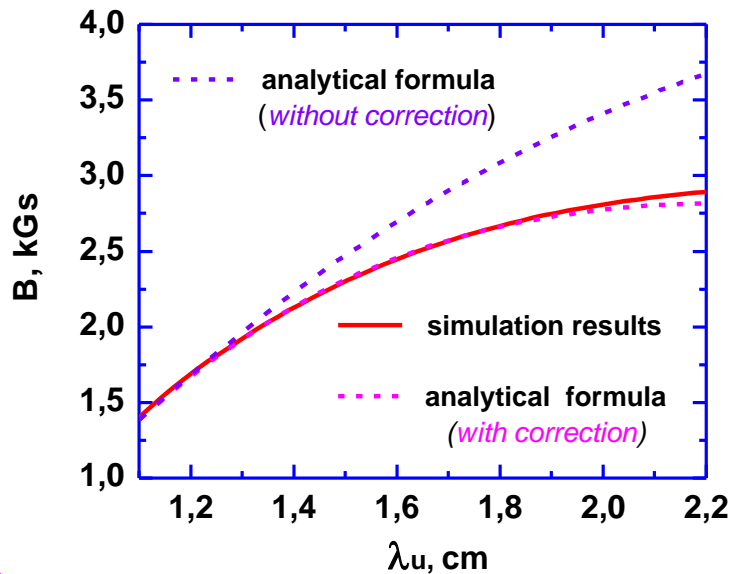




**Magnetic field decreases due to the break**

*Introduce correction factor*

$$\kappa = 1 - \frac{\pi}{\tanh\left(\frac{\pi g}{\lambda_u}\right)} \frac{1}{\text{sinc}\left(\frac{\pi t}{\lambda_u}\right)} \left(\frac{\Delta}{\lambda_u}\right)^2$$



*Compare resulting analytical formula to computer simulations*

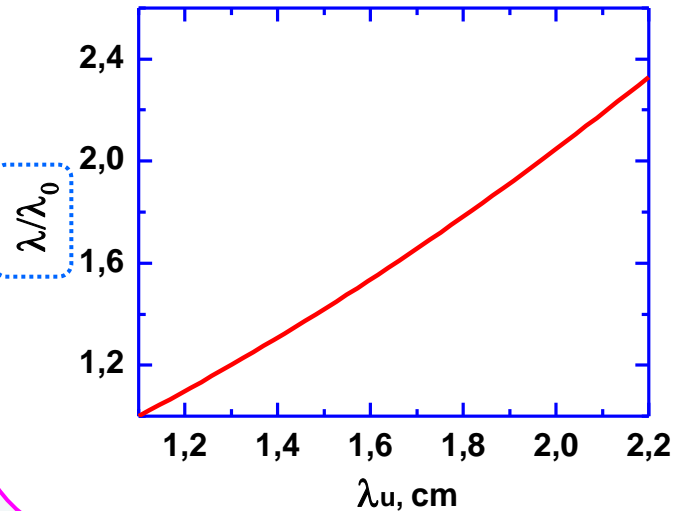
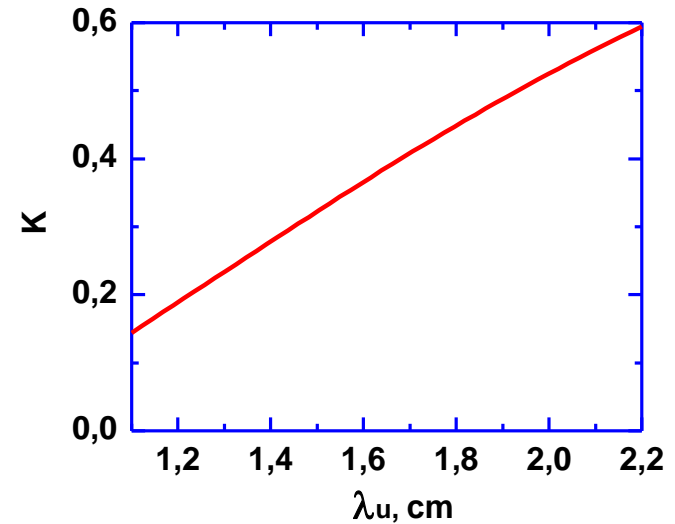
# Undulator Simulation Example

## Undulator parameters

Minimal period, cm	1.1
Maximal period, cm	2.2
Undulator gap, cm	1
Magnet width along longitudinal axes, cm	0.4
Magnet height, cm	1.3

$$\frac{\lambda}{\lambda_0} = \frac{\lambda_u}{\lambda_{u0}} \frac{2 + K^2}{2 + K_0^2}$$

## Simulation results





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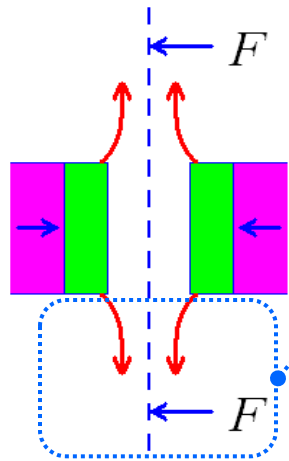
*b. Possible design scheme*

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## a. Calculation of repulsing force



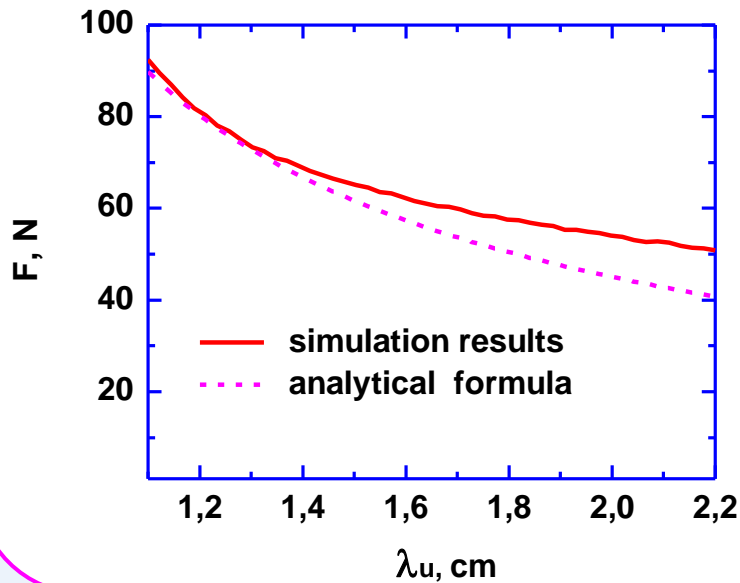
The force is caused by magnetic field pressure

$$\chi(x) = \int_0^1 \ln \left( \frac{x + s^2}{1 + xs^2} \right)^2 \frac{1}{1 - s^2} ds$$

$$F = \frac{H_c^2 \lambda_u (\lambda_u)_{\min} P}{8\pi 2t \pi^3} \chi \left( \frac{1 - \sin\left(\frac{\pi t}{\lambda_u}\right)}{1 + \sin\left(\frac{\pi t}{\lambda_u}\right)} \right)$$

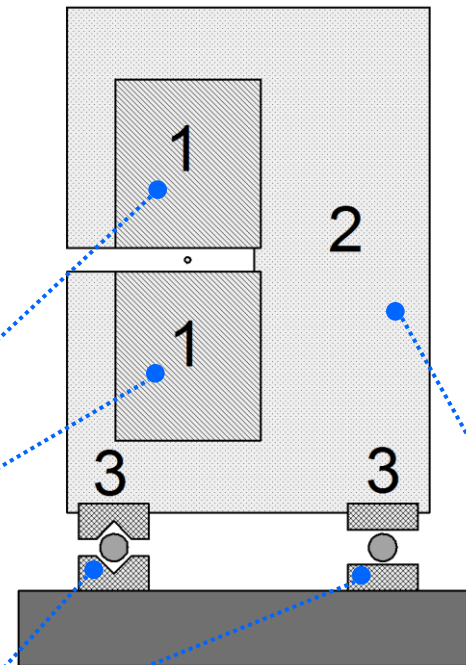
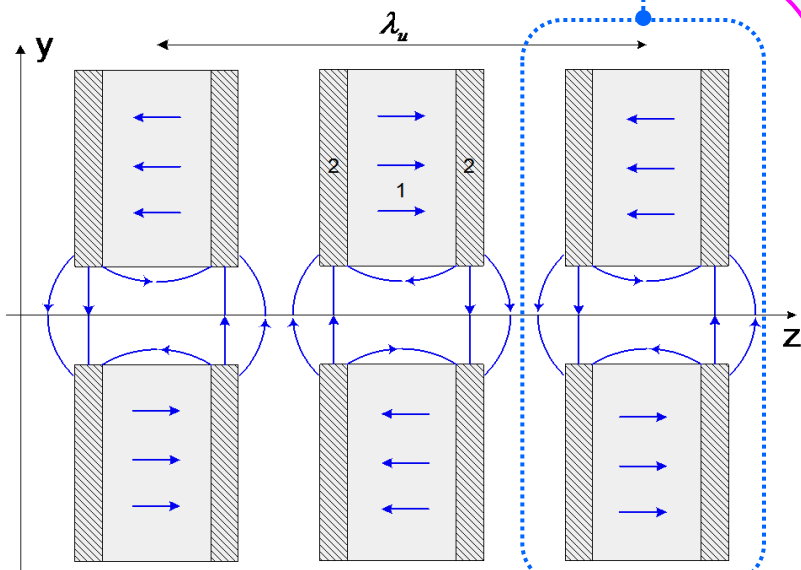
Replace by simple fitting formula

$$F = \frac{H_c^2 (\lambda_u)_{\min} P}{8\pi 2\pi} \arcsin \left( \frac{2t}{\lambda_u} \right)$$



## ***b.*** Possible design scheme

Side view of a half-period of the fixed-gap VPU

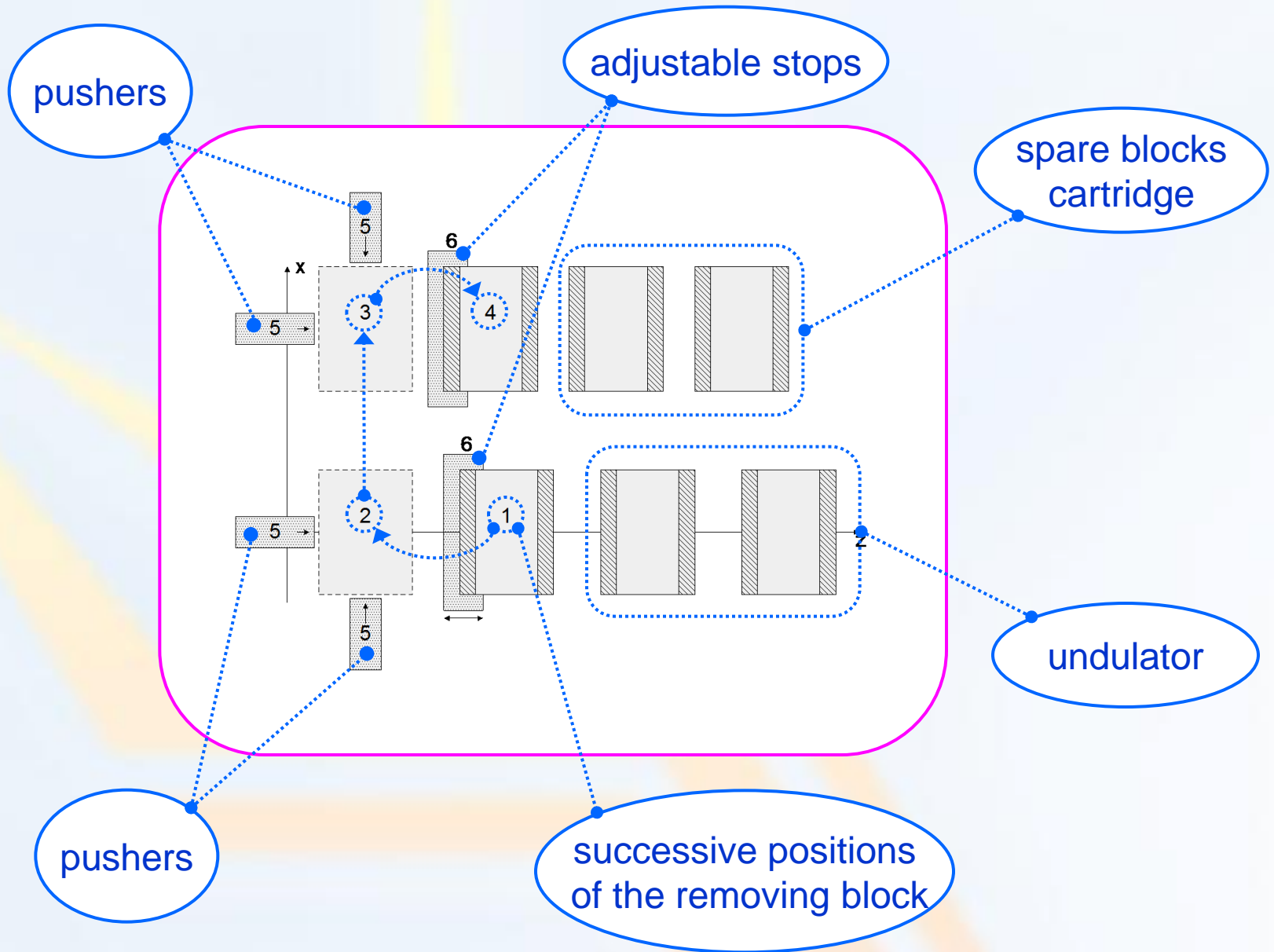


poles

slideway bearing

support plate

# Variable Period Number Option



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## **a. Generation of spontaneous radiation**

The same minimal gap & the same radiation wavelength

### **Variable gap**

Large  $K > 1$  is required to adjust wavelength

One needs larger undulator period

**One needs larger beam energy**

### **Variable period**

One can use undulator with  $K \sim 0.1$

The undulator period can be smaller

**One can use smaller beam energy**

## a. Generation of spontaneous radiation

$$g = 1 \text{ cm}, \lambda_0 = 0.1 \text{ nm}$$

**Variable gap**

$$K_{\max} = \sqrt{2}$$

$$\lambda_u = 2.4 \text{ cm}$$

$$E = 5.5 \text{ GeV}$$

**Variable period**

$$K = 0.1$$

$$(\lambda_u)_{\min} = 1.1 \text{ cm}$$

$$E = 3.7 \text{ GeV}$$

## ***b. High gain X-ray FEL application***

Minimal gap is limited by wakefields

### ***Variable gap***

For short wavelength one increases the gap

The K parameter decreases significantly

**The gain length increases dramatically**

### ***Variable period***

The gap is fixed, one decreases the period

Decreasing of K is less, the period is shorter

**The gain length increases much less**



# FEL simulation example

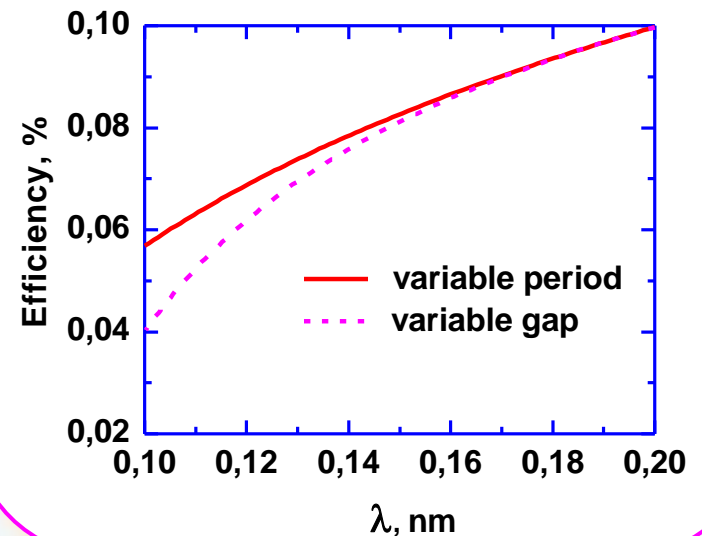
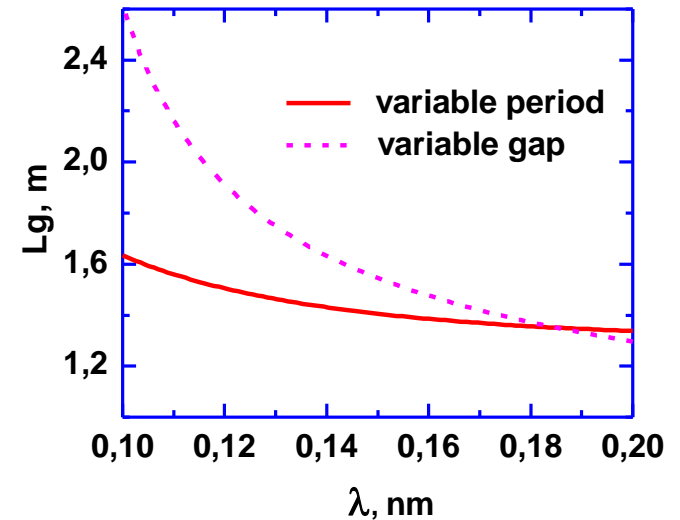
## Beam parameters

Electron energy, GeV	5.84
Beam current, kA	2
Normalized emittance, $\mu\text{m}$	0.2
Energy spread, %	0.01

## Undulator parameters

VPU minimal period, cm	1.6
VPU maximal period, cm	2.14
Minimal gap, cm	0.6
VGU period, cm	2.07
VGU maximal gap, cm	1.1

## Simulation results



# Conclusion

- 1. We considered the new design of permanent magnet undulators which allows to change undulator period.**
- 2. Variable period undulators have many advantages compared to conventional undulators.**
- 3. Application of variable period undulators can open new prospects for further improvements of accelerator-based radiation sources.**



**Thank you for your  
attention !**

**The end.**