Coherence properties of the radiation from x-ray free electron lasers

E.A. Schneidmiller, M.V. Yurkov
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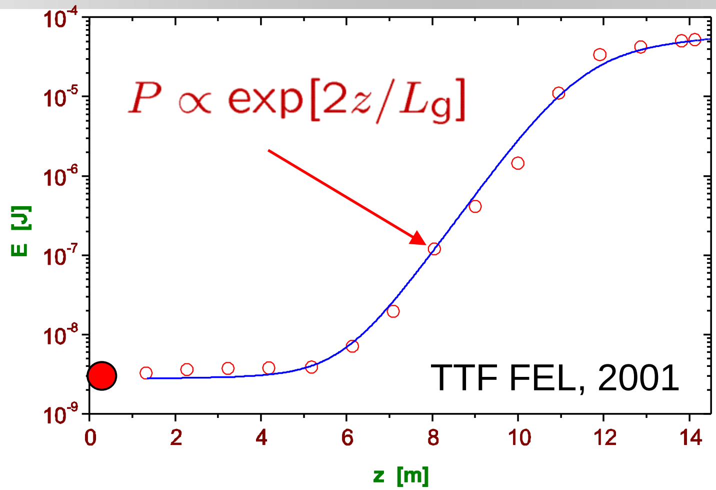
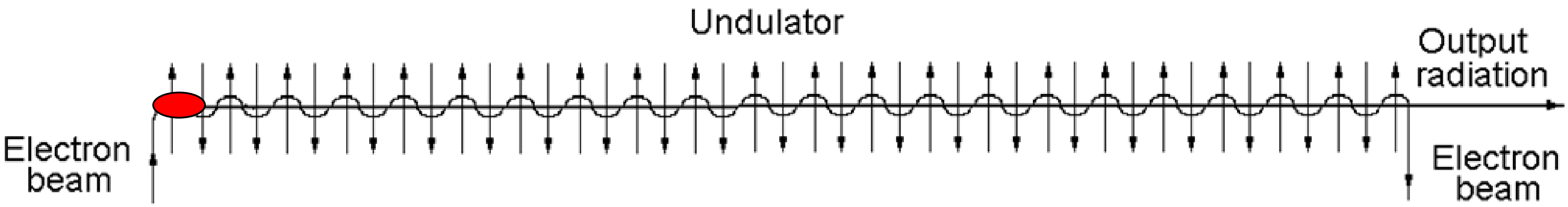
- Statistical properties.
- Longitudinal and transverse coherence.
- Higher harmonics.

Qualitative look at the radiation properties

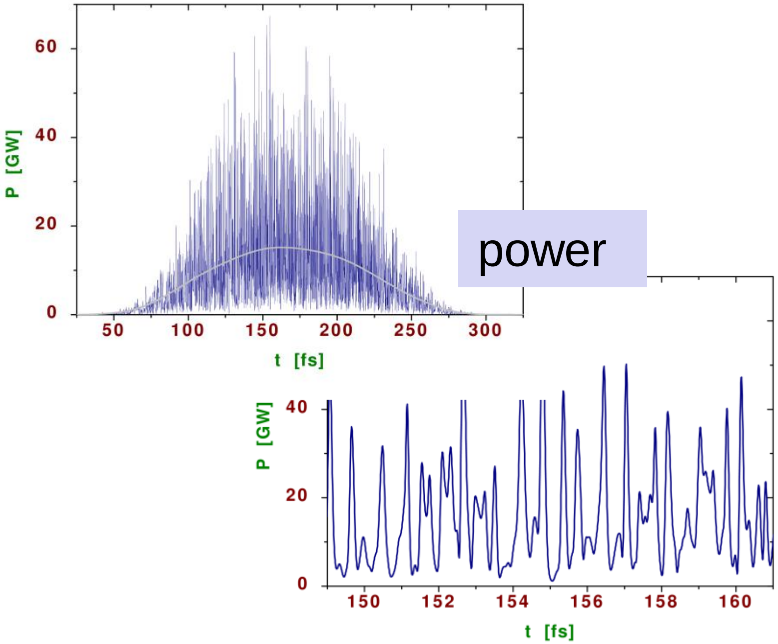


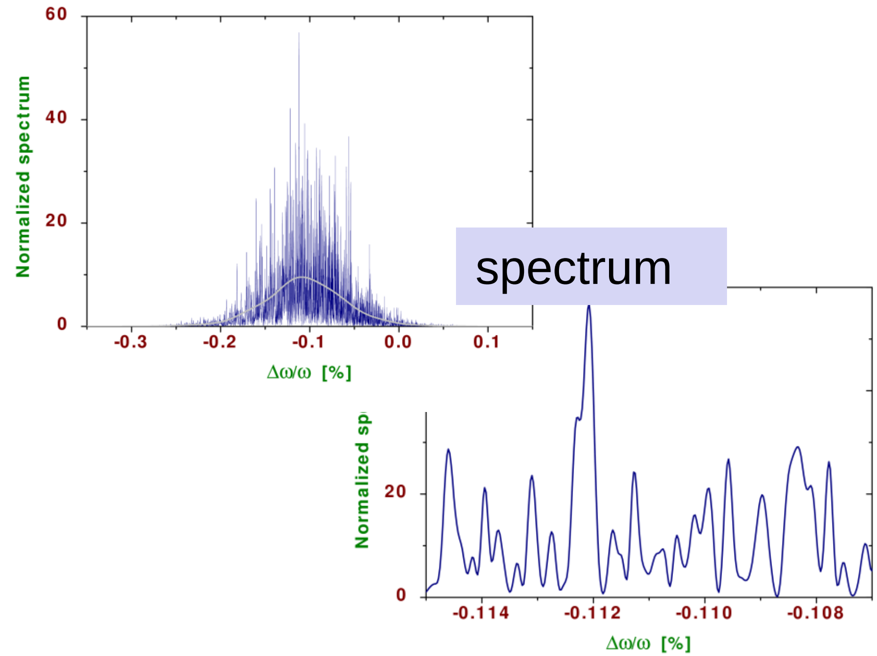


- An object of our analysis is an X-ray free electron laser (XFEL) – single pass FEL amplifier starting from shot noise in the electron beam. It is frequently named as SASE FEL (Self Amplified Spontaneous Emission FEL).
- Amplification process starts from the shot noise in the electron beam.
- The FEL collective instability in the electron beam produces an exponential growth of the modulation of the electron density on the scale of undulator radiation wavelength, and exponential growth of the radiation power.

Ya.S. Derbenev, A.M. Kondratenko, and E.L. Saldin, Nucl. nstrum. and Methods 193(1982)415

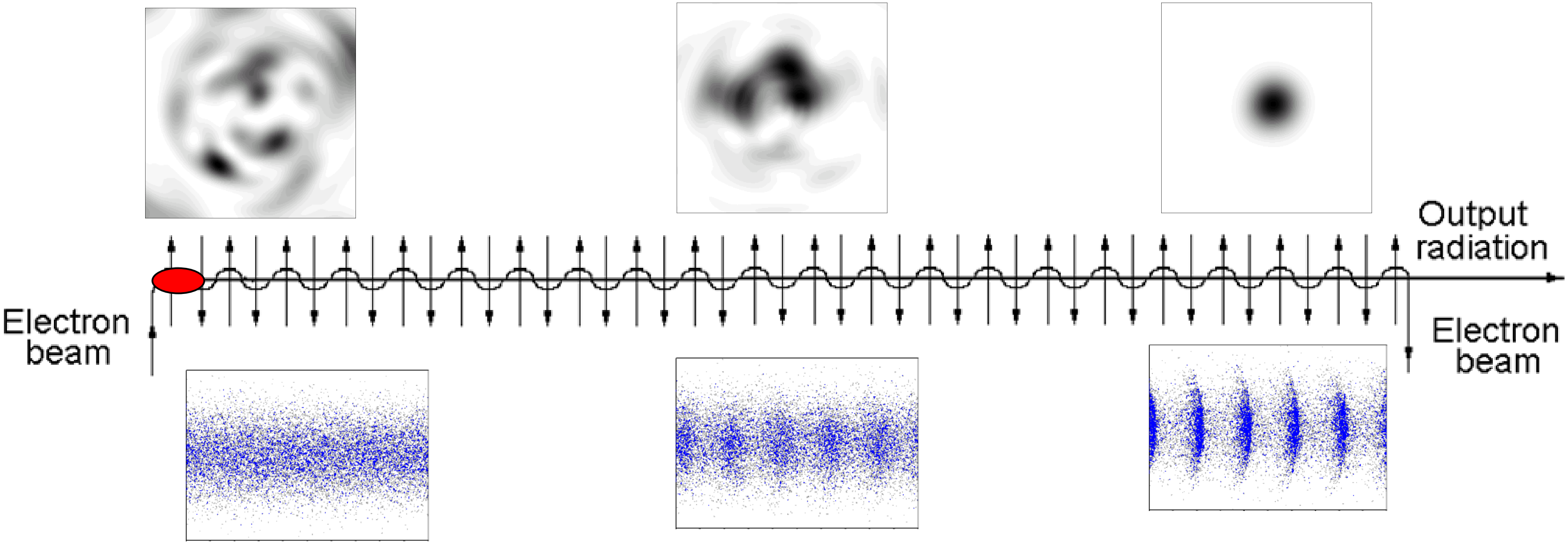
Qualitative look at the radiation properties





- Radiation generated by SASE FEL consists of wavepackets (spikes). Typical duration of the spike in the time domain is about coherence time τ_c.
- Spectrum also exhibits spiky structure. Spectrum width is inversely proportional to the coherence time, $\Delta\omega \sim 1/\tau_c$. Typical width of a spike in a spectrum is inversely proportional to the pulse duration T.

Qualitative look at the radiation properties

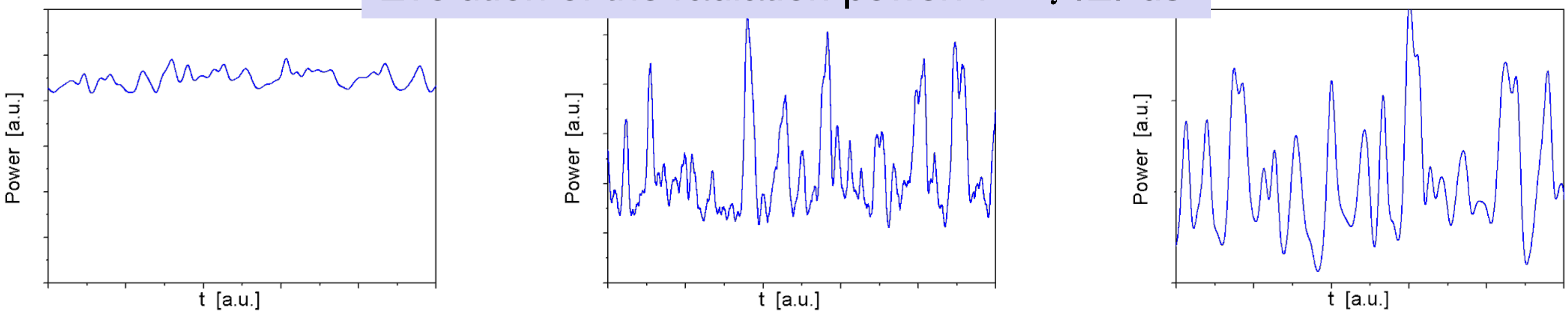


- Longitudinal coherence is formed due to slippage effects (electromagnetic wave advances electron beam by one wavelength $\lambda = \lambda_u (1+K^2)/(2\gamma^2)$ while electron beam passes one undulator period λ_u. Typical figure of merit is relative slippage of the radiation with respect to the electron beam on a scale of field gain length ➔ coherence time $\tau_c \sim \lambda L_g/(c\lambda_u)$.
- Transverse coherence is formed due to diffraction effects. Typical figure of merit is ratio of the transverse area of the electron beam to the diffraction expansion of the radiation on a scale of field gain length, $\sigma^2/(\lambda L_g)$.
- Amplification process selects narrow band of the radiation, coherence time is increased, and spectrum is shrinked. Transverse coherence is improved due to the mode selection process:

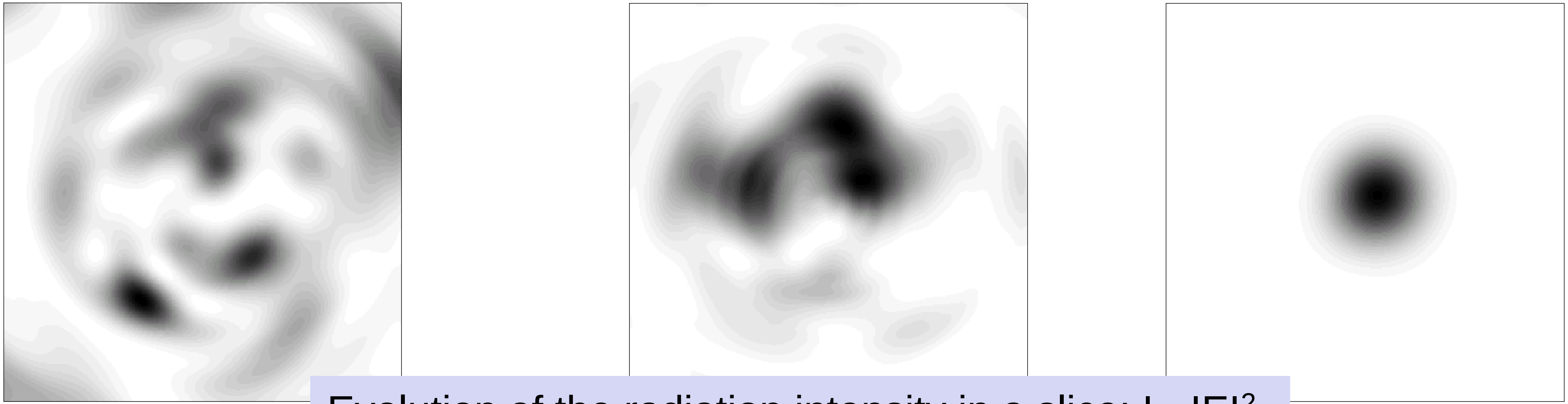
$$E_x + iE_y = \int d\omega \exp[i\omega(z/c - t)] \times \sum_{n,k} A_{nk}(\omega, z)\Phi_{nk}(r, \omega) \exp[\Lambda_{nk}(\omega) z + in\phi]$$

Statistics and probability distributions

Evolution of the radiation power: $P = \int |E|^2 ds$



$z = 0.1\ z_{sat}$ $z = 0.5\ z_{sat}$ $z = z_{sat}$

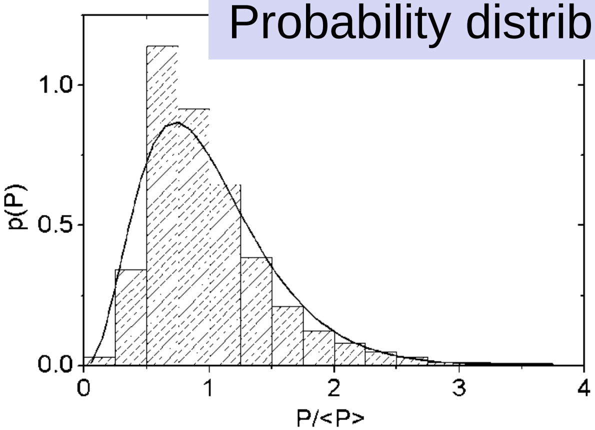


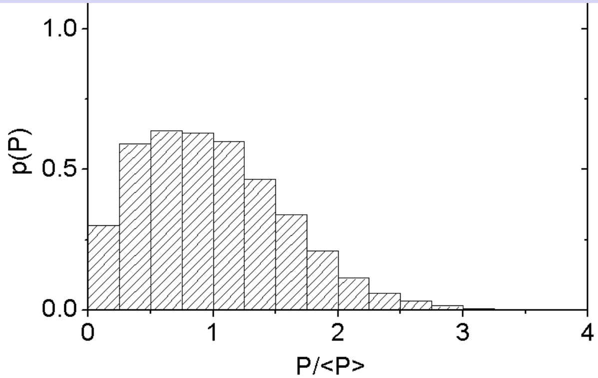
Evolution of the radiation intensity in a slice: $I = |E|^2$

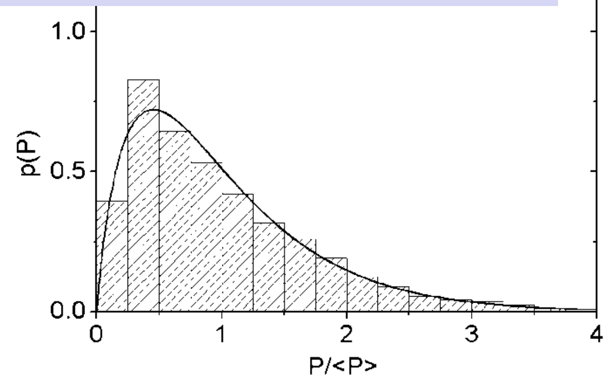
Statistics and probability distributions

Linear regime | Saturation | Deep nonlinear regime

Probability distributions of the instantaneous radiation power: $P = \int |E|^2 ds$

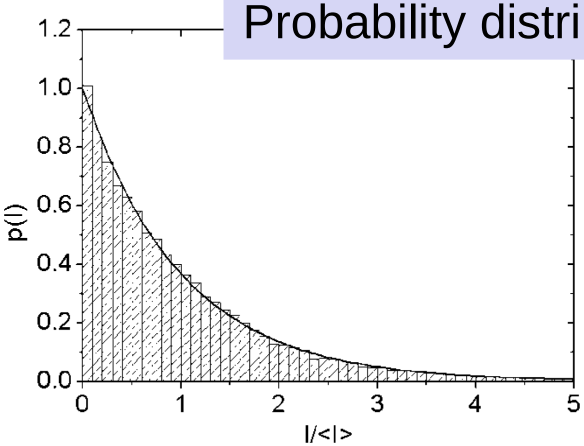


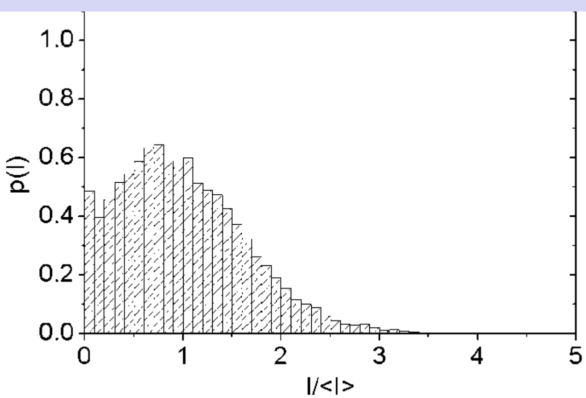


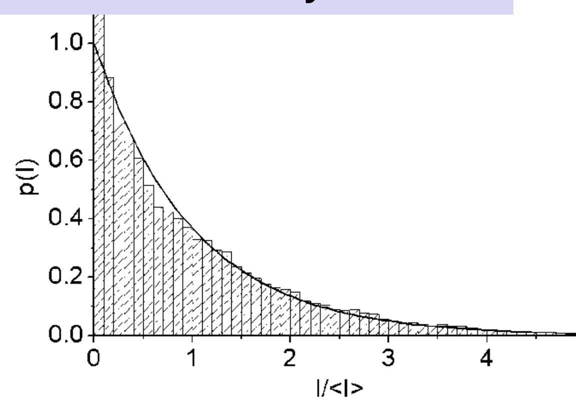


- Probability distributions look more elegant and seem to be described by relatively simple functions, at least in the linear and deep nonlinear regime.

Probability distributions of the instantaneous radiation intensity $I = |E|^2$







Statistics and probability distributions

- The origin of this simplicity is fundamental. Statistical properties of the shot noise in the electron beam define statistical properties of the output radiation from a SASE FEL.
- Fluctuations of the electron beam current density serve as input signals in a SASE FEL. These fluctuations always exist in the electron beam due to the effect of shot noise.
- The electron beam current consists of moving electrons randomly arriving at the entrance of the undulator: $I(t) = (-e)\sum_{k=1}^{N}\delta(t-t_k)$. Fourier harmonic of the current is just a sum of complex phasors:

$$\bar{I}(\omega) = \int\limits_{-\infty}^{\infty} e^{i\omega t} I(t)dt = (-e)\sum_{k=1}^{N} e^{i\omega t_k}$$

- The Fourier transformation of the input current, $\bar{I}(\omega)$, is the sum of large number of complex phasors with random phases $\phi_k = \omega t_k$. Thus, harmonics of the electron beam current are described with gaussian statistics.
- FEL amplifier operating in the linear regime is just linear filter, $\bar{E}(\omega) = H_A(\omega-\omega_0)\bar{I}(\omega)$, which does not change statistics.
- This kind of radiation is usually referred to as completely chaotic polarized light, a well known object in the field of statistical optics.

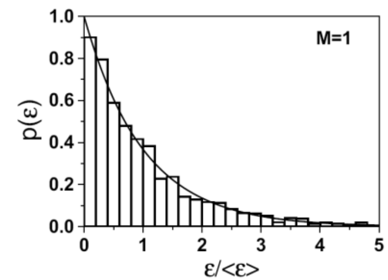
E.L. Saldin, E.A. Schneidmiller, M.V. Yurkov, The Physics of Free Electron Lasers

Statistics and probability distributions

Radiation from SASE FEL operating in the high gain linear regime possesses all the features of completely chaotic polarized light:

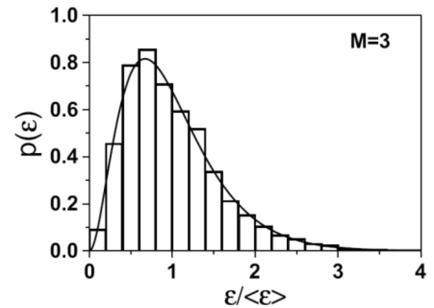
- The higher order correlation functions are expressed via the first order correlation function. For instance,

$$g_2(t-t') = 1 + |g_1(t-t')|^2 , \qquad g_2(\Delta\omega) = 1 + |g_1(\Delta\omega)|^2$$



- The probability density distribution of the instantaneous radiation intensity $I = |E|^2$, or of the radiation energy filtered by narrow-band monochromator follows the negative exponential distribution:

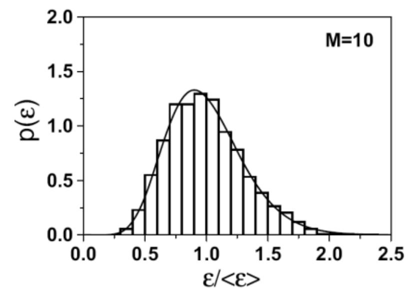
$$p(I) = \frac{1}{<I>} \exp\left(-\frac{I}{<I>}\right)$$



- The probability density function of the integrals of the instantaneous power density (instantaneous radiation power P, or pulse energy) follows the gamma distribution

$$p(P) = \frac{M^M}{\Gamma(M)} \left(\frac{P}{<P>}\right)^{M-1} \frac{1}{<W>} \exp\left(-M\frac{P}{<P>}\right) ,$$

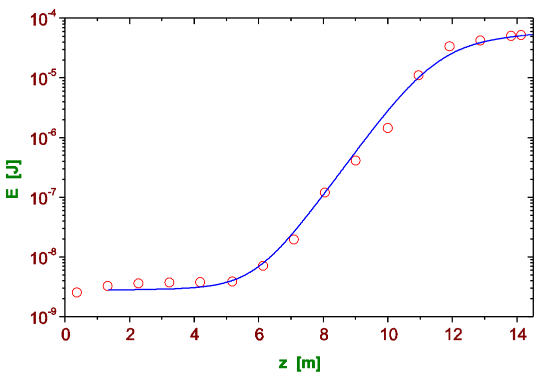
$$M^{-1} = \sigma_P^2 = <(P-<P>)^2> / <P>^2$$

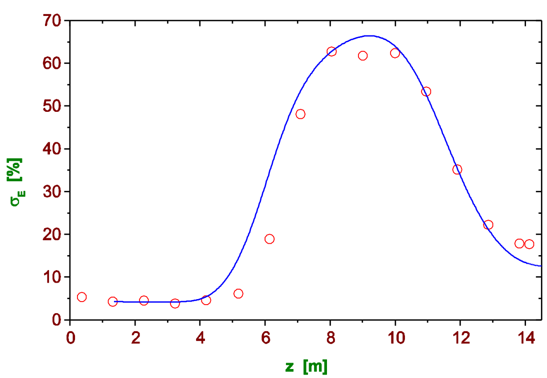


- For completely chaotic polarized light parameter M has a clear physical interpretation – it is the number of modes.

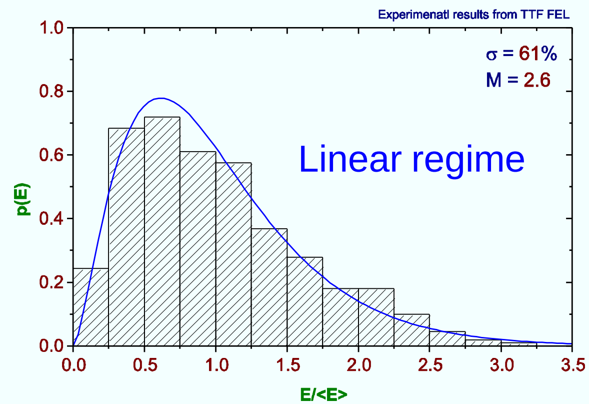
Statistics and probability distributions: Experimental results from TTF FEL/FLASH

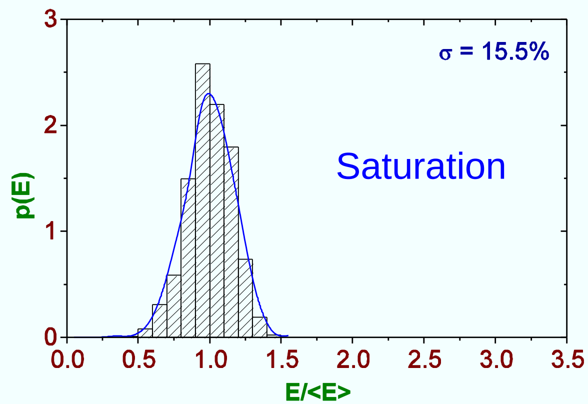
Gain curve and dispersion of the radiation energy



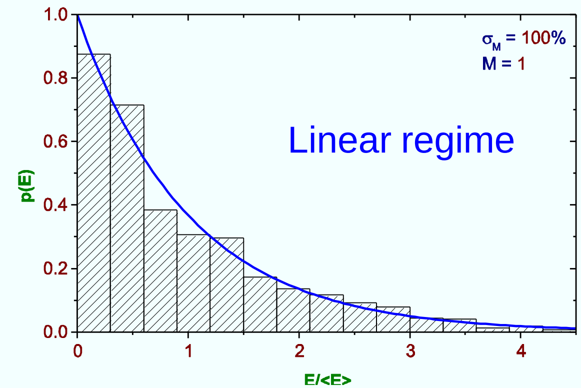


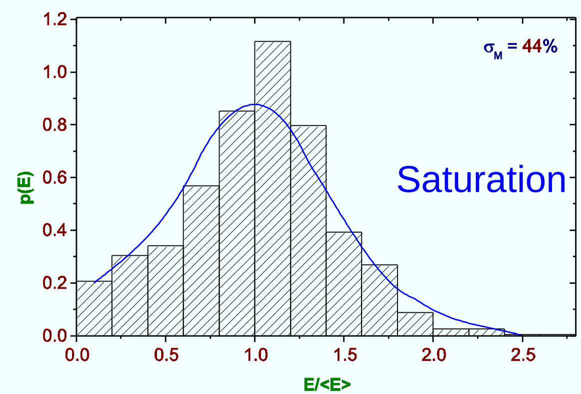
Probability distribution of the energy in the radiation pulse





Probability distribution of the energy after narrow band monochromator





V. Ayvazyan et al., Nucl. Instrum. and Methods A 507 (2003)368

Strict definitions of statistical characteristics

- The first order time correlation function and coherence time:

$$g_1(\vec{r}, t - t') = \frac{\langle \tilde{E}(\vec{r}, t)\tilde{E}^*(\vec{r}, t')\rangle}{\left[\langle |\ \tilde{E}(\vec{r}, t)\ |^2\rangle\langle |\ \tilde{E}(\vec{r}, t')\ |^2\rangle\right]^{1/2}}\ , \qquad \tau_c = \int\limits_{-\infty}^{\infty} |g_1(\tau)|^2 \, \mathrm{d}\,\tau\ .$$

- The first-order transverse correlation function and degree of transverse coherence:

$$\gamma_1(\vec{r}_\perp, \vec{r'}_\perp, z, t) = \frac{\langle \tilde{E}(\vec{r}_\perp, z, t)\tilde{E}^*(\vec{r'}_\perp, z, t)\rangle}{\left[\langle |\tilde{E}(\vec{r}_\perp, z, t)|^2\rangle\langle |\tilde{E}(\vec{r'}_\perp, z, t)|^2\rangle\right]^{1/2}}\ .$$

$$\zeta = \frac{\int\int |\gamma_1(\vec{r}_\perp, \vec{r'}_\perp)|^2 \langle I(\vec{r}_\perp)\rangle\langle I(\vec{r'}_\perp)\rangle \, \mathrm{d}\,\vec{r}_\perp \, \mathrm{d}\,\vec{r'}_\perp}{\left[\int \langle I(\vec{r}_\perp)\rangle \, \mathrm{d}\,\vec{r}_\perp\right]^2}\ .$$

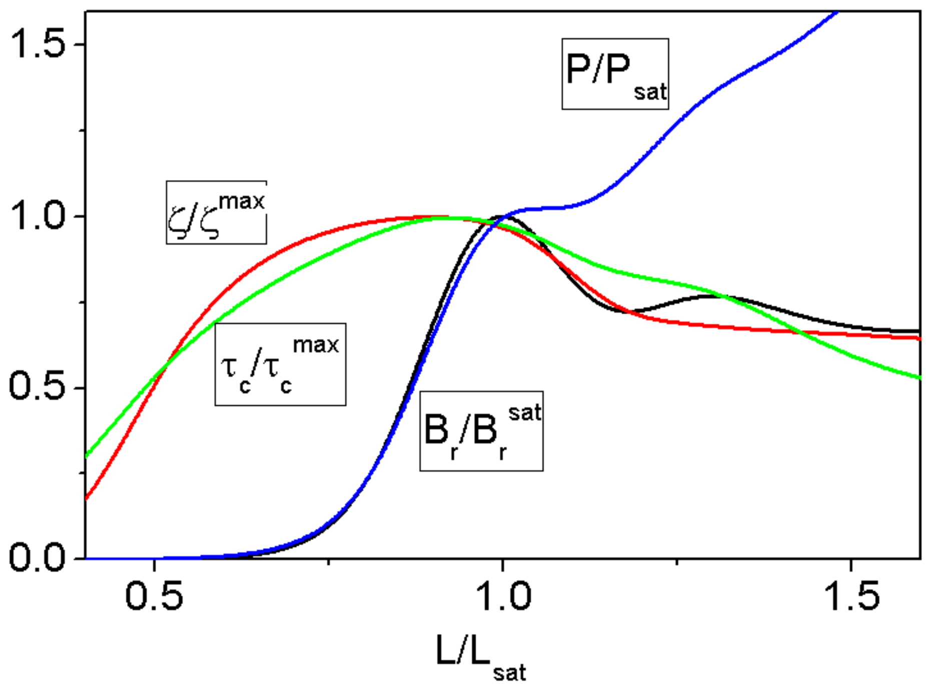
- Degeneracy parameter – the number of photons per mode (coherent state):

$$\delta = \dot{N}_{ph} \tau_c \zeta\ .$$

- Peak brilliance is defined as a transversely coherent spectral flux:

$$B_r = \frac{\omega \, \mathrm{d}\,\dot{N}_{ph}}{\mathrm{d}\,\omega} \frac{\zeta}{\left(\frac{\lambda}{2}\right)^2} = \frac{4\sqrt{2}c}{\lambda^3}\,\delta\ .$$

Qualitative look at the evolution of the radiation properties in XFEL



Radiation power

Degree of transverse coherence

Coherence time

Brilliance

- Radiation power continues to grow along the undulator length.
- Degree of transverse coherence and coherence time reach their maximum values in the end of exponential regime.
- Brilliance reaches maximum value at the saturation point.

Optimized XFEL

- Radiation of XFEL operating in the high gain exponential regime can be presented as a set of self-reproducing radiation modes:

$$\tilde{E} = \int \mathrm{d}\omega \exp[i\omega(z/c - t)] \times \sum_{n,k} A_{nk}(\omega, z)\Phi_{nk}(r,\omega)\exp[\Lambda_{nk}(\omega)z + in\phi] \ .$$

- XFEL is optimized for maximum gain of fundamental (TEM_{00}) beam radiation mode. Gain length and optimum beta function (for negligible energy spread):

$$L_{\mathrm{g}} \simeq 1.67 \left(\frac{I_A}{I}\right)^{1/2} \frac{(\epsilon_n \lambda_{\mathrm{w}})^{5/6}}{\lambda^{2/3}} \frac{(1+K^2)^{1/3}}{K A_{JJ}} \ ,$$

$$\beta_{\mathrm{opt}} \simeq 11.2 \left(\frac{I_A}{I}\right)^{1/2} \frac{\epsilon_n^{3/2} \lambda_{\mathrm{w}}^{1/2}}{\lambda K A_{JJ}} \ .$$

- Application of similarity techniques to the FEL equations gives elegant result: for the case of negligible energy spread characteristics of optimized XFEL written down in the normalized form are functions of two parameters, ratio of geomtrical emittance to the wavelength, and number of electrons in the volume of coherence:

$$\hat{\epsilon} = 2\pi\epsilon/\lambda \ , \qquad N_{\mathrm{c}} = I L_{\mathrm{g}} \lambda/(e \lambda_{\mathrm{w}} c) \ .$$

- Dependence of the FEL characterestics on N_{c} is very slow, in fact, logarithmic. Approximately, with logarithmic accuracy they depend only on $\hat{\epsilon}$.

E.L. Saldin, E.A. Schneidmiller, M.V. Yurkov, Opt. Commun. 235(2004)415, 281(2008)1179; 281(2008)4727.

Optimized XFEL at saturation

Saturation length:

$$\hat{L}_{\mathrm{sat}} = \Gamma L_{\mathrm{sat}} \simeq 2.5 \times \hat{\epsilon}^{5/6} \times \ln N_{\mathrm{c}} \ ,$$

FEL efficiency:

$$\hat{\eta} = P/(\bar{\rho} P_{\mathrm{b}}) \simeq 0.17/\hat{\epsilon} \ ,$$

Coherence time and rms spectrum width:

$$\hat{\tau}_{\mathrm{c}} = \bar{\rho}\omega\tau_{\mathrm{c}} \simeq 1.16 \times \sqrt{\ln N_{\mathrm{c}}} \times \hat{\epsilon}^{5/6} \ , \quad \sigma_{\omega} \simeq \sqrt{\pi}/\tau_{\mathrm{c}} \ .$$

Degree of transverse coherence:

$$\zeta_{\mathrm{sat}} \simeq \frac{1.1\hat{\epsilon}^{1/4}}{1 + 0.15\hat{\epsilon}^{9/4}} \ ,$$

Degeneracy parameter:

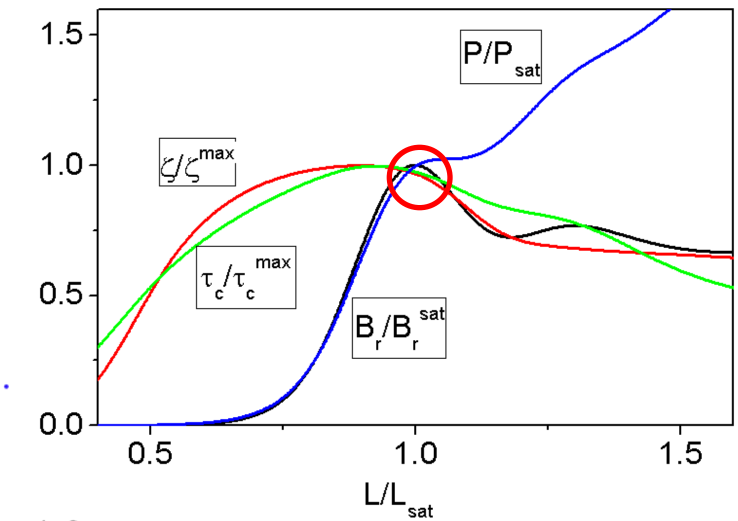
$$\hat{\delta} = \hat{\eta}\zeta\hat{\tau}_{\mathrm{c}}$$

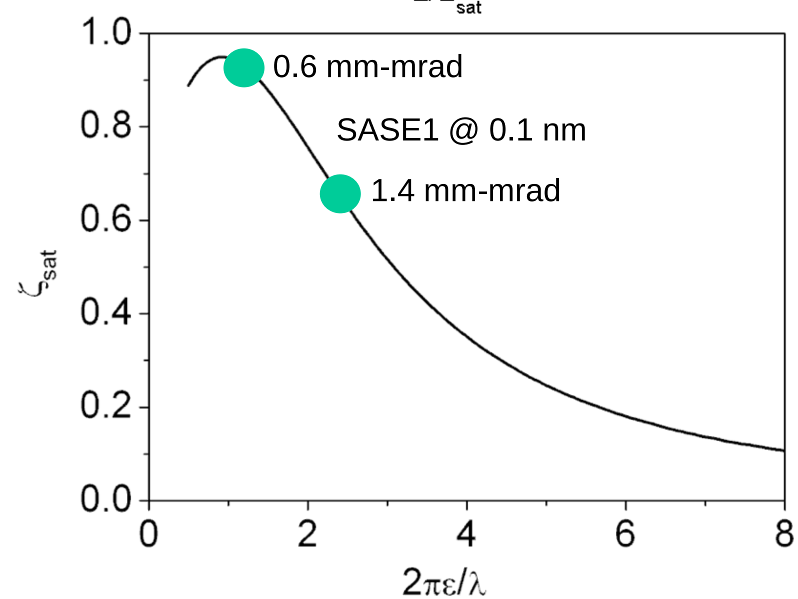
Brilliance:

$$B_r = \frac{\omega d\dot{N}_{ph}}{d\omega} \frac{\zeta}{\left(\frac{\lambda}{2}\right)^2} = \frac{4\sqrt{2}c}{\lambda^3} \frac{P_{\mathrm{b}}}{\hbar\omega^2} \hat{\delta} \ .$$

Normalizing parameters:

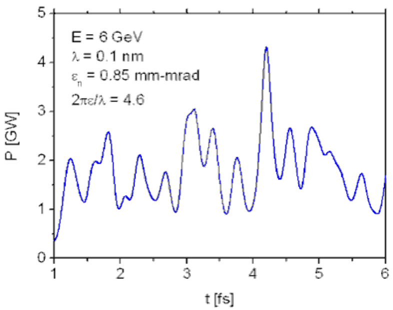
$$\Gamma = \left[\frac{I}{I_{\mathrm{A}}} \frac{8\pi^2 K^2 A_{\mathrm{JJ}}^2}{\lambda\lambda_{\mathrm{w}}\gamma^3}\right]^{1/2} \ , \quad \bar{\rho} = \frac{\lambda_{\mathrm{w}}\Gamma}{4\pi} \ .$$

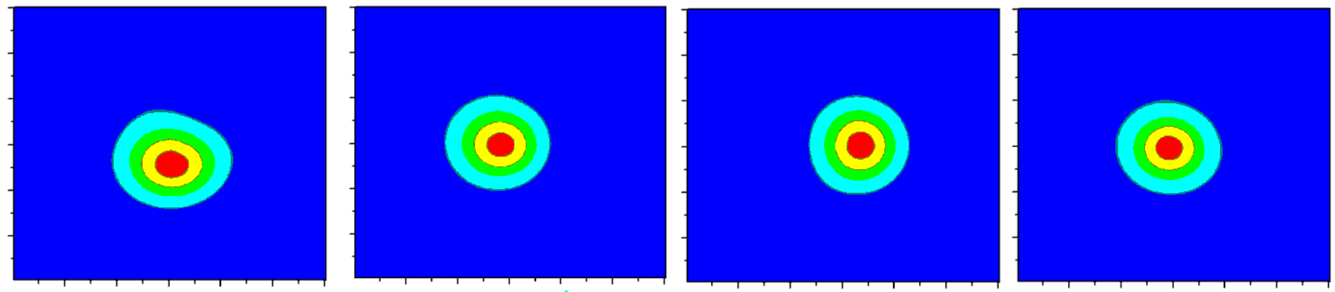




E.L. Saldin, E.A. Schneidmiller, M.V. Yurkov, Opt. Commun. 281(2008)1179; 281(2008)4727 ; NJP 12 (2010) 035010

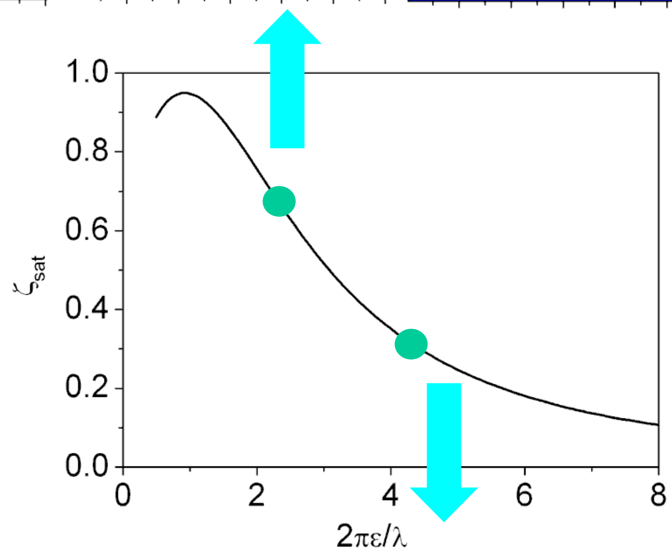
Qualitative look at the transverse coherence





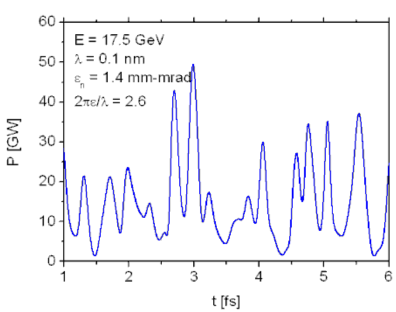
$2\pi\varepsilon/\lambda = 2.5$, $\zeta = 0.65$

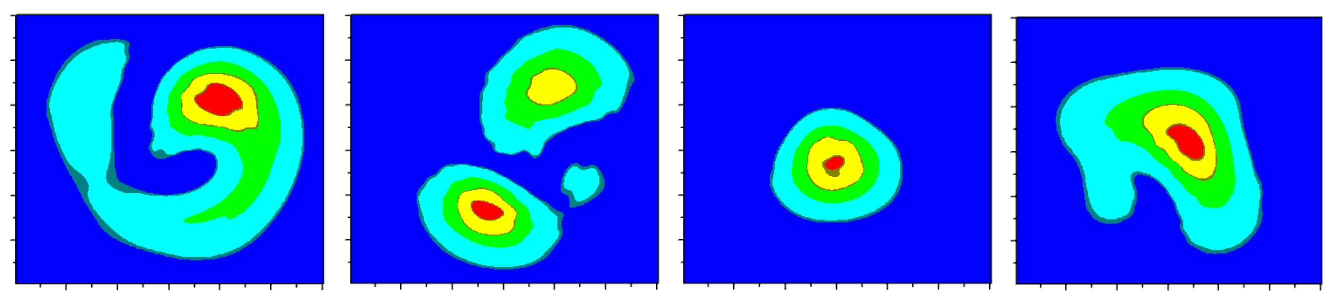
t = 2 fs, 2.7 fs, 3 fs, 4.1 fs



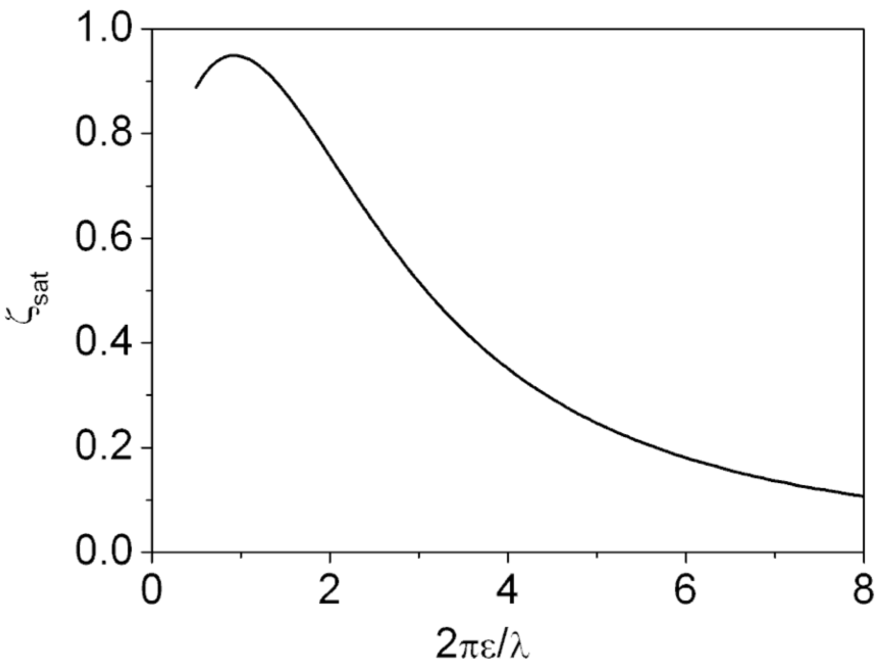
$2\pi\varepsilon/\lambda = 4.5$, $\zeta = 0.4$

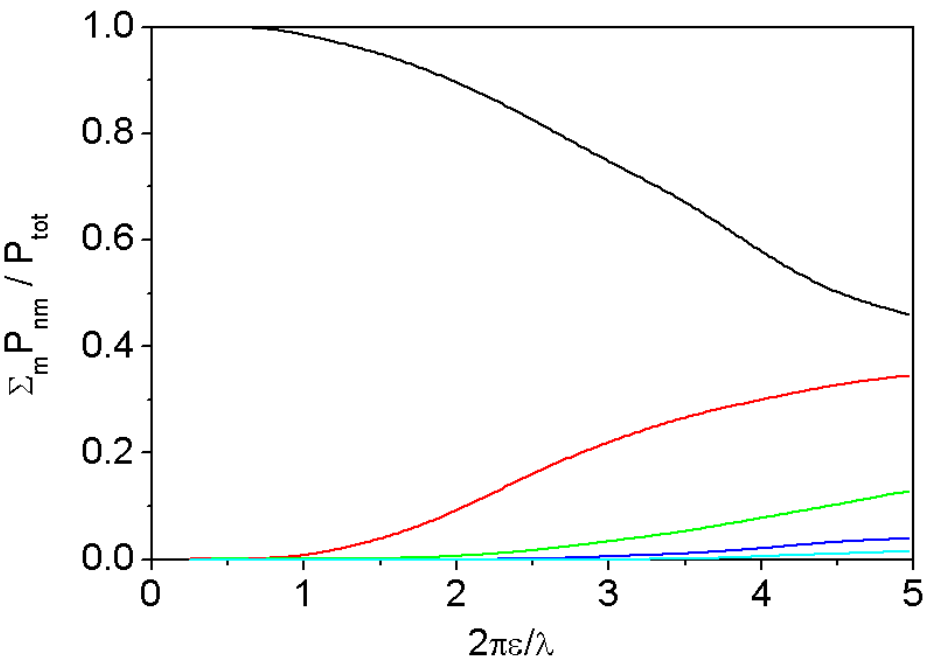
t = 1.7 fs, 2.4 fs, 3.1 fs, 4.2 fs





Transverse coherence

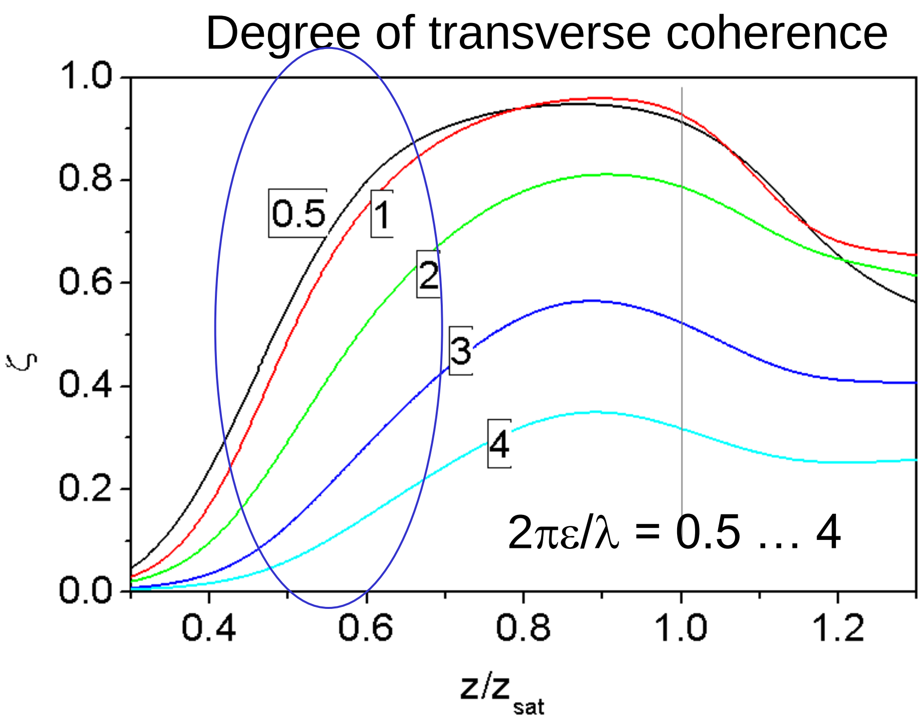


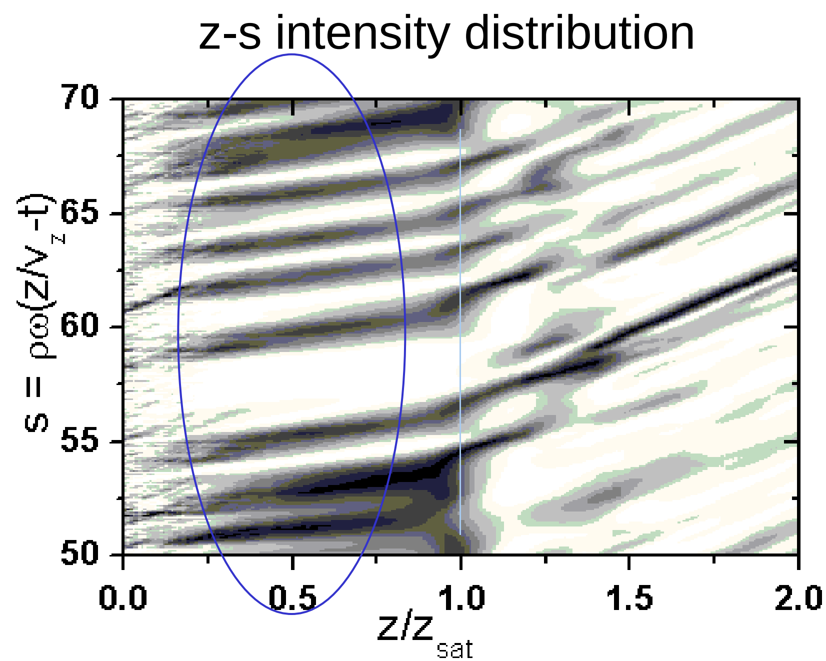


$$E_x + iE_y = \int \mathrm{d}\omega \exp[i\omega(z/c - t)] \times \sum_{n,k} A_{nk}(\omega, z)\Phi_{nk}(r,\omega) \exp[\Lambda_{nk}(\omega) z + in\phi]$$

- Contribution to the total saturation power of the radiation modes with higher azimuthal indexes 1, 2, 3, 4… grows with the emittance.
- For large emittances the degree of transverse coherence in the saturation degrades due to poor mode selection and falls down as inverse sqare of emittance: $\zeta \propto (\ln N_c/\hat{\epsilon})^2$.

Transverse coherence

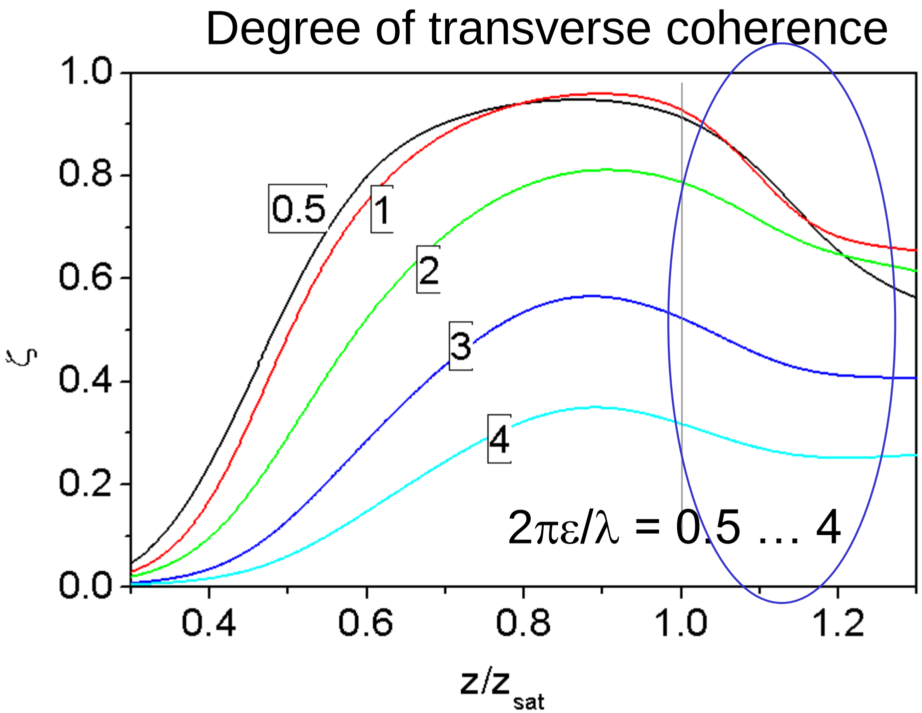


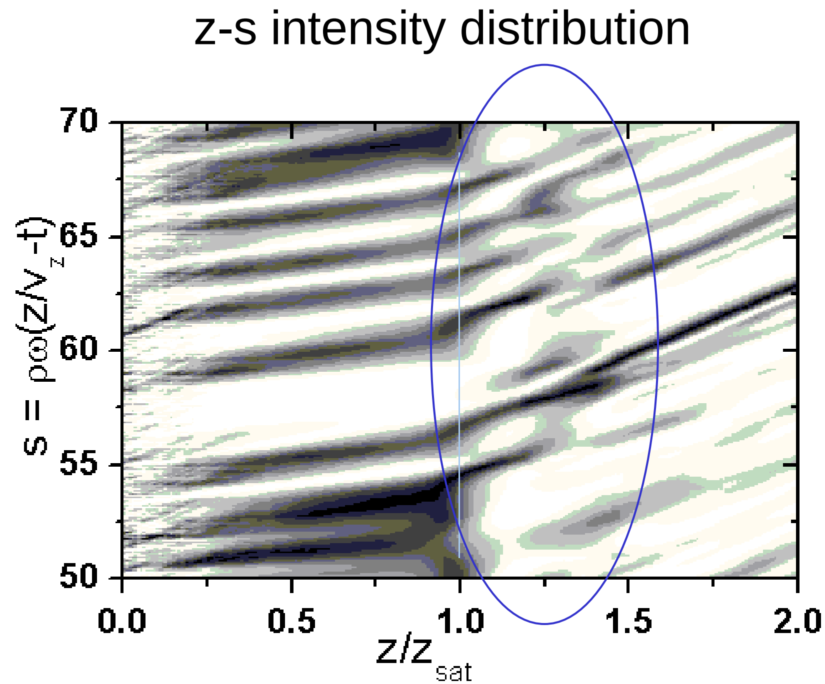


- For small emittances the degree of transverse coherence visibly differs from unity. This happens due to poor longitudinal coherence: radiation spikes move forward along the electron beam, and interact with those parts of the beam which have different amplitude/phase.
- Longitudinal coherence develops slowly with the undulator length (in fact as $z^{1/2}$), thus preventing full transverse coherence.
- The gain is higher for smaller emittances ➔ coherence time is smaller ➔ more stronger effect of transverse coherence degradation.

E.L. Saldin, E.A. Schneidmiller, M.V. Yurkov, Opt. Commun. 186(2000)185

Transverse coherence





- Poor longitudinal coherence is also responsible for the fast degradation of the transverse coherence in the nonlinear regime.
- In the linear exponential regime group velocity of spikes (ds/dz) is visibly less than the velocity of light due to strong interaction with the electron beam. In the nonlinear regime group velocity of spikes approaches velocity of light due to weak interaction with the electron beam.
- Radiation spikes move forward faster along the electron beam and start to interact with those parts of the beam which were formed due to interaction with different wavepackets.
- This process develops on the scale of the field gain length.

XFEL with planar undulator: odd harmonics

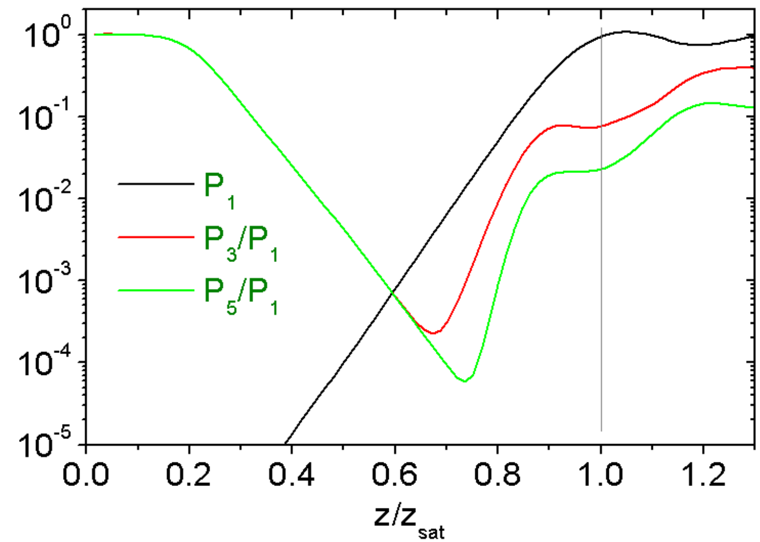
- In the saturation a universal dependency holds for the ratio of the power in the higher harmonics with respect to the fundamental one:

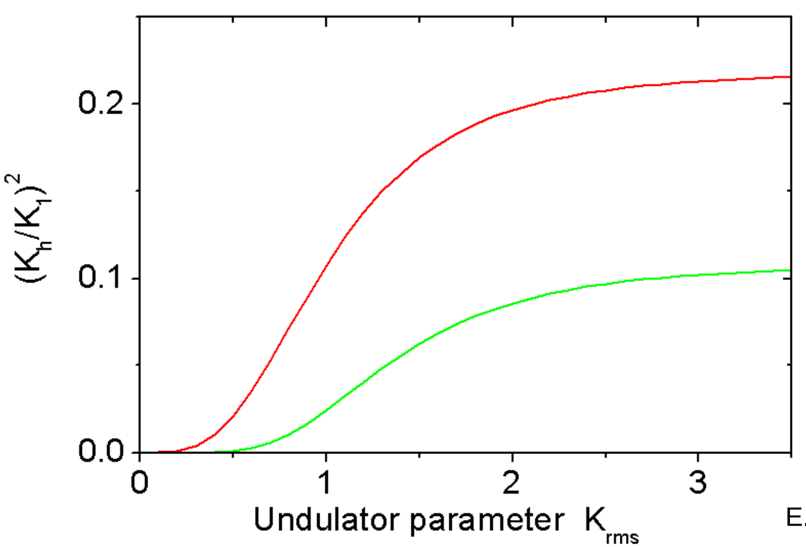
$$\frac{\langle W_3 \rangle}{\langle W_1 \rangle}\Big|_{\mathrm{sat}} = f_3(\hat{\Lambda}_{\mathrm{T}}^2) \times \frac{K_3^2}{K_1^2} \,,$$

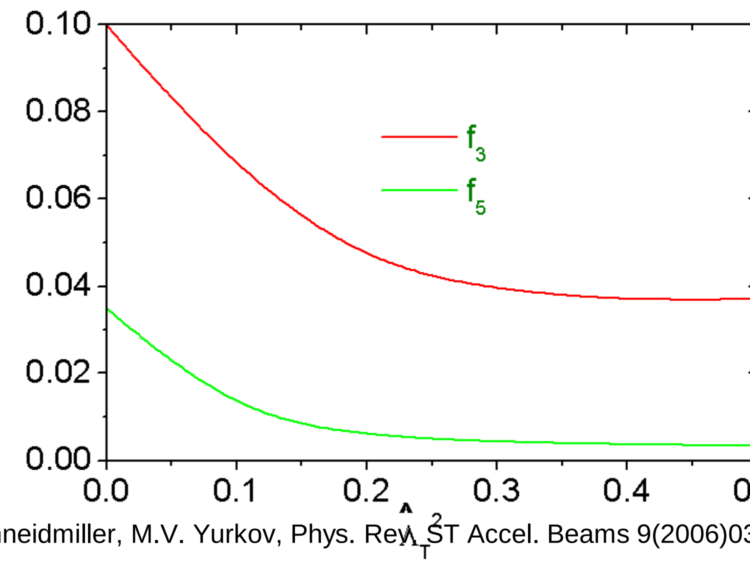
$$\frac{\langle W_5 \rangle}{\langle W_1 \rangle}\Big|_{\mathrm{sat}} = f_5(\hat{\Lambda}_{\mathrm{T}}^2) \times \frac{K_5^2}{K_1^2} \,.$$

$$K_h = K(-1)^{(h-1)/2}[J_{(h-1)/2}(Q) - J_{(h+1)/2}(Q)]$$

$$Q = K^2/[2(1+K^2)]$$







E.L. Saldin, E.A. Schneidmiller, M.V. Yurkov, Phys. Rev. ST Accel. Beams 9(2006)030702

XFEL with planar undulator: odd harmonics

- The statistics of the high-harmonic radiation from the SASE FEL operating in the linear regime changes significantly with respect to the fundamental harmonic (e.g., with respect to Gaussian statistics).

- The probability density function of the fundamental intensity W:

$$p(W) = \langle W \rangle^{-1} \exp(-W/\langle W \rangle)$$

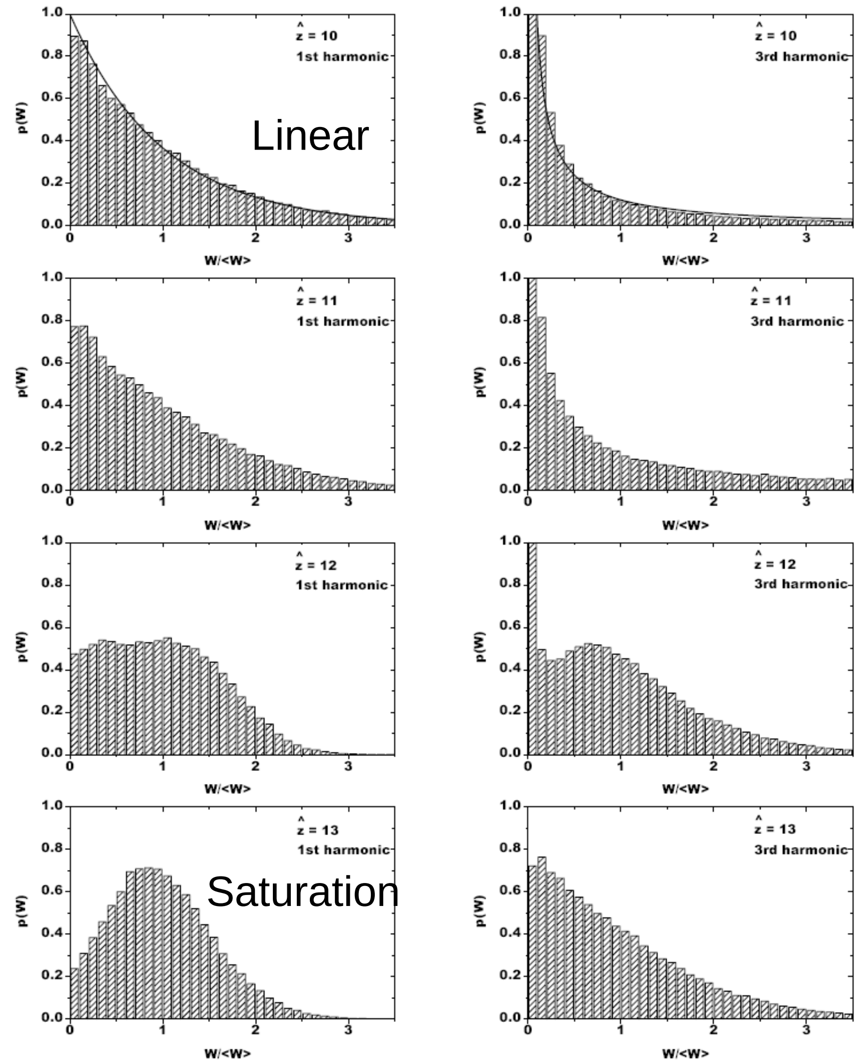
  is subjected to a transformation $z = (W)^n$.

- The probability density function $p(z)$ is:

$$p(z) = \frac{z}{n\langle W \rangle} z^{(1-n)/n} \exp(-z^{1/n}/\langle W \rangle) \ .$$

- Probability distribution of the instantaneous power of higher harmonics in saturation regime is close to the negative exponential distribution.

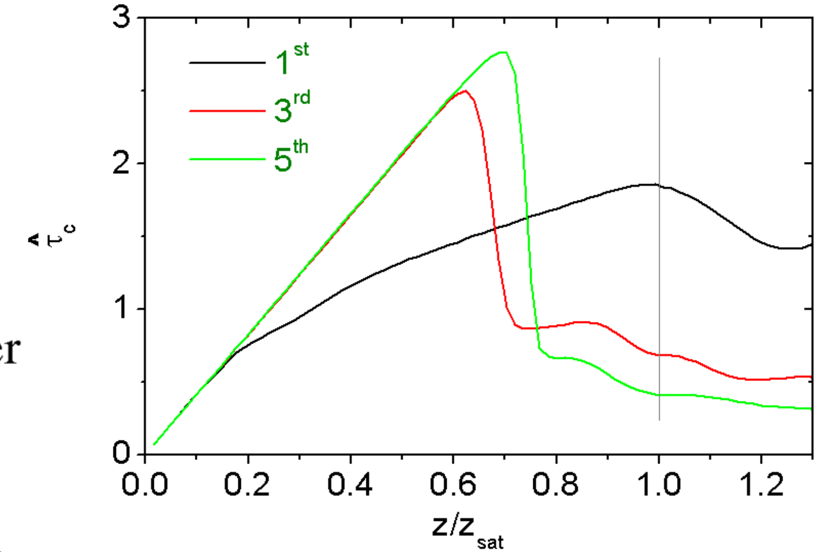
Evolution of probability distributions for the 1st and the 3rd harmonics



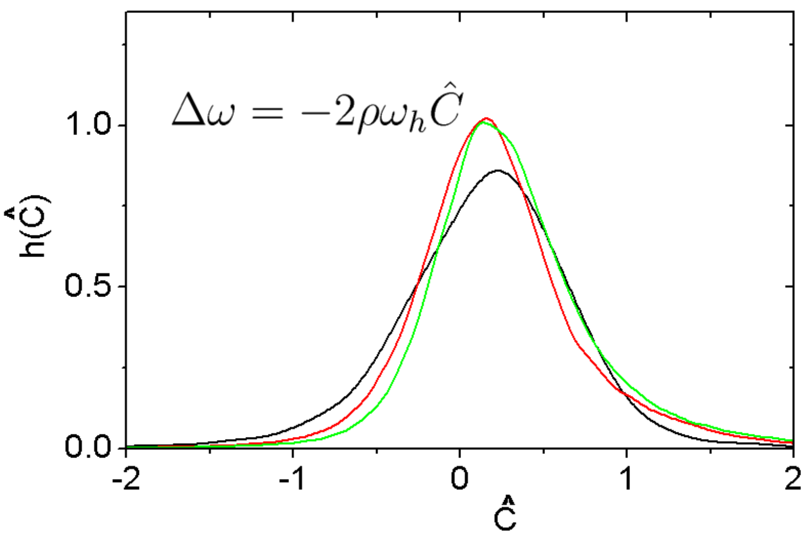
XFEL with planar undulator: odd harmonics

- Contributions of the higher odd harmonics to the FEL power for SASE FEL operating at saturation are universal functions of the undulator parameter K.
- Power of higher harmonics is subjected to larger fluctuations than that of the fundamental one.
- Probability distributions of the instantaneous power of higher harmonics in saturation regime is close to the negative exponential distribution.
- The coherence time in saturation falls inversely proportional to harmonic number: $\tau_c \propto 1/h$.
- Relative spectrum bandwidth remains constant with harmonic number.

Coherence time: 1^{st}, 3^{rd}, 5^{th}



Average spectra: 1^{st}, 3^{rd}, 5^{th}



Summary

- Statistical properties of the fundamental harmonic of the radiation from SASE FEL operating in the linear regime are described with gaussian statistics and correspond to those of completely chaotic polarized light.
- Statistics of the high-harmonic radiation from the SASE FEL operating in the linear regime changes significantly with respect to the gaussian statistics.
- Parameters of an optimized XFEL in the saturation are universal functions of the only parameter $2\pi\varepsilon/\lambda$ and are described by simple physical dependencies.
- The best properties in terms of transverse coherence are achieved for $2\pi\varepsilon/\lambda \sim 1$. At small values of the emittance the degree of transverse coherence is reduced due to strong influence of poor longitudinal coherence on a transverse one. At large values of the emittance the degree of transverse coherence degrades due to poor mode selection.
- XFEL driven by low energy (or, bad emittance) electron beam suffers from bad transverse coherence. Asymptotically degree of transverse coherence scales as $\zeta \propto (\ln N_c/\hat{\epsilon})^2$.

Thank you for your attention!