FEL Simulations: History, Status and Outlook

Sven Reiche, PSI

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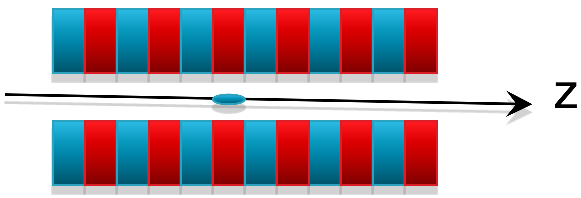
Thanks to

- Brian McNeil
- Bill Fawley (Author of GINGER)
- Luca Giannessi (Author of PERSEO)

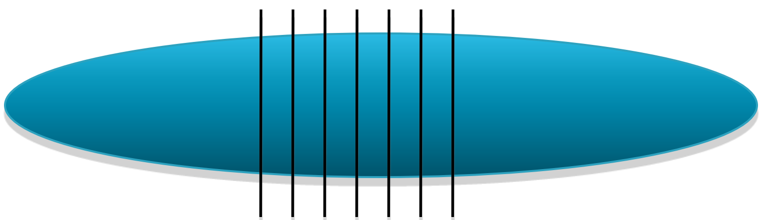
The Modeling Frame

- Undulator Field (~200 m)



Longitudinal position is independent variable. Undulator field and focusing become "time-dependent"

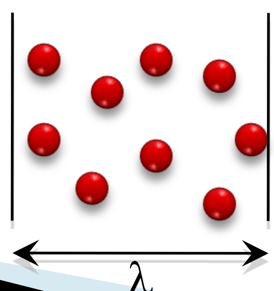
- Electron Beam (~50 μm)



Co-moving frame:

$$s = z - c\beta_0 t$$

- Electron Slice (~1 Å)



Slice thickness λ defines reference wavelength, which is not necessarily the resonant wavelength. Though both should be close to avoid strong drifts in slice:

$$\beta_0 = \frac{k}{k + k_u}$$

The FEL Equations (period-averaged)

- Particle Motion

$$\frac{d}{dz}\theta = (k + k_u)\beta_z - k$$ *(Ponderomotive Phase)*

$$\frac{d}{dz}\gamma = -k\frac{f_c K}{2\gamma}\left(ue^{i\theta} + cc\right) + \frac{e}{imc^2}\left(\tilde{E}e^{i\theta} - cc\right)$$

$$\frac{d}{dz}\vec{r}_\perp = \frac{\vec{p}_\perp}{\gamma}$$

$$\frac{d}{dz}\vec{p}_\perp = \underline{M}(z)\cdot\vec{r}_\perp$$

- Field Equation

Radiation field

$$\left[\nabla_\perp^2 + 2ik\left(\frac{\partial}{\partial z} + \frac{\partial}{c\partial t}\right)\right]u = i\frac{e^2\mu_0}{m}\sum_j \frac{f_c K}{\gamma_j}e^{-i\theta_j}$$

Space charge field

$$\left[\nabla_\perp^2 - \frac{k^2}{\langle\gamma_z\rangle^2}\right]\tilde{E} = i\frac{e}{\varepsilon_0}\frac{k}{\langle\gamma_z\rangle^2}\sum_j \delta\left(\vec{r} - \vec{r}_j\right)e^{-i\theta_j}$$

Limitations to the Current Model

- Resonant Behavior
  - Significant change in beam parameters occurs only over many periods, which allows to drop fast oscillating terms in the equation.

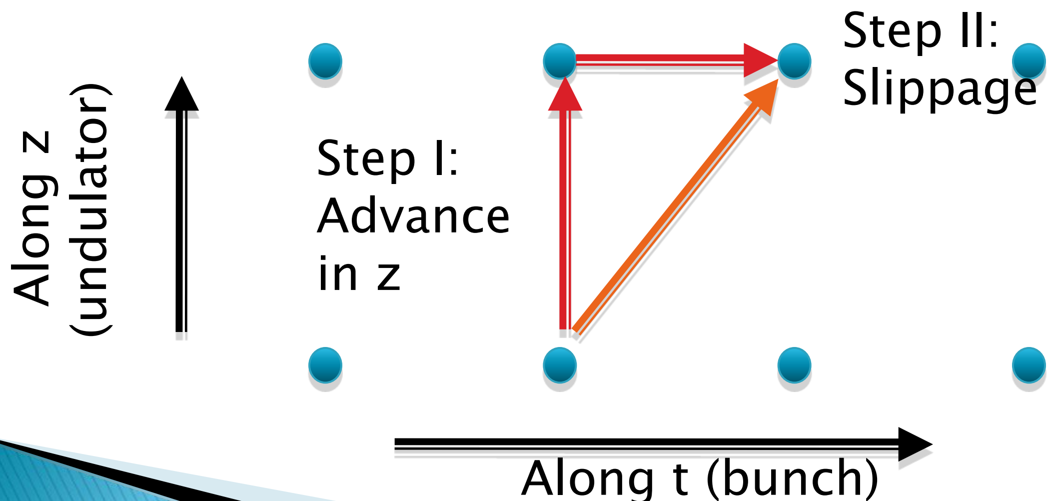
- Non-rapid evolution of energy modulation and radiation field
  - Particle motion averaged over undulator period and/or radiation wavelength.
  - Slow Varying Envelope Approximation of field equation.

- External effects (wakefields, undulator field taper) added *ad hoc* to the FEL equations.

Time-Dependent Simulations

$$\left[\nabla_\perp^2 + 2ik\left(\frac{\partial}{\partial z} + \frac{\partial}{c\partial t}\right)\right]u = i\frac{e^2\mu_0}{m}\sum_j \frac{f_c K}{\gamma_j} e^{-i\theta_j}$$

- Most codes evolved from “steady-state”, single frequency algorithm, where the time-derivative is dropped.
- Time-dependence is added by hand by enforcing the slippage.



In co-moving frame: $c\Delta t=(\lambda/\lambda_u)\Delta z$

Time-Dependent Simulations

- Advantage:
  - Allows for sequential progression through bunch (loop along bunch and undulator)
  - Very modest memory demand (in particular when inner loop is along undulator)
  - Fixed number of particles per slice, allows for very efficient parallelization of the codes

- Limitations:
  - Sampling of electron bunch parameters are at best on the radiation wavelength (steady-state algorithm)
  - No exchange of particles among slices
  - Suppression of gain towards the limits of the bandwidth due to underlying steady-state model.

Bunching and Harmonics

- Harmonics are described by their own equation and bandwidth are disjointed.
- Fundamental challenge is the particle distribution resolution on sub-wavelength scale to provide the correct shot noise on all harmonics:

$$\langle b_n \rangle = 0 \qquad \langle |b_n|^2 \rangle = \frac{1}{N_e}$$

bunching factor: $b_n = \frac{1}{N_m} \sum_{j=1}^{N_m} e^{in\theta_j}$

- Pure random distribution requires $N_m = N_e$
- To have control on the particle fluctuations on all harmonics the method of beamlets is used:

6D macro particle with multiple internal degrees of bunching

Shot Noise Algorithm

- Most common algorithm by W.Fawley:
  - Duplicate macro particles and distribute evenly over the phase (beamlet = set of N_b macro particles):

$$\theta_j = \theta_0 + \frac{j}{N_b} 2\pi$$

  - Apply Fourier series for n harmonics:

$$\theta_j \rightarrow \theta_j + \sum_{k=1}^{n} a_k \cos\left(k\theta_j + \phi_j\right)$$

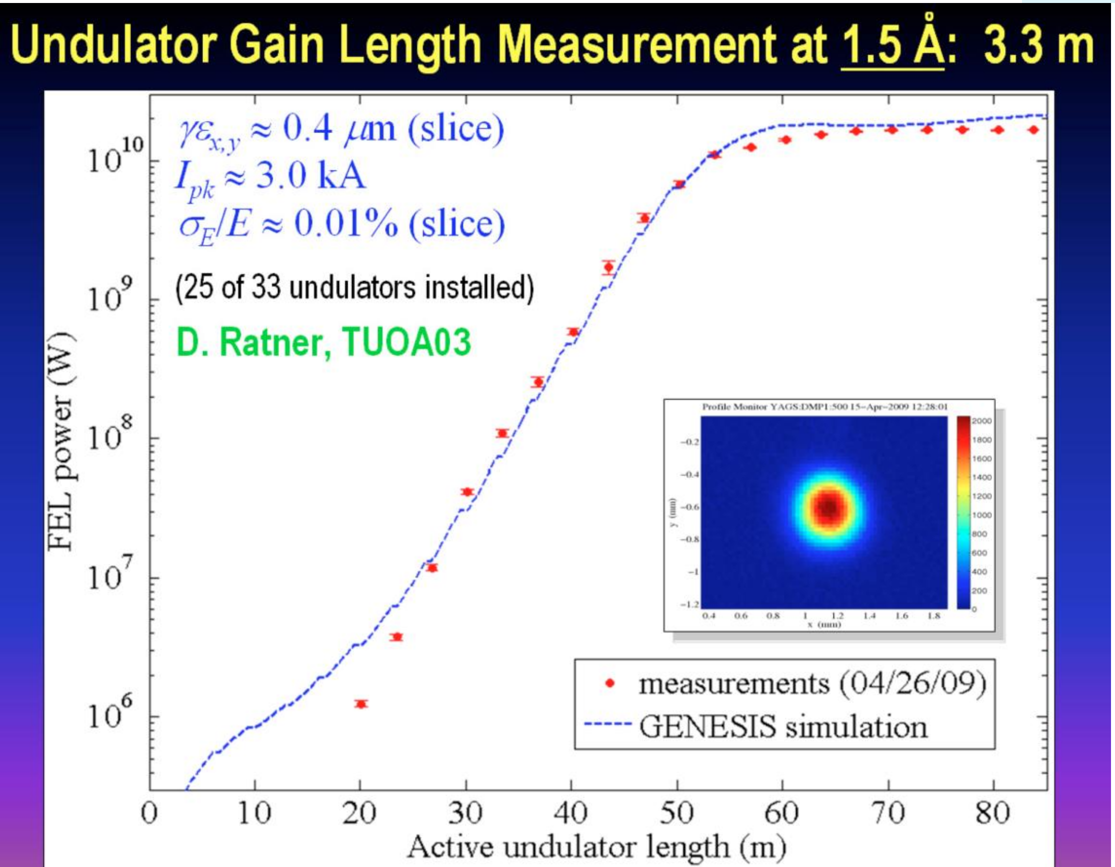
W. Fawley, PRSTAB 5 (2002) 070701

- Algorithm requires $N_b > 2n$, which leads to large particle numbers for HGHG cascades (e.g. FERMI, NLS, SwissFEL with n ~200)

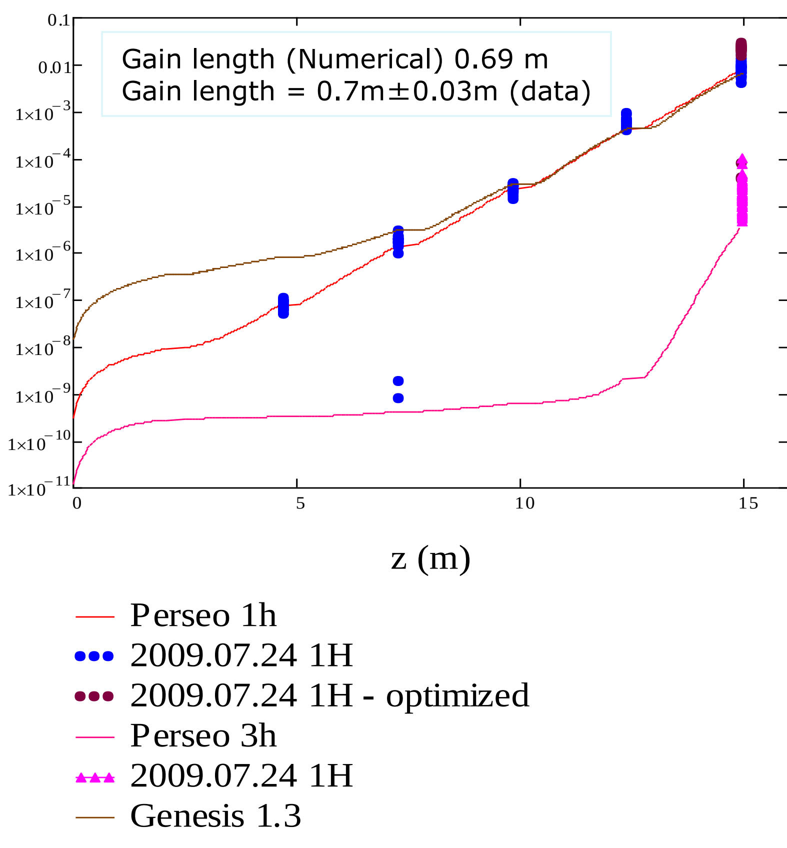
Current Status of FEL Codes

- Codes have been successfully benchmarked against experimental results of SASE FELs (e.g. LEUTL, SDL, FLASH, VISA, LCLS, SPARC)

LCLS: courtesy P. Emma

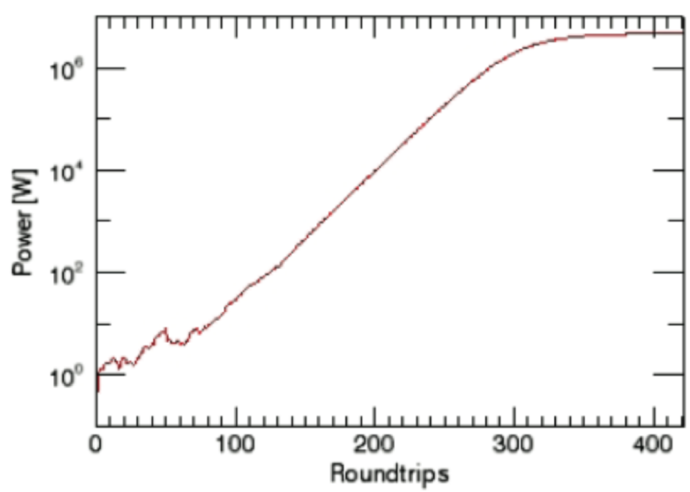


SPARC: courtesy M. Ferrario



Time Consumptions of FEL Codes

- Thanks to parallelization of the codes runs times are reasonable for the most demanding single-pass SASE FELs
  - Example: LCLS with $4 \cdot 10^8$ gridpoints and $6 \cdot 10^8$ macro particles requires about 200 CPU hours per run.
- Emerging concepts are pushing the grid points and particle numbers to larger values
  - Example: 1 Å wavelength X-ray Free-electron Laser Oscillator.

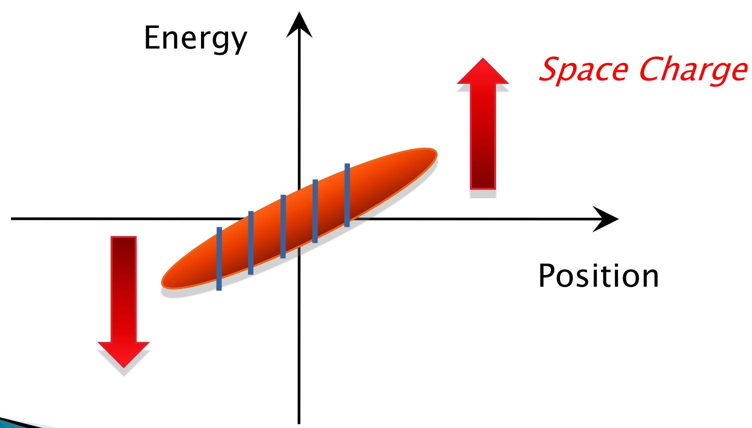


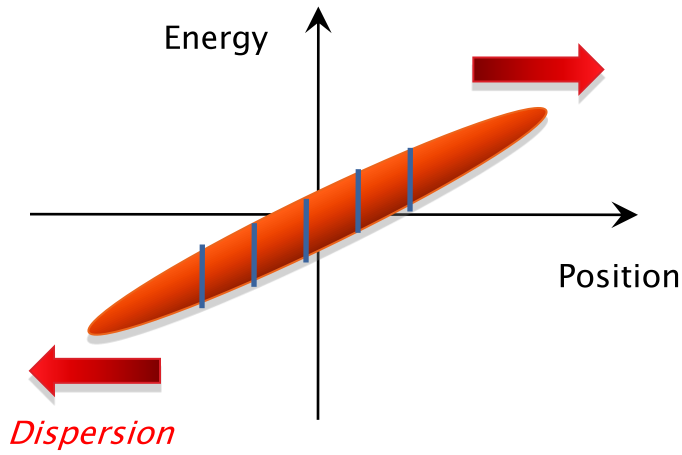
1 Round trip = 1 LCLS run + wavefront propagation

K.J. Kim, PRL100, 244802 (2008)

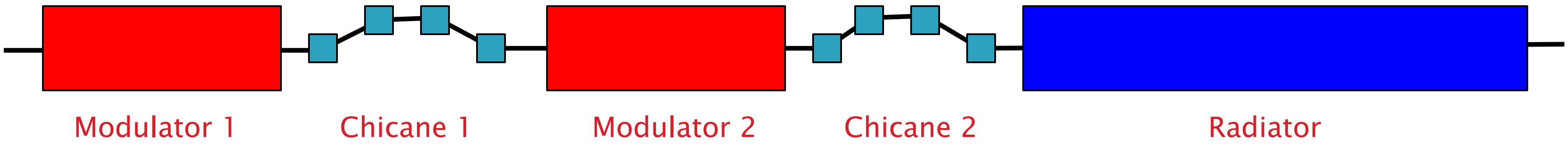
Modeling Challenges: TT FEL

- Plasma Injector to generate 1 GeV, 1 micron long bunch with peak current of up to 10-20 kA.
- Strong space charge effects and dispersion of undulator stretches bunch in longitudinal phase space.
- Micro bunching will be stretched (like an accordion), shifting the wavelength of bunching



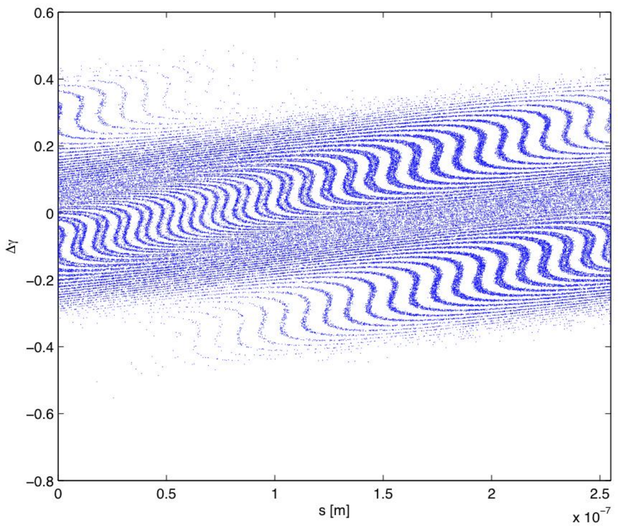


Modeling Challenges: EE-HG



D. Xiang and G. Stupakov, PR STAB 12, 030702 (2009)

- Echo Enabled Harmonic Generation induces a high harmonic current modulation as a seed for a FEL (starting from the coherent emission of the current modulation).



Currently considered the most promising method to seed X-ray FELs

Modeling Challenges: EE-HG

- Dynamic in the two chicane very important because it limits the efficiency of the seeding scheme.
- Limiting factors are:
  - Coherent Synchrotron Radiation
  - Quantum fluctuation in incoherent synchrotron radiation
  - R_{51}, R_{52} and higher order terms of chicane

- FEL requires a large particle number to resolve sub-wavelength structures but they cannot model chicanes
- CSR codes cannot handle large number of particles in FEL code dumps after modulator 1 and 2

Modeling strategies are currently explored

Addressing the Challenges

- No averaging over undulator period or radiation wavelength
- Convenience of slicing and sequential progress through bunch have to be given up. Full bunch needs to be in memory.
- Non-average codes improves upon:
  - No restriction in bandwidth of the FEL
  - Variation of electron beam parameters and radiation field on a scale smaller than the FEL wavelength (avoids the SVEA approximation, which suppresses CSE effects)
  - Allows for a simplified broadband shot noise algorithm
  - Electron motion over many wavelengths (chirp, space charge)

The Problem to Solve…

- Particle Tracker almost unchanged, except longitudinal position is not expressed as a phase and oscillation of particle is included in trajectory.

- Longitudinal discretization is now included in the effective finite-different/finite-element field equation:

$$\left[\nabla_\perp^2 + 2ik\left(\frac{\partial}{\partial z} + \frac{\partial}{c\partial t}\right)\right]u = i\frac{e^2\mu_0}{m}\sum_j q_j \delta\left(\vec{r}_\perp - \vec{r}_{\perp,j}\right)\delta\left(t - t_j\right)\frac{K}{\gamma_j} e^{-i\frac{k_u}{1-\beta_z}(ct-z)}$$

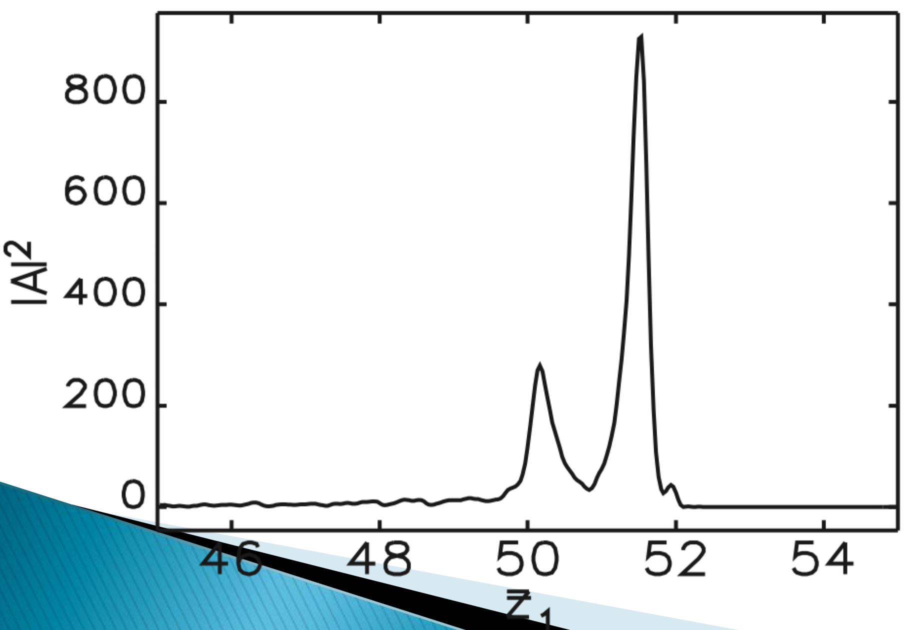
- New numerical methods have to be applied to solve field equation

Example of Non-Average Code

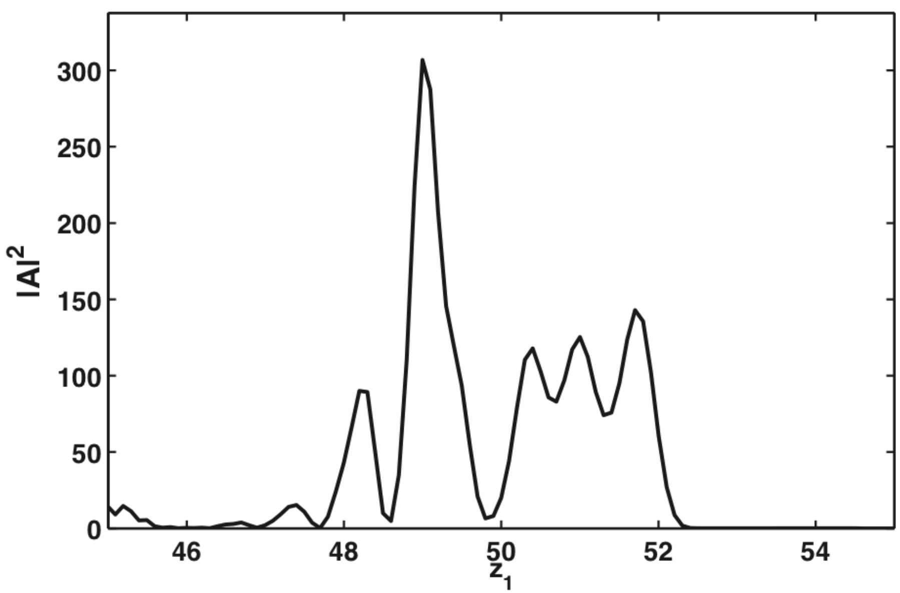
- Group by Brian McNeil

- Study of superradiant regime of FELs, which typically exhibits a spike narrowing process, violating the SVEA approach

Non-averaged Code



SVEA Code



Courtesy of B. McNeil

Towards the Future…

- Unlike established FEL codes, which were developed for single processor and then ported to parallel computer, new codes are utilizing the computer architecture from the start.

- Highest detailed model requires about $5 \cdot 10^9$ particles and radiation modes/gridpoint, corresponding to about 500 Gbyte distributed memory.

Can be provided by currently existing parallel computers

Thank you for your attention