The effect of undulator harmonics field on Free-Electron Laser harmonic generation

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Outline

- Introduction
- Analysis
  - in an ideal undulator
  - in an actual undulator
  - 3rd harmonic case
- Summary

INTRODUCTION

- Using the higher harmonic:

  a way for FEL => shorter λs.

$$\lambda_{sn} = \frac{\lambda_u}{2n\gamma^2}(1+a_u^2)$$

- For a planar undulator with $B_u \sin(k_u z)$,

  the electron's non-uniform axial motion (β_z)

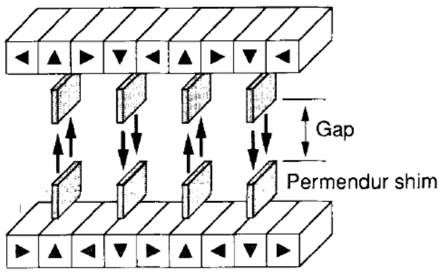
  => the odd harmonics radiations on axis

  n=1,3,5,….

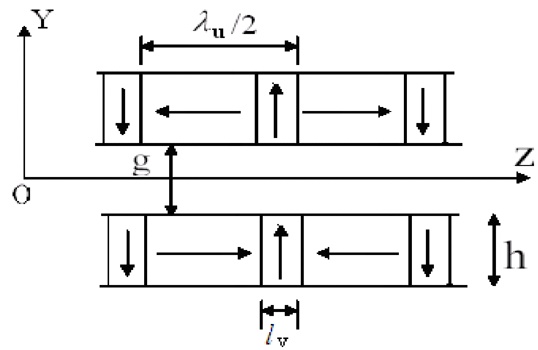
- The harmonic radiation can be enhanced by $\Uparrow B_{un}$ *

Some methods for this were proposed, *eg*.:

putting high permeability shims inside the undualtor **



optimizing magnetic blocks size in a standard Harbch undulator ***



- Here,we analysis the effect of B_n on FEL harmonic generation

*M.J. Schmitt and C.J. Elliott, IEEE J. Quantum Electron.QE-23 (1987) 1552.
**M. Asakaw *et al*. Nucl. Instr. and Meth. in Phys. Res. A 358 (1995) 399-402, A 375 (1996) 416-319
***Qi-ka Jia , "Undulator Harmonic field enhancement analysis", Proceedings of IPAC10, WEPD033/3165

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ANALYSIS

In a ideal planar undulator

$$\vec{B}_u = \hat{B}_u \sin(k_u z)$$

the e-s oscillate*

at odd harmonics in the transverse direction

$$\beta_x \approx -4 \sum_{n=1,3,5,...} \left(\frac{K}{4\gamma}\right)^n \frac{(-1)^{(n-1)/2}}{[(n-1)/2]!} \cos[n k_u \bar{z}]$$

at even harmonics in the axial direction

$$\beta_{II} \approx \bar{\beta}_{II} + 2 \sum_{n=2,4,..}^{\infty} \left(\frac{K}{4\gamma}\right)^n \frac{(-2)^{\frac{n}{2}}}{[(n-2)/2]!} \cos[n k_u \bar{z}]$$

=> radiations

on-axis odd harmonics

even harmonics off-axis

*Qika Jia, "Harmonic motion of electron trajectory in planar undulator," PAC09-WE5RFP088

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For FEL

the *n*th harmonic optical field equation and the phase equation in 1-D mode:

$$\frac{d}{dz}\tilde{a}_{sn} \approx \frac{r_e n_e a_u [J,J]_n \lambda_{sn}}{\gamma}\left\langle e^{-in\phi}\right\rangle$$

$$\frac{d^2\phi}{dt^2} = -\frac{c^2}{\gamma^2} 2a_u k_u \operatorname{Re}\sum_n [J,J]_n k_{sn}\tilde{a}_{sn} e^{in\phi}$$

$\phi=(k_s+k_u)z-\omega_s t$

the coupling coefficient:

$$[J,J]_n = (-1)^{\frac{n-1}{2}}\left[J_{\frac{n-1}{2}}\left(\frac{na_u^2}{2(1+a_u^2)}\right) - J_{\frac{n+1}{2}}\left(\frac{na_u^2}{2(1+a_u^2)}\right)\right]$$

the harmonic generation can be charactered by the coupling coefficients*

- Small signal gain in low gain FEL

$$g_n = -n\left(\frac{[J,J]_n}{[J,J]_1}\right)^2 (4\pi N\rho)^3 \left\langle \frac{\partial}{\partial x} \sin c^2 \frac{x}{2} \right\rangle_{\phi_0'}$$

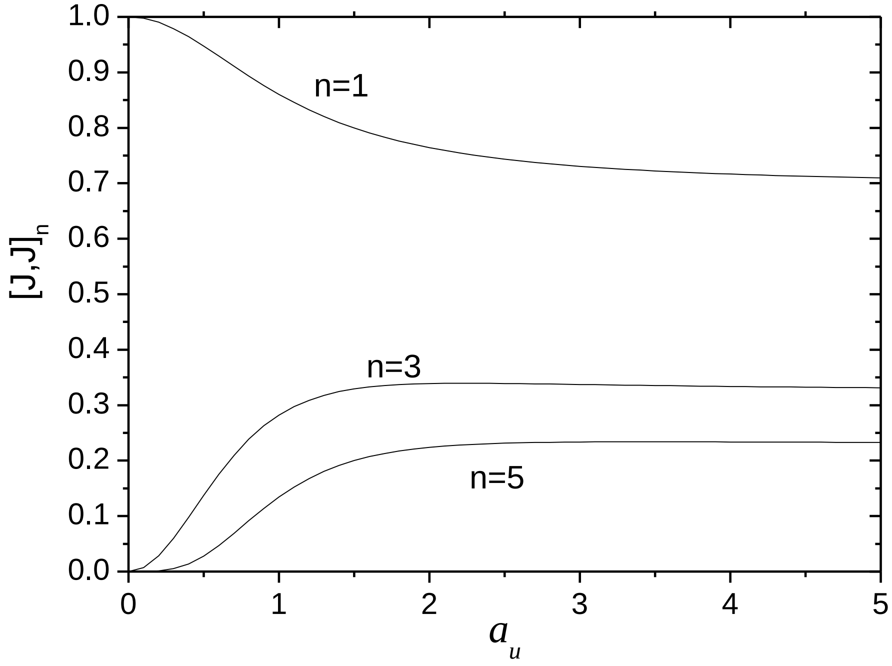
- nonlinear harmonic generation in high gain FEL

$$\frac{P_n}{\rho P_e} \square \left(\frac{n^{n-1}[J,J]_n}{n![J,J]_1}\right)^2 \left(\frac{P_{10}}{9\rho P_e}\right)^n e^{n\frac{z}{L_g}}$$

harmonic saturation power: $\frac{P_{ns}}{P_{1s}} \approx \frac{(n+1)^n}{2(n*n!)^2}\left(\frac{[J,J]_n}{[J,J]_1}\right)^2$

*Qi-ka Jia， "An analysis of nonlinear harmonic generation in high gain free electron laser" IEEE J.Quantum Electron. 43, 833 (2007)

the harmonic coupling coefficient with undulator deflection parameter



harmonic generation $\propto ([J,J]_n/[J,J]_1)^2$

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For actual planar undulators

$$B_u = \sum_m B_{um} \sin(mk_u z) \qquad \vec{a}_u = \sum_m \hat{a}_{um} \cos(mk_u z)$$

m: all or part odd numbers,
due to the symmetry of the magnetic structure

,

generally $B_{um} << B_{u1}$, $a_{um} = \frac{B_{um}}{mB_{u1}} a_{u1} << a_{u1}$

- **Resonance condition**

$$\lambda_{sn} = \frac{\lambda_u}{2n\gamma^2}(1 + \sum_m a_{um}^2)$$

$$a_{um}^2 = \hat{a}_{um}^2 / 2, \ (rms)$$

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•Electron motion

Using relations $\beta_\perp^2 = \frac{\vec{a}_u^2}{\gamma^2}$, $\beta_{II} = 1 - \frac{1}{2}(\frac{1}{\gamma^2} + \beta_\perp^2)$

give

$$z = \overline{z} - \{\sum_m \frac{\xi_m}{k_s}\sin(2k_u\overline{z}) + \sum_{m\neq l}[\frac{\xi_{ml+}}{k_s}\sin((m+l)k_u\overline{z}) + \frac{\xi_{ml-}}{k_s}\sin((m-l)k_u\overline{z})]\}$$

$$\overline{z} = \overline{\beta}_{\parallel} ct,\quad \overline{\beta}_{\parallel} = 1 - \frac{1}{2\gamma^2}(1 + \sum_m a_{um}^2)$$

where

$$\xi_m = \frac{a_{um}^2}{2m(1+\sum a_{ui}^2)} = \frac{r_m^2}{m}\xi_1\ ,\ \xi_{ml\pm} = \frac{a_{um}a_{ul}}{(m\pm l)(1+\sum a_{ui}^2)} = \frac{2r_m r_l}{m\pm l}\xi_1$$

because $r_m = \frac{a_{um}}{a_{u1}} = \frac{B_{um}}{mB_{u1}} << 1$, $\xi_1 = \frac{a_{u1}^2}{2(1+\sum a_{ui}^2)} < \frac{a_{u1}^2}{2(1+a_{u1}^2)} < \frac{1}{2}$

ξ_m , $\xi_{ml\pm} << 1$, for $m,l \neq 1$

Only the terms related with the fundamental are dominant therefore

$$z \approx \overline{z} - \{\frac{\xi_1}{k_s}\sin(2k_u\overline{z}) + \sum_{m\neq 1}[\frac{\xi_{m+}}{k_s}\sin((m+1)k_u\overline{z}) + \frac{\xi_{m-}}{k_s}\sin((m-1)k_u\overline{z})]\}$$

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• the phase equation

$$\phi'' = \frac{2k_u}{\gamma^2}\sum_{n,l} k_{sn} a_{sn} a_{ul} \{\cos[(nk_s + lk_u)z - n\omega_s t + \phi_n] + \cos[(nk_s - lk_u)z - n\omega_s t + \phi_n]\}$$

substituting expression of z

$$\phi'' = \frac{2k_u}{\gamma^2}\sum_{n} k_{sn} a_{sn} a_{u1} f_n \,\mathrm{Re}\, e^{-i(n\phi + \varphi_{sn})}$$

$$f_n = \mathrm{Re}\sum_l \frac{a_{ul}}{a_{u1}}\left[e^{i(n-l)k_u \bar{z}} + e^{i(n+l)k_u \bar{z}}\right] e^{in\xi\sin(2k_u \bar{z})} \prod_{m\neq 1} e^{in\{\xi_{m+}\sin[(m+1)k_u \bar{z}] + \xi_{m-}\sin[(m-1)k_u \bar{z}]\}}$$

In the exponential of f_n, many terms are small and oscillate fast,
a average over undulator period will eliminate these small contribution terms.

•the *n*th harmonic optical field equation

$$\frac{d}{dz}\tilde{a}_{sn} \approx \frac{r_e n_e a_{u1} \lambda_{sn}}{\gamma} f_n \left\langle e^{-in\phi} \right\rangle$$

the modified coupling coefficient: $[J, J]_n \rightarrow f_n$

the third harmonic field case

the most important harmonic

$$B_u = B_{u1}\sin(k_u z) + B_{u3}\sin(3k_u z)$$

in this case:

$$z = \bar{z} - \frac{\zeta_1}{k_{s1}}\sin(2k_u\bar{z}) - \frac{\zeta_2}{k_{s1}}\sin(4k_u\bar{z}) \qquad \xi_1 = \frac{a_{u1}(a_{u1}+a_{u3})}{2(1+a_{u1}^2+a_{u3}^2)}\ ,\ \xi_2 = \frac{a_{u1}a_{u3}}{4(1+a_{u1}^2+a_{u3}^2)}$$

We have

$$f_n = \mathrm{Re}\sum_l \frac{a_{ul}}{a_{u1}}[e^{i(n-l)k_u\bar{z}} + e^{i(n+l)k_u\bar{z}}]\sum_{h_1}\sum_{h_2} J_{h_1}(n\zeta_1)J_{h_2}(n\zeta_2)e^{i(h_1+2h_2)2k_u\bar{z}}$$

n,l=1,3.

Taking average over undulator period

the dominant product term in the sum is that with $h_1 + 2h_2 = -(n \pm l)/2$

$$f_n = \sum_l \frac{a_{ul}}{a_{u1}}\{\sum_{\substack{h_1,h_2,\\ h_1+2h_2=-\frac{n+l}{2}}} J_{h_1}(n\zeta_1)J_{h_2}(n\zeta_2) + \sum_{\substack{h_1,h_2,\\ h_1+2h_2=-\frac{n-l}{2}}} J_{h_1}(n\zeta_1)J_{h_2}(n\zeta_2)\}$$

for the small arguments, only zero order Bessel function contribute

Because $\zeta_2 << \zeta_1 < 1/2$, it can be further simplified by taking h_2=0 :

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the modified coupling coefficient:

$$[J,J]_1 \rightarrow f_1 = J_0(\zeta_2)\{[J_0(\zeta_1) - J_1(\zeta_1)] + \frac{a_{u3}}{a_{u1}}[J_2(\zeta_1) + J_1(\zeta_1)]\}$$

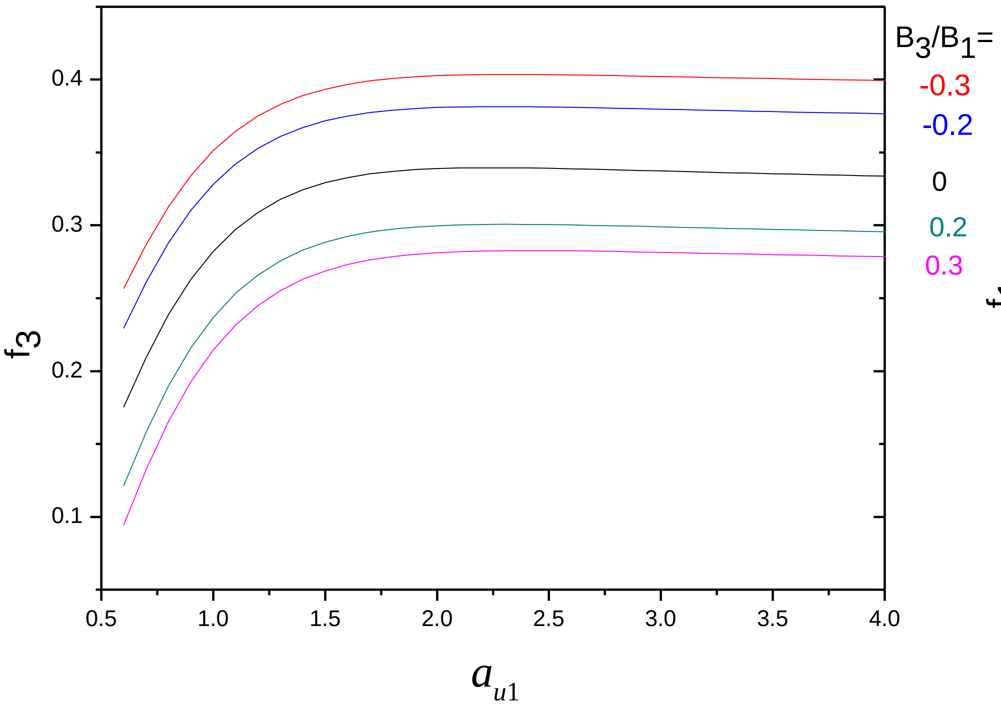
$$[J,J]_3 \rightarrow f_3 = J_0(3\zeta_2)\{[J_2(3\zeta_1) - J_1(3\zeta_1)] + \frac{a_{u3}}{a_{u1}}[J_0(3\zeta_1) - J_3(3\zeta_1)]\}$$

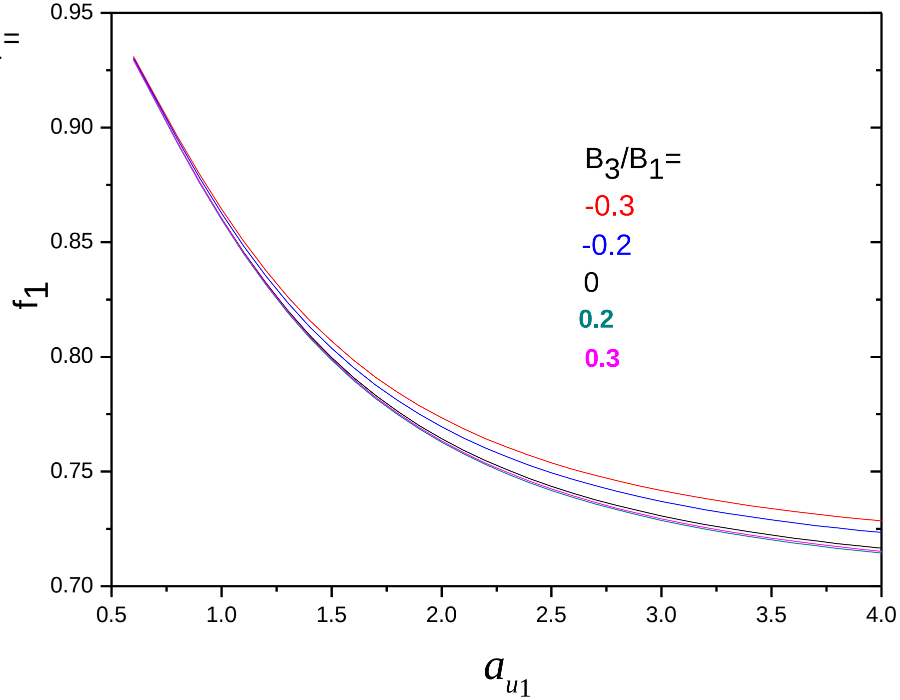
3rd harmonic generation $\propto (f_3 / f_1)^2$

enhancement of the 3rd harmonic

$$R_3 = (\frac{f_3 / f_1}{[J,J]_3 / [J,J]_1})^2$$

numerical calculation result





Modified coupling coefficient due to 3rd harmonic magnetic field

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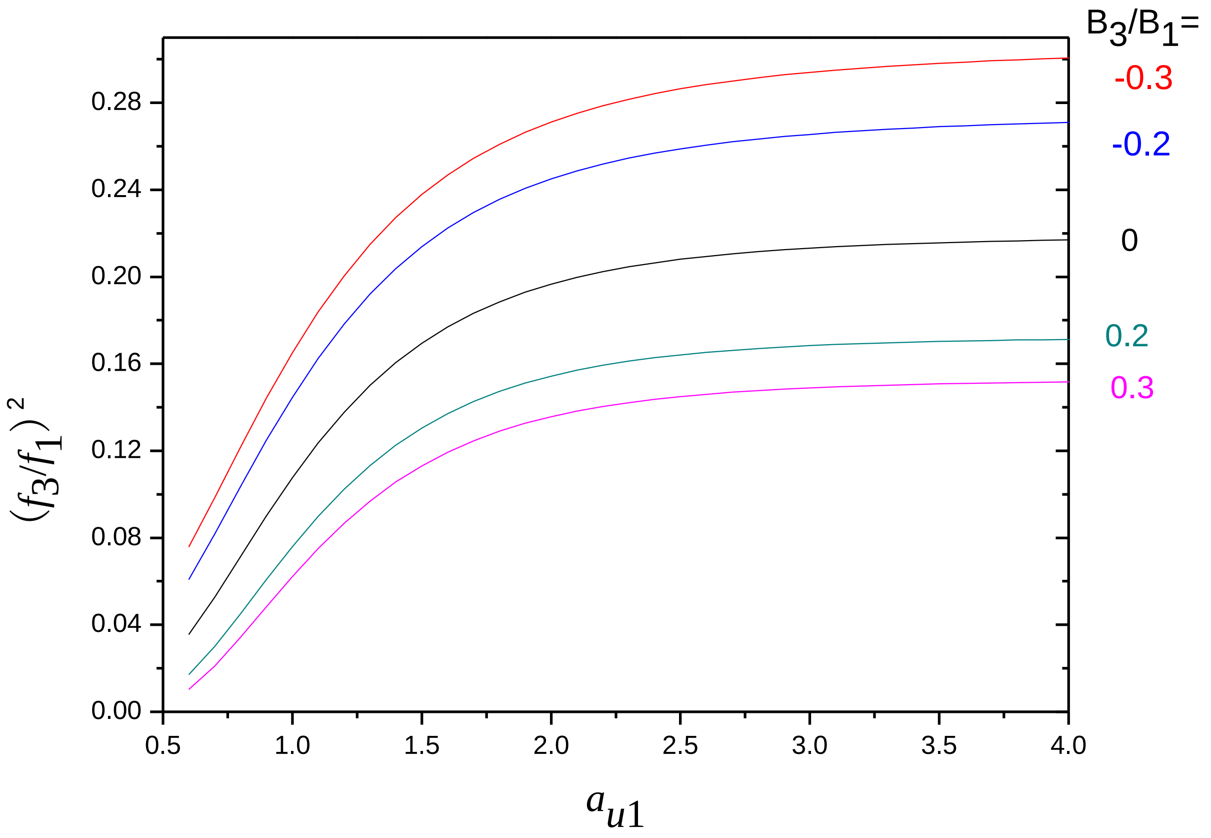
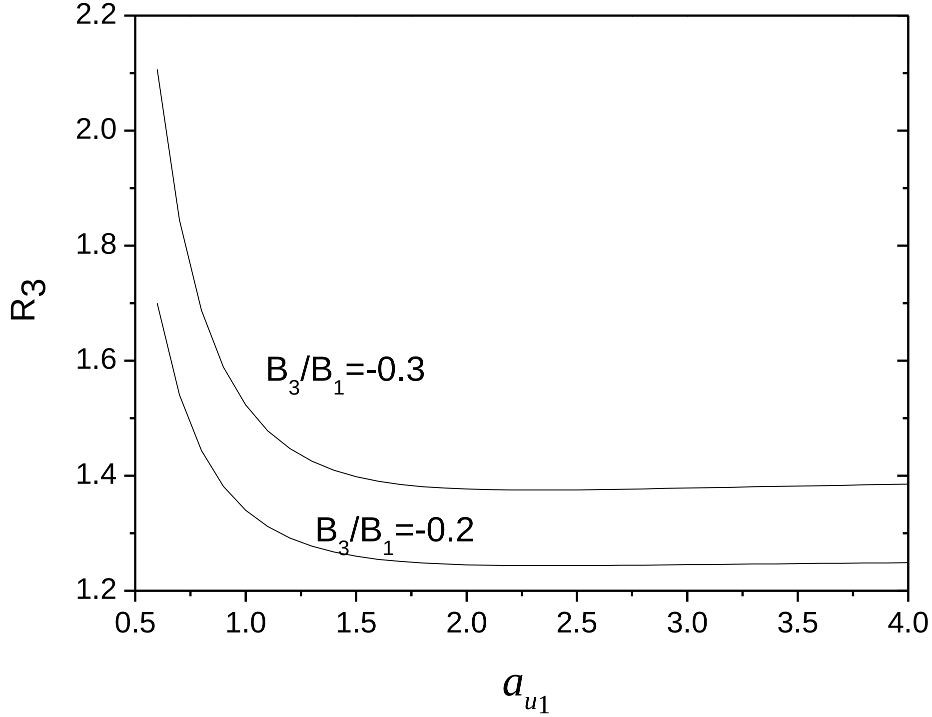
3rd harmonic generation $\propto (f_3/f_1)^2$



the effect of B_3 on FEL harmonic generation

The enhancement of the FEL 3rd harmonic radiation



$$R_3 = \left(\frac{f_3 / f_1}{[J,J]_3 / [J,J]_1}\right)^2$$

argument: a_{u1}, a_u has a little difference:

$$a_u^2 = a_{u1}^2 [1 + (\frac{a_{u3}}{a_{u1}})^2]$$ => same resonant wavelength

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- 3rd harmonic emission can be distinctly enhanced
  by 3rd harmonic field with an opposite sign to fundament field
- the larger magnetic harmonics fractions
  => the larger radiation enhancement,
  $|B_{u3}/B_{u1}|\uparrow \Rightarrow a_{s3}\uparrow$
- fundamental radiation has been less affected
- for given B_{u3}/B_{u1}
  the weaker a_u => larger enhancement of a_{s3}

With $B_{u3}/B_{u1} = -0.3$:
the 3rd-harmonic radiation are enhanced ~40%
maximally doubled for $K(=\sqrt{2}a_u) \sim 1$

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SUMMARY

- effects of undulator harmonics field on the harmonic coupling coefficients and FEL harmonic generation are analysed
- For the case 3rd magnetic field present, analytical expression is given for coupling coefficients, is easy to calculate and can be used to predict the enhancement of FEL HG
- 3rd emission increase with 3rd magnetic field that has an opposite sign to B_1
- fundamental emission has been less affected.
- next work: further study by simulation

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Thank you