DEPENDENCE OF GAIN ON CURRENT IN THE COHERENT SMITH-PURCELL EXPERIMENT AT CESTA

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Abstract

At FEL 2009, we presented experimental results on coherent Smith-Purcell obtained at CESTA in the microwave frequency domain. Those results strongly supported the two-dimensional theory proposed by Andrews and Brau some years ago, and were consistent with simulations performed with the PIC code "MAGIC". That experiment used a large current, 200 A, for a grating of width 10 cm. In a follow-up experiment, emittance slits were used to reduce the current to as low as 2 A, with a quite thin, flat, and wide beam. The gain as a function of current and also of vertical beam position was measured in detail. In particular, the start current for our set-up was found. In parallel, 2-D simulations of the experiment with "MAGIC" were compared with the experimental results. Very good agreement between simulations and experiment is obtained. This lends confidence that simulations of a scaled-down version of our experiment will be a reliable guide for Terahertz frequency coherent Smith Purcell experiments. Such simulations suggest that coherent Smith-Purcell radiation in the range 100-200 GHz should be feasible.

INTRODUCTION

When, in 1953, Smith and Purcell (SP) [1] sent a 300 keV electron beam along a grating of period L = 1.67 m, they observed incoherent visible light that satisfied the condition,

$$\lambda = L(1/\beta - \cos\theta)/|n| ,$$

where λ denotes the wavelength of radiation produced at angle θ with respect to the beam, L is the period, n is the order of diffraction and $\beta = v/c$ is the usual relative velocity. Since its discovery, SP radiation has been the subject of much theoretical and experimental work. Recently there has been renewed interest in coherent SP free-electron devices that might be used as compact, tunable sources for coherent THz radiation. Much of this was inspired by the work of Andrews and Brau (henceforth AB) [2], who established the dispersion relation between frequency ω and axial wave number k for the evanescent wave on a lamellar grating. Their analysis was two-dimensional (2-D), i.e., no dependence in the direction along the grooves. Assuming the beam to be uniform plasma moving above the grating, they calculated the gain of the beam-wave interaction, finding a result similar to that which had been established long ago by Pierce for traveling wave tubes [3]. In particular, they predicted that the gain would be proportional to the cube root of the current. However, they also pointed out two essential features which earlier analyses had overlooked. At sufficiently low beam energy, the intersection of the beam line, $\omega = vk$, occurs on the downward sloping portion of the dispersion relation, as in a backward wave oscillator. This allows for feedback even if there were no reflection from the grating ends. Secondly, they noted that the frequency of the evanescent wave is always less that the minimum allowed SP frequency. The evanescent wave then emits omni-directional radiation upon reaching the ends of the grating. If, however, the bunching at that frequency is strong, higher harmonics will appear in the current, and these may correspond to allowed SP frequencies. When this happens, what Andrews, Boulware, Brau and Jarvis call interbunch coherence occurs [4], where the emissions from successive bunches interfere so as to produce coherent radiation only at harmonics of the bunching frequency. For each harmonic the radiation satisfies the SP relation between angle and frequency, and thus is emitted only in a small angular range.

Support for this theory was provided by simulations using Particle-in-Cell (PIC) codes [5, 6], that used the commercial code "MAGIC" [7]. Kumar and Kim [8] considered a sheet electron beam of zero thickness moving above the grating, rather than uniform plasma, but still found results similar to those of AB. They gave an estimate of the threshold or start current needed to overcome losses and produce gain. Except for the results found by Skrynnik and co-workers [9] at extremely low energy, 2-5 keV, none of the numerous observations of SP radiation offered support to the AB theory. However, quite recently, two experiments have confirmed it. The first [10] performed using a narrow beam of milli-ampere intensity, observed the evanescent wave at the expected sub-SP frequency. The second, [11] using an intense (200 A), wide (10 cm) and thin (5 mm) beam, was able to observe not only the evanescent wave (4.6 GHz), but also its second (9.2 GHz) and third (13.8 GHz) harmonics. The beam bunching was observed directly with a current monitor and also with a magnetic probe placed at the end of a groove. This was intended to be a demonstration experiment, and the frequencies involved were of order several GHz. Far more efficient devices operate in this frequency range, but this experiment did indeed confirm key aspects of the AB theory. The results were also quite consistent with the 2-D "MAGIC" simulations presented in reference 5.

Here we report on an improved version of that experiment, which addresses a critical issue, the dependence of the gain on current. By varying the beam current we have found the threshold value needed to produce bunching and exponential growth. New simulations have also been performed, whose results are in good agreement with our experimental observations.

EXPERIMENTAL SET-UP

The experimental configuration has been described in reference 11. The grating dimensions were those of the simulations reported in reference 5, except that the overall width was 10 cm. The essential parameters are summarized in the Table . In order to have a system in which the 2-D approximation may be valid, we used a wide sheet beam (10 cm), produced by a thin copper cathode. In our previous experiment, we were unable to vary significantly the beam current. However, in the new one, we have added a slit at the grating entrance, which is visible in Fig. (1).

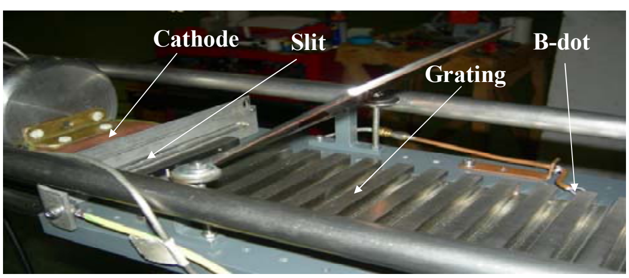

Figure 1: Photograph of set-up.

With this disposition, the current of the electron beam can be varied from 0 to the maximum value (280 A), just by changing the slit thickness. The pulsed-power generator that drives the diode operates in single-shot mode. Both the diode and the grating are enclosed in a cylindrical vacuum chamber, which is surrounded by a pulsed solenoid that provides a uniform axial magnetic field. As may be seen in Fig. 1, a B-dot probe is placed at the end of a groove in the grating. This probe is used to measure the B_x field component of the grating evanescent mode.

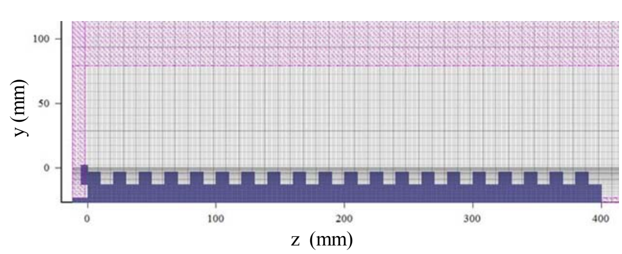

Figure 2: Simulation area.

In Fig. (2), we show the "MAGIC" 2-D geometry we have used to simulate the experiment. The cathode and grating area has a fine mesh size of 100 µm. The rose-colored regions represent absorbing surfaces that prevent reflection, at least for normal incidence.

EXPERIMENTAL RESULTS

We have used fits to the experimental voltage

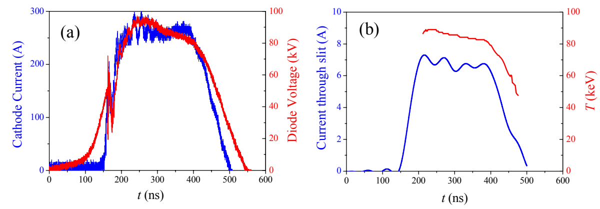

Figure 3: (a) I (before slit) and V vs. t (ns), (b) I (after slit) and T vs. t (ns).

and current curves as input conditions in "MAGIC". The voltage applied to the diode is measured with a capacitive sensor, while the post-slit current is measured by a Rogowski coil downstream from the grating on the current return stalk. Typical $V(t)$ and pre-slit $I(t)$ waveforms are displayed in Fig. (3a). The maximum current (slit wide open) is 280 A at 95 kV, for a pulse of duration 300 ns (FWHM). Fig. (3b) shows the collected current after passage above the grating when the slit thickness is set to 100 µm. The time-averaged current I_0 (blue) is 7 A. Because the voltage varies slightly during the interaction, we expect to observe a variation of the bunching frequency by performing a sliding FFT. This measurement turns out to give the actual beam kinetic energy (red curve in Fig. (3b)), because the CSP interaction occurs at the intersection between the beam line (slope β) and the grating dispersion relation $\omega(k)$. This time variation is small, and the FFT spectra of both the collected current and the B-dot probe signals strongly peak near 4.6 GHz. They are shown in Fig. 4, and the former has been normalized arbitrarily so that the peak values of both curves coincide.

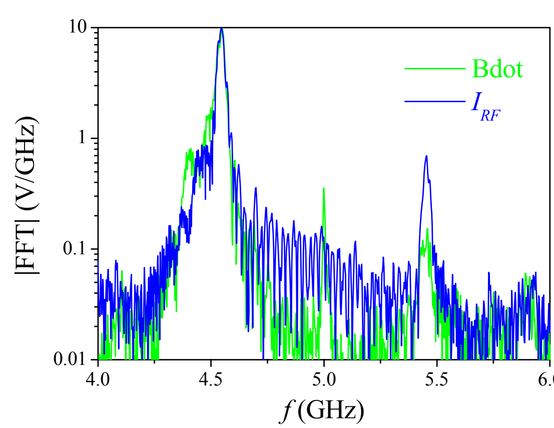

Figure 4: FFTs of B-dot signal (green) and RF current I.

The exponential growth rate of the interaction,

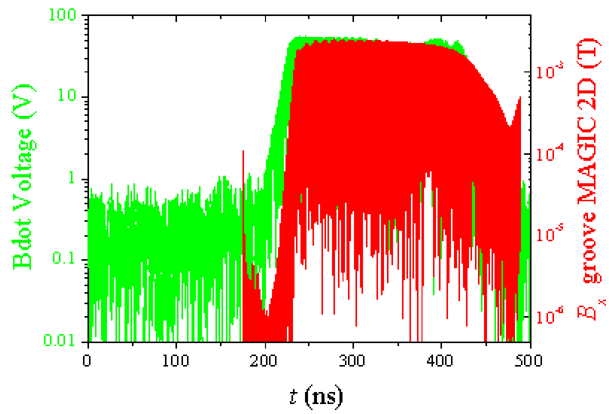

Figure 5: Log scale plot of filtered B-dot signal (green) and “MAGIC” 2-D simulation (red)

Im(ω), can be obtained by measuring the slope of the envelope of the RF part of the B-dot signal in a log-scale plot. This is indicated in Fig. (5), for the shot whose current was shown in Fig. (3b). The measured signal (green curve) is the time derivative of the B_x component of the grating mode, after filtering around the nominal 4.6 GHz frequency. At early time, one sees a noisy signal, from which then emerges a clearly exponentially growing signal. Finally, saturation is reached at a level two orders of magnitude above noise. For comparison, the results of a 2-D "MAGIC" simulation (red curve) of the magnetic field in the groove are shown. The simulation predicts a growth rate about twice that of the experiment, which we consider acceptable agreement. We note that the analysis of the RF signal of the evanescent wave, as detected by a horn that is placed in the far field region in the experimental hall, gives a similar growth rate. We have decided to base our analysis of gain *vs.* current on the B-dot probe signals only.

The main result of this work is given in Figs. (6a). The thickness of the slit was varied by moving only the upper

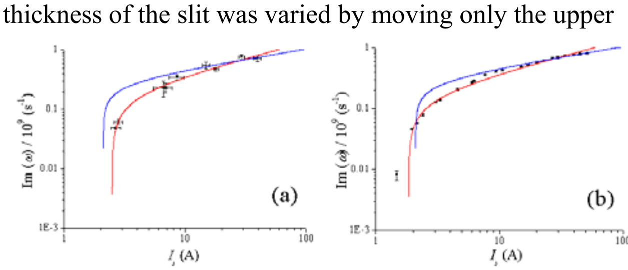

Figure 6: Log-log plot of Im(ω) vs current I_s.(a) experiment, (b) Magic 2-D simulation.

lip, while the lower remained at a constant distance (3 mm) from the grating surface. The analysis is complicated by the fact that for current values I_0 > 10 A, the exponential growth starts during the rise time (50 ns) of the current pulse. Consequently, in Figure (3a), we have plotted the gain, not as a function of I_0, but rather as a function of I_s, the value of the current at the time when the growth starts. Each point represents the average values of Im(ω) and I_s for five identical shots, and the corresponding statistical error bars are shown. Two fits to these data are shown. The red curve assumes that the data may be fitted with a simple three-parameter expression,

$$\operatorname{Im}\omega = \alpha\theta\left(I_s - I_{th}\right)\left(I_s - I_{th}\right)^{\gamma}$$

where θ denotes the Heaviside step function, I_{th} the threshold current in Amperes, and α and γ are free parameters. The blue curve is similar, except that γ is fixed at the theoretically favored value, 1/3. We find for the best fit (reduced $\chi^2 = 1.23$) :

$$\begin{aligned} \alpha &= \left(0.119 \pm 0.011\right)\cdot 10^9\,\mathrm{s}^{-1}, \\ I_{th} &= \left(2.48 \pm 0.09\right)A, \\ \gamma &= 0.53 \pm 0.05. \end{aligned}$$

Since this is an empirical formula, we attach no deep significance to α and γ, but we note that the threshold or start current is of the order of 2 A. The fit with the imposed exponent of 1/3 is better at higher currents, but not as good at lower values. In principle, it should be easy to distinguish between these choices, but in practice, we can't greatly increase the current I_s, since the gain starts long before the maximum of the current is reached.

We have also performed 2-D "MAGIC" simulations for 23 different values of beam current; the results are shown in Figure (6b). In 2-D simulations, the relevant current is really a linear current density, whose dimension is A/m. Given our grating width of 10 cm, the true linear current density is 10 I_s. The simulations were performed with a beam thickness of 100 µm and we checked that the gain remained unchanged provided the slit thickness didn't exceed 2 mm. We have taken into account the error we make when estimating the slope of the "MAGIC" result presented in Figure (2d). The blue curve is the same asymptotic behavior we used in Figure (3a). The red curve is a fit of these "MAGIC" predictions with the same function we used to fit the experimental data. We find:

$$\begin{aligned} \alpha &= \left(0.119 \pm 0.011\right)\cdot 10^9\,\mathrm{s}^{-1}, \\ I_{th} &= \left(1.84 \pm 0.01\right)A, \\ \gamma &= 0.53 \pm 0.01. \end{aligned}$$

Compared to the experimental fit, the only difference is a smaller value of the threshold current, I_{th}. The agreement between figures (6a) and (6b) is quite good. In both curves, the start current is approximately 2 A, the asymptotic behaviors are similar ($I^{1/3}$) but, if we fit the whole range of current, our best fit is with $I_s^{0.5}$.

Another interesting result is illustrated in Fig. 7, where the saturation time is plotted as a function of I_s. The black squares are the experimental points, the red squares are 2-D "MAGIC" predictions along with a three-parameter fit (red curve) of the form: $t_{sat} = t_1 e^{-I_s/I_{s0}} + t_2$. We find: t_1 = 391 ns , t_2 = 35 ns, and I_{st} = 2.3 A . For high currents, I_s > 10 A, saturation of the interaction is fast and occurs during the 50 ns rise-time. For smaller currents,

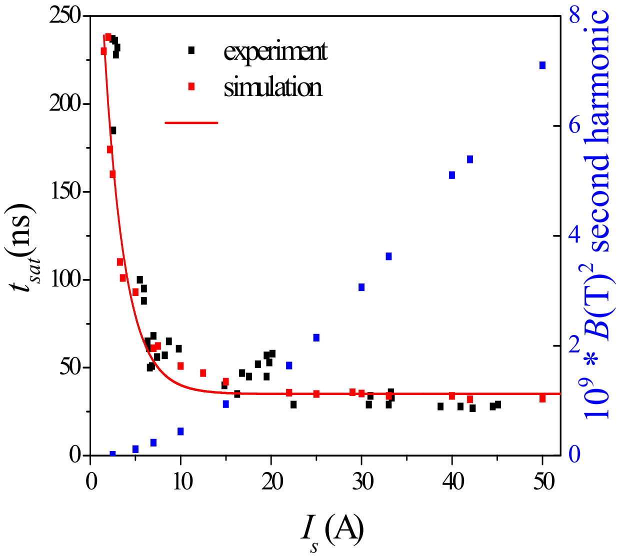

Figure 7: Left scale, Saturation time t_{sat} vs I_s, Right scale, Simulated B^2 vs I_s

I_s < 3 A, saturation is not actually reached because the current pulse drops off too soon. On the right-hand scale of this figure we show as blue squares the simulated value at saturation of B_x^2, filtered at the second harmonic (9.2 GHz**)**, and observed at a point situated approximately 8 cm above the middle of the grating. This quantity is proportional to the power radiated at the second harmonic, and it is seen to exhibit an approximately quadratic dependence on current. Although this is simulated and not measured directly, the general agreement between measurements and simulations leads us to believe that this is also true experimentally.

CONCLUSIONS

We have addressed here some important issues for a CSP-FEL. First we have measured a threshold current needed for the beam bunching. Of course, it depends on the details of the experimental set-up, such as slit position, beam-grating distance, etc. Secondly we have obtained a gain curve that is consistent with the expected asymptotic behavior, $I_s^{1/3}$, at large currents. But at smaller currents, a somewhat better fit is obtained with $(I_s- I_{th})^{1/2}$.

Although this experiment is not in the more interesting Thz range, the underlying physics doesn't depend on the operating frequency. The agreement between our measurements and our simulations encourages us to believe that for a sufficiently wide grating (w >> l), a high frequency coherent sp experiment could be designed with the help of 2-d simulations. In particular, the start current could be found to optimize the beam parameters for a table top Thz experiment.

Table 1: Experimental Parameters

parameters	value
beam kinetic energy	95 kev
peak current	0 - 280 a
pulse duration	300 ns
beam thickness	0.01-2 mm
beam-grating distance	3 mm
grating period	2 cm
grating groove depth	1 cm
grating groove width	1 cm
grating width	10 cm
number of periods	20
external magnetic field	0.3-0.5 t

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