# CONTROL OF INSTABILITY INDUCED BY A DETUNING IN FEL OSCILLATOR

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*Abstract*

In FEL oscillators, a desynchronisation (so-called detuning) between the electron-bunch passage frequency and the repetition rate of the laser can lead to instabilities, characterised by erratic longitudinal shape of the emitted light pulses. We show that these instabilities can be controlled using a simple feedback system which consist in re-injecting in the cavity a part of the emitted light. Analytical, numerical and experimental studies on the UVSOR-II storage ring have been performed, and show that the energy needed to achieved the control can be extremely weak, in practical higher than the noise level.

## MODEL AND DYNAMICS OF STORAGE RING FEL WITH A DESYNCHRONISATION

### *Model*

For these studies, an adapted modelling corresponds to a longitudinal description of the laser pulse. The dynamics of the laser pulse is modelled by a master equation which describes the envelop of the laser pulse $e_n(\theta)$ at each round trip number $n$ of the laser, as shown in Figure 1 [1]. $\theta$ being the temporal coordinate associated to the longitudinal coordinate.

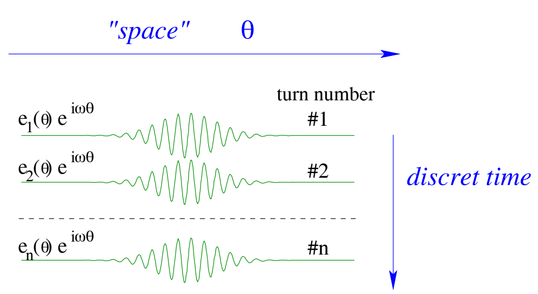


Figure 1: Schematic representation of the pulse $e_n(\theta)e^{i\omega\theta}$ observed at each cavity turn. $e_n(\theta)$ represents the pulse envelop at the turn number $n$; $\omega$ is the angular frequency of the laser. The hypothesis of slow variation at each turn of $e_n(\theta)$ permits to use a continuous time T [2]. The variable becomes $e(\theta, T)$.

Making the assumption that the evolution of the laser pulse is slow compared to the cavity round trip time, the evolution of the envelop of the pulse $e(\theta, T)$ is given by [1]:

$$\begin{aligned} e_T(\theta, T) &= -e - ve_\theta + Gf(\theta)(e + e_{\theta\theta}) \quad (1)\\ &+\sqrt{\eta}\xi(\theta, T) \quad (2) \end{aligned}$$

With $T$ the slow time associated to the number of cavity round trip, in unit of $\tau_c$, the lifetime of the photons in the cavity. $\theta$ is the fast time, in unit of $t_u = \frac{\pi}{\sqrt{2}\Delta\omega_g}$, with $\Delta\omega_g$ the FEL gain bandwidth. $v$ is the drift parameter, it describes the desynchronisation between $T_e$ and $T_L$: $v = \frac{T_L - T_e}{T_L}\frac{\tau_c}{t_u}$. In this representation, a desynchronisation between the two periods $T_L$ and $T_e$ acts as a constant drift of velocity $v$, which tends to push the pulse toward one extremity of the system (cf. Fig. 2). Spontaneous emission is taken into account in the white noise term $\xi(\theta, T)$, with $< \xi^*(\theta', t')\xi(\theta, t) >= \delta(\theta - \theta')\delta(t - t')$, $\eta$ representing the noise level.

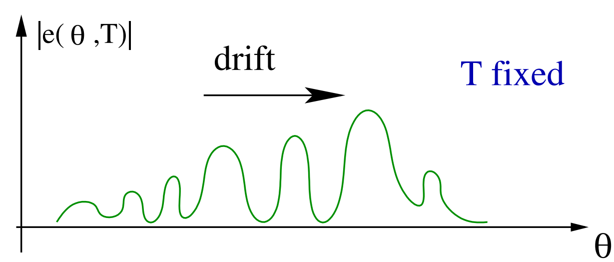


Figure 2: Schematic example of a laser pulse submitted to a constant drift. The shape has been taken non regular because the drift can lead the system turbulent.

The gain dynamics depends of the type of accelerator and of the insertion devices. For storage ring, it can be expressed as [3, 4, 5]:

$$f(\theta) = e^{-\frac{\theta^2}{2\sigma_b^2}} \quad (3)$$

$$G = \frac{A}{\sigma}e^{-\frac{(\sigma^2-1)}{2}} \quad (4)$$

$$\frac{d\sigma^2}{dT} = \frac{1}{T_s}(1 - \sigma^2 + \int_0^L |e(\theta, T)|^2 d\theta), \quad (5)$$

with $\sigma_b$ the length of the electron bunch, in unit of $t_u$, $\sigma$ the electron bunch energy spread, $A$ the maximum gain, in unit of the losses of the cavity, $T_s$ is the synchrotron decay time, in unit of $\tau_c$ and $L$ the cavity round trip time, in unit of $t_u$. For numerical simulations, values are taken close to these of UVSOR-II operation condition values [6]: $T_s = 263$, $A = 2.17$, $\sigma_b = 900$, $t_u = 100$ fs, $\tau_c = 20$ ms. The noise level is fixed at $\eta = 10^{-10}$.

### *FEL Dynamics in Function of the Drift Velocity*

Figure 3 shows results of the integration of the equations (2) to (5) for two values of the drift velocity: $v = 0$ and $v = 1$. It is represented in (a) and (c) the laser pulse intensity $|e(\theta, T)|^2$ and in (b) and (d) the norm of the spectrum $|\tilde{e}(k, T)|^2$ $\left(\tilde{e}(k, T) = \int_{-\infty}^{+\infty} e(\theta, T) e^{ik\theta} d\theta\right)$. For a zero or a weak drift velocity, the system is stable (Fig. 3a, b). For higher desynchronisation value, the laser pulses have an erratic behaviour (Fig. 3c). Structures also appear in the spectrum (Fig. 3d) [1]. These behaviours have also been observed experimentally [3, 7, 8, 9, 1].

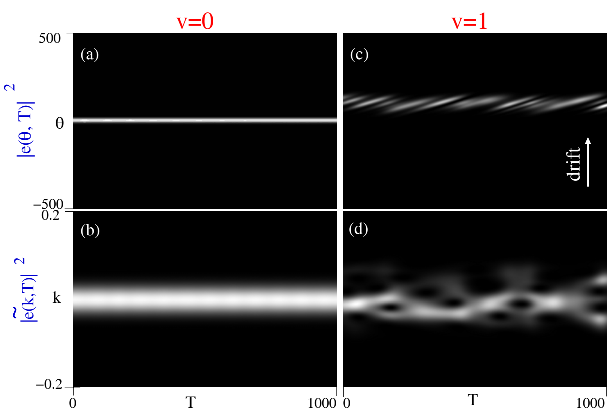


Figure 3: (a),(c): spatio-temporal diagram of the intensity $|e(\theta, T)|^2$, (b),(d): spectro-temporal diagram of the norm of the spectrum $|\tilde{e}(k, T)|^2$, for two values of the drift velocity: (a),(b) $v = 0$, (c),(d) $v = 1$.

Concerning the origin of the turbulent behaviour, it is known that this type of system has always a stable stationary solution, either a gaussian like shape or the uniform solution with zero amplitude ($e(\theta, T) = 0$) [10, 11]. However if the drift velocity is sufficiently high, small localised perturbations, while drifting toward one extremity of the system, can be strongly amplified, and can become macroscopic before disappearing at one limit of the system. This is the so-called "transient growth" scenario, which arises in a general manner in spatio-temporal systems submitted to a drift [12, 13]. As noise from spontaneous emission is always present, this scenario is repeated at infinity, creating some so-called "noise sustained structures" (Fig. 3 b,d) [14, 15].

## CONTROL OF THE INSTABILITIES

### *Principle*

The feedback method we test consists in reinjecting with a delay a part of the output pulses in the cavity thanks to a external mirror [16] (Fig. 4). The delay of the feedback can be changed easily in changing the position of the mirror.

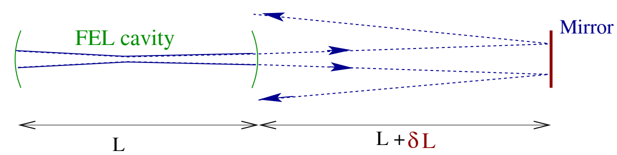


Figure 4: Schematic representation of the control principle: a small part of the output pulses is reinjected in the cavity, with a small shift $\delta L$ compared to the cavity length $L$.

This technique has been previously applied to "classical" active mode locked laser (the so-called "Coherent Photon Seeding" technique [17, 18, 19, 20]), which can present similar dynamics [21]. With the feedback, the equation (2) becomes :

$$
\begin{aligned}
e_T(\theta, T) &= -e - v e_\theta + G f(\theta)(e + e_{\theta\theta}) && (6)\\
&\quad + \alpha e(\theta - a, T) + \sqrt{\eta}\xi(\theta, T), && (7)
\end{aligned}
$$

and the equations (3),(4), (5) remain identical. $\alpha^2$ is the fraction of power reinjected in the cavity and $a$ is the delay, proportional to $\delta L$ (Fig. 4).

### *Numerical Results*

Figure 5 shows numerical results of the integration of the equations (7), and (3) to (5), with a feedback applied at $T = 1000$.

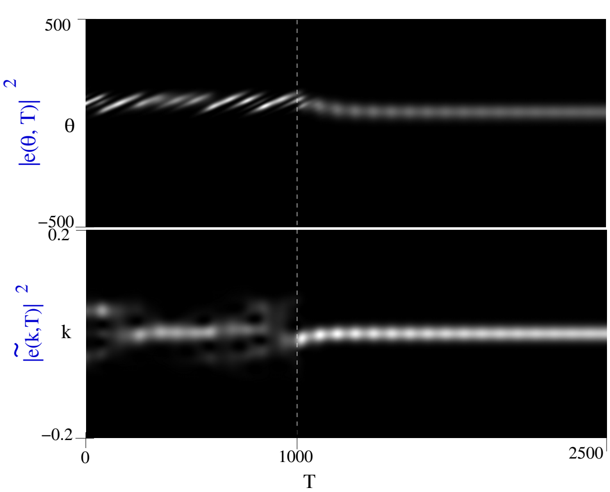


Figure 5: (a) Spatio-temporal and (b) spectro-temporal evolution of the laser pulses when a feedback is applied at time $T = 1000$. Parameters: $v = 1$, for $T < 1000$ $\alpha = 0$ and for $T \geq 1000$, $\alpha = 10^{-2}$ and $a = 50$.

Figure 5 shows clearly that the feedback, with these parameters ($v = 1$, $\alpha = 10^{-2}$, $a = 50$) and after a transient, permits to suppress in a large manner the fluctuations. Noise sustained structures in the spectrum are also suppressed. Repeated numerical simulations show that the system converges every time toward the same new stationary solution.

### Experimental Results

Experimental results obtained at the storage ring of UVSOR-II are shown in Figure 6 [16]. The laser pulse intensity is recorded with a double sweep streak-camera (Hamamastu C5680) and the spectrum is obtained using a planar Perrot-Fabry and a linear CCD camera.

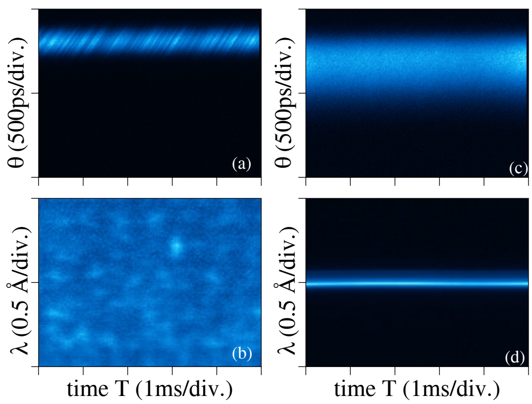


Figure 6: Experimental feedback-induced erratic regime suppression in a free electron laser (fraction of reinjected power $\alpha^2 \simeq 0.5 \times 10^{-8}$, spatial shift $a = 130$ ps). Upper and lower Figures are $|e(\theta,T)|^2$ and the spectrum versus time $|\tilde{e}(k,T)|^2$, respectively. (a), (b) Without feedback. (c), (d) With feedback. Each streak camera recording is synchronised with its corresponding spectrum.*C. Evain et. al., Phys. Rev. Lett. 102, 134501 (2009). Copyright (2009) by the American Physical Society.*

Without feedback, the behaviour of the laser is unstable (Fig. 6a), and the spectrum is too large to be recorded entirely (Fig. 6b). With feedback, fluctuations are in large part suppressed (Fig. 6c), and this is associated with a spectrum width narrowed (Fig. 6d). The fraction of the power which has been reinjected in the cavity (in the $TEM_{00}$ mode) can be calculated and is found to be very low: $\alpha^2 \simeq 0.5 \times 10^{-8}$ [16].

### Analytical Study: Convective-absolute Threshold

In a spatio-temporal system submitted to a drift, like a FEL oscillator with a detuning, the threshold of turbulent-regular behaviour is usually near the threshold of absolute-convective instability [14, 22]. In this manner, the knowledge of the convective-absolute threshold expression in the FEL system with feedback can provide information about the efficiency of the feedback. In the case of global coupling, like in storage ring FEL [Eq. (5)], the convective-absolute threshold does not coincide exactly with the threshold of turbulent-regular behaviour [16], however it can give qualitative information about the efficiency of the feedback in function of the parameters.

The convective-absolute threshold permits to separate two asymptotic behaviours of the linearised system in studying the local stability of the uniform solution $e(\theta,T) = 0$, which is the stationary stable solution when the desynchronisation is strong. The strategy consists first in perturbing locally the uniform solution (for example at $\theta = 0$ and $T = 0$), and then in studying the behaviour of system at the point where the perturbation has been applied. There are two possible behaviours : if the drift is strong, the perturbation is taken far from the point where it has been applied and $\lim_{T\to\infty} e(\theta = 0, T) \to 0$. The system is said to be in a linear convective regime [22]. The other possibility is that the perturbation grows at the point where it has been applied and $\lim_{T\to\infty} e(\theta = 0, T) \to \infty$. In this case, the system is said to be in an absolute linear regime [22]. Generally, a convective regime is associated to a turbulent behaviour whereas an absolute regime is associated to a regular behaviour, which means in this system that a convective (resp. absolute) regime is generally associated to a non-efficient (resp. efficient) control of the instabilities.

It is possible to find an expression of the convective-absolute threshold of the linearised system [16]:

$$\alpha = \frac{e^{\beta a}}{a}(2A\beta + v), \tag{8}$$

with $\beta$ given by the largest solution of:

$$A\beta^2 + 2\beta(\frac{A}{a} + \frac{v}{2}) + \frac{v}{a} + A - 1 = 0. \tag{9}$$

The Figure 7 shows the convective-absolute threshold in function of the feedback delay $a$ and of the feedback gain $\alpha$.

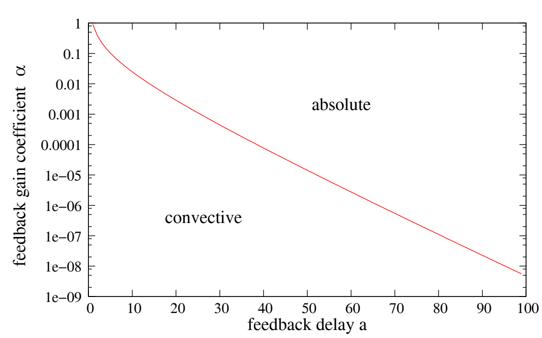


Figure 7: Convective-absolute areas in function of the feedback delay and of the feedback gain coefficient $\alpha$. Parameters: $v = 4$, $A = 2.17$.

We see that when the delay $a$ is increased, the gain coefficient $\alpha$ needed to be in the absolute area tends to zero, that is the quantity of the laser pulse to be reinjected in the cavity to control the instabilities can be very small. This is consistent with the experimental results where we have calculate a coefficient of $\alpha^2 = 0.5 \times 10^{-8}$. In practice, a value higher that the noise level should be sufficient to stabilise the system [16].

## CONCLUSION

In a FEL oscillator, a desynchronisation between the laser repetition rate and the electron bunch passage frequency can lead to instabilities. This turbulent behaviour can be control by a simple feedback method, which consists in reinjecting a small part of the output laser pulse in the cavity thanks to a external mirror. Numerical, experimental and analytical studies show that the quantities necessary to control the turbulent regime is very weak, in practice higher that the noise level. This study is applied to storage ring FEL oscillator, but this feedback method can be applied more generally to finite spatio-temporal systems submitted to a drift [16].

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