

# FEASIBILITY STUDY OF SHORT-WAVELENGTH AND HIGH-GAIN FELS IN AN ULTIMATE STORAGE RING

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## Abstract

Z. Huang et al. studied FELs in the EUV and soft x-ray regions in PEP-X storage ring and showed that three orders of magnitude improvement in the average brightness is possible at these radiation wavelengths [1]. We studied the feasibility of high-gain FELs in the wavelength range from 0.10 nm to 1.86 nm in an ultimate storage ring. If a 1 mA bunch current is obtained, more than 300 times power amplification is possible in wavelengths longer than 1.86 nm and even at a 0.90 nm wavelength one order of magnitude improvement can be expected. If a bypass is used, saturation can be achieved and about 1GW FEL power can be obtained at a 1.8 nm wavelength.

## INTRODUCTION

Since the first observation of synchrotron radiation from an accelerator, many accelerators have been constructed as synchrotron radiation sources. During this time, average synchrotron radiation brightness increased exponentially with time and that of third generation light sources reached the level of  $\sim 10^{20}$  ph/s/mm<sup>2</sup>/mrad<sup>2</sup>/0.1% BW. After the development of third generation light sources, linac-based fourth generation light sources were constructed to generate an x-ray free electron laser (XFEL). Their average brightness is on the level of  $\sim 10^{25}$  ph/s/mm<sup>2</sup>/mrad<sup>2</sup>/0.1%BW.

Though the linac-based light sources can generate high brightness and coherent radiation, storage-ring-based light sources are still attractive due to factors such as their high beam stability, availability of many beam lines, and variety of bunch patterns. Thus, it is important to increase the average brightness of the storage rings. For this purpose, ultimate storage rings were studied [2]-[6] and were shown to increase the average brightness to  $10^{22}\sim 10^{23}$  ph/s/mm<sup>2</sup>/mrad<sup>2</sup>/0.1%BW. However, this is still low compared to the linac-based XFELs.

Z. Huang et al studied high-gain FELs in a storage ring and applied them to the PEP-X storage ring [1]. They showed that three orders of magnitude improvement is possible in the wavelength range from 3.3 nm to 30 nm. If average brightness improvement is possible at wavelengths shorter than 3.3 nm, it is very useful for storage-ring-based light sources. The PEP-X energy is 4.5 GeV, the emittance is 50 pm, and the energy spread is  $1.14\times 10^{-3}$  with a damping wiggler. The ultimate storage ring studied by Tsumaki et al. [4] has 6 GeV beam energy, 34.5 pm natural emittance, and  $0.89\times 10^{-3}$  energy spread. The higher energy, the smaller emittance, and smaller energy spread of the ultimate storage ring have the capability of the shorter wavelength high-gain FELs than that in the PEP-X storage ring. Thus, we studied the

possibility of short-wavelength high-gain FELs in the ultimate storage ring.

## ULTIMATE STORAGE RING

The main parameters of the ultimate storage ring [4] are shown in Table 1. The length of the straight section is 6 m and the values of the horizontal and vertical betatron functions in the straight section are 25 m and 5 m, respectively. These values are not adequate for long undulators for FEL use, but it is easy to change the straight section length and the betatron function values without changing the main parameters of the storage ring. Thus, the parameters shown in Table 1 were used to calculate the FEL parameters except for the emittance which changes with the bunch current. For 1 mA bunch current the emittance grows to 37 pm with 100 % coupling [4]. Bunch lengthening due to intrabeam scattering for 1 mA bunch current with 100 % coupling is less than 0.01 % and this effect was not taken into account.

Table 1: Main Parameters of Storage Ring

Parameter	Symbol	Value
Energy	$E$	6 GeV
Circumference	$L$	1999 m
Natural emittance	$\epsilon_{x0}$	$34.5\times 10^{-12}$ mrad
Cell type		10 bend achromat
Number of cells	$N_c$	32
Damping time	$\tau_x$	23.3 ms
	$\tau_y$	23.3 ms
	$\tau_e$	11.6 ms
Momentum compaction	$\alpha$	$8\times 10^{-6}$
Energy spread	$\sigma_{e0}$	$8.92\times 10^{-4}$
Bunch length	$\sigma_{l0}$	1.23 mm

## ANALYSIS

### Analysis Method

The analysis method was the same as that of Z. Huang et al. [1]. The method is described briefly. The rate-of-change of energy spread under the existence of FEL interaction is written as

$$\frac{d\sigma^2}{dt} = \frac{\sigma_0^2}{\tau_e} - \frac{\sigma^2}{\tau_e} + \frac{\Delta(\sigma^2)}{T_0}, \quad (1)$$

$$\sigma_0 = \sigma_{e_0}/\rho, \sigma = \sigma_e/\rho,$$

where  $t$  is the time variable,  $\tau_e$  is the longitudinal damping time,  $\sigma_{e0}$  is the equilibrium energy spread in the absence of FEL interaction,  $\sigma_e$  is the energy spread under the existence of FEL interaction,  $T_0$  is the revolution time,  $\rho$  is the FEL Pierce parameter [7] [8], and

$$\rho = \left[ \frac{1}{8\pi} \frac{\hat{I}}{I_A} \left( \frac{K[JJ]}{1+K^2/2} \right)^2 \frac{\gamma\lambda_r^2}{\Sigma_x} \right]^{1/3}, \quad (2)$$

where  $\hat{I}$  is the bunch peak current,  $I_A=17$  kA is the Alfvén current,  $K$  is the undulator parameter,  $[JJ]$  is the Bessel function factor,  $\gamma$  is the beam energy in units of an electron mass,  $\lambda_r$  is the FEL resonant wavelength and  $\Sigma_x=2\pi\sigma_x^2$  is the cross section of the beam. Since the bunch length increases proportionally with the energy spread in the lowest order approximation and peak current reduces in inverse proportion to the bunch length, we assumed  $\hat{I} \propto \sigma_{e0}/\sigma_e$ . The first term on the right side of Eq. (1) is the radiation excitation term, the second term shows the radiation damping, the last term is the FEL interaction, and

$$\Delta(\sigma^2) \approx \frac{2P}{\rho P_{beam}}, \quad (3)$$

$$P \approx P_n \exp(Z/L_G), \quad (4)$$

$$P_n \approx \frac{\sqrt{2\pi}\rho^2\gamma mc^3}{\lambda_r}, \quad (5)$$

$$L_G = L_{G0}(1+\Lambda), \quad (6)$$

$$L_{G0} = \frac{\lambda_u}{4\pi\sqrt{3\rho}}, \quad (7)$$

where  $P_{beam}$  is the electron beam power,  $m$  is the electron mass,  $c$  is the speed of light,  $\lambda_u$  is the undulator period, and  $\Lambda$  is the gain length degradation factor [9] and is expressed as

$$\Lambda = a_1\eta_d^{a_2} + a_3\eta_e^{a_4} + a_5\eta_\gamma^{a_6} + a_7\eta_e^{a_8}\eta_\gamma^{a_9} + a_{10}\eta_d^{a_{11}}\eta_\gamma^{a_{12}} + a_{13}\eta_d^{a_{14}}\eta_e^{a_{15}} + a_{16}\eta_d^{a_{17}}\eta_e^{a_{18}}\eta_\gamma^{a_{19}}, \quad (8)$$

$$\eta_d = \frac{L_{G0}}{2k_r\sigma_x^2}, \quad (9)$$

$$\eta_e = \frac{4\pi k_\beta L_{G0}\varepsilon}{\lambda_r}, \quad (10)$$

$$\eta_\gamma = \frac{4\pi L_{G0}\sigma_e}{\lambda_u}, \quad (11)$$

where  $a_1 \sim a_{19}$  are the fitting parameters,  $k_r=2\pi/\lambda_r$ ,  $k_\beta=1/\beta$ , and  $\varepsilon$  is the beam emittance. In steady state, setting  $d\sigma^2/dt=0$  and solving the above equations for  $\sigma_e$  numerically, we obtained the energy spread, the FEL power, and the gain length.

## Undulator

The undulator parameters shown in Table 2 were assumed. The parameters for 0.1 nm and 0.18 nm wavelengths are the same as those for the XFEL in SPring-8 [10] and that for 0.49 nm wavelength is the same as that used in the SPring-8 storage ring [11]. The 0.9 nm wavelength undulator is an assumed rather than an actual one. The parameter for a 1.86 nm wavelength is the same as that used in the calculation for the FEL in the PEP-X storage ring [1].

Table 2: Undulator Parameters

Wavelength $\lambda_r$	Period $\lambda_u$	Undulator Parameter $K$
0.10 nm	15 mm	1.3
0.18 nm	18 mm	1.9
0.49 nm	37 mm	2.3
0.90 nm	45 mm	3.0
1.86 nm	50 mm	4.3

## ANALYTICAL RESULTS

### Betatron Function Dependence

Since the FEL power depends on the betatron function of the undulator section, we studied the betatron function dependence of the FEL power first. In the calculation, 100 m undulator length, 1 mA bunch current, and the undulators listed in Table 2 were assumed. Bunch current of 1 mA corresponds to a 648 A peak current. Taking beam instability into consideration, this value seems to be very optimistic for a low momentum compaction storage ring. However, the theoretical prediction sometimes shows a lower threshold current of instability than that of actual machines. This is because the actual machines are very complex but theory deals only with instability under simple conditions. Thus we assumed 1 mA bunch current.

For the 1 mA bunch current and 100 % coupling of betatron oscillation, the transverse emittances are 17 pm but they increase to 37 pm due to intrabeam scattering. However, the emittance does not increase to this value in an actual machine. This is because an actual machine used as a synchrotron radiation source has many undulators, which reduce the emittance and energy spread. Furthermore, using a damping wiggler and properly choosing its maximum field strength makes it possible to further reduce the emittance and energy spread. Accordingly, two emittance cases were considered: 17 pm emittance, and 37 pm emittance. The actual machine's emittance will range from 17-37 pm.

Betatron function dependences of FEL power were obtained by solving the equations described in the previous section and are shown in Figs. 1 and 2. The highest power values were obtained from 2 m to 4 m for a 17 pm emittance beam and 2 m to 5 m for a 37 pm emittance beam.

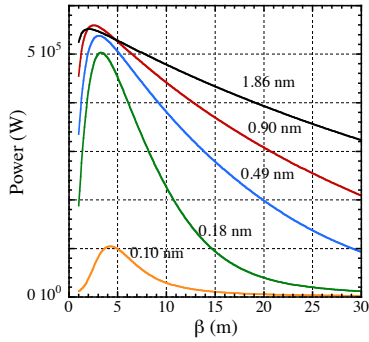


Figure 1: Betatron function dependence of FEL power for 17 pm emittance.

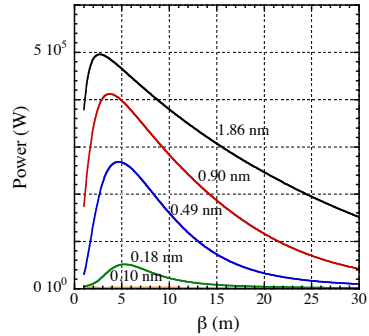


Figure 2: Betatron function dependence of FEL power for 37 pm emittance.

### Magnet Lattice and Betatron Function

Average betatron functions from 2-5 m make it possible to obtain higher FEL power. Because of its simple arrangement, a FODO cell was chosen as the lattice in the undulator section. The shorter the cell length is, the smaller is the average betatron function we can obtain. In this case, however, the undulator's occupation ratio in the undulator section becomes small and the effective undulator length under a constant section length becomes short. We determined a 3.5 m cell length and 3 m undulator length in a cell. As a result, the average betatron function value became 6.7 m.

### Undulator Length Dependence

Undulator length dependence of FEL parameters was calculated assuming a 6.7 m betatron function value. The calculated results of FEL power and energy spread for a 17 pm emittance beam are shown in Fig. 3. FEL power increases rapidly with the undulator length. Increasing rates for longer wavelengths are larger than those for shorter wavelengths in the short undulator region, but the each FEL power level reaches about  $10^6$  W in the longer undulator region and they have almost the same value. This can be attributed to the energy spread increase due to FEL interaction. If the undulator length is short, FEL power remains small and the energy spread due to FEL interaction is also small and the gain length does not degrade. As the power increases with the undulator length, the energy spread due to FEL interaction increases and the gain length degrades rapidly. For longer undulator

regions, the gain length increases in proportion to the undulator length and the power begins to saturate.

Figure 4 shows FEL power and energy spread for a 37 pm emittance beam. As can be seen, energy spread and FEL power are small compared to the 17 pm emittance case. Considering the shorter wavelength, this is quite significant. In the long undulator length region FEL power was found to saturate at a value of about  $10^6$  W which is almost the same as that obtained for a 17 pm emittance beam.

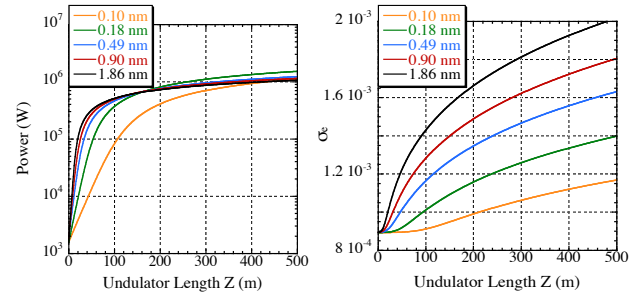


Figure 3: Undulator length dependence of FEL power and energy spread for 17 pm emittance beam.

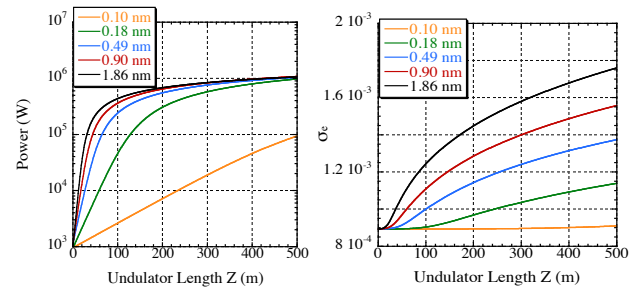


Figure 4: Undulator length dependence of FEL power and energy spread for 37 pm emittance.

## SIMULATION

Numerical simulations using SIMPLEX [12] were performed using the parameters shown in Table 3. Thirty undulators each of them 3 m in length, were assumed supposing about 100 m length undulator section. Since the undulator unit length was 3 m, the total undulator length is 90 m and the total length of the undulator section including the focusing magnets is 105 m. Figure 5 shows simulation results for 17 pm and 37 pm emittance beams. For a 1.86 nm wavelength and a 17 pm emittance beam, FEL power reaches  $4 \times 10^5$  W and it is about 600 times power amplification. For 0.90 nm and 0.49 nm wavelengths, they are about 100 and 30 times power amplification, respectively. Even at a 0.18 nm wavelength, FEL power increased about fivefold. For a 37 pm emittance beam, however, amplification was observed only for the 1.86 nm and 0.90 nm wavelength cases and FEL power levels are about  $1 \times 10^5$  W and  $3 \times 10^3$  W, respectively. These correspond to about 300-fold and sixfold power amplification, respectively. As described in the previous section, the emittance in an actual machine is better than 37 pm. This means that we can expect about

300-600 times power amplification at a 1.8 nm wavelength, about 6-100 times amplification at a 0.9 nm wavelength, and less than 30-fold amplification at a 0.49 nm wavelength.

If a bypass is used at the undulator section, it is possible to use an electron beam with an energy spread that does not take FEL interaction into account. Numerical simulations were performed for this case and the results are shown in Fig. 6. For a 17 pm emittance beam and 1.86 nm and 0.90 nm wavelengths, saturation was achieved and the FEL power reached about  $10^9$  W. For a 37 pm emittance beam, FEL power nearly reached saturation for a 1.86 nm wavelength. Thus, if a 1 mA bunch current is obtained, we can achieve a fully lased FEL at a 1.86 nm wavelength in the ultimate storage ring.

Table 3: Parameters used in FEL Simulation

Parameter	Value			
Bunch current $I_b$	1 mA			
Bunch length $\sigma_{l0}$	1.23 mm			
Betatron function $\beta_x$ , ( $=\beta_y$ )	6.72 m			
Emittance $\epsilon_x$ , ( $=\epsilon_y$ )	$17.3 \times 10^{-12}$ m $\cdot$ rad			
	$37.3 \times 10^{-12}$ m $\cdot$ rad			
Energy spread $\sigma_{e0}$	$0.892 \times 10^{-3}$			
Wavelength $\lambda_r$ (nm)	0.18	0.49	0.90	1.86
Energy spread at equilibrium for 17pm emittance $\sigma_e$	$0.99 \times 10^{-3}$	$1.14 \times 10^{-3}$	$1.25 \times 10^{-3}$	$1.40 \times 10^{-3}$
Energy spread at equilibrium for 37pm emittance $\sigma_e$	$0.90 \times 10^{-3}$	$0.98 \times 10^{-3}$	$1.08 \times 10^{-3}$	$1.21 \times 10^{-3}$

## SUMMARY

We studied the viability of the high-gain FELs in wavelengths ranging from 0.10 nm to 1.86 nm, assuming 1 mA bunch current in an ultimate storage ring. Analytical results showed that the achievable maximum FEL power in the storage ring is on the order of  $10^6$  W.

Assuming a 90 m effective length undulator, we carried out the simulations using SIMPLEX. For a 17 pm emittance beam, about 600 times power amplification was found to be possible at a 1.86 nm wavelength and even at 0.49 nm about 30 times power amplification was achieved. For the case of a 37 pm emittance beam, 300-fold power amplification was achieved at a 1.86 nm wavelength and sixfold power amplification was achieved for a 0.90 nm wavelength. No power evolution was observed at the 0.49 nm wavelength. Since emittance in an actual machine is

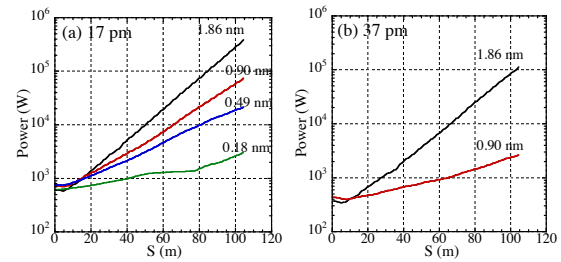


Figure 5: Power evolution for 17 pm and 37 pm emittance beams.

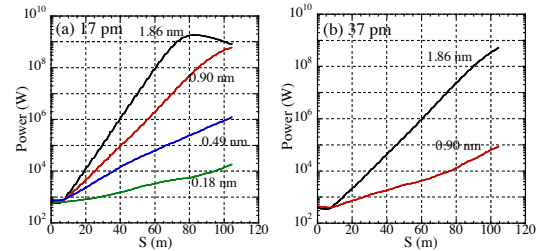


Figure 6: Power evolution for 17 pm and 37 pm emittance beams with a bypass.

expected to be less than 37 pm due to damping by undulator radiation, more than 300 times power amplification is expected in wavelengths longer than 1.86 nm and even at the 0.90 nm wavelength, power amplification more than one order is expected.

We also carried out simulations with a bypass. It was found that if a bypass is used, FEL saturation becomes possible at 1.86 nm and 0.90 nm wavelengths for a 17 pm emittance beam. Saturation is very nearly achieved for a 37 pm emittance beam at 1.86 nm wavelength. Saturated FEL power was about 1 GW.

These results show that FEL in the ultimate storage ring is promising for wavelengths longer than 0.9 nm and that full lasing is possible at the 1.86 nm wavelength when using a bypass.

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