

## VARIABLE-PERIOD PERMANENT MAGNET UNDULATORS

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### Abstract

To change the wavelength of undulator radiation, variation of undulator magnetic field amplitude is frequently applied. Another option is changing the undulator period. A scheme for such an undulator is described. It provides a possibility of changing both the period and number of periods. For a set of undulator sections (like in the x-ray FELs), mechanical motion of periods eliminates the necessity of phase shifters between the undulator sections. Magnetic field calculations for some interesting undulator parameters were performed. Numerous advantages of new undulators (fixed gap, strong dependence of undulator radiation wavelength on the period, relatively low field amplitude variation, and variable number of periods) look very attractive. Prospects for this new type of undulators are discussed.

### INTRODUCTION

Contemporary storage-ring-based x-ray sources and free electron lasers use magnetic undulators. For most of such applications it is necessary to tune the wavelength of the undulator radiation in the forward direction

$$\lambda = \lambda_u \frac{1 + K^2}{2\gamma^2}, \quad (1)$$

where  $\lambda_u$  is the undulator period,  $\gamma$  is the electron energy divided by its rest energy  $mc^2$ . The undulator parameter  $K$  takes into account the reduction of the longitudinal velocity of an electron due to the curvature of the wiggling trajectory. For a planar undulator with sinusoidal longitudinal dependence of the vertical magnetic field,  $K = eB_0\lambda_u / (2\sqrt{2}\pi mc^2)$ , and  $B_0$  is the field amplitude.

Typically, variation of particle energy is impossible (on a storage ring with many undulators) or complicated (because of focusing and trajectory change). A frequently used way of undulator radiation wavelength tuning is variation of the field amplitude  $B_0$ . Serious drawbacks of this technique are too low field value at short wavelengths and too high one at long wavelengths. This circumstance is in particular an obstacle for using undulators with very short period, because the tuning range of such undulators is very small. Wavelength tuning by field amplitude variation is especially difficult in x-ray FELs [1]. A decrease of field amplitude in these devices may cause a significant increase in the gain length. Moreover, the field amplitude must be the same in different sections of a long undulator, with a precision better than 0.1%. Tuning the gaps of all undulator sections with such precision is a significant technological challenge.

Another solution for wavelength tuning is variation of the undulator period  $\lambda_u$ . The "simplest" way to do it is to replace one undulator with another [2, 3]. In such a

design, fine wavelength tuning is provided by gap variation. Different electromagnetic variable-period undulators (VPU) were discussed in papers [4 - 6].

### SPLITTED-POLE UNDULATOR

Let us consider first the hybrid permanent magnet undulator [7, 8]. We propose dividing all poles (iron or permendure) into two halves, as shown in Fig. 1.

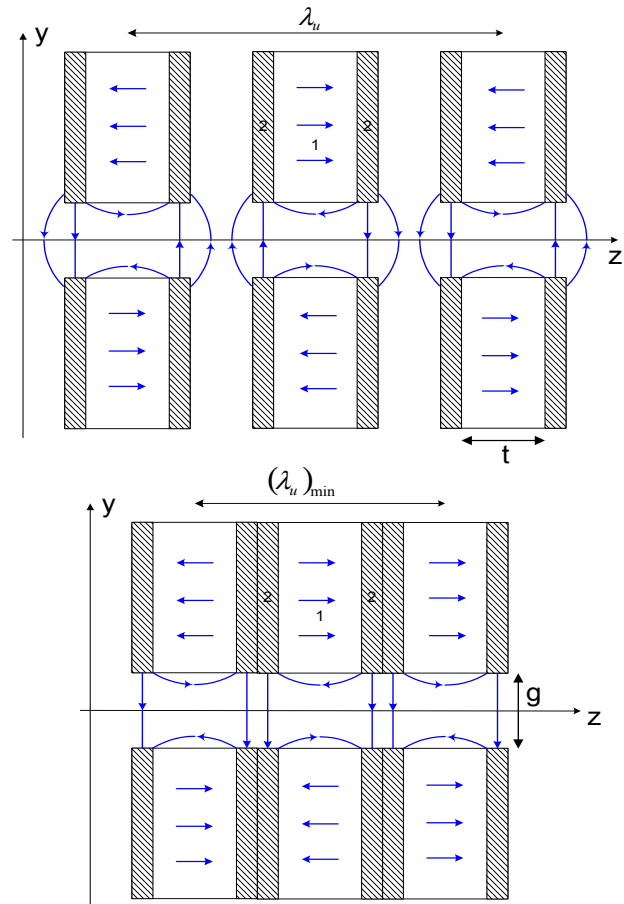


Figure 1: The hybrid permanent magnet undulator with splitted poles (side view). (1) magnet blocks, (2) iron (or permendure). Lines of magnetic induction are shown with arrows. The lower scheme represents the minimum period, when the splitted-pole undulator becomes a conventional hybrid undulator.

It is easy to estimate the magnetic field amplitude  $B_0$  in this undulator. For magnetic blocks high enough, magnetic induction in them is almost zero, and magnetic field intensity is close to the coercivity  $H_c$ . Then the scalar magnetic potential of a pole is  $H_c t / 2$  ( $t$  is the thickness of the permanent magnet, see Fig. 1). For a minimum period, when the undulator is a conventional hybrid undulator, a plot of scalar magnetic potential at the gap boundary  $y = g/2$  is shown in Fig. 2.

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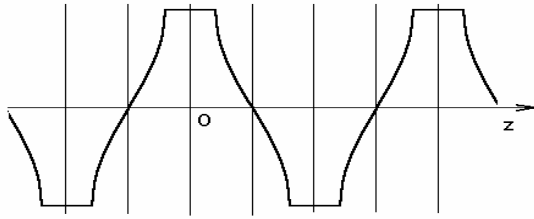


Figure 2: Potential at the upper boundary.

To find the first harmonics of the potential, one can approximate this curve with a trapezoidal one. Then the amplitude of the first harmonics of the potential is

$$B_0 \frac{\lambda_u}{2\pi} \sinh \frac{\pi g}{\lambda_u} = H_c \frac{2\lambda_u}{\pi^2} \sin \frac{\pi}{\lambda_u}. \quad (2)$$

Eq. (2) leads to a simple estimate of the field amplitude:

$$B_0 = \frac{4}{\pi} H_c \frac{\sin \frac{\pi}{\lambda_u}}{\sinh \frac{\pi g}{\lambda_u}} \rightarrow H_c \frac{\sin \frac{\pi}{\lambda_u}}{\sinh \frac{\pi g}{\lambda_u}}. \quad (3)$$

Due to the finite height (and width) of the magnetic blocks the magnetic field intensity in them is less than  $H_c$ . Therefore, we have replaced the factor  $4/\pi$  in Eq. (3) with unity. To take into account the breaks between the undulator poles which appear when one increases the period, this amplitude has to be multiplied by the factor

$$\kappa = 1 - \frac{\pi}{\tanh\left(\frac{\pi g}{\lambda_u}\right)} \frac{1}{\text{sinc}\left(\frac{\pi}{\lambda_u}\right)} \left(\frac{\Delta}{\lambda_u}\right)^2, \quad (3')$$

where  $\Delta$  is the break length.

The modified Eq. 3 gives a very good approximation for a general case with an increased period, as demonstrated in Fig. 3.

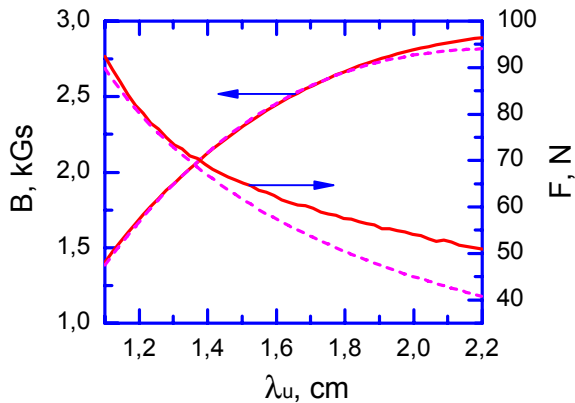


Figure 3: Dependence of the first harmonic amplitude and repulsion force on the undulator period. The solid curves are for the simulation results, the dashed curves are for the analytical expressions.

A similar consideration may be applied to a pure permanent magnet undulator shown in Fig. 4.

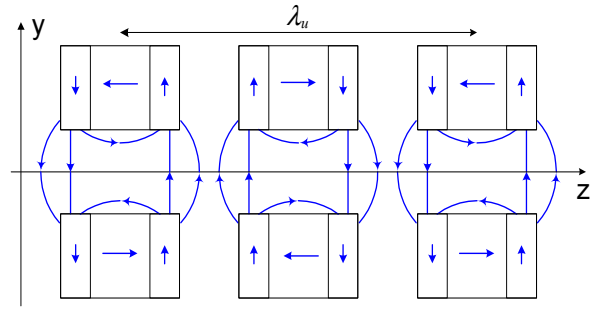


Figure 4: The pure permanent magnet undulator with splitted "poles" (side view). Lines of magnetic induction are shown with arrows.

### MECHANICAL DESIGN

As shown in Fig. 1, the two half-poles and permanent magnet block are assembled into a rigid unit. An interesting feature of this design is the strong longitudinal repulsion between the units. To estimate it one can calculate the magnetic field pressure in the  $x$ - $y$  plane in the middle of a splitted pole. The field there has no longitudinal ( $z$ ) component, and it is maximal near the edges (top, bottom, left, and right) of the pole halves. A rough estimate of the repulsion force for a pole of a perimeter  $P$  is

$$F = \frac{H_c^2}{8\pi} \frac{(\lambda_u)_{\min} P}{2\pi^2} \arcsin\left(\frac{2t}{\lambda_u}\right). \quad (4)$$

Comparison of this expression with the simulation results is presented in Fig. 3.

For a short-period undulator it is of the order of 0.1 kN. Moreover, forces between different undulator half-periods are equal with high accuracy. Therefore, it is enough to fix the end units only.

This feature opens the possibility of variations of the number of periods during wavelength tuning. An example of such design is shown in Fig. 5.

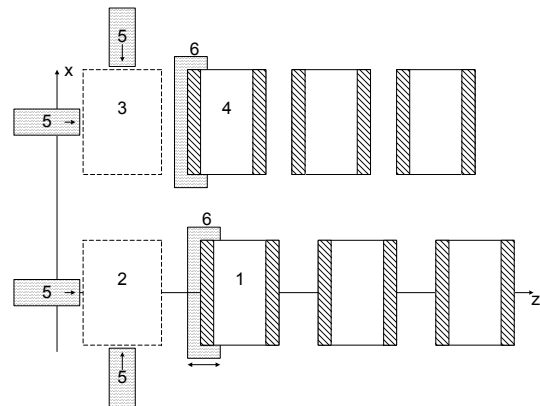


Figure 5: One of the possible schemes of variation of the number of periods (top view). (1) – (4) successive positions of the block being removed, (5) pushers, (6) adjustable stops. The lower blocks are in the undulator, the upper ones, in the cartridge for spare blocks.

When the undulator period increases, the pushers move the extra blocks to the side cartridge. The “feeders” may be placed near the undulator ends. Thus, the number of periods may also be variable, and the full length allocated for the undulator is used. The longitudinal displacement of the undulator ends provides the possibility of phasing undulators in long undulator chains (as it takes place in a high gain x-ray FEL) without dedicated phase shifters.

### ADVANTAGES OF THE VARIABLE-PERIOD UNDULATORS

To compare the VPU with the variable-amplitude undulator, we have to consider particular applications. The simplest one is generation of spontaneous radiation. For a conventional undulator, a significant  $K$  value is required for wavelength tuning. Indeed, according to Eq. (1) one needs  $K > 1$  to increase the wavelength twice. Taking into account the field estimate of Eq. (3) and assuming  $t = \lambda_u/3$ , we get

$$\frac{eH_c \lambda_u \sqrt{3}}{4\sqrt{2}\pi m c^2 \sinh \frac{\pi g}{\lambda_u}} > 1. \quad (5)$$

For  $H_c = 13$  kOe and  $g = 1$  cm it gives  $\lambda_u > 2.4$  cm.

For the VPU one can allow  $K = 0.1$  and have  $(\lambda_u)_{\min} = 1.1$  cm for the same parameters. Eqs. (3, 3') show that it is necessary to increase this period up to  $\lambda_u = 2$  cm to double the wavelength. For this period  $K = 0.4$ .

If we discuss the same wavelengths for these two undulators, the required particle energies differ by a factor of 1.45. For storage rings such energy decrease is very significant. It reduces the beam emittance and synchrotron radiation losses and, therefore, the RF power. Moreover, low- $K$  undulators cause very low tune shifts.

Another prospective application is a high-gain x-ray FEL. As the required beam peak current in such an FEL is rather high, the gap of the undulator is limited by wakefields. If the wavelength of such an FEL is changed via raising the undulator gap from its minimal value, the gain length increases dramatically. The reason is not only the decrease in the particle-wave interaction due to a lower  $K$ , but the decrease in the wavelength and, correspondingly, the ratio of the wavelength to the beam emittance. If this ratio is not large, a finite longitudinal velocity spread and transverse beam sizes lead to a significant increase in the gain length.

Another problem of the variable-gap undulators are extremely tight tolerances for the gap difference in different sections of the long undulators.

The VPU, on contrary, can be optimized at the minimum wavelength and minimum gap. Then, for the shortest wavelength the undulator gap (and therefore the period) is less than the maximum gap of the variable-gap undulator. Therefore, the use of VPU decreases the gain length and required beam energy of high-gain FELs. An example of the gain length dependence on the radiation wavelength is shown in Fig. 6.

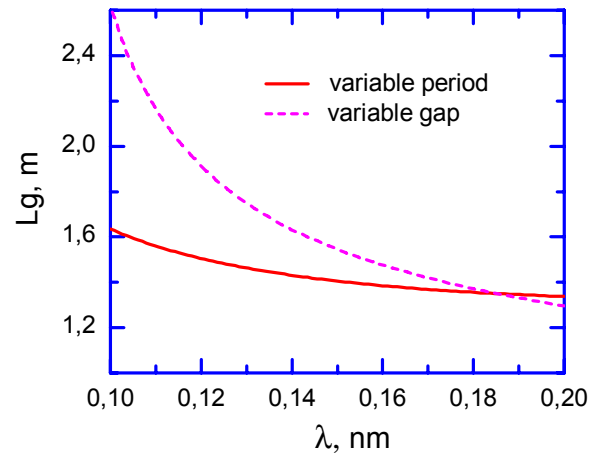


Figure 6: The dependence of the gain length on the radiation wavelength.

The calculations were performed with the M. Xie formula [9] for a minimum gap of 6 mm, electron energy of 5.8 GeV, peak current of 2 kA, relative energy dispersion of  $10^{-4}$ , and normalized emittance of 0.2 micron. At the left edge of Fig. 6, the VPU period is minimal, and the variable gap period is more than 6 mm. At the right edge, the gap of the variable gap undulator is 6 mm.

Another advantage of the VPU is a precisely fixed gap (see Fig. 7). Tolerances for the period variations along the undulator are not so tight. Moreover, as it was mentioned above, the phase shifters may be eliminated.

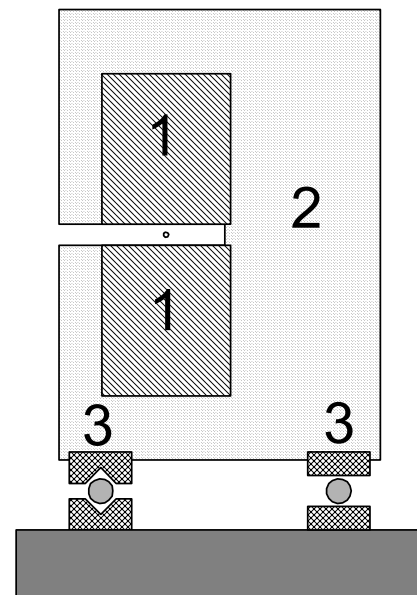


Figure 7: Side view of a half-period of the fixed-gap VPU. (1) poles, (2) plate, (3) ball-bearing guideways.

Further performance improvement of VPU is possible with the use of cooling to low temperatures [10]. This option is rather easy due to the small transverse sizes of VPU.

## CONCLUSION

A new design of permanent magnet undulators, VPU, was considered in the paper. It opens new prospects for further improvements of accelerator-based radiation sources.

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