# BEAM QUALITY AND TRANSPORT STABILITY SIMULATIONS IN ECHO ENABLED HARMONIC GENERATION

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### Abstract

The method of Echo Enabled Harmonic Generation (EEHG) [1, 2] is a possible method of achieving coherent short wavelengths in an FEL amplifier. In this paper the effects of noise variations and some of the important parameters affecting the stability of the final harmonic bunching of the electron beam are investigated numerically.

## **INTRODUCTION**

The so-called echo effect has been studied for many years in phenomena such the photon echo [3], spin echoes [4], and the echo effect in circular accelerators [5]. The EEHG scheme for improving temporal coherence in FELs was recently proposed in [1] and developed in [2]. The EEHG scheme allows high harmonics of a seed laser to be generated with high frequency up-conversion efficiency. The scheme consists of two modules of an undulator and dispersive section, both of which energy modulate and then disperse the beam. The effect of this is to give the beam a complicated phase space structure containing high harmonic components. The notation set out in [1, 2] gives the energy modulation parameters as  $A_1 = \Delta E_1 / \sigma_E$  and  $A_2 = \Delta E_2 / \sigma_E$ , where  $\sigma_E$  is the rms energy spread of the beam and  $\Delta E_1$  and  $\Delta E_2$  are the values of energy modulation imparted on the beam in the first and second undulators respectively. The dispersive parameters in the first and second dispersive sections are  $B_1 = R_{56}^{(1)} k_1 \sigma_E / E_0$  and  $B_2 = R_{56}^{(2)} k_2 \sigma_E / E_0$ respectively, with  $R_{56}$  the standard of dispersion term.

### **PARAMETER ANALYSIS**

The four main parameters of EEHG scheme are the two energy modulation parameters  $A_1$ ,  $A_2$  and the dispersive parameters  $B_1$  and  $B_2$ . These parameters have varying degrees of control over the bunching factor. A study of the bunching factor with respect to these parameters was reported in [6]. This paper takes that approach one step further through analysing the bunching factor stability with respect to two simultaneously varying parameters. The bunching factor was calculated at the centre of the beam using  $b = |N^{-1}\sum \exp(-iazk_1)|$  where a = 24 (the harmonic factor).



Figure 1: The bunching factor as a function of the two energy modulation parameters  $A_1$  and  $A_2$ .

Figure 1 shows the dependence of the bunching factor on the energy modulation parameters  $A_1$  and  $A_2$ . The bunching factor shows remarkable stability in relation to the first energy modulation parameter  $A_1$ . Moderate adjustment to the second energy modulation parameter  $A_2$  can result in the bunching factor being significantly reduced. Figure 1 shows three distinct peaks in bunching parameter with variation of  $A_2$ ; the first peak occurs at  $A_2 = 1$  and the second peak at  $A_2 = 1.2$ . Between the peaks the bunching is significantly reduced. The stability of the EEHG scheme to variation in the first modulation section is considered in Fig. 2.



Figure 2: The bunching factor as a function of the first energy modulation parameter  $A_1$  and the first dispersive parameter  $B_1$ .

The bunching factor at the 24<sup>th</sup> harmonic is very stable with variation in the parameters  $A_1$  and  $B_1$  of the first EEHG section. The bunching factor is also relatively insensitive to variation of the parameters  $A_1$  and  $B_2$  (as shown in Fig. 3), the first undulator parameter and second dispersive parameter. In the cases of Fig. 2 and 3, a change in one parameter could be compensated through a change in the other. The bunching factor as a function of  $A_2$  and  $B_2$  is shown in Fig. 4, and as a function of  $A_2$  and  $B_1$  in Fig. 5. In both cases there are three distinct peaks, potentially allowing a choice of bunching factor is not aligned with either axis, implying that a change in parameter  $B_2$  can be compensated for by adjusting parameter  $B_1$ .



Figure 3: The bunching factor as a function of the first energy modulation parameter  $A_1$  and the second dispersive parameter  $B_2$ .



Figure 4: The bunching factor a function of the second energy modulation parameter  $A_2$  and the second dispersive parameter  $B_2$ .



Figure 5: The bunching factor as a function of the second energy modulation parameter  $A_2$  and the first dispersive parameter  $B_1$ .



Figure 6: The bunching factor as a function of the first dispersive parameter  $B_1$  and the second dispersive parameter  $B_2$ .

#### **EEHG NUMERICAL MODELLING**

EEHG simulations were carried out using a 1D code written in MATLAB. These simulations are based on the notation set out in [1, 2]. A dimensionless energy deviation variable is defined as  $p = (E - E_0)/\sigma_E$ . The beam goes through four manipulations. Firstly the beam is given an energy modulation in the first undulator which modifies the energy coordinates as  $p' = p + A_1 \sin(k_1 z)$ . The first dispersive section modifies the longitudinal coordinates as  $z' = z + pB_l/k_l$ . These two steps are repeated in the second EEHG section but the values of  $A_2$  and  $B_2$  are generally significantly lower. This is to prevent the delicate microstructure of the beam generated in the first EEHG section from being washed out. The following parameters were used in the parameter analysis simulations: the energy modulation parameters were  $A_1 = 5$ ,  $A_2 = 1$  and the dispersive parameters

 $B_1 = 27.01$ ,  $B_2 = 1.14$ . The beam parameters were  $E_0 = 1.2$  GeV (average beam energy), rms energy spread of  $\sigma_E = 150$  keV and bunch length of 12 µm. The two seed lasers were chosen to have the same wavelength of 240 nm. These parameters generate bunching at the 24<sup>th</sup> harmonic (10 nm) of the initial seed laser, with a bunching factor of  $b_{24} = 0.1249$ . The phase space plot of beam at the exit of the last undulator (Fig. 7) reveals the beam's delicate microstructure which contains the higher harmonic components.



Figure 7: The second undulator and dispersive section generates a large density modulation at the harmonic in the beam's longitudinal plane.

The bunching factor for the  $24^{\text{th}}$  harmonic of a 240 nm seed laser (10 nm output) is  $b_{24} = 0.1249$ . The electron bunch is now sent to a radiator tuned to the resonant wavelength of 10 nm.

#### Non-Linear Energy Chirp

The effects of a linear energy chirp have been previously analysed in [6]. Here the effects of a nonlinear chirp are analysed. To do this, a crude approximation to an electron beam distribution generated from start-to-end simulations of the UK New Light Source [7] was constructed and used in simulations.

The non-linear energy chirp (shown in Fig. 8) has the following approximate parameters:  $E_0 = 2.206 \text{ GeV}$ and  $\sigma_E = 150 \text{ keV}$ . The energy chirped bunch was sent through an EEHG simulation with the following parameters,  $A_1 = 3$ ,  $A_2 = 1$ ,  $B_1 = -26.83$  and  $B_2 = -1.14$ . This gives a low bunching factor of 0.0090, as might reasonably be expected.



Figure 8: Approximation of a realistic electron beam distribution with non-linear energy chirp, based on [7].

In [6] it was shown that effects of a linear energy chirp can be compensated for by adjusting parameter  $B_2$ . This same technique can be applied to a non-linear energy chirp. The bunching factor is increased to 0.07 (as shown in Fig. 9) when parameter  $B_2$  is adjusted  $(B_2 = -1.118)$ . When dispersive parameters  $B_1$  and  $B_2$ are set to 26.83 and 1.14 respectively, the bunching factor value is initially 0.05 and can be increased to 0.07 upon adjusting parameter  $B_2$ . Interestingly, the bunching factor variation with  $B_2$  (Fig. 10) for the case when positive parameters are chosen is significantly different for the case when negative parameters are used (Fig. 9). This is due to the slope of the energy chirp (Fig. 8); when the parameters are negative the chirp causes bunch compression giving narrow peaks. However when the parameters are positive the chirp causes bunch expansion giving much wider peaks.



Figure 9: Bunching degradation due to an initial energy chirp can be compensated for by adjusting parameter  $B_2$  (from -1.14 to -1.118).



Figure 10: Bunching factor variation with  $B_2$  for the case of positive dispersive parameters,  $B_1$  and  $B_2$ .

### **CONCLUSIONS**

The EEHG scheme was modelled and electron bunch phase space evolution through the system was shown. Non-ideal effects were considered and a technique for overcoming energy chirps was demonstrated. Stability analysis was carried out for the energy modulation and dispersive parameters of the two modulator sections through simultaneously varying these parameters in pairs. This analysis revealed ways in which parameter variation can be compensated and also the potential to loosen the constraints on parameter selection. This type of analysis will be crucial when designing and fine tuning an EEHG FEL.

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