ONE-DIMENSIONAL FREE-ELECTRON-LASER EQUATIONS WITHOUT THE SLOWLY VARYING ENVELOPE APPROXIMATION

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Abstract

A set of one-dimensional equations have been deduced in the time-domain for describing the free-electron laser radiation without using the Slowly Varying Envelope Approximation. They are valid for arbitrarily short electron bunches and for current distributions with ripples on the scale of the wavelength. We demonstrate that, under the assumption that the backward low frequency wave be negligible, these equations can be reduced to the usual 1-d equations but with a different definition of the bunching term. Few numerical examples are presented, showing that for long homogeneous bunches the new set of equations gives results in agreement with the usual FEL theory, and that short or pre-bunched electron beams can decrease the lethargy.

INTRODUCTION

The usual equations describing the Free-Electron Laser radiation process [1-3] have been deduced in the framework of the Slowly Varying Envelope Approximation (SVEA), requiring that all the characteristic lengths L describing the electron beam (i.e. the length of the beam L_b, the characteristic lengths of the gradients, the cooperation length L_c) are very much larger than the wavelength λ of the radiation. Under this hypothesis, the second order radiation equation can be simplified in a first order one, the low frequency resonant backward wave is disregarded, the shorter interval of length that can be resolved being just the wavelength λ. This model in its 1-d version [1],[2] and in the 3-d extension [3], has been at the basis of the development of various numerical codes [4]-[9] which have been extensively and successfully used in the project and in the interpretation of almost all FEL experiments.

Few works [10]-[13] tried to reintroduce the backward wave into the model, associating to the particle equations not one, but a couple of independent radiation equations, written for two wave packets centered respectively on two different single resonances correlated only by the electrons. In this way, a further characteristic length, the wavelength of the second FEL resonance λ_{low}, is introduced, and the set of equations holds under the further assumption $L>>\lambda_{low}$. This system of equations do not answer to the expectations for a tool suitable for studying short or pre-bunched beams, because, even if it is true that the usual application of the SVEA necessarily excludes all backward waves, the radiation field is supposed to be the superposition of two wave packets both structured with slowly varying amplitudes. The same method has been applied for the insight of the radiation harmonic, where, however, if the SVEA is valid for the fundamental, than it surely holds also for all the harmonics.

Superradiance and coherent spontaneous emission produced in short bunches have been the object of the studies of Refs [14]-[15]. In the first paper, an approach in the frequency-domain is developed, while a time-domain model is presented in the second one. In both approaches, the radiation is assumed to have a broad bandwidth around the high frequency resonance, but the contribution of other parts of the spectrum is neglected, together with the second derivatives in the radiation equation.

In this paper, we derive a set of 1-d FEL equations from the Maxwell-Lorentz system, without using the SVEA, valid therefore for arbitrarily short electron bunches and for current longitudinal distributions with ripples on the scale of the wavelength. No hypotheses on the spectrum are assumed and, therefore, all the spectrum is globally taken into account. Furthermore we develop a code which solves these equations. We demonstrate that, under the assumption that the backward low frequency wave is negligible, these equations can be reduced to the usual SVEA-like 1-d equations but with a different definition of the bunching term, retrieving a model very similar to that of Ref. [15]. Few numerical examples are presented, showing that for long homogeneous bunches the new set of equations gives results in agreement with the usual FEL theory, and that short or pre-bunched electron beams can decrease the lethargy.

THE MODEL EQUATION

The From the Maxwell-Lorentz system, the following set of one-dimensional equations can be deduced, describing the dynamics of a beam of N_p electrons in an helical undulator and the consequent FEL radiation:

$$\frac{dz_j(t)}{dt}=\frac{c}{\bar{\gamma}_j(t)}P_j(t)$$

$$\frac{dP_j(t)}{dt}=-\frac{ca_{w0}}{\sqrt{2}\gamma_0^2}\frac{1}{\bar{\gamma}_j(t)}\left[\frac{\partial}{\partial z}(A(z,t)e^{ik_w z}+cc)\right]_{z=z_j(t)} \quad (1)$$

$$\left(\frac{\partial^2}{\partial t^2}-c^2\frac{\partial^2}{\partial z^2}\right)A(z,t)=-\frac{4\pi e^2 n_S a_{w0}}{\sqrt{2}m\gamma_0}\sum_{j=1}^{N_p}\frac{e^{-ik_W z_j(t)}}{\bar{\gamma}_j(t)}\delta(z-z_j(t))\equiv S(z,t)$$

In (1) z_j is the longitudinal coordinate, $P_j(t) = \beta_{j||}(t)\gamma_j(t)/\gamma_0$ is the normalized momentum,

$$\bar{\gamma}_j(t) = \gamma_j(t)/\gamma_0 \approx \left(\frac{1+a_{w0}^2}{\gamma_0^2} + P_j^2(t)\right)^{1/2}$$

and γ_j the Lorentz factor of the j-th electron. γ_0 is the average value of $\gamma_j(t{=}0)$ and n_s is the superficial electron density. Furthermore, the undulator vector potential is described by the expression:

$$\mathbf{A}_w(z) = \frac{a_{w0}}{\sqrt{2}}(e^{-ik_w z}\hat{\mathbf{e}} + cc) \qquad (2)$$

with a_{w0} the wiggler parameter, k_w the wiggler wave number, and $\hat{\mathbf{e}} = (\mathbf{e}_x + i\mathbf{e}_y)/\sqrt{2}$.

The radiation potential vector is represented by:

$$\mathbf{A}(z,t) = A(z,t)\hat{\mathbf{e}} + cc. \qquad (3)$$

Both $\mathbf{A}$ and $\mathbf{A}_w$ have been written in the Coulomb gauge and normalized with respect to mc^2/e.

Only few standard hypotheses have been done: in particular, we have supposed that expression (2) is valid everywhere, not only on the z-axis, we have disregarded space charge, we have assumed that the transverse components of the generalized momenta vanish, and that the radiation potential $|A(z,t)|$ is smaller than a_{w0} during all the radiation process.

Then we simply write the radiation as the sum of two terms:

$$A(z,t) = A_p(z,t)+A_r(z,t) \qquad (4)$$

The set of equations

$$\begin{aligned}(\frac{\partial}{\partial t} + c\frac{\partial}{\partial z})A_p(z,t) &= Q(z,t)\\ (\frac{\partial}{\partial t} - c\frac{\partial}{\partial z})A_r(z,t) &= -Q(z,t)\end{aligned} \qquad (5)$$

is completely equivalent to the last of equations (1) if $Q(z,t)$ is solution of the equation :

$$\frac{\partial}{\partial z}Q(z,t) = -\frac{1}{2c}S(z,t) \qquad (6)$$

From the definition of S, and solving (6) with the condition $\lim_{z\to-\infty}Q(z,t) = 0$ for each $t \geq 0$, one obtains:

$$\begin{aligned}Q(z,t) &= \frac{4\pi e^2 n_S a_{w0}}{2\sqrt{2}mc\gamma_0}\sum_{j=1}^{N_P}\frac{e^{-ik_W z_j(t)}}{\bar{\gamma}_j(t)}\int_{-\infty}^{z} dz'\delta(z'-z_j(t))\\ &= \frac{4\pi e^2 n_S a_{w0}}{2\sqrt{2}mc\gamma_0}\sum_{j<z}\frac{e^{-ik_W z_j(t)}}{\bar{\gamma}_j(t)}\end{aligned} \qquad (7)$$

where the last sum is performed over all the electrons of the beam behind the point z.

Our equations are therefore :

$$(\frac{\partial}{\partial t} + c\frac{\partial}{\partial z})A_p(z,t) = \frac{4\pi e^2 n_S a_{w0}}{2\sqrt{2}mc\gamma_0}\sum_{j<z}\frac{e^{-ik_W z_j(t)}}{\bar{\gamma}_j(t)} \qquad (8)$$

$$(\frac{\partial}{\partial t} - c\frac{\partial}{\partial z})A_r(z,t) = -\frac{4\pi e^2 n_S a_{w0}}{2\sqrt{2}mc\gamma_0}\sum_{j<z}\frac{e^{-ik_W z_j(t)}}{\bar{\gamma}_j(t)} \qquad (9)$$

$$\frac{dz_j(t)}{dt} = \frac{c}{\bar{\gamma}_j(t)}P_j(t) \qquad (10)$$

$$\frac{dP_j}{dt} = \frac{-ca_{w0}}{\sqrt{2}\gamma_0^2\bar{\gamma}_j(t)}\left[\frac{\partial}{\partial z}((A_p + A_r)e^{ik_w z} + cc)\right]_{z=z_j} \qquad (11)$$

The emitted power P for unit of surface S can be written as:

$$\frac{dP}{dS} = \frac{m^2c^5}{2\pi e^2}\left[\left|\frac{\partial A_p}{\partial z}\right|^2 - \left|\frac{\partial A_r}{\partial z}\right|^2\right] \qquad (12) \quad .$$

The relation between these no SVEA equation and the usual SVEA ones can be made by introducing the hypothesis that

$$A(z,t) = M(z,t)e^{i(kz-ckt)} \qquad (13)$$

with $k = k_w\beta_0/(1-\beta_0)$ and $\beta_0 \approx (1-(1+a_{w0}^2)/\gamma_0^2)^{1/2}$ and by obtaining the following equation:

$$\begin{aligned}&(\frac{\partial}{\partial t} + c\frac{\partial}{\partial z})M + \frac{i}{2ck}(\frac{\partial^2}{\partial t^2} - c^2\frac{\partial^2}{\partial z^2})M =\\ &-i\frac{4\pi e^2 n_S a_{w0}}{2\sqrt{2}mck\gamma_0}\sum_{j=1}^{N_P}\frac{e^{-i\theta_j(t)}}{\bar{\gamma}_j(t)}\delta(z-z_j(t))\end{aligned} \qquad (14)$$

Where $\theta_j(t) = (k+k_W)z_j(t)-ckt$ are the phase angles of the electrons in the ponderomotive wave.

With an average operation $< >$ on the wavelength λ and with the cancellation of all second derivatives, which have been supposed smaller than the other terms due to the required slowly variation of the amplitude $M(z,t)$, the usual SVEA model equation is retrieved:

$$\begin{aligned}&(\frac{\partial}{\partial t} + c\frac{\partial}{\partial z}) < M(z,t) > =\\ &-i\frac{4\pi e^2 n_S a_{w0}}{2\sqrt{2}mck\gamma_0 L_m}\sum_{s(L_m)}\frac{e^{-i\theta_s(t)}}{\bar{\gamma}_s(t)} \equiv b(z,t)\end{aligned} \qquad (15)$$

Here the right hand side term $b(z,t)$ is the usual bunching factor defined in the framework of the SVEA and, accordingly, the sum is extended over all the electrons inside an interval $z-L_m/2 < z_s(t) < z+L_m/2$, with the average length $L_m>\lambda$.

NUMERICAL RESULTS

Equations (8)-(11) have been integrated numerically starting from noise and the results compared with those obtained by using the SVEA model described by equation

(15). Typical results are presented in Fig. 1 and in Fig. 2 . Common values of the parameters are: $N_e=2.10^8$ (N_e total number of electrons), $\gamma_0=100$, a_{w0} = 1.47 , λ_w = 2.8 cm, r_b=50 μm (r_b is the beam radius).. With these numbers, the resonant wavelength is λ = 4.42 μm. The case presented in Fig 1 describes the physics of a long bunch with L_b=800 μm (I=12 A), while the second one in Fig. 2, which can be classified as a situation representative of short bunches, has L_b=20 μm (I=480 A). In both Figures, red curves give the average power vs z/L_g , solutions of the equations (8)-(12), while the blue curves are the usual SVEA results, obtained by integrating equation (15)-(17).

For long bunches, the growth of the power and the saturation values shown by the two curves in Fig. 1 are very similar with discrepancies within few percents due to the different numerical schemes, while for shorter bunches, the details of the growth are also qualitatively different and the saturation power is larger in the non SVEA case by roughly a factor between 2 and 3.

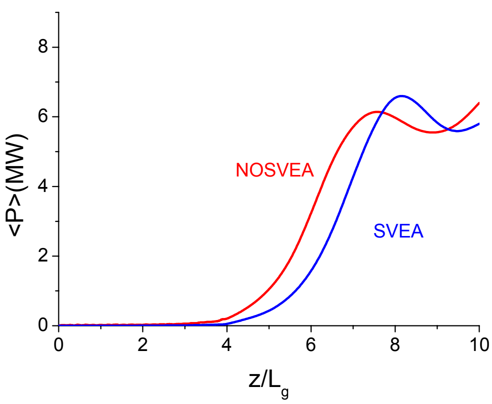

Figure 1: Average power <P> vs z/L_g in the case SVEA (solution of equation 15) and NOSVEA (solution of equations (8)-(11)) for L_b=800 μm.

Another different particular is that the lethargy presented by the radiation emitted short bunches in the treatment without the SVEA is shorter, and the radiation present a very early growth characterized by oscillations whose spatial periodicity is the wiggler wavelength.

This large amount of spontaneous emission shown by short beams with large gradients was first observed in Refs [16] and studied in detail in [14] and [15].

We have already shown that from the general equations (8)-(11) it is possible to deduce the SVEA equation (15), by admitting that the average value <M(z,t)> of the function M(z,t), defined in (13) be a slowly varying function both in space and time. These conditions are sufficient, but not necessary for reducing (8) in (15).

In fact, starting again by (8) in the form:

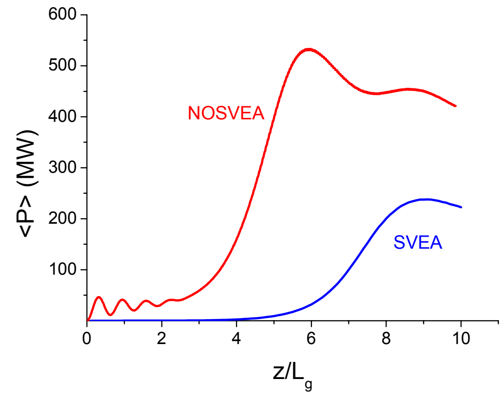

Figure 2: Average power <P> vs z/L_g in the case SVEA (solution of equation 15) and NOSVEA (solution of equations (8)-(11)) for L_b=20 μm.

$$\left(\frac{\partial}{\partial t}+c\frac{\partial}{\partial z}\right)A_p(z,t)=$$
$$\frac{4\pi e^2 n_S a_{w0}}{2\sqrt{2}mc\gamma_0}\sum_{j=1}^{N_P}\frac{e^{-ik_W z_j(t)}}{\bar{\gamma}_j(t)}\int_{-\infty}^{z}dz'\delta(z'-z_j(t)) \qquad (16)$$

and introducing a new quantity :

$$N_p(z,t)=-\frac{i}{k}\frac{\partial A_p(z,t)}{\partial z}e^{i(kz-ckt)} \qquad (17)$$

we find without any restriction:

$$\left(\frac{\partial}{\partial t}+c\frac{\partial}{\partial z}\right)N_p(z,t)=\frac{-i4\pi e^2 n_S a_{w0}}{2\sqrt{2}mck\gamma_0 L_m}\sum_{s(L_m)}\frac{e^{-i\theta_s(t)}}{\bar{\gamma}_s(t)} \qquad (18)$$

which is valid in the only limit that the length L_m used in the process of average be sufficiently small to permit to indentify the average value <$N_p(z,t)$> with the punctual value $N_p(z,t)$. By comparing (18) and (8), we conclude that $N_p(z,t)$ and the slowly varying envelope M(z,t) satisfy similar equations with the only difference of the dimension of the length of average, which, usually, when the equation is used in the SVEA limit, is indeed the wavelength, but, in general, can be reduced according to the dimension of the shorter characteristic length in the problem.

Furthermore, since the quantities $N_p(z,t)$ and M(z,t) appear in the particle equation in the same way, then they can be considered indeed equal if they satisfy the same initial conditions.

This last circumstance is satisfied in all cases when the regressive term $N_r(z,t)$ can be considered negligible and when: $\frac{k_W}{k}\approx\frac{1+a_{w0}^2}{2\gamma_0^2}<<1$.

In Fig 3 and 4 we present the solution of equation (18) for various values of the average length L_m for the same parameters of Fig 1 and 2 respectively.

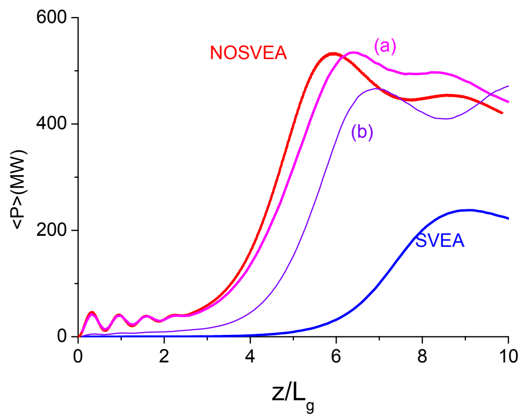

Figure 3: Average power vs z/L_g in the same cases of Fig. 1 and (a) solution of equation (18) with L_m=0.25 λ, and (b) L_m=0.75 λ.

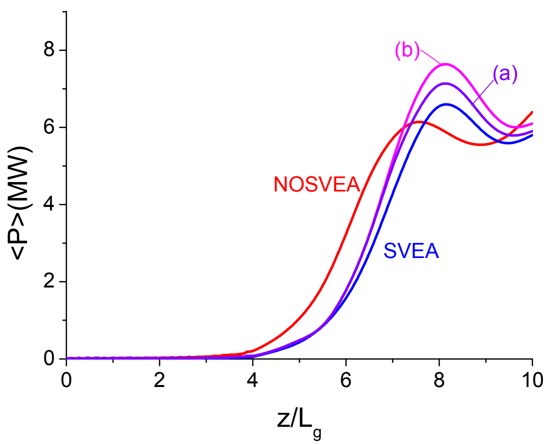

Figure 4: Average power vs z/L_g in the same cases of Fig. 2 and (a) solution of equation (18) with L_m=0.5 λ, and (b) L_m=0.1 λ.

The numerical evidencies show that the code based on equation (20) gives results that, in the case of short bunches, are more and more similar to the no SVEA results as the average length L_m is decreased (see Fig. 3, where curve (a) is made with L_m=0.25 λ, and (b) with L_m=0.75 λ). In the case of long bunches, all the cases (SVEA, no SVEA, and different average lengths L_m, i.e., (a) made with L_m=0.5 λ, and (b) with L_m=0.1 λ) give very similar results. In cases where the backward waves are not important and after a careful weight of the dimensions involved, the code based on equations (18)-(20), can be substituted to that based on (8)-(12), with saving of computer time.

CONCLUSIONS

We have written a set of equations valid outside the SVEA limits, which can be used to investigate the FEL radiation when the electron bunch is short or presents density gradients on the wavelength scale.
We have developed a numerical code for integrating these equations and the results have been compared with those of the usual SVEA approach.
We have confirmated that short bunches present a strong initial spontanous emission, shorter lethargy and larger saturation values.

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