FEL-IFEL, A CROSSED FIELD WIGGLER SCHEME FOR ENERGY TRANSFER BETWEEN TWO ELECTROMAGNETIC WAVES

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Abstract

A combination of two planar magnetic wigglers with orthogonal fields and a shared electron beam is proposed for energy transfer between two different electromagnetic waves. It is shown that one of the wigglers can acts as an IFEL accelerator by extracting energy from a seed wave, while simultaneously another wiggler works as a FEL and amplifies its corresponding resonant frequency. The equation of motion in the small signal gain (SSG) regime for this FEL-IFEL structure is studied. It is shown that the bunching process occurs for the electron beam in two different scales, corresponding to two different ponderomotive waves. It is concluded finally that, in principle, it is possible to use a FEL-IFEL scheme for energy exchange between two electromagnetic waves and retain an electron beam in resonance with two different electromagnetic waves simultaneously.

INTRODUCTION

A combination of two planar magnetic wigglers with orthogonal fields and a shared electron beam is proposed for energy transfer between two different electromagnetic waves. It is shown that one of the wigglers can acts as an IFEL accelerator by extracting energy from a seed wave, while simultaneously another wiggler works as a FEL and amplifies its corresponding resonant frequency. The equation of motion in the small signal gain (SSG) regime for this FEL-IFEL structure is studied. It is shown that the bunching process occurs for the electron beam in two different scales, corresponding to two different ponderomotive waves. It is concluded finally that, in principle, it is possible to use a FEL-IFEL scheme for energy exchange between two electromagnetic waves and retain an electron beam in resonance with two different electromagnetic waves simultaneously.

FEL-IFEL STRUCTURE

Consider a wiggler which is composed of two planar wigglers that its total vector potential can be presented as

$$\vec{A}_w = \frac{B_{1w}}{k_{1w}}\sin(k_{1w}z)\hat{x} + \frac{B_{2w}}{k_{2w}}\sin(k_{2w}z)\hat{y} \qquad (1)$$

where $B_{1,2w}$ magnetic field amplitudes and $k_{1,2w} = 2\pi/\lambda_{1,2w}$ are and wavenumbers associated to the wiggler periods $\lambda_{1,2w}$ of two magnetic fields. Each of planar wigglers interacts with an electromagnetic wave which has the same polarity as the wiggler field. Therefore, the total vector potential of the radiation wave has the form

$$\vec{A}_r = \frac{E_{10}}{k_1}\sin[k_1(z-ct)+\psi_1]\hat{x} + \frac{E_{20}}{k_2}\sin[k_2(z-ct)+\psi_2]\hat{y} \qquad (2)$$

where $E_{1,20}$, $k_{1,2}$ and $\psi_{1,2}$ are amplitudes, wavenumbers and phases of two waves, respectively. The Hamiltonian of the system can be written as

$$H = \left[(\vec{p}-e\vec{A})^2 c^2 + m^2c^4\right]^{1/2} \qquad (3)$$

Then, from the canonical equations of motion we get to the following expressions for the transverse components of electron velocity

$$\beta_x = -\frac{K_1}{\gamma}\sin(k_{1w}z) - \frac{K_{1r}}{\gamma}\sin[k_1(z-ct)+\psi_1] \qquad (4)$$

$$\beta_y = -\frac{K_2}{\gamma}\sin(k_{2w}z) - \frac{K_{2r}}{\gamma}\sin[k_2(z-ct)+\psi_2] \qquad (5)$$

Here $K_{1,2} = \frac{eB_{1,2w}}{k_{1,2w}mc}$ are the FEL parameters corresponding to each magnetic field, $K_{1,2r} = \frac{eE_{1,20}}{k_{1,2}mc^2}$ and γ is the electron beam relativistic factor.

Now we introduce the ponderomotive phase of each e-wave as

$$\theta_i = (k_{iw}+k_i)z - kct \qquad (6)$$

Then, the z-component of electron velocity, after some manipulation and using the Bessel function expansion of sin function, becomes

$$\beta_z = \beta_0 + \sum \frac{F_{1i}K_iK_{ir}}{2\gamma^2}\cos(\theta_i+\psi_i)$$

where $F_{1i} = J_0(\xi_i) - J_1(\xi_i)$, $\xi_i = \dfrac{K_1^2 + K_2^2}{4[1+(K_1^2+K_2^2)/2]}$

and $\beta_0 = 1 - \dfrac{1}{2\gamma^2}\left[1 + \dfrac{1}{2}\sum (K_i^2 + K_{ir}^2)\right]$.

The time derivative of γ, which determines the rate of energy transfer of electron beam, now can be calculated as follows,

$$\dot{\gamma} = \frac{e\vec{\beta}\cdot\vec{E}}{mc} = -\sum_{i=1}^{2} \frac{F_{1i}K_i K_{ir}}{2\gamma} k_{ir} \sin(\theta_i + \psi_i)\,. \tag{7}$$

Introducing the resonant energy

$$\gamma_{Ri}^2 = \frac{k_i}{2k_{iw}}\left(1 + \frac{K_1^2 + K_2^2}{2}\right) \tag{8}$$

and two detuning parameters for e-beam as $\eta_1 = \dfrac{\gamma - \gamma_{R1}}{\gamma_{R1}}$

and $\eta_2 = \dfrac{\gamma - \gamma_{R2}}{\gamma_{R2}}$ we find for the ponderomotive phase

$$\dot{\theta}_i \approx 2k_{iw}\eta_i\,. \tag{9}$$

In an ideal situation where e-beam is in resonant with both em-waves, the following equality must hold

$$\dot{\eta}_1 = \dot{\eta}_2 \approx -\sum_{i=1}^{2} \Omega_i^2 \sin\theta_i \tag{10}$$

where $\Omega_i^2 = \dfrac{k_i F_{1i} K_i K_{ir}}{2\gamma_{Ri}^2}$. Therefore, the pendulum equation is now a set of two coupled pendulum equations for θ as

$$\ddot{\theta}_j = -2k_{jw}\sum \Omega_i^2 \sin\theta_i \text{ (j=1,2).} \tag{11}$$

Equivalently, we can solve the following set of first order equation for the phase-space evolution of electron beam,

$$\dot{\theta}_1 = 2k_{1w}\eta_1 \tag{12}$$

$$\dot{\theta}_2 = 2k_{2w}\eta_2 \tag{13}$$

$$\dot{\eta}_1 = -\sum_{i=1}^{2} \Omega_i^2 \sin\theta_i \tag{14}$$

Here, η_1 and η_2 differ only in a constant. Now we can solve this set of equations numerically. The results for the phase-space evolution of a set of initially uniform distributed electron beam after pass through a wiggler with $\lambda_{1w} = 10\lambda_{2w}$ is shown in Fig. 1.

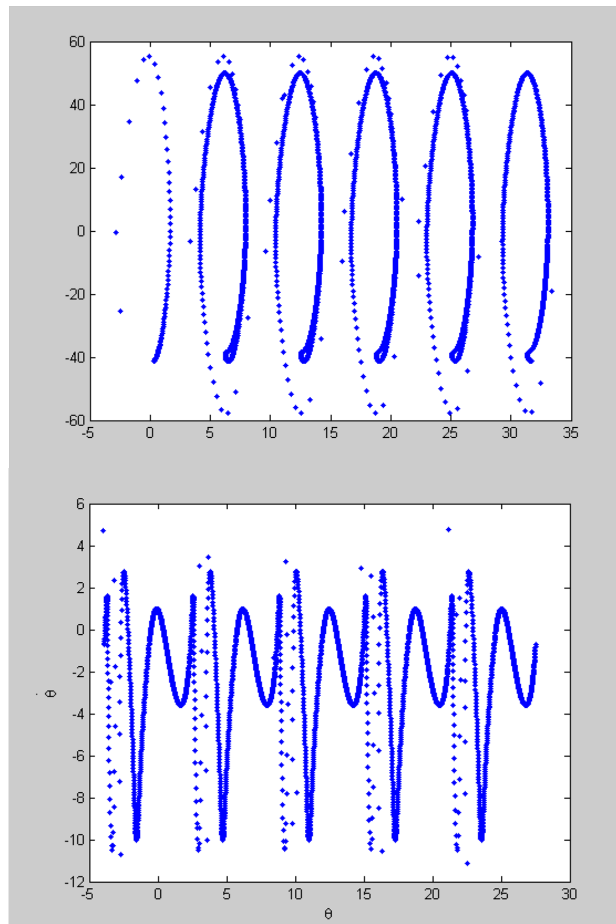

Figure 1: Phase-space evolution of electron beam in scales of two ponderomotive waves.

As one can see here, the bunching occurs for both ponderomotive phases. This means that both phenomena, namely FEL and IFEL, can occur in this system. The bunching factor that defines as

$$b = abs\left[\frac{1}{Np}\sum_{n=1}^{Np} \exp(in\theta)\right] \tag{15}$$

is also depicted in Fig. 2, for two types of bunching.

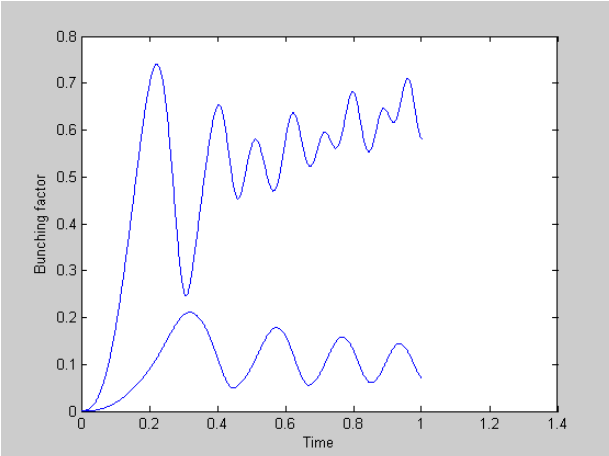

Figure 2: Bunching factor versus interaction time in FEL-IFEL structure.

This illustration shows that there is an optimize condition in which the bunching in both scales is considerable.

CONCLUSION

In this work we have done an elementary study about possibility of using FEL and IFEL structures together for energy exchange between two electromagnetic waves. Our investigations for bunching process in such configuration show that it is possible, essentially, to have electron bunching in such structure such that both FEL and IFEL phenomenon occur. More studies about growth rate and saturation mechanism are future works that can be considered for future.

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