THE PHYSICS OF FEL IN AN INFINITE ELECTRON BEAM*

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Abstract

We solve linearized Vlasov-Maxwell FEL equations for a 3-D perturbation in an infinite electron beam with Lorentzian energy distributions using paraxial approximation. We present analytical solutions for various initial perturbations and discuss the effect of optical guiding in such system.

INTRODUCTION

Developing complete theoretical model of Coherent electron Cooling (CeC) [1] is important for gaining insights into the physics of the processes, studying the scaling law and benchmarking simulation codes. Deriving analytical formula under certain assumptions is one of the key-stone in this process. For instance, the modulation process can be described by a close form solution obtained for an infinite electron beam with kappa-2 velocity distribution [2]. This solution is applicable to a realistic case when the transverse Debye radii are much smaller than the transverse size of electron beam.

In this work, we try to derive an analytical 3-D solution for the FEL amplification process under assumption of infinitely wide electron beam. 1D FEL theories has been applied to the amplification process in CeC, naturally assuming an infinite electron beam and longitudinally propagating radiation fields, i.e. $k_\perp = 0$ [3]. While 1D FEL theory provides closed-form analytical solutions for certain energy distributions, the diffraction effects are ignored. Hence, the transverse profile of the amplified modulation can not be obtained. In present day analytical 3D FEL theory, applied to specific spatial profiles of electron beam [4-6], the solutions are usually expanded into infinite number of modes determined by specific boundary conditions. In the high gain limit, the transverse profile of the electron modulation is determined by the mode with largest growth rate. However, for FEL with nominal or relatively short length, transient effect may not be ignored and thus presents difficulties in analytical evaluation of the amplification.

In order to incorporate the diffraction effects into analytical solution capable of describing the transient effect, we investigate the FEL amplification process for an infinite electron beam. The results derived under this assumption are applicable if the electron beam size is much larger than that of the amplified current modulation. Similarly to 1D FEL model, we assume the unperturbed electron spatial density is a constant and electrons are moving along their trajectory determined by the undulator field with no transverse dynamic effects from the radiation field or space charge. However, we allow the radiation to propagate with an angle with respect to the longitudinal direction, i.e. $k_\perp \neq 0$. Starting from the self-consistent paraxial field equation, we arrive to a third order ordinary differential equation (ODE). Analytical solutions are obtained for various initial conditions and the effect of optical guiding is discussed.

EQUATION OF MOTION

We use standard assumptions that the amplitude of the radiation field varies slowly with respect to the undulator period and that fast oscillation terms can be dropped. The paraxial equation on the amplitude of the radiation field is [4]

$$\begin{aligned}&\nabla_\perp^2 \tilde{E}(z,r_\perp,C) + 2i\frac{\omega}{c}\frac{\partial}{\partial z}\tilde{E}(z,r_\perp,C)\\ &= ij_0(r_\perp)\int_0^z dz'\left\{\frac{2\pi e}{c^2}\theta_s^2\omega\tilde{E}(z',r_\perp,C)\right.\\ &\left.+\frac{4\pi e}{\omega}\left[\nabla_\perp^2\tilde{E}(z',r_\perp,C)+2i\frac{\omega}{c}\frac{\partial}{\partial z}\tilde{E}(z',r_\perp,C)\right]\right\}\\ &\times\int_{-\infty}^{\infty} dP\frac{dF}{dP}\exp\left[i\left(C+\frac{\omega}{\gamma_z^2\varepsilon_0 c}\right)(z'-z)\right]\end{aligned} \tag{1}$$

where $\tilde{E}(z,r_\perp,C)$ is the complex amplitude of the radiation field, ω is the radiation frequency, C is the detuning, ε_0 is the nominal electron energy, P is the electron energy deviation, θ_s is the electron deflection angle, $F(P)$ is the energy distribution function and $j_0(r_\perp)$ is the transverse spatial distribution of the unperturbed electron beam. Assuming $j_0(r_\perp)=j_0$, the Fourier transformation of eq. (1) with respect to transverse spatial coordinates x and y yields

$$\begin{aligned}&i\frac{k_\perp^2 c}{2\omega}\tilde{E}(z,k_\perp,C)+\frac{\partial}{\partial z}\tilde{E}(z,k_\perp,C)\\ &=\frac{\pi e j_0\theta_s^2}{c}\int_0^z dz'\left\{\tilde{E}(z',k_\perp,C)+\frac{4ic}{\theta_s^2\omega}\left[i\frac{k_\perp^2 c}{2\omega}\tilde{E}(z',k_\perp,C)\right.\right.\\ &\left.\left.+\frac{\partial}{\partial z}\tilde{E}(z',k_\perp,C)\right]\right\}\int_{-\infty}^{\infty} dP\frac{dF}{dP}\exp\left[i\left(C+\frac{\omega P}{\gamma_z^2\varepsilon_0 c}\right)(z'-z)\right]\end{aligned} \tag{2}$$

Inserting the definition

$$\tilde{R}(z,k_\perp,C)\equiv e^{i\frac{k_\perp^2 c}{2\omega}z}\tilde{E}(z,k_\perp,C), \tag{3}$$

into eq. (2), we get the following:

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$$\frac{\partial}{\partial z}\tilde{R}(z,k_\perp,C)=\frac{\pi e j_0\theta_s^2}{c}\int_0^z dz' e^{i\frac{k_\perp^2 c}{2\omega}(z-z')}$$
$$\times\left\{\tilde{R}(z',k_\perp,C)+\frac{4ic}{\theta_s^2\omega}\frac{\partial}{\partial z'}\tilde{R}(z',k_\perp,C)\right\} \quad (4)$$
$$\times\int_{-\infty}^{\infty}dP\frac{dF}{dP}\exp\left[i\left(C+\frac{\omega P}{\gamma_z^2\varepsilon_0 c}\right)(z'-z)\right]$$

Using normalized variables defined in [4] and [7], eq. (4) becomes

$$\frac{\partial}{\partial\hat{z}}\tilde{R}(\hat{z},\hat{k}_\perp,\hat{C})$$
$$=\int_0^{\hat{z}}d\hat{z}' e^{i\hat{k}_\perp^2(\hat{z}-\hat{z}')}\left\{\tilde{R}(\hat{z}',\hat{k}_\perp,\hat{C})+i\hat{\Lambda}_p^2\frac{\partial}{\partial\hat{z}'}\tilde{R}(\hat{z}',\hat{k}_\perp,\hat{C})\right\}, \quad (5)$$
$$\times\int_{-\infty}^{\infty}d\hat{P}\frac{d\hat{F}}{d\hat{P}}\exp\left[i(\hat{C}+\hat{P})(\hat{z}'-\hat{z})\right]$$

where $\hat{z}\equiv z\Gamma$, $\hat{C}\equiv C/\Gamma$, Γ is the gain parameter:

$$\Gamma\equiv\left[\frac{\pi j_0\theta_s^2\omega}{c\gamma_z^2\gamma I_A}\right]^{1/3},$$

$I_A\equiv m_e c^3/e$ is the Alven current, $\hat{P}\equiv P/(\rho\varepsilon_0)$, $\rho\equiv\gamma_z^2\Gamma c/\omega$ is the pierce parameter, $\hat{k}_\perp\equiv\sqrt{\rho}k_\perp/(\sqrt{2}\gamma_z\Gamma)$, $\hat{\Lambda}_p$ is the space charge parameter defined as

$$\hat{\Lambda}_p\equiv\frac{1}{\Gamma^2}\left[\frac{4\pi j_0}{\gamma_z^2\gamma I_A}\right]^{1/2},$$

and $\hat{F}(\hat{P})$ is the energy distribution function satisfying

$$\int_{-\infty}^{\infty}\hat{F}(\hat{P})=1\cdot$$

In order to proceed further, we assume Lorentzian energy distribution, i.e.

$$\hat{F}(\hat{P})=\frac{1}{\pi\hat{q}}\frac{1}{1+\frac{\hat{P}^2}{\hat{q}^2}}\cdot \quad (6)$$

Inserting eq. (6) into eq. (5) results in the following:

$$\frac{\partial}{\partial\hat{z}}\tilde{R}(\hat{z},\hat{k}_\perp,\hat{C})=-i\int_0^{\hat{z}}d\hat{z}'(\hat{z}'-\hat{z})\exp\left[i(\hat{C}-\hat{k}_\perp^2)(\hat{z}'-\hat{z})-|\hat{q}(\hat{z}'-\hat{z})|\right].$$
$$\times\left\{\tilde{R}(\hat{z}',\hat{k}_\perp,\hat{C})+i\Lambda_p^2\frac{\partial}{\partial\hat{z}'}\tilde{R}(\hat{z}',\hat{k}_\perp,\hat{C})\right\} \quad (7)$$

It has been demonstrated that integro-differential equation with the form of (7) can be reduced to a third-order ODE (see Chapter 6.3.3 and 6.3.4 of [7]). Eq. (7) is transformed into

$$\frac{d^3}{d\hat{z}^3}\tilde{R}(\hat{z})+2\left(i\hat{C}_{3d}+\hat{q}\right)\frac{d^2}{d\hat{z}^2}\tilde{R}(\hat{z})$$
$$+\left[\hat{\Lambda}_p^2+\left(i\hat{C}_{3d}+\hat{q}\right)^2\right]\frac{d}{d\hat{z}}\tilde{R}(\hat{z})-i\tilde{R}(\hat{z})=0, \quad (8)$$

where we defined a new variable as:

$$\hat{C}_{3d}\equiv\hat{C}_\perp-\hat{k}_\perp^2. \quad (9)$$

The solution of (8) is the sum of three eigen-modes, i.e.

$$\tilde{R}(\hat{z})=\sum_{i=1}^{3}A_i(\hat{C},\hat{k}_\perp)e^{\lambda_i\hat{z}}, \quad (10)$$

where the eigen-value λ_i are determined by

$$\lambda^3+2(i\hat{C}_{3d}+\hat{q})\lambda^2+\left[\hat{\Lambda}_p^2+(i\hat{C}_{3d}+\hat{q})^2\right]\lambda-i=0 \quad (11)$$

and $A_i(\hat{C},\hat{k}_\perp)$ are determined by initial conditions at the FEL entrance. From eq. (3) and eq. (10), the complex amplitude of the radiation field is given by

$$\tilde{E}(\hat{z},\hat{k}_\perp,\hat{C})=e^{-i\hat{k}_\perp^2\hat{z}}\sum_{i=1}^{3}A_i(\hat{C},\hat{k}_\perp)e^{\lambda_i(\hat{C}_{3d})\hat{z}}. \quad (12)$$

The slowly varying radiation field amplitude is related to the current modulation via [4]

$$\left(\nabla_\perp^2+2i\frac{\omega}{c}\frac{\partial}{\partial z}\right)\tilde{E}(z,\vec{r}_\perp,C)=\frac{-4\pi i\theta_s\omega}{c^2}\tilde{j}(z,\vec{r}_\perp,C)\cdot \quad (13)$$

In the transverse wave vector domain, their relation is as follows

$$\tilde{j}_1(\hat{z},\hat{C},\hat{k}_\perp)=-\frac{c\Gamma}{2\pi\theta_s}\left[i\hat{k}_\perp^2+\frac{\partial}{\partial\hat{z}}\right]\tilde{E}(\hat{z},\hat{C},\hat{k}_\perp)\cdot \quad (14)$$

Inserting eq. (12) into (14) leads to

$$\tilde{j}_1(\hat{z},\hat{C},\hat{k}_\perp)=-\frac{c\Gamma}{2\pi\theta_s}e^{-i\hat{k}_\perp^2\hat{z}}\sum_{i=1}^{3}A_i(\hat{C},\hat{k}_\perp)\lambda_i(\hat{C}_{3d})e^{\lambda_i(\hat{C}_{3d})\hat{z}}. \quad (15)$$

From eq. (12) and eq. (15), the coefficients A_i can be explicitly written as the following matrix form

$$\begin{pmatrix}A_1\\A_2\\A_3\end{pmatrix}=\begin{pmatrix}1&1&1\\\lambda_1&\lambda_2&\lambda_3\\\lambda_1(\lambda_1-i\hat{k}_\perp^2)&\lambda_2(\lambda_2-i\hat{k}_\perp^2)&\lambda_3(\lambda_3-i\hat{k}_\perp^2)\end{pmatrix}^{-1}\begin{pmatrix}\tilde{E}\\\tilde{j}_1\\\frac{\partial}{\partial\hat{z}}\tilde{j}_1\end{pmatrix}_{\hat{z}=0} \quad (16)$$

where $\tilde{E}(0,\hat{C},\hat{k}_\perp)$, $\tilde{j}_1(0,\hat{C},\hat{k}_\perp)$ and $\frac{\partial}{\partial\hat{z}}\tilde{j}(0,\hat{C},\hat{k}_\perp)$ are initial conditions defined by a specific problem. In the following section, we will carry out calculations for excitation by external field with various pulse profiles.

EXCITATION WITH EXTERNAL FIELD

If the initial seeding of FEL is solely from external field, the coefficients A_i can be derived from eq. (16) as

$$A_i(\hat{C},\hat{k}_\perp)=\frac{\varepsilon_{ijk}\lambda_j\lambda_k}{\lambda_i^2-\lambda_i\lambda_j-\lambda_i\lambda_k+\lambda_j\lambda_k}\tilde{E}_{ext}(\hat{C},\hat{k}_\perp), \quad (17)$$

where $i,j,k=1,2,3$ and ε_{ijk} is the Levi-Civita symbol. For simplicity, we assume the transverse profile of the external field is Gaussian:

$$\tilde{E}_{ext}(\hat{C},\hat{k}_\perp)=\tilde{E}_{ini}F_\omega(\hat{C})\exp(-\hat{k}_\perp^2\hat{\sigma}_\perp^2), \quad (18)$$

where $\hat{\sigma}_\perp$ is a parameter describing the transverse range of the external field, $F_\omega(\hat{C})$ is a function describing the frequency content of the external field and $\tilde{E}_{ini}$ is a parameter determining the strength of the excitation.

Instantaneous Pulse

For an instantaneous excitation at the FEL entrance $\hat{z}=0$ described by Dirac delta-function $\delta(t)$, $F_\omega(\hat{C})$ is just a constant independent of $\hat{C}$. Without loss of generality, we assume

$$F_\omega(\hat{C})=1. \tag{19}$$

Thus the external field is written as

$$\tilde{E}_{ext}(\hat{k}_\perp)=\tilde{E}_{ini}\exp\left(-\hat{k}_\perp^2\hat{\sigma}_\perp^2\right), \tag{20}$$

and the coefficients A_i is

$$A_i(\hat{C}_{3d},\hat{k}_\perp)=\tilde{E}_{ini}A_{\omega,i}(\hat{C}_{3d})\exp\left(-\hat{k}_\perp^2\hat{\sigma}_\perp^2\right) \tag{21}$$

with

$$A_{\omega,i}(\hat{C}_{3d})\equiv\frac{\varepsilon_{ijk}\lambda_j\lambda_k}{\lambda_i^2-\lambda_i\lambda_j-\lambda_i\lambda_k+\lambda_j\lambda_k}. \tag{22}$$

Inserting eq. (21) into eq. (15) generates

$$\tilde{j}_1(\hat{z},\hat{C},\hat{k}_\perp)=-\frac{c\Gamma\tilde{E}_{ini}}{2\pi\theta_s}e^{-i\hat{k}_\perp^2\hat{z}-\hat{k}_\perp^2\hat{\sigma}_\perp^2} \times\sum_{i=1}^{3}A_{\omega,i}(\hat{C}_{3d})\lambda_i(\hat{C}_{3d})e^{\lambda_i(\hat{C}_{3d})\hat{z}}. \tag{23}$$

The current density modulation is given by the inverse Fourier transformation of (23) with respect to k_x, k_y and $\hat{C}$, i.e.

$$\tilde{j}_1(\vec{x},t)=\frac{c^2\Gamma^2\gamma_z^2\tilde{E}_{ini}}{8\pi^4\theta_s}e^{i\hat{k}_w\hat{z}}e^{i2\gamma_z^2\hat{k}_w(\hat{z}-c\hat{t})}\int_{-\infty}^{\infty}dk_x e^{ik_x x}\int_{-\infty}^{\infty}dk_y e^{ik_y y} \times e^{-i\hat{k}_\perp^2\hat{z}-\hat{k}_\perp^2\hat{\sigma}_\perp^2}\int_{-\infty}^{\infty}\sum_{i=1}^{3}A_i(\hat{C}_{3d})\lambda_i(\hat{C}_{3d})e^{\lambda_i(\hat{C}_{3d})\hat{z}}e^{-i2\gamma_z^2(\hat{z}-c\hat{t})\hat{C}}d\hat{C} \tag{24}$$

Since the Jacobian

$$J=\left|\frac{\partial(\hat{C},\hat{k}_x,\hat{k}_y)}{\partial(\hat{C}_{3d},\hat{k}_x,\hat{k}_y)}\right|=1, \tag{25}$$

changing in eq. (24) the integration variable from $\hat{C}$ to $\hat{C}_{3d}$ and integrating over k_x and k_y gives:

$$\tilde{j}_1(\vec{x},t)=\frac{c^2\Gamma^4\gamma_z^4\tilde{E}_{ini}}{4\pi^3\theta_s\rho}\frac{e^{i\hat{k}_w\hat{z}}e^{i2\gamma_z^2\hat{k}_w(\hat{z}-c\hat{t})}}{\sqrt{(\hat{\sigma}_x^2-i\xi)(\hat{\sigma}_y^2-i\xi)}}e^{-\frac{1}{4}\left(\frac{\hat{x}^2}{\hat{\sigma}_x^2-i\xi}+\frac{\hat{y}^2}{\hat{\sigma}_y^2-i\xi}\right)} \times\int_{-\infty}^{\infty}\sum_{i=1}^{3}A_{\omega,i}(\hat{C}_{3d})\lambda_i(\hat{C}_{3d})e^{\lambda_i(\hat{C}_{3d})\hat{z}}e^{-i2\gamma_z^2(\hat{z}-c\hat{t})\hat{C}_{3d}}d\hat{C}_{3d} \tag{26}$$

where

$$\xi(z,t)\equiv-\Gamma\left[2\gamma_z^2(z-ct)+z\right]. \tag{27}$$

As seen from eq. (26), the longitudinal evolution of the current density is identical to that in the 1D FEL theory, while the transverse evolution is described by a Gaussian function.

Gaussian Pulse

Consider an excitation of Gaussian pulse with finite duration, i.e.

$$F_\omega(\hat{C})=e^{-\hat{C}^2\hat{\sigma}_t^2}. \tag{28}$$

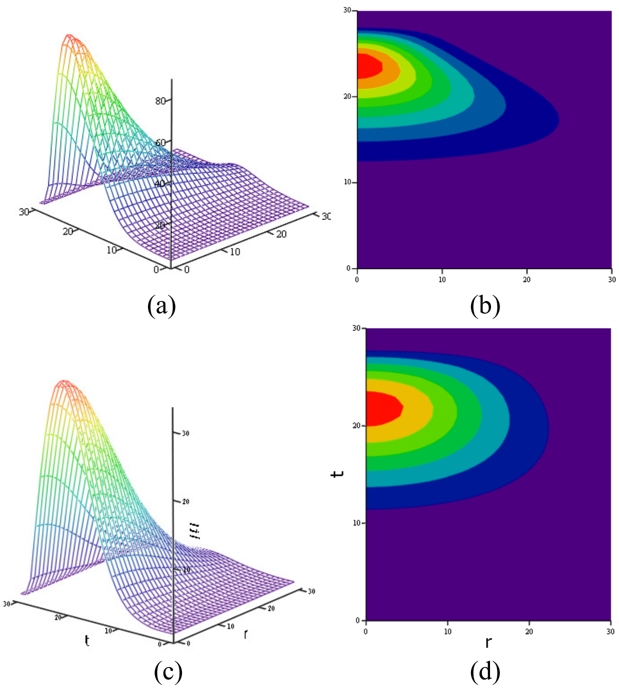


Figure 1 Amplitude of current density modulation at $\hat{z}=6$: a) surface plot of density as a function of the radius and time for $\hat{\sigma}_x=1$; (b) contour plot of (a); (c) surface plot for $\hat{\sigma}_x=2$; (d) contour plot of (c). By technical reasons the r and t differ from $\hat{r}$ and $2\gamma_z^2c\hat{t}$. The scaling is: $\hat{r}=\frac{4r}{15}$, $2\gamma_z^2c\hat{t}=-0.6+\frac{7.6t}{30}$.

Inserting (17), (18) and (28) into eq. (15) and conducting the inverse Fourier transformation results in

$$\tilde{j}_1(x,y,z,t)=\frac{c^2\Gamma^4\gamma_z^4}{4\pi^3\theta_s\rho}\tilde{E}_{ini}e^{i\hat{k}_w\hat{z}}e^{i2\gamma_z^2\hat{k}_w(\hat{z}-c\hat{t})}\int_{-\infty}^{\infty}d\hat{C}_{3d}I(r,\hat{C}_{3d}) \times\sum_{i=1}^{3}A_i(\hat{C}_{3d})\lambda_i(\hat{C}_{3d})e^{\lambda_i(\hat{C}_{3d})\hat{z}}e^{-i2\gamma_z^2(\hat{z}-c\hat{t})\hat{C}_{3d}} \tag{29}$$

where

$$I(r,\hat{C}_{3d})=\int_0^{\infty}e^{-(x+\hat{C}_{3d})^2\hat{\sigma}_t^2}e^{-(\hat{\sigma}_\perp^2-i\xi(\hat{z},t))x}J_0(\hat{r}\sqrt{x})dx. \tag{30}$$

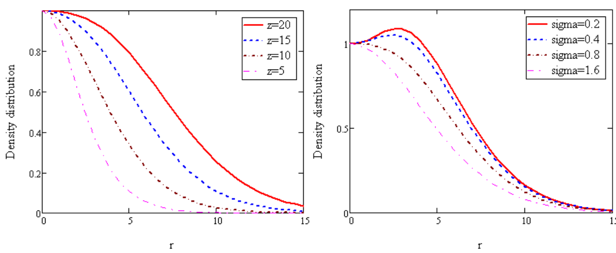
In general, the integral in (30) should be evaluated numerically. Fig.1 shows two distributions obtained by numerical integration of eq. (29) for $\hat{\sigma}_x=1$ and $\hat{\sigma}_x=2$ for initial Gaussian pulse with $\hat{\sigma}_t=0.1$. While the initial spot sizes differ by a factor of two, after six gain lengths the sizes become essentially identical. This is clear indication that the FEL system works in the diffraction dominated regime.

Monochromatic Wave

In case of a monochromatic wave at the FEL resonant frequency:

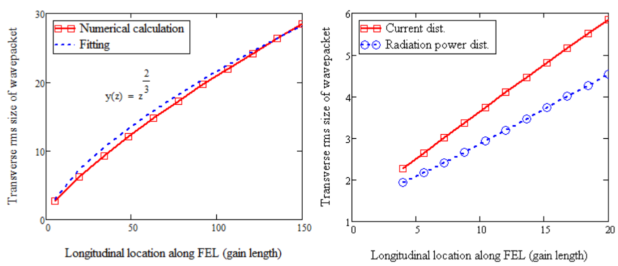
$$F_\omega(\hat{C})=\delta(\hat{C}). \tag{31}$$

and eq. (15), (17), (18) and (31) give:



(a) (b)

Figure 2. Profiles of current density for the monochromatic wave. They are normalized to their value at $\hat{r}=0$. (a) profiles for $\hat{\sigma}_\perp=1$ at various locations along FEL; (b) profiles $\hat{z}=15$ for various initial sizes, $\hat{\sigma}_\perp$.



(a) (b)

Figure 3. Transverse rms size of the density modulation and radiation power along the length of FEL. Only the growing mode is taken into account. (a) the rms size of the density modulation and the radiated power for $\hat{\sigma}_\perp=1$. (b) asymptotic dependence of the rms size of density modulation for initial size $\hat{\sigma}_\perp=0.3$;

$$\tilde{j}_1(\vec{x},t)=\frac{c^2\Gamma^4\gamma_z^4}{2\pi^3\theta_s\rho}e^{i\hat{k}_w\hat{z}}e^{i2\gamma_z^2\hat{k}_w(\hat{z}-c\hat{t})} \sum_{i=1}^{3}\int_0^\infty A_i\left(-\hat{k}_\perp^2\right)\lambda_i\left(-\hat{k}_\perp^2\right)J_0\left(\hat{k}_\perp\hat{r}\right)e^{\lambda_i\left(-\hat{k}_\perp^2\right)\hat{z}-i\hat{k}_\perp^2\hat{z}-\hat{k}_\perp^2\hat{\sigma}_\perp^2}\hat{k}_\perp d\hat{k}_\perp \quad (32)$$

where $\hat{k}_\perp=\sqrt{\hat{k}_x^2+\hat{k}_y^2}$ and we assume $\hat{\sigma}_x=\hat{\sigma}_y=\hat{\sigma}_\perp$ for simplicity.

Fig. 2 shows results of numerical integration of eq. (32) for various $\hat{z}$ and $\hat{\sigma}_\perp$. Fig. 2 (a) suggests that the transverse size keeps growing along the FEL. In order to investigate the nature of this growth, we studied the dependence of the transverse rms spread of the density modulation and that of the radiation power as function of the length along the FEL. As shown in fig.3 (b), the rms size of the density modulation grows near-linearly during few tens of the gain length (i.e. in a case of any practical FEL). This dependence would switch into one~ $z^{2/3}$ in an extremely large $\hat{z}\sim 100$, but this unphysical area is off interest.

DISCUSSION

Because the amplification of the plane wave in an FEL with infinitely wide beam depends on its propagation angle via changing the detuning from the FEL resonance (9): $\hat{C}_{3d}\equiv\hat{C}_\perp-\hat{k}_\perp^2$. One can expect that this will confine effective amplification to a narrow cone alone that axis of the FEL and, therefore, some optical guiding of the optical beam. Our numerical studies and analytical estimates showed that in a typical FEL this effect results in near-linear RMS size grows. Even though the growth of the transverse beam size is smaller than in free space case, the optical guiding FEL effect by an infinite electron beam is much smaller and is different from that by a beam with finite size.

SUMMARY

We obtained results resembling some aspects of the 1D theory, especially for the longitudinal dynamics. However, we successfully incorporated diffraction into the evolution of the transverse density modulation profile, which is of critical importance for studying the transverse coherence of the electron beam in CeC. For few selected initial conditions, the spatial domain solutions were expressed through 1D or 2D integrals, which can be readily numerically integrated.

We showed that while FEL dispersion provides some optical guiding, it is very different from that provided by the finite size electron beams.

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