SUB-RADIANCE AND THE COHERENCE LIMITS OF FEL*

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Abstract

The e-beam and radiation wave dynamics in the radiating and non-radiating beam transport sections of Free Electron Lasers are analyzed in the collective regime by use of a single transverse mode linear response formulation. This is employed to derive conditions for coherent operation of seeded high gain FELs. It is shown that the level of incoherent self-amplified spontaneous emission (SASE) radiation power can be controlled by adjusting the plasma oscillation phase in the non-radiating beam transport sections preceding the FEL, and that at short wavelengths the FEL coherence is limited by energy noise (rather than current shot-noise), and ultimately by quantum noise.

INTRODUCTION

Exciting progress in the technology of Free Electron Lasers (FEL) holds promise for the development of high brightness x-ray radiation sources at X-ray wavelengths with brightness six to ten orders of magnitude higher than that of other radiation sources in this wavelength regime. These capabilities have just been demonstrated recently at a wavelength of 1.5 Å in LCLS [1].

It would be highly desirable to operate FELs with temporal coherence and high spectral brightness. However, this is hampered by the temporal incoherence of SASE, which is considered to be ultimately limited by the current shot-noise [2], [3]. In recent years a number of schemes were developed to overcome the coherence limitation of FELs due to shot-noise. These include schemes of seed radiation injection, which have been demonstrated at UV wavelengths [4] based on High Harmonic Generation (HHG) of an intense femtoSecond laser beam in a gas. Another seeding scheme is based on prebunching the e-beam by consecutive Harmonic Generation and High Gain amplification (HGHG) in wiggler structures, which has been demonstrated in the visible [5]. In these schemes coherence is expected to be achieved if the coherent harmonic signal (of radiation or current modulation) is strong enough to significantly exceed the shot noise (SASE) power.

In this context, as efforts persist to produce temporally coherent X-UV FELs of extremely high spectral brightness, SASE radiation is no longer a desired FEL output, but rather a source of noise which hinders the attainment of full temporal coherence with a coherently seeded FEL amplifier. In this paper we address this problem using a linear response formulation to describe the e-beam noise evolution in the beam transport line preceding the FEL [6]. The linear model is also employed to describe the subsequent coherent and incoherent radiation power-generation in the FEL wiggler [7] - [10]. Using the combined analysis of the accelerator transport line and the FEL wiggler we present a scheme for suppression of SASE radiation noise based on controlling the input current shot-noise of the e-beam at the entrance to the wiggler. The formulation results in the conditions for suppression of the radiation noise and the expressions for the ultimate coherence limits achievable in FEL.

SINGLE TRANSVERSE MODE LINEAR RESPONSE FORMULATION

The schemes for reduction of beam noise below the current shot-noise level is based on "smoothing" the e-beam current (or density) fluctuations by means of space-charge force repulsion. This occurs around the point where a quarter period of the e-beam plasma oscillation takes place during the transit time along the e-beam transport line [10]: $\omega'_{pr}L/v_{z0}= \pi/2$, where we define the longitudinal beam plasma oscillation frequency as [7]:

$$\omega_{pr}^{'\,2} = r_p^2\, e^2 n_0 \big/ \varepsilon_0 m \gamma_0 \gamma_{z0}{}^2 \tag{1}$$

Here $r_p \leq 1$ is the finite width beam plasma reduction factor

For the analysis of the e-beam modulation and noise dynamics we introduce here a relativistic extension of Chu's kinetic voltage parameter [6], [11], [12]

$$\tilde{V} = -\tilde{v}_z \frac{m}{e}\frac{d\gamma_0(z)}{dv_{z0}} = -\frac{m}{e}\gamma_0\gamma_{z0}^2 v_{z0}\tilde{v}_z = -\left(\frac{mc^2}{e}\right)\tilde{\gamma}(z) \tag{2}$$

This expansion near the average beam parameters is valid also for sections of axial acceleration and sections with transverse magnetic force: $\gamma_{z0}^2(z) = \gamma_0{}^2(z)/(1+a_\perp^2(z))$, $a_\perp(z) = -(e/mc)\int_0^z B_\perp(z')dz'$.

The solution of the FEL linear response problem can be expressed in terms of the radiation mode amplitude $\tilde{C}_q(L_w)$, the beam-current $\tilde{i}(L_w) = \int \tilde{J}_z(\mathbf{r})d^2\mathbf{r}_\perp$, the kinetic voltage $\tilde{V}(L_w)$ and a general frequency transfer matrix:

$$\begin{pmatrix} \tilde{C}_q(L_w) \\ \tilde{i}(L_w) \\ \tilde{V}(L_w) \end{pmatrix} = \underline{\underline{\tilde{\mathbf{H}}}}_{FEL} \begin{pmatrix} \tilde{C}_q(0) \\ \tilde{i}(0) \\ \tilde{V}(0) \end{pmatrix} = \begin{pmatrix} H^{EE} & H^{Ei} & H^{EV} \\ H^{iE} & H^{ii} & H^{iV} \\ H^{VE} & H^{Vi} & H^{VV} \end{pmatrix} \begin{pmatrix} \tilde{C}_q(0) \\ \tilde{i}(0) \\ \tilde{V}(0) \end{pmatrix} \tag{3}$$

The explicit expression for the components of the transfer matrix of a uniform wiggler section can be derived in all linear gain regimes and given in [8].

THE ACCELERATOR AND BEAM TRANSPORT SECTION

If the transport section is composed only of fast acceleration and drift sections, the 3 x 3 transfer matrix from the cathode to the FEL wiggler may be modeled by [6]

$$\underline{\underline{\tilde{\mathbf{H}}}}_T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\phi_p & -i\sin\phi_p / W_d \\ 0 & -iW_d \sin\phi_p & \cos\phi_p \end{pmatrix} \exp(i\varphi_b) \tag{4}$$

In our previous analysis of noise dynamics in a drifting electron beam [6] we showed, based on Eq.4, that at a drift distance

$$L_d = \pi / 2\theta_{pr} \tag{5}$$

the initial beam velocity noise turns into current noise and vice versa, and then:

$$\overline{\left|\breve{i}(L_d,\omega)\right|^2} = \overline{\left|\breve{V}(z_i,\omega)\right|^2} / W_d^2 = N^2 \overline{\left|\breve{i}(L_d,\omega)\right|^2} \tag{6}$$

$$N^2 \equiv \overline{\left|\breve{V}(z_i,\omega)\right|^2} \Big/ \overline{\left|\breve{i}(z_i,\omega)\right|^2} W_d^2 \; ; \tag{7}$$

Usually, the noise of high quality relativistic electron beams, which are used in FELs, is dominated by current shot-noise. Namely: $N^2 \ll 1$.

Under this condition, transporting the beam through a quarter plasma wavelength oscillation length (5) reduces the current shot-noise by a factor N^2 (7), and since the SASE radiation power in FEL is believed to be dominated by the input current shot-noise, this would enable suppression of SASE radiation power. However, since there is continued noise evolution dynamics also within the FEL interaction region, it is necessary to solve for the noise dynamics evolution in the combined system of the beam transport section and the wiggler in order to get more accurate expressions for the radiation noise suppression and the optimal drift length for SASE output minimization power.

COHERENT AND INCOHERENT RADIATION IN THE COMBINED E-BEAM TRANSPORT AND FEL SECTIONS

Consider now a general FEL structure that consists of a non-radiative section (acceleration and drift sections) and a radiating wiggler (Fig. 1). Based on the solution of the FEL linear response problem (3) for *coherent radiation* seed injection and for *coherent beam prebunching* schemes we get respectively:

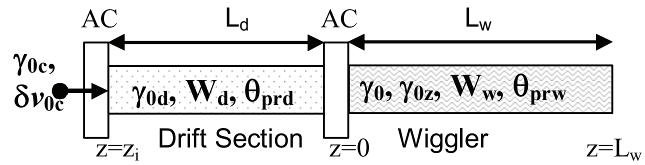

Figure 1: An FEL system composed of a free drift e-beam section, acceleration sections (AC) and a wiggler section.

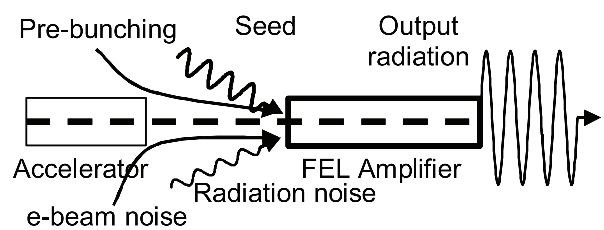

Figure 2: Coherent and incoherent radiation and beam modulation input signals at the FEL amplifier input. The amplified signals output power is partially coherent.

$$\left[P_s(L_w)\right]_{coh} = P_q \left|\tilde{C}_q(L_w)\right|^2 = P_q \left|\tilde{H}_{FEL}^{EE}\right|^2 \left|\tilde{C}_q(0)\right|^2 \tag{8}$$

$$\left[P_s(L_w)\right]_{prebunch} = P_q \left|\tilde{H}_{FEL}^{Ei}(\omega)\tilde{i}(0) + \tilde{H}_{FEL}^{Ev}(\omega)\tilde{V}(0)\right|^2 \tag{9}$$

where $z = 0$ is the wiggler entrance point and L_w its length.

For the *incoherent* radiation power calculation we need to keep a transfer matrix that includes both the FEL ($\underline{\underline{\tilde{\mathbf{H}}}}_{FEL}$) and accelerator ($\underline{\underline{\tilde{\mathbf{H}}}}_T$) sections (starting from the "cathode" position $z = z_c$ or more correctly – from the drift section entrance point): $\underline{\underline{\tilde{\mathbf{H}}}}_{TOT} = \underline{\underline{\tilde{\mathbf{H}}}}_{FEL} \underline{\underline{\tilde{\mathbf{H}}}}_T$.

The total incoherent spectral power at the FEL output is then:

$$\left(\frac{dP(L_w,\omega)}{d\omega}\right)_{incoh} = \frac{2P_q}{\pi}\Big\{\left|\tilde{H}_{FEL}^{EE}\right|^2 \overline{\left|\breve{C}_q(0,\omega)\right|^2} + \left|\tilde{H}_{TOT}^{Ei}(\omega)\right|^2 \overline{\left|\breve{i}(z_c,\omega)\right|^2} + \left|\tilde{H}_{TOT}^{EV}(\omega)\right|^2 \overline{\left|\breve{V}(z_c,\omega)\right|^2} + 2\,\mathrm{Re}\left(\tilde{H}_{TOT}^{Ei}\tilde{H}_{TOT}^{EV*}\right)\mathrm{Re}\left(\overline{\breve{i}\,\breve{V}_c^*}\right) - 2\,\mathrm{Im}\left(\tilde{H}_{TOT}^{Ei}\tilde{H}_{TOT}^{EV*}\right)\mathrm{Im}\left(\overline{\breve{i}\,\breve{V}_c^*}\right)\Big\} \tag{10}$$

At the entrance to the drift section, we assume that various dissipative processes that increase the e-beam energy spread and emittance render the velocity noise to be *uncorrelated* with the electron current shot noise [6]. At this no-correlation point, $\mathrm{Im}\left(\overline{\breve{i}\,\breve{V}_c^*}\right) = 0$, and single particle analysis produces the following expressions for the beam noise parameters:

$$\overline{|\breve{i}|^2} = eI_b ; \tag{11}$$

$$\overline{|\breve{V}|^2} = \left(mc^2\delta\gamma\right)^2 / eI_b ; \tag{12}$$

$$\overline{\breve{i}\,\breve{V}^*} = mc^2\delta\gamma\delta\beta_{zc} / \beta_{zc} \tag{13}$$

Here $\delta\gamma \equiv \gamma_{z0c}^2\gamma_{0c}\beta_{z0c}\delta\beta_{zc}$, where $\beta_{z0c} = \left\langle \beta_{zj} \right\rangle_j$ and $(\delta\beta_{zc})^2 = \left\langle (\beta_{zj} - \beta_{z0c})^2 \right\rangle_j$ are averages over the electron beam axial velocity β_{z0j} distribution at the start point ($\delta\beta_{zc} < \beta_{z0c}$). The beam energy spread is fundamentally limited by the cathode temperature (for thermionic cathode): $\delta\gamma = \delta\gamma_c = k_B T_c / mc^2$, where T_c is the cathode temperature and k_B - Botlzman constant. However, in practice, after transport through the e-gun and accelerator sections the energy spread is increased or even intently heated [13]. The effective ("slice") energy spread $\delta\gamma$ after acceleration is hard to measure, but is at least 3 orders of magnitudes larger than the cathode temperature energy spread.

To deal further with the incoherent FEL radiation, instead of the output power (10), it is convenient to define an incoherent (noise) effective radiation input power (Noise Equivalent Radiation Power - NERP), which lumps all effective incoherent input signal sources, and is composed of the e-beam shot-noise contributions (of current, velocity and kinetic power) and the quantum spontaneous emission and background black-body radiation at the FEL entrance (see Fig. 2):

$$\left(dP_{in} / d\omega\right)^{eq} = \left(dP_{in}(L_w) / d\omega\right)_{incoh} \Big/ \left|\tilde{H}_{FEL}^{EE}\right|^2 \tag{14}$$

CONVENTIONAL FEL THEORY

In conventional FEL theory it is customary to assume $\underline{\underline{\tilde{\mathbf{H}}}}_{TOT} = \underline{\underline{\tilde{\mathbf{H}}}}_{FEL}$ (no plasma oscillation dynamics in the beam transport sections preceding the FEL). Substituting then (11) – (13) into (10), (14), one obtains that the e-beam NERP (see Fig. 2) is composed of three contributions (current noise, velocity noise and kinetic power noise):

$$\left(dP_{in} / d\omega\right)_{conv}^{i} = \frac{eI_b Z_0}{16\pi A_{em}} \left(\frac{a_w}{\gamma\beta_z\Gamma}\right)^2 \tag{15}$$

$$\left(dP_{in} / d\omega\right)_{conv}^{V} = \left(\frac{mc^2\delta\gamma_{ef}}{eI_b}\right)^2 \left(\frac{\theta_{pr}}{W_w\Gamma}\right)^2 \left(\frac{dP_{in}}{d\omega}\right)_{conv}^{i} \tag{16}$$

$$\left(dP_{in} / d\omega\right)_{conv}^{iV} = \frac{2}{\pi}\sqrt{3}mc^2\delta\gamma_{ef}\delta\beta_{zc} / \beta_{zc} \tag{17}$$

In addition, there may be an effective input radiation noise due to spontaneous emission and finite temperature (T) background black-body radiation [19]:

$$\left(dP / d\omega\right)_{in}^{E} = \hbar\omega / \left(1 - e^{-\hbar\omega/k_B T}\right) \tag{18}$$

FELs operate in practice only in the cold beam regime [7] $\delta\beta_{z0} / \beta_{z0} << \Gamma / k_0$, where $\delta\beta_{z0} = \delta\gamma_{ef} / \gamma_{z0}^2\gamma_0\beta_{z0}$ is the axial velocity spread in the FEL. Therefore (16), (17) are negligible relative to (15). This justifies in retrospect the non-obvious common neglect of velocity noise in conventional SASE-FEL theory.

Thus under the assumptions of conventional FEL 1-D linear theory, neglecting all noise contributions except current shot-noise, the FEL coherence condition for seed radiation and prebunching schemes simplify to:

$$\left[P_s(0)\right]_{coh} >> \frac{eI_b Z_0}{16\pi A_{em}} \left(\frac{a_w}{\gamma\beta_z\Gamma}\right)^2 \Delta\omega , \tag{19}$$

$$\overline{\left|\tilde{i}_s(0)\right|^2} >> eI_b\Delta\omega \tag{20}$$

Here $\Delta\omega$ is the frequency bandwidth of the incoherent power [7]. If filtering is employed, then $\Delta\omega$ is the filter bandwidth. In a pulse of duration t_p the bandwidth is Fourier transform limited: $\Delta\omega \approx \pi/t_p$.

SASE POWER CONTROL AND SUPPRESSION

We now show that by proper control of the e-beam plasma dynamics in the non-radiating sections of the transport line it may be possible to reduce the current shot noise below the velocity noise, so that the incoherent power of the FEL would not be limited by shot noise, but by the slice energy spread of the beam.

Assume there is a non-radiative section between the starting point (non-correlation) and the wiggler entrance that contains a fast acceleration section to any middle energy level (γ_{0d}), a long enough drift section and a second fast acceleration stage, which accelerates the beam to the energy level (γ_0) required for the wiggler radiation (Fig. 1). In this case the effective input power derived from (14, 10) is modified due to the dynamics of energy transfer and correlation between the beam dynamic parameters ($\breve{i}$ and $\breve{V}$) in the drift section. The modified current, kinetic voltage and kinetic power noise effective radiation input power can be written then in terms of the corresponding conventional (no drift section) effective input radiation noise power (15) – (17). Using Eq.8 and

Eq.4, one obtains general expressions for the current noise reduction factor, which depends only on a "noise reduction parameter".

$$S = W_d \theta_{prw} / W_w \Gamma \approx \left(\gamma_{0d}^3 \beta_{0d}^3 / \gamma_0 \gamma_{0z}^2 \beta_{0z}^3\right)^{\frac{1}{2}} \left(\theta_{prw} / \Gamma\right) \tag{21}$$

and the plasma phase increment in the drift section $\phi_{pd} = \theta_{pd} L_d$.

It turns out (because of the continued interaction in the wiggler), that maximum radiation noise suppression does not take place exactly at the condition of maximum current noise suppression (6) but at a slightly smaller phase shift ($S << 1$):

$$\phi_{pd} = \pi / 2 - \sqrt{3} S / 2 \tag{22}$$

At this condition one finds that the NERP due to the beam current shot-noise is reduced by a factor $(S/2)^2 << 1$ relative to the conventional case equation (17).

At this point one needs to examine the contribution of the other noise sources to the total NERP of the FEL. The next noise source of importance is the velocity noise, and since this noise grows in the drift section when the current noise diminishes, it turns out that the contribution to the NERP due to velocity noise is bigger relative to the conventional case expression (18) by a factor $(1/S)^2 >> 1$. Also in this case, the total NERP is suppressed relative to the basic case (no drift section), but the new value of the total NERP of the FEL depends on the ratio between the two parameters N and S, both << 1.

In the case that the noise factor is smaller than the noise suppression factor $2N \langle\langle S$ the current shot-noise contribution to the NERP is still dominant, and given by :

$$\left(\frac{dP}{d\omega}\right)^{eq} = \left(\frac{S}{2}\right)^2 \left(dP_{in} / d\omega\right)^i_{conv} = \frac{2}{\pi} \frac{\theta_{prw}}{\Gamma} \frac{W_d^2}{W_w} \left(eI_b\right) \tag{23}$$

In the opposite case S<<N, the velocity noise contribution to the NERP becomes dominant, given by:

$$\left(\frac{dP}{d\omega}\right)^{eq} = \left(\frac{1}{S}\right)^2 \left(dP_{in} / d\omega\right)^V_{conv} = \frac{2}{\pi} \frac{\Gamma}{\theta_{prw}} \frac{W_w}{W_d^2} \frac{\left(mc^2 \delta\gamma\right)^2}{eI_b} \tag{24}$$

In this case, the NERP of the FEL depends on the beam energy spread (note though that the FEL still operates in the cold beam gain regime).

It remains to be seen up to what high frequencies the noise suppression scheme can be exploited. Preliminary estimates, based on presently available beam quality parameters suggest that noise can be suppressed up to the UV spectral range with present state of the art technology.

It is still of fundamental physics interest to have an expression for the ultimate NERP of FEL, after the current shot-noise is suppressed and assuming that lower beam energy spread $\delta\gamma$ cam be attained. In this limit one needs to consider also the radiation-noise contribution (20). At long wavelengths down to FIR, this term becomes $\left(dP_{in} / d\omega\right)^E = k_B T$. At high frequencies of X-UV it becomes $\hbar\omega$ and may exceed the beam energy spread. The fundamental limit of FEL coherence is then the quantum noise limit:

$$\left(dP_{in} / d\omega\right)^{eq} = \hbar\omega \tag{25}$$

This limit may be considered the equivalent of the Schawlow-Townes limit for atomic laser oscillators [15]. We note, in conclusion, that in *prebunching* schemes like HGHG, the noise suppression scheme may be still effective even for very short wavelengths lasing, since the main contribution to the high frequency shot-noise in this scheme originates from harmonic generation and high gain amplification of the shot-noise at the *fundamental harmonic frequency*, where shot-noise suppression is presently plausible.

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