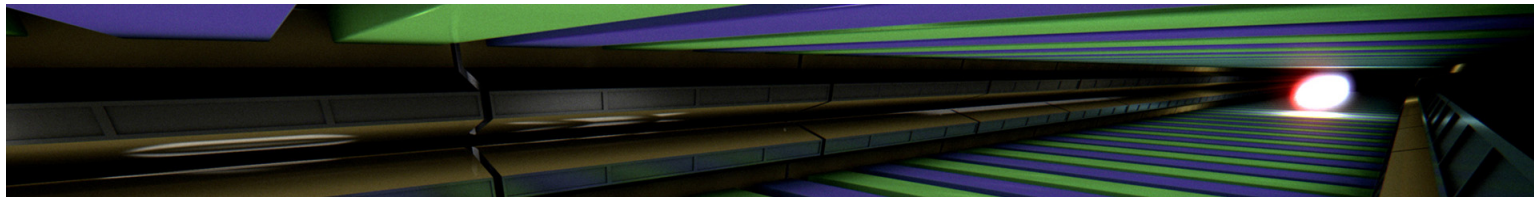


Theory of Edge Radiation. Foundations and applications

Gianluca Geloni
DESY & European XFEL



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Theory of Edge Radiation. Part 1 - Foundations and basic applications

Gianluca Geloni, Vitali Kocharyan, Evgeni Saldin, Evgeni Schneidmiller and Mikhail Yurkov

THOB04 (This Talk) - Nuclear Inst. and Methods in Physics Research, A 605 (2009), pp. 409-429

Theory of Edge Radiation. Part 2 - Advanced applications

Gianluca Geloni, Vitali Kocharyan, Evgeni Saldin, Evgeni Schneidmiller and Mikhail Yurkov

WEPC02- Nuclear Inst. and Methods in Physics Research, A 607 (2009), pp. 470-487

Integration of the optical replica ultrashort electron bunch diagnostics with the high-resolution coherent optical transition radiation imager

Gianluca Geloni, Petr Ilinski, Evgeni Saldin, Evgeni Schneidmiller and Mikhail Yurkov

WEPC46 - DESY 069-2009 – To be published

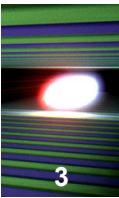
Method for the determination of the three-dimensional structure of ultrashort relativistic electron bunches

Gianluca Geloni, Petr Ilinski, Evgeni Saldin, Evgeni Schneidmiller and Mikhail Yurkov

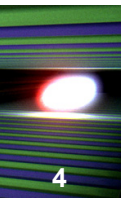
WEPC47 - DESY 069-2009 – To be published



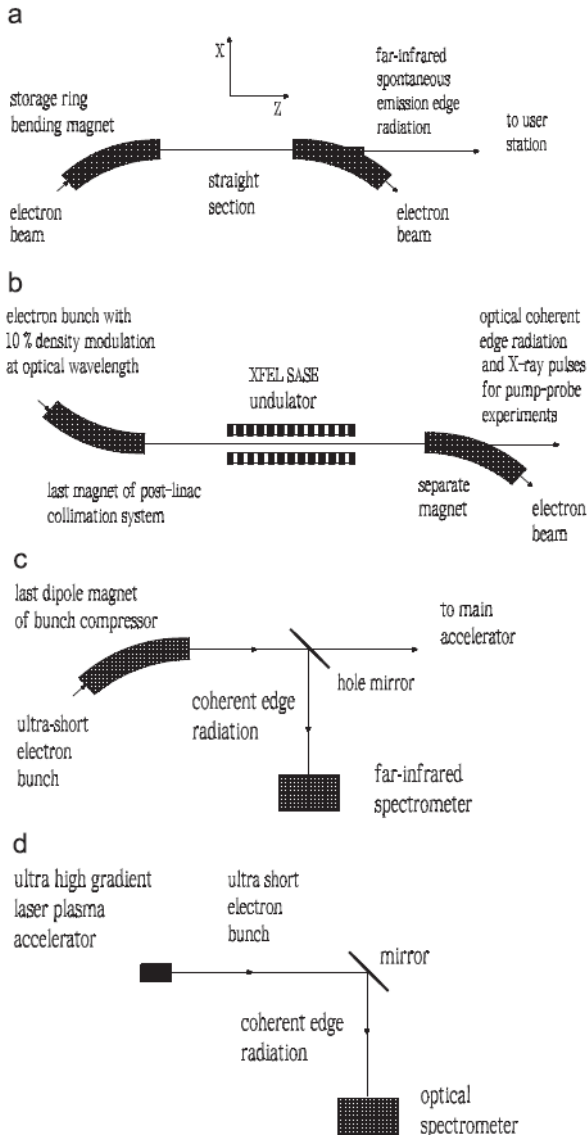
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- Basic Theory of edge radiation (single electron)
- Edge Radiation and Optical Transition Radiation
- Coherent OTR for diagnostic purposes: Integration with the Optical Replica Synthesizer (ORS: THOB02)
- Coherent OTR for diagnostic purposes: towards three-dimensional structure determination of ultrashort relativistic electron bunches



Basic Theory of Edge Radiation (single electron)

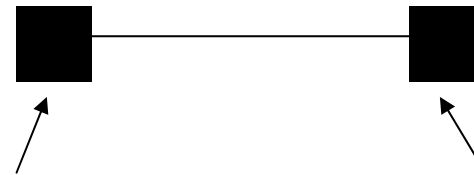


$$\left(\nabla^2 + \frac{2i\omega}{c} \frac{\partial}{\partial z}\right) \vec{E} = \frac{4\pi e}{v_z(z)} \exp\left[i\omega\left(\frac{s(z)}{v} - \frac{z}{c}\right)\right] \left[\frac{i\omega}{c^2} \vec{v}_\perp(z) - \vec{\nabla}_\perp\right] \delta(\vec{r} - \vec{r}_0(z))$$

Paraxial Maxwell Equations in the Space-frequency domain

$$\vec{E} = \vec{E}_\perp \exp[-i\omega z/c]$$

Sources: $\bar{\rho}(\vec{r}, z, \omega)$ $\vec{j}(\vec{r}, z, \omega)$

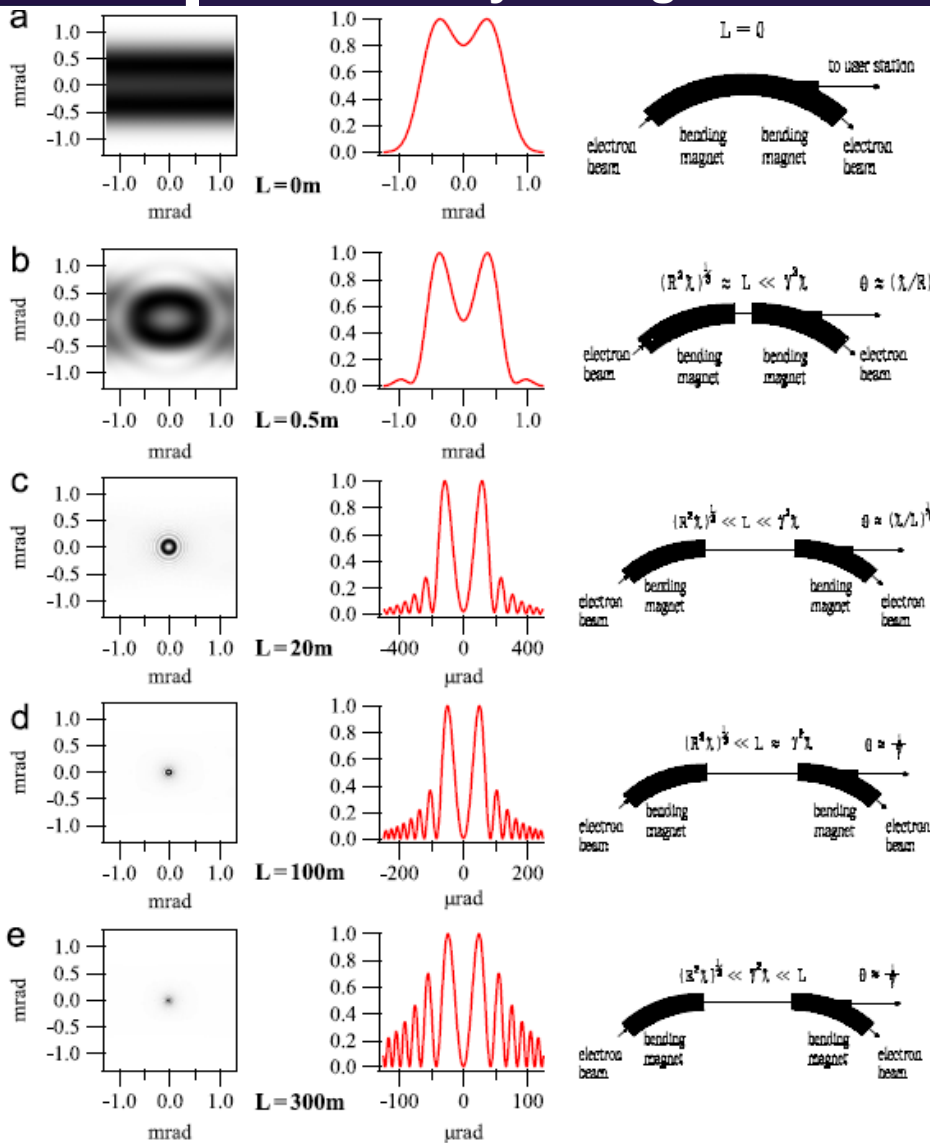
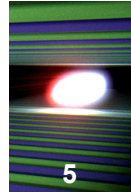


Sources begin to exist here

Sources end to exist here

The way sources begin and cease to exist can include Negligible/non negligible modifications to the field...

Basic Theory of Edge Radiation (single electron)



Synergy with SRW

Similarity techniques can be used to study ER

$$\delta \equiv \frac{\sqrt[3]{R^2 \lambda}}{L}, \quad \phi \equiv \frac{L}{\gamma^2 \lambda}$$

$\delta \ll 1$ – can neglect bending magnet contributions
 $\phi \delta \ll 1 - \lambda \gg \lambda_c$
 → Theory of edge radiation

Red: L=0.5m, R=400m, λ=400 nm, @ 17.5 GeV

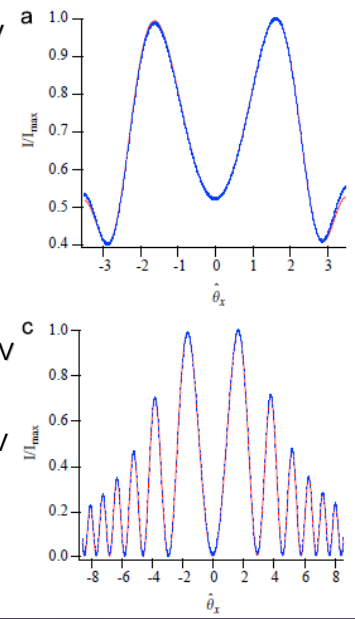
Blue: L=1 m, R=800m, λ=800, nm @ 17.5 GeV

$$\delta \simeq 0.43 \text{ and } \phi \simeq 6.7 \times 10^{-3}$$

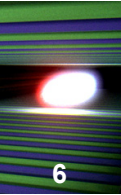
Red: L=300m, R=400m, λ=400 nm, @ 17.5 GeV

Blue: L=150 m, R=800m, λ=800, nm @ 8.5 GeV

$$\delta \ll 1 \text{ and } \phi \simeq 4$$



Basic Theory of Edge Radiation (single electron)



Space-frequency domain

SR beams from single electrons are similar to laser-beams

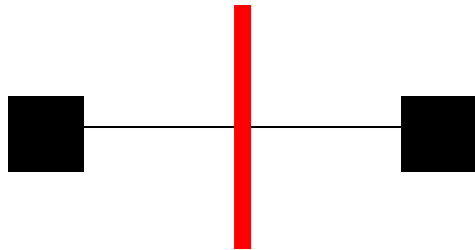


• Calculate the far-zone field

- The far field completely characterizes radiation at a virtual source = “waist”
- Fresnel propagation formula propagates the field from the virtual source

Two equivalent pictures for ER

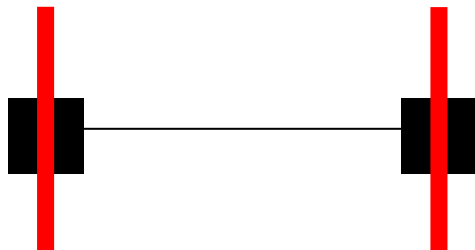
One source = one laser-like beam



$$\vec{E}(0, \vec{r}) = -\frac{\omega^2 e L}{2\pi c^3} \int d\vec{\theta} \vec{\theta} \operatorname{sinc} \left[\frac{\omega L}{4c} \left(\theta^2 + \frac{1}{\gamma^2} \right) \right] \exp \left[\frac{i\omega}{c} \vec{r} \cdot \vec{\theta} \right]$$

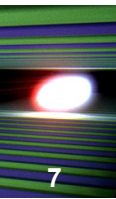
Better applied for $\phi = \frac{L}{\gamma^2 \lambda} \ll 1 \rightarrow \vec{E}(0, \vec{r}) = -i \frac{4\omega e}{c^2 L} \vec{r} \operatorname{sinc} \left(\frac{\omega r^2}{cL} \right)$

Two sources = two laser-like beam

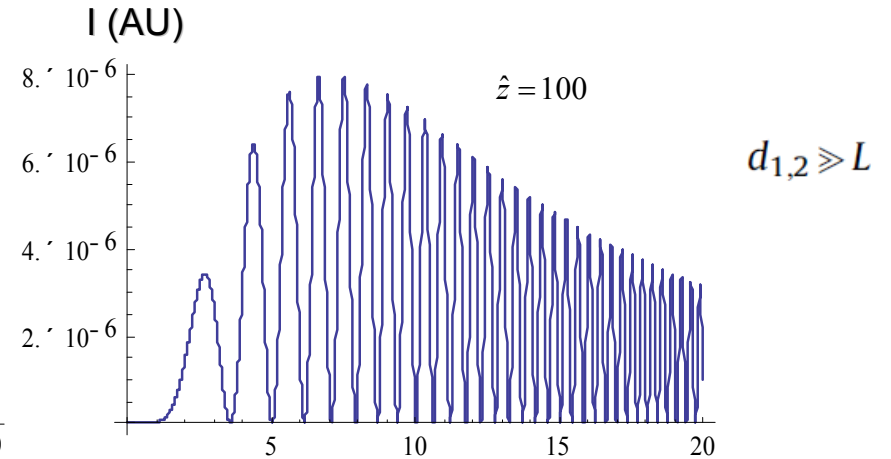
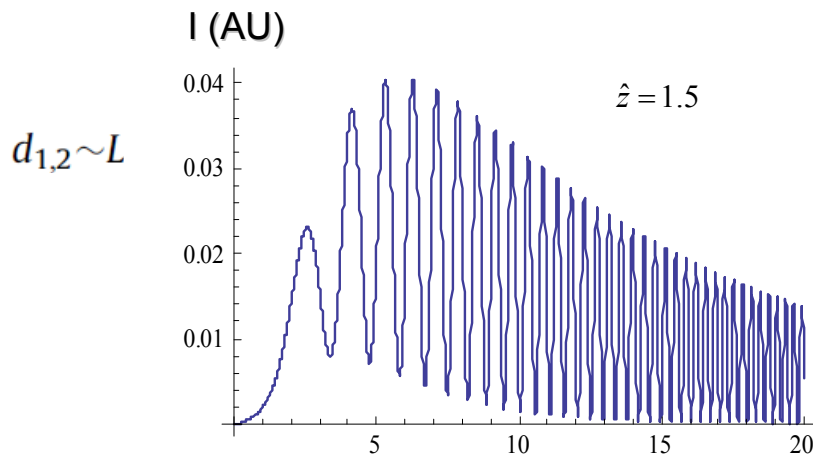
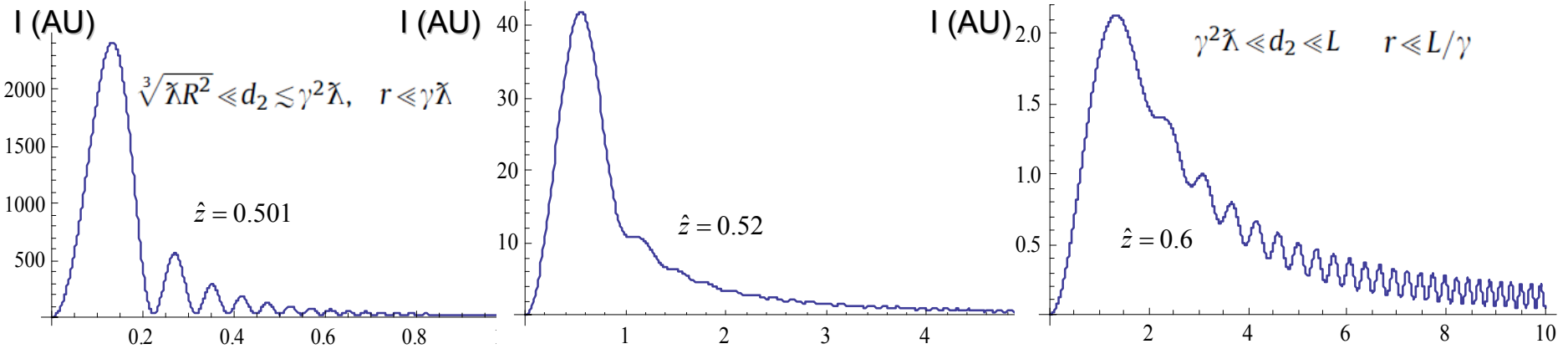


$$\vec{E}_{s1,s2} \left(\mp \frac{L}{2}, \vec{r} \right) = \pm \frac{2\omega e}{c^2 \gamma} \exp \left[\mp \frac{i\omega L}{4\gamma^2 c} \right] \frac{\vec{r}}{r} K_1 \left(\frac{\omega r}{c\gamma} \right)$$

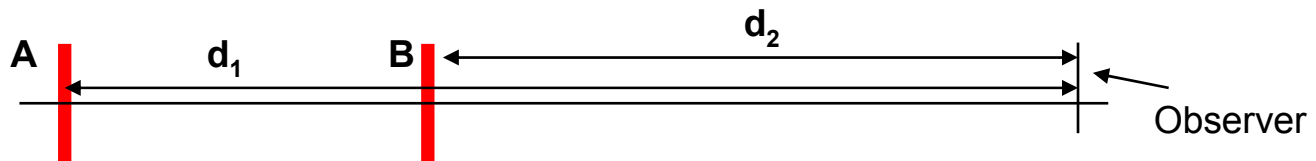
Better applied for $\phi = \frac{L}{\gamma^2 \lambda} \gg 1$

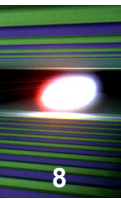


Basic Theory of Edge Radiation (single electron)



$\phi = 50$

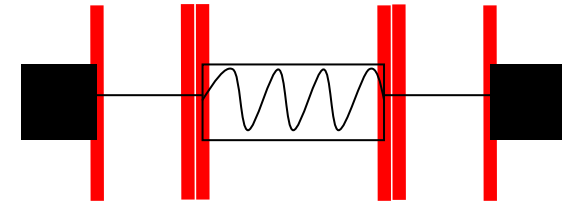




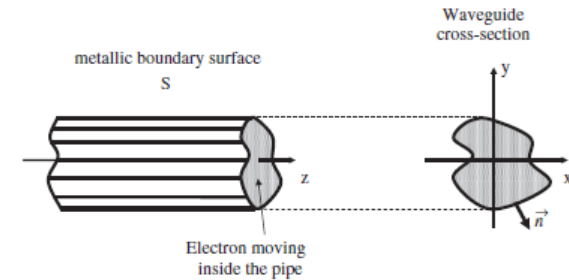
WEPC02

Many possible applications:

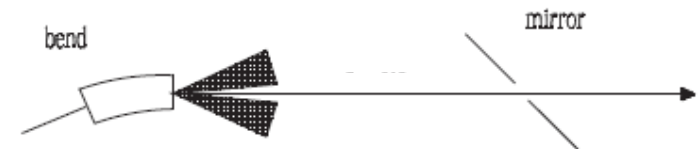
Transition Undulator Radiation



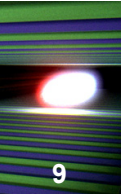
Edge radiation in a waveguide



Extraction of edge radiation from a mirror
(Transition radiation setup)



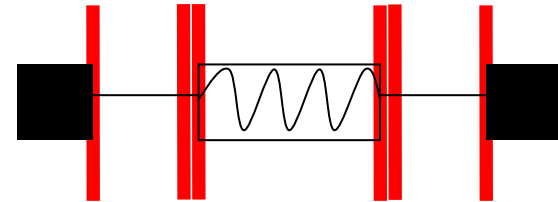
Coherent edge radiation



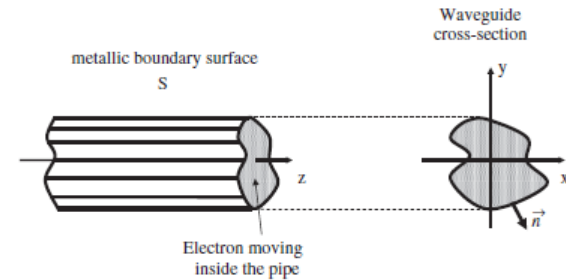
WEPC02

Many possible applications:

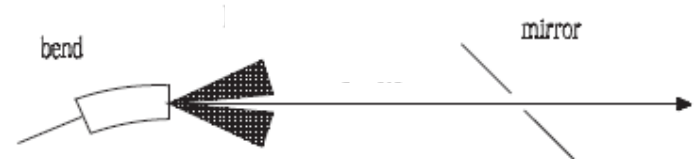
Transition Undulator Radiation



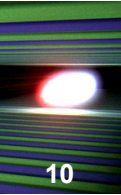
Edge radiation in a waveguide



Extraction of edge radiation from a mirror
(Transition radiation setup)

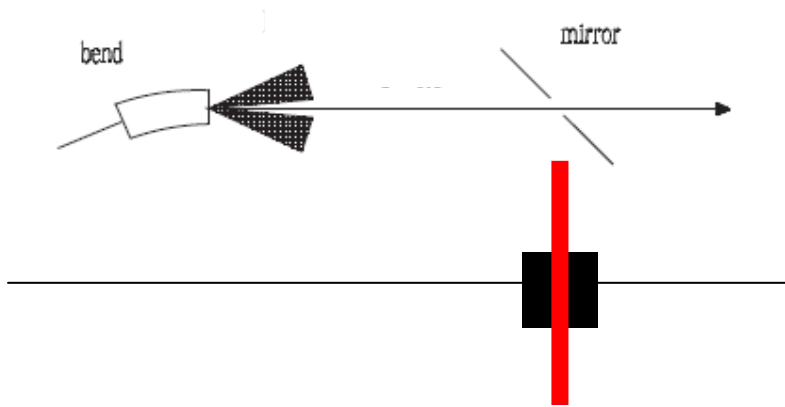


Coherent edge radiation



WEPC02

Typical treatment of TR uses the Ginzburg-Frank approach



$$\vec{E}(\vec{r}) = -\frac{2\omega e \vec{r}}{c^2 \gamma r} K_1\left(\frac{\omega r}{c\gamma}\right)$$

...what about the other edge? And the contribution from the bend?

THz Radiation

E=500 MeV, R~2 m, L~15 cm, λ=1200 μm

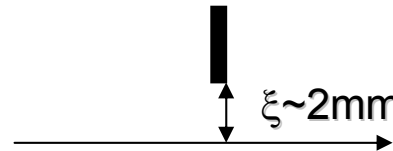
$$\Rightarrow \delta = \frac{\sqrt[3]{R^2 \lambda}}{L} \sim 0.6$$

Contribution from bend
Cannot be neglected!

High-Energy, Optical diffraction radiation

E=7 GeV, L~5 m, λ=800 nm

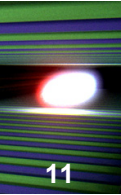
→ γ²λ/(2π)~30m i.e. φ~0.2



ξ~γλ/(2π)
No exponential suppression

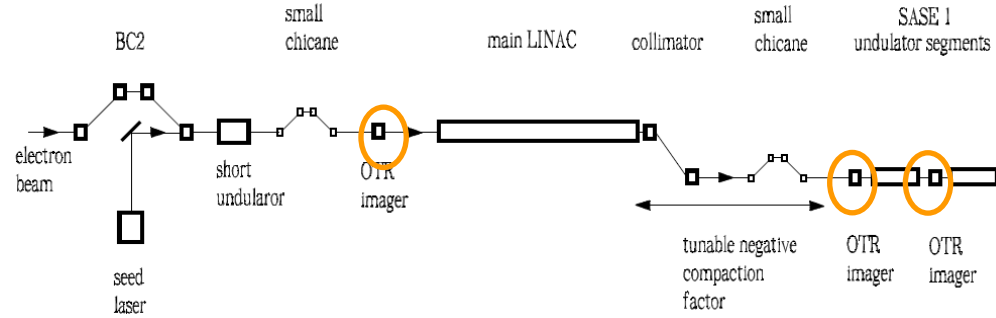
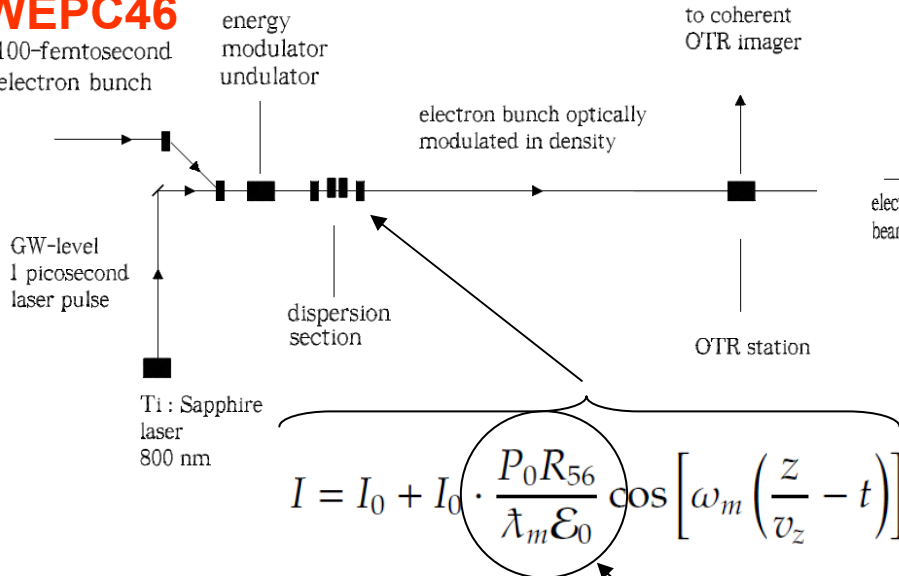
$$\vec{E}\left(\frac{L}{2}, \vec{r}\right) = -\frac{2e \vec{r}}{c r^2} \exp\left[\frac{i\omega r^2}{2cL}\right]$$

Integration of the OTR imager with the ORS



WEPC46

100-femtosecond electron bunch



$$I = I_0 + I_0 \cdot \frac{P_0 R_{56}}{\lambda_m \mathcal{E}_0} \cos \left[\omega_m \left(\frac{z}{v_z} - t \right) \right]$$

Seed Laser:

$\lambda = 800 \text{ nm}$

$E = 1 \text{ mJ}$

Duration (FWHM) = 1 ps

Rayleigh length = L_w

Waist at the undulator center

$r(\text{waist}) = 20 r(\text{bunch}) = 600 \mu\text{m}$

Modulator undulator:

$L_w = \beta = 1 \text{ m} ; N_w = 5 ; \lambda_w = 20 \text{ cm}$

Energy modulation: 500 keV

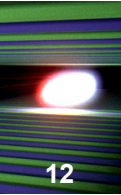
Energy of e^- @ modulation: 2 GeV

Modulation energy $P_0 = 500 \text{ keV}$

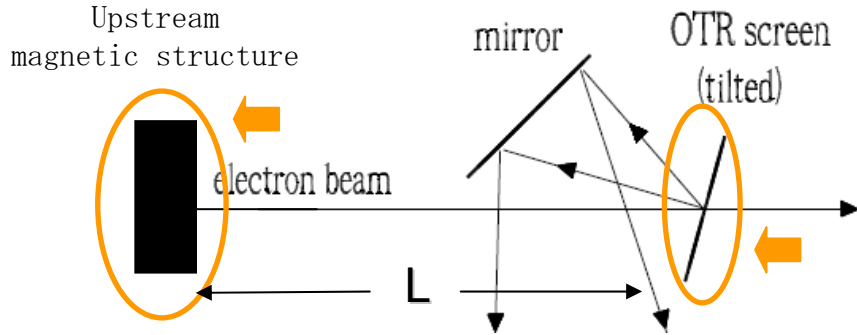
Dispersion: $R_{56} = 50 \mu\text{m}$

Initial density modulation $a = 10\%$

Integration of the OTR imager with the ORS

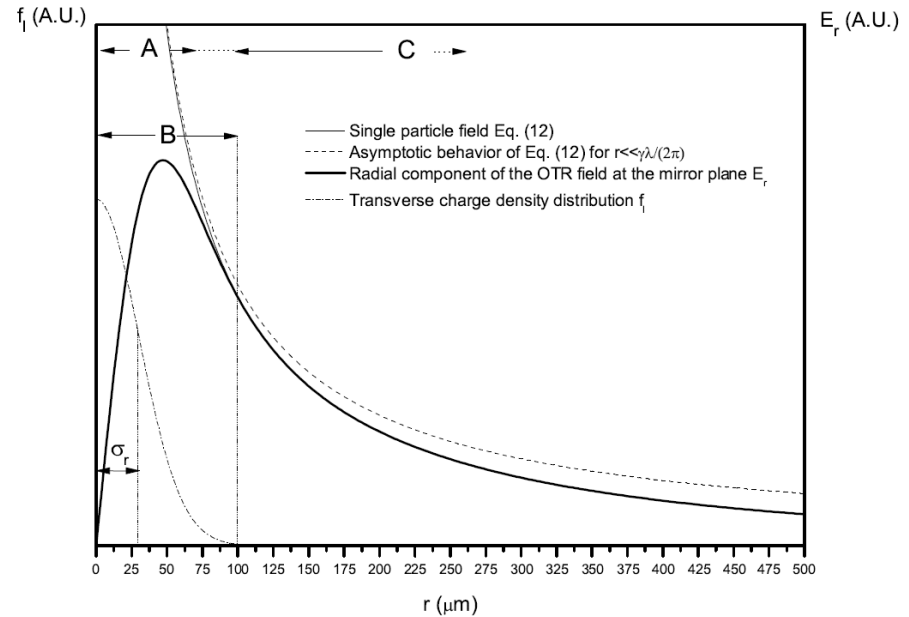


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Can neglect upstream source when

$$\phi \equiv \frac{L}{\gamma^2 \lambda} \quad \phi \gg 1$$



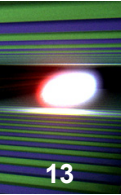
For us: $\phi \gtrsim 1$ $\phi \sim 1$ But we are interested in $\sigma_c \ll \gamma\lambda/(2\pi)$

because the total field is a convolution in space of the FT of the charge density distribution and the FT of the single-electron field

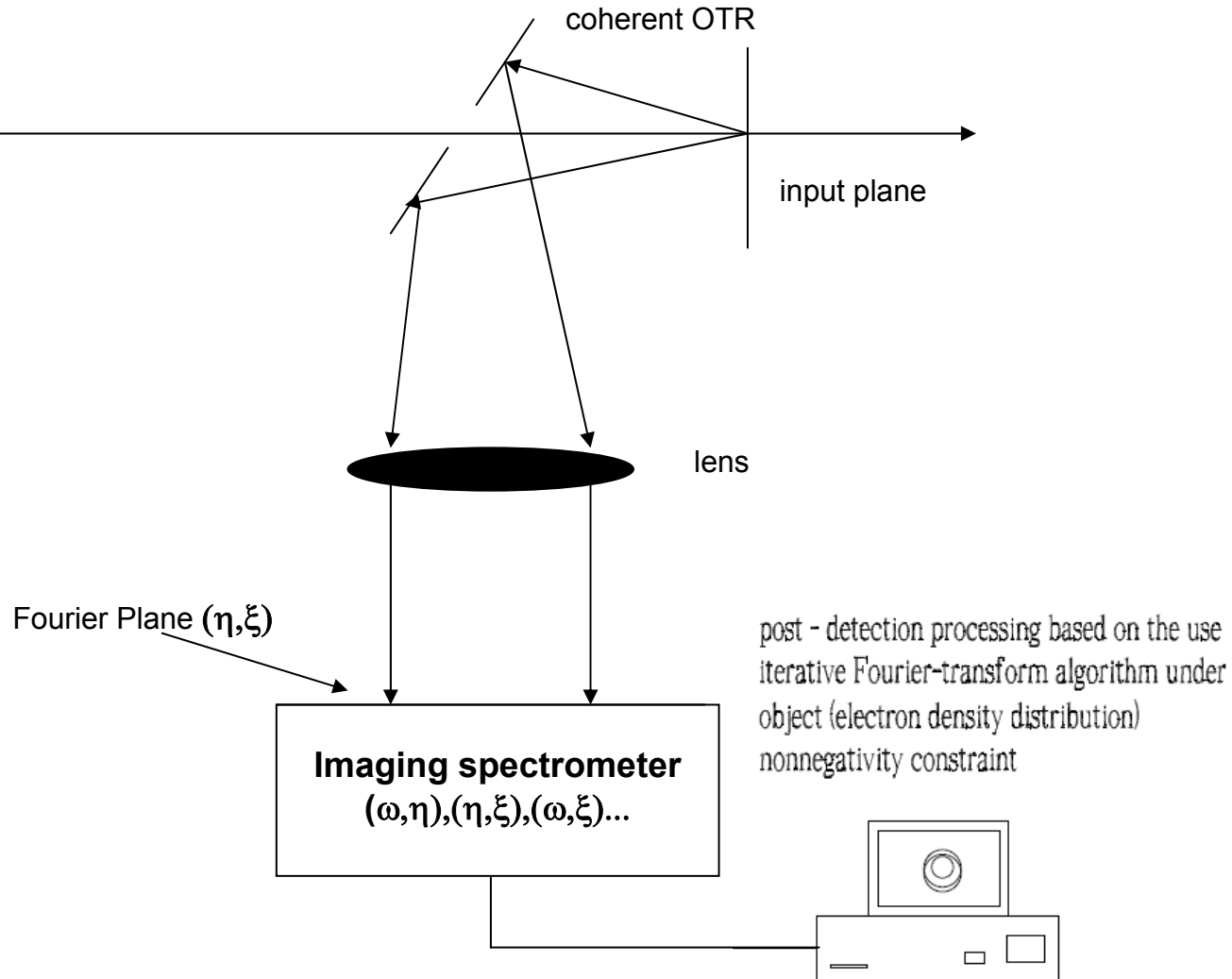
It can be shown that in this case $L_f(\sigma_c) \sim \sigma_c^2 / \lambda \ll \gamma^2 \lambda$

➔ For our parameter choice the G-F description holds for the field (envelope)

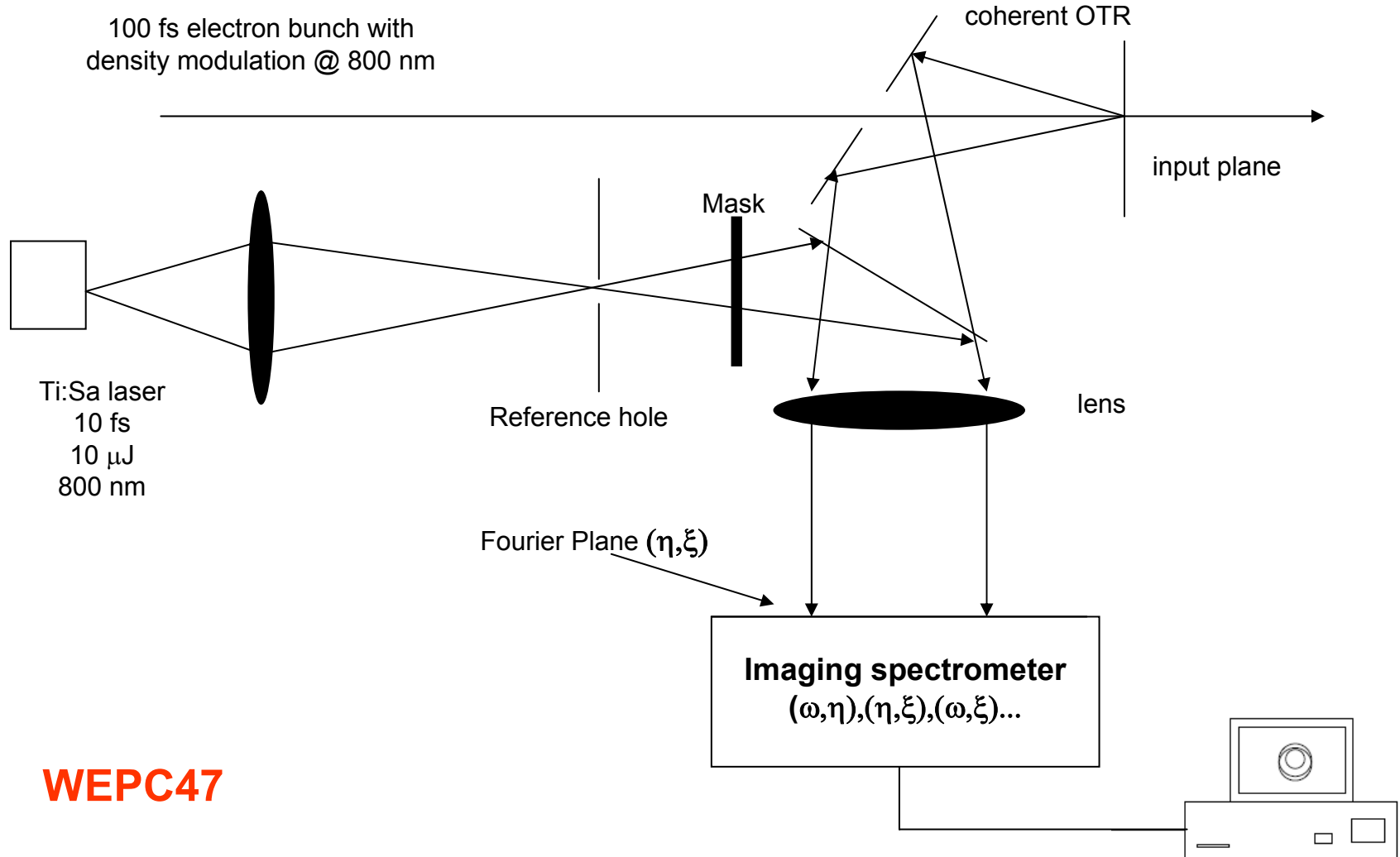
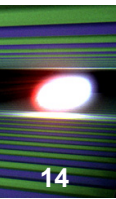
$$\vec{E}(\vec{r}) = -\frac{2\omega e \vec{r}}{c^2 \gamma r} K_1\left(\frac{\omega r}{c\gamma}\right)$$



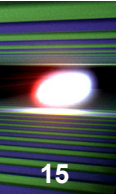
100 fs electron bunch with
density modulation @ 800 nm



WEPC47



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- Theory of Edge Radiation important for many FEL-related applications
- Based on two main parameters : δ and ϕ
- Undulator transition radiation, ER in a waveguide, Extraction from a mirror... many possible applications
- In particular, Transition radiation \rightarrow XFEL diagnostics
- New field in e-beam diagnostics using coherent (vs. incoherent) Optical Transition Radiation
- Main advantage exploited: large coherent photon number
- Using imaging spectrometers based on
 - Diffraction Imaging
 - Holographytowards the 3D structure determination of ultrashort electron beams!